Cross Correlators

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3rd IRAM mm interferometry school

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- Cross correlators are the brain of correlation interferometers.
- They calculate the correlation function (continuum correlator),
- and derive astronomical spectra (spectral cross correlator).

Basic Principles

- The output of a correlation interferometer is the visibility function. For a monochromatic, stationary (with respect to the averaging time) signal, it is

$$
\begin{aligned}
R(\boldsymbol{b}\cdot\boldsymbol{s}) &= \langle V_1(t)\cdot V_2(t)\rangle \\
&= E_1 E_2 \cdot \langle \cos(\omega t+\phi_1)\cdot\cos(\omega t+\phi_2)\rangle \\
&= \frac{1}{2} E_1 E_2 \cdot \cos(\Delta\phi)
\end{aligned}
$$

- The visibility function is recorded as a time series, which variation is ideally only due to the Earth's rotation, the source structure, and atmospheric perturbations.

Examples for time dependence

- Time scale for a phase variation by 1° due to source structure (point source at 100 GHz, ? ? = 10" offset from phase reference center, E-W baseline of 180 m during transit):

  10 minutes.

- Time scale for phase variations due to atmospheric perturbations (depending on conditions and baseline):

  1 sec - several hours.

basic principles (continued)

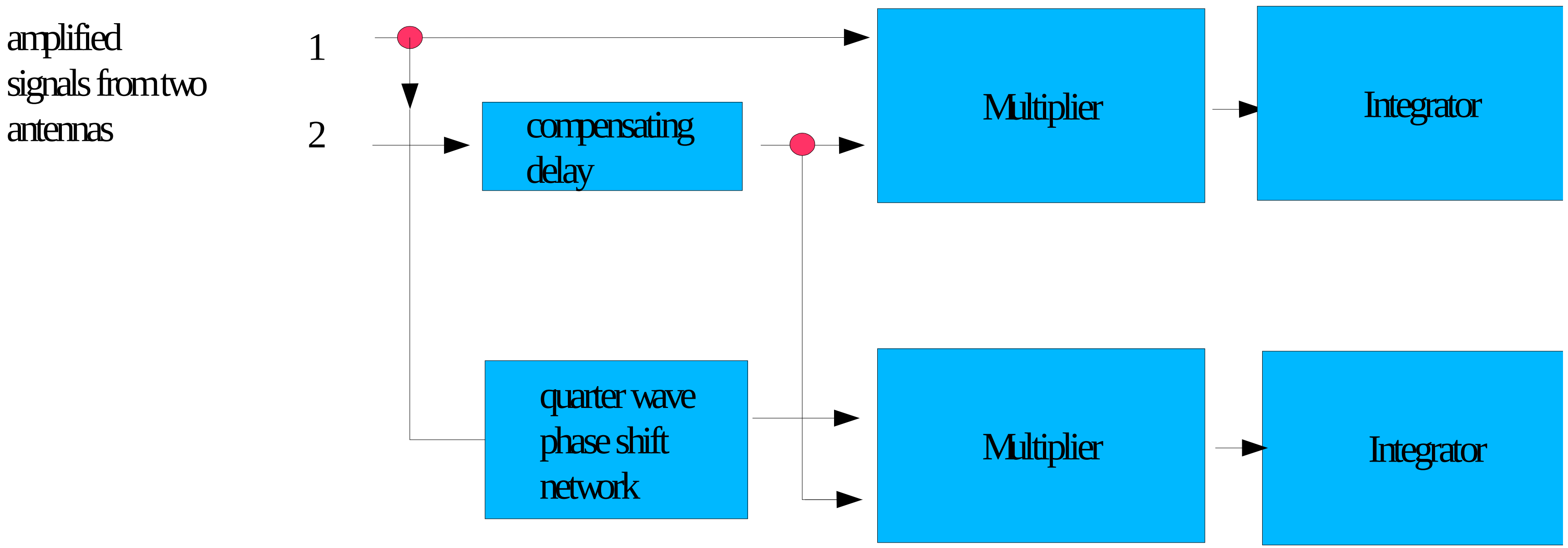
- Problem: cannot distinguish amplitude from phase.
- First solution: add a quarter wave phase shift before the correlation.

$$R(\boldsymbol{b}\cdot\boldsymbol{s}) = E_1 E_2 \cdot \langle \cos(\omega t + \phi_1) \cdot \sin(\omega t + \phi_2) \rangle$$
$$= \frac{1}{2} E_1 E_2 \sin(\Delta \phi)$$

- Record cosine and sine parts separately (continuum correlator).

basic principles (continued)

Architecture of a continuum correlator



basic principles (continued)

If we want to recover spectral information, we have 2 solutions at hand ...

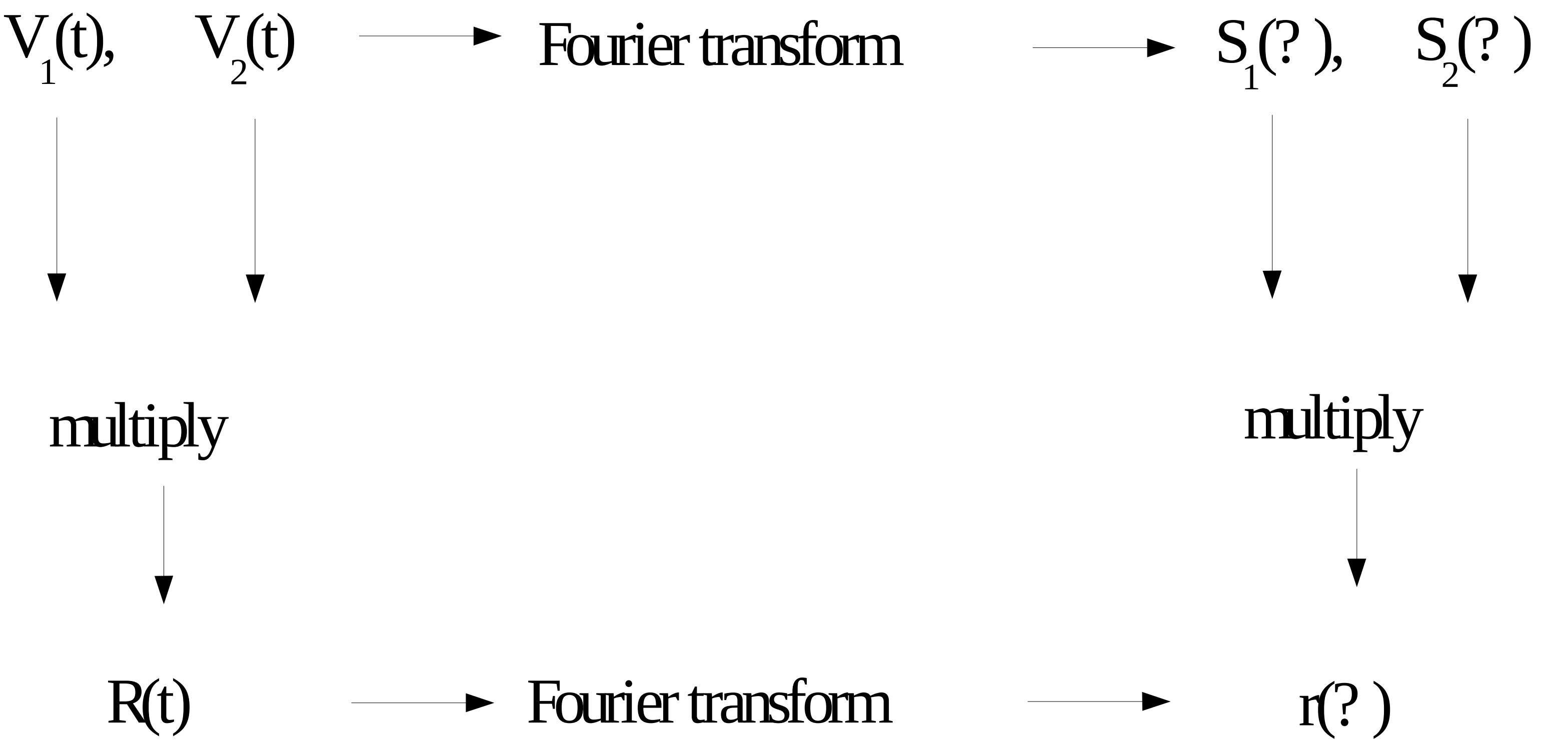
- quarter wave phase shift for all frequencies within the intermediate frequency range, select quasi - monochromatic channels with a filterbank
- Drawback: need a separate filterbank for all baselines. Analog components are unstable and difficult to match.

basic principles (continued)

- Second solution: in order to digitize the signal, we have to sample it in time -> use Fourier transform relations between time and frequency.
- Use of Fourier transform relations in digital spectroscopy became possible thanks to the **F**ast **F**ourier **T**ransform (FFT).
- Direct Fourier Transform of N samples: process of order N^2, FFT of order $N \log_2 N$.
- CPU time for nsec cycle computer and 2^{13} samples: 0.3 msec vs. 5? sec.

basic principles (continued)

FX vs XF correlation



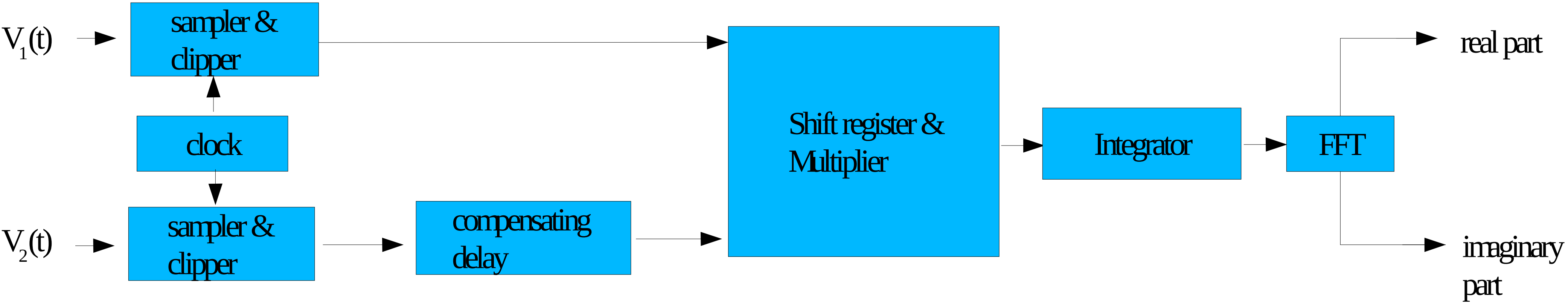
basic principles (continued)

- XF technology: needs 2 processes, one (rapid) with N real multiplications (for the cross correlation), one with N $\log_2$N complex multiplications (for the FFT, for each baseline).
- FX technology: needs 2 processes, one with 2 x N $\log_2$N complex multiplications (for the FFT of each antenna's signal), one with with N complex multiplication.

=> XF vs. FX is a trade-off between the bandpass and number of baselines. At the Plateau de Bure, XF is the more economic choice.

basic principles (continued)

Architecture of a spectroscopic cross correlator



- The continuum correlator determined

$$R(\mathbf{b}\cdot\mathbf{s}) = \langle V_1(t)\cdot V_2(t)\rangle = \frac{1}{2}E_1E_2\cos(\Delta\phi)$$ and the sine part.

- The spectroscopic correlator calculates

$$R(\mathbf{b}\cdot\mathbf{s},\Delta t) = \langle V_1(t+\Delta t)\cdot V_2(t)\rangle = \frac{1}{2}E_1E_2\cos(\omega\Delta t+\Delta\phi)$$ for a range of time lags ? t.

- The FFT yields the cross power spectrum, $r(\mathbf{b}\cdot\mathbf{s},\nu) = S_1(\nu)\cdot S_2(\nu)^*$ (Wiener-Khinchin theorem)

- The real part of the cross power spectrum is the FT of the even component of $R(\mathbf{b}\cdot\mathbf{s},\Delta t)$, the imaginary part that of the odd one.

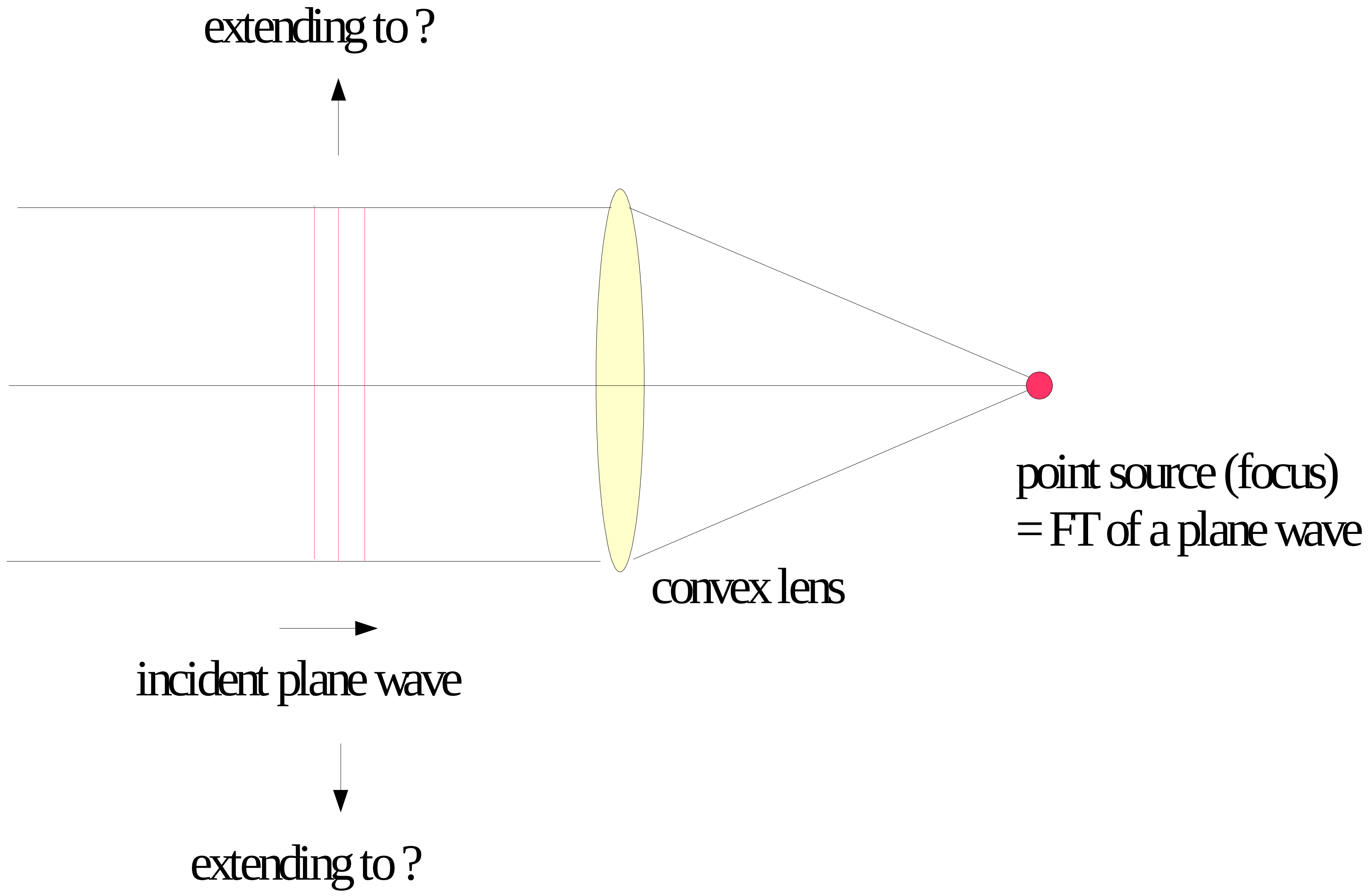
The correlator in practice

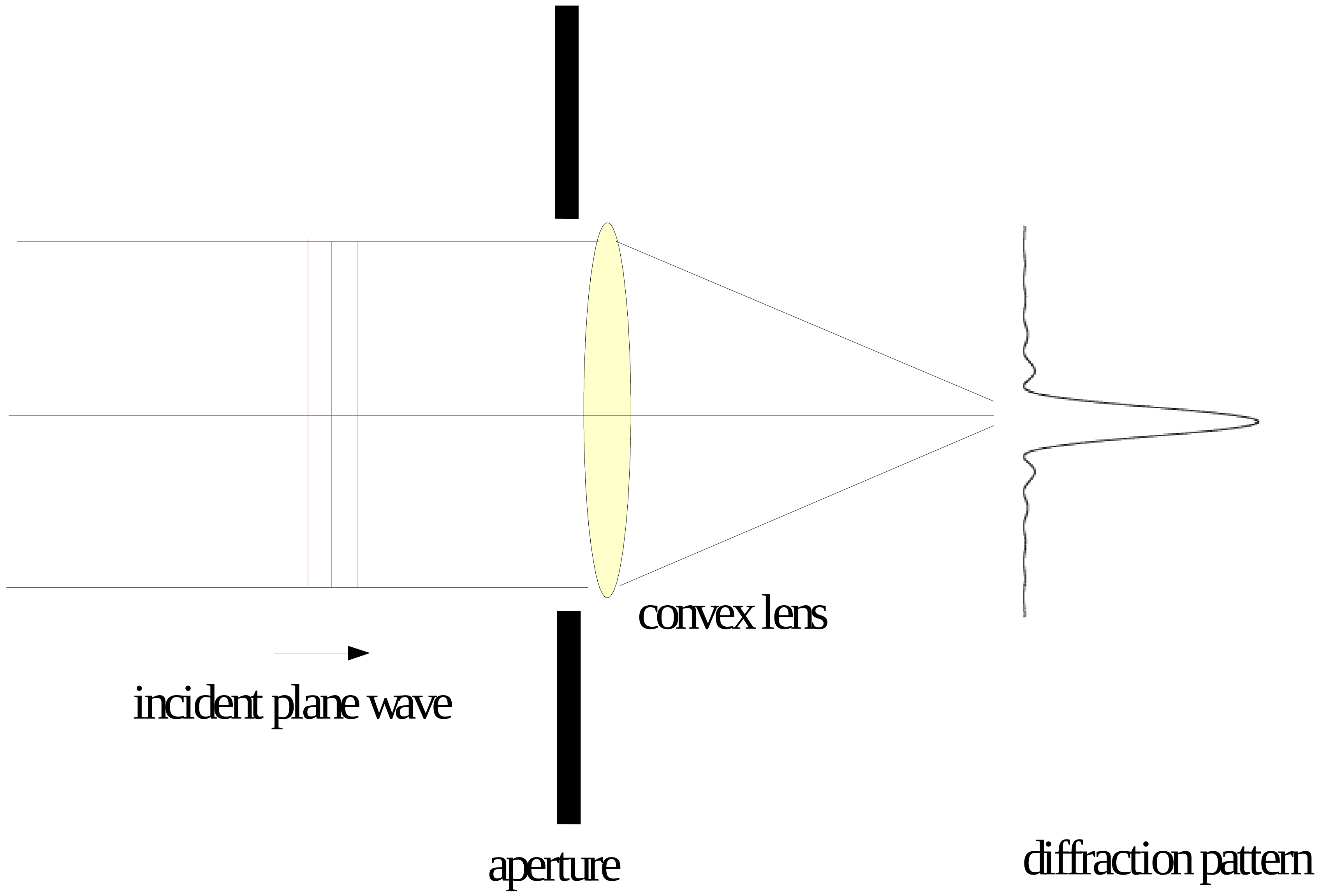
Nothing is perfect...

Complications:

- Digitization (i.e. sampling & clipping) leads to a (small) loss of information.
- The signals are limited in frequency.
- The time series of the cross correlation product are finite.

There is an anologon to the last two complications in classical optics...


extending to ?
point source (focus)
= FT of a plane wave
convex lens
incident plane wave
extending to ?


convex lens
incident plane wave
aperture
diffraction pattern

The correlator in practice

Consequence of the finite intermediate frequency bandwidth

frequency space: multiplication with a rectangular bandpass function

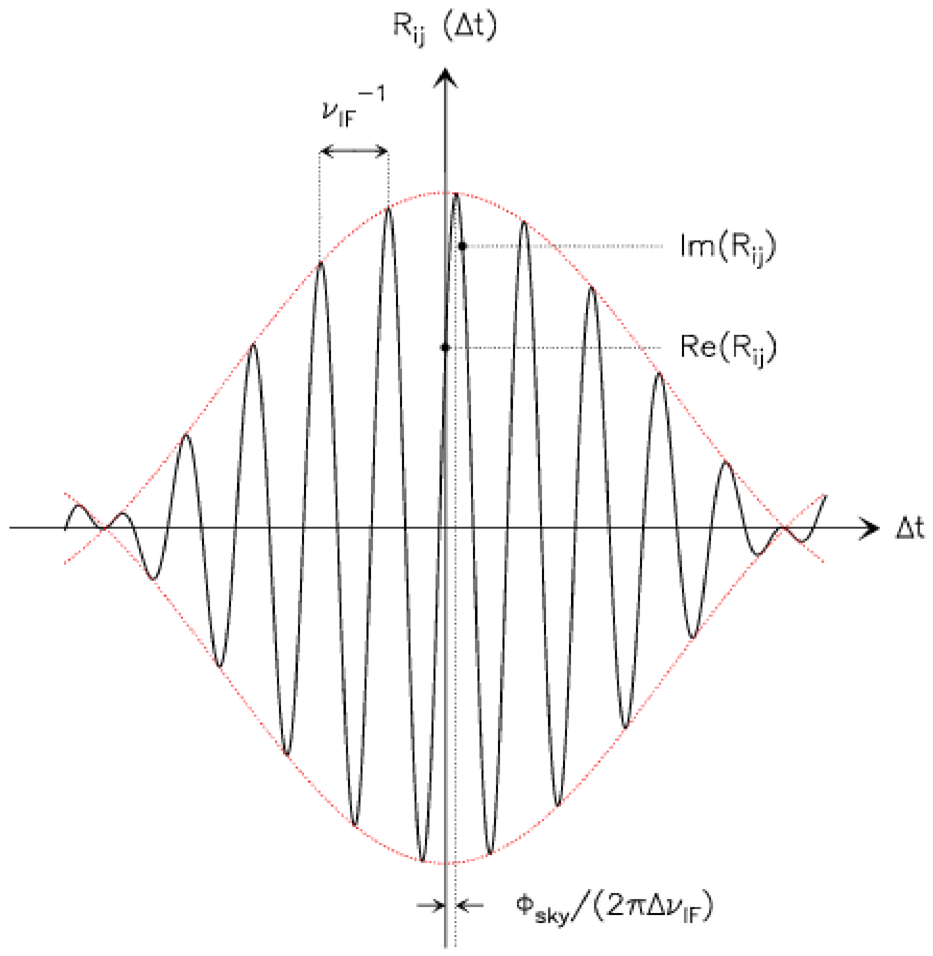
? ? Fourier transform

delay space: convolution with a sinc function

→ A large bandwidth requires accurate delay tracking.

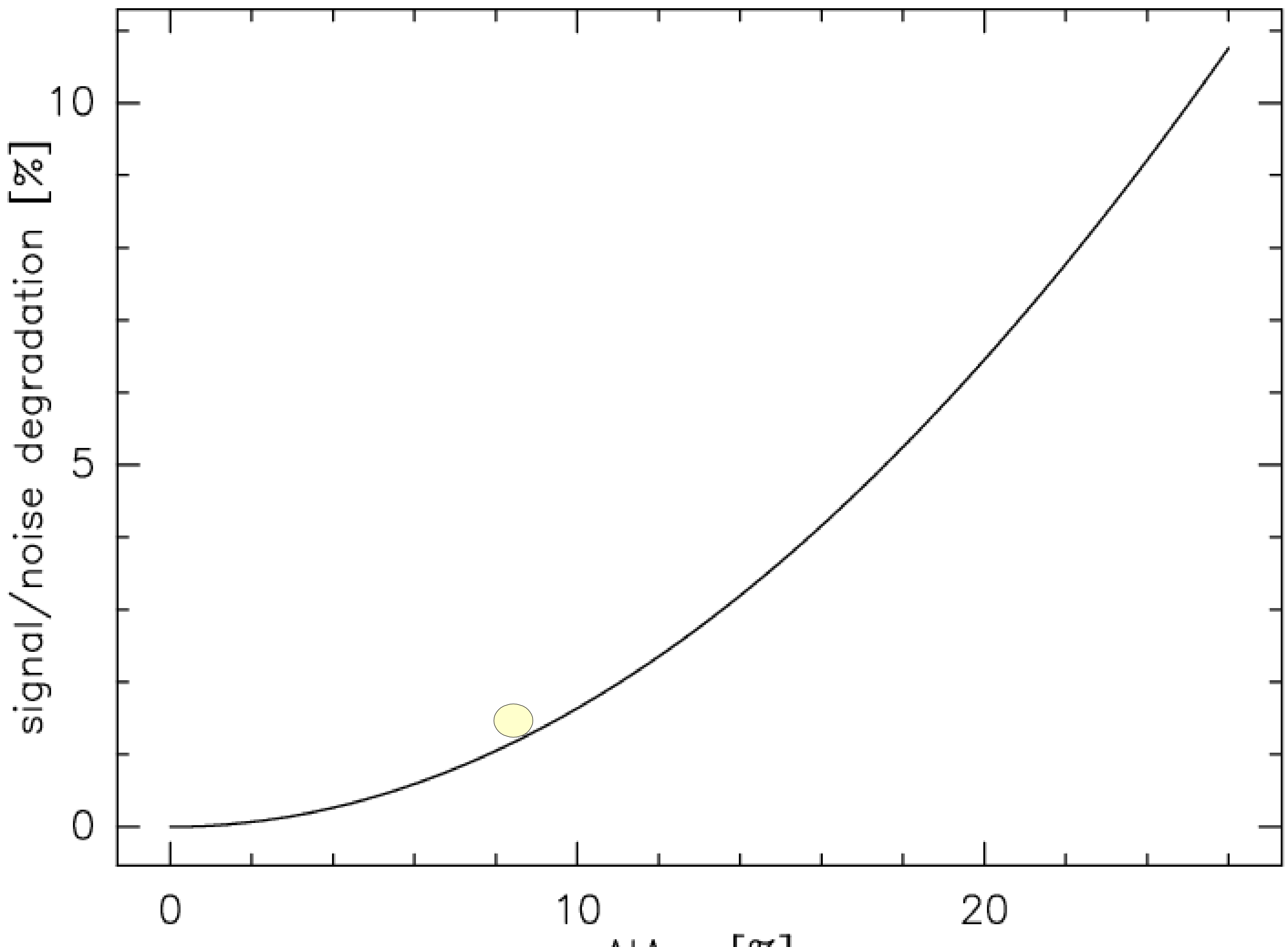
The correlator in practice (continued)

Response of a cross correlator to a rectangular bandpass shape



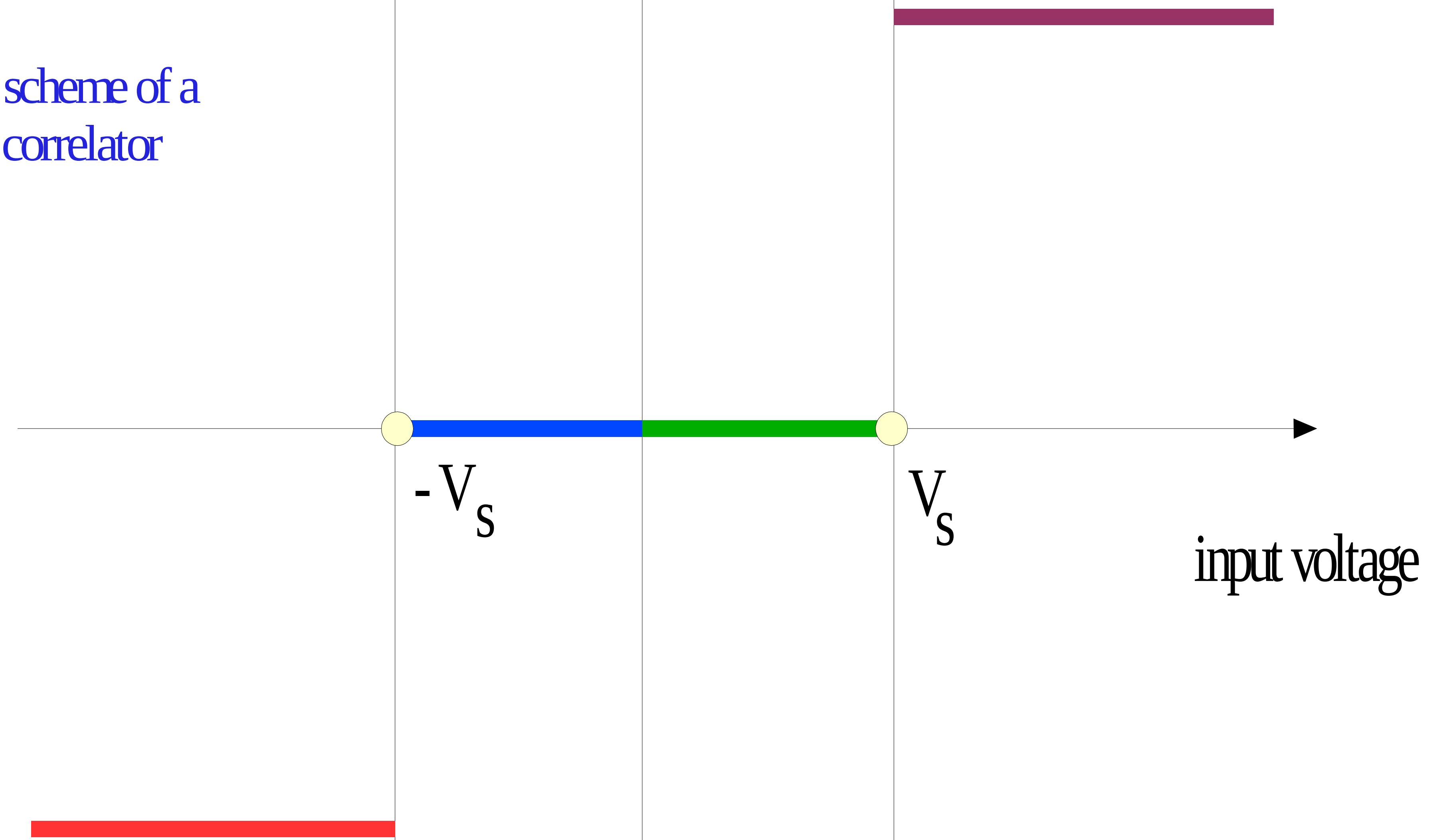
The correlator in practice (continued)

Example: 1 % loss of sensitivity for a delay tracking error of 0.16 nsec and 500 MHz IF bandwidth.



The correlator in practice (continued)

Digitization scheme of a 4 level 2 bit correlator



Sign bit	1	1	0	0
Magnitude bit	1	0	0	1

The correlator in practice (continued)

Simple case

Signal 1 in static register, signal 2 in shift register.

Disadvantage:

needs maximum time lag to get all channels.

Signal 2	1	2	3	4	5	6	7	8
Signal 1	8	9	10	11	12	13	14	15

$\Delta T = 7$

The correlator in practice (continued)

Technical implementation

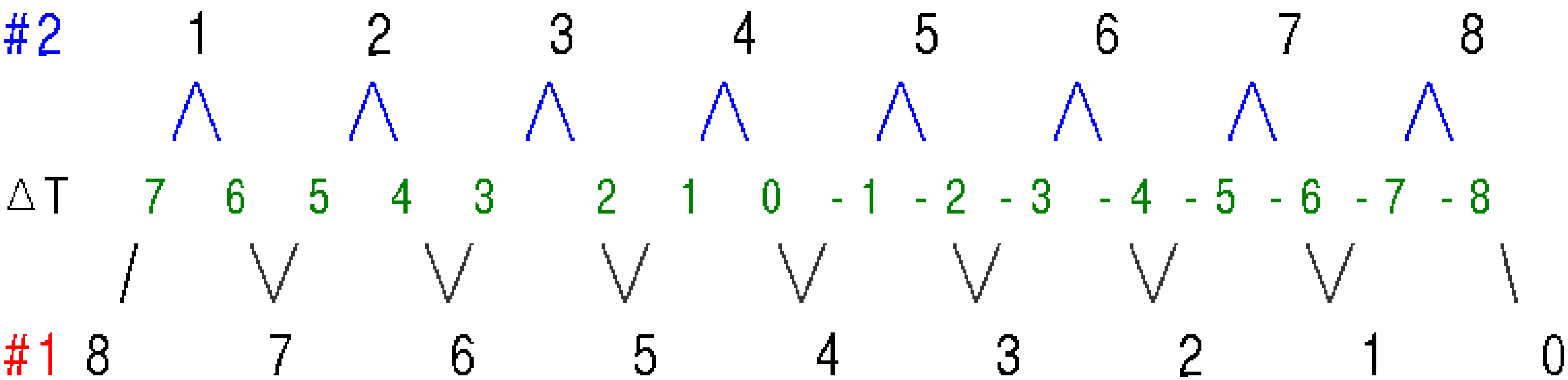
Signal 1 and 2 are in opposed shift registers.

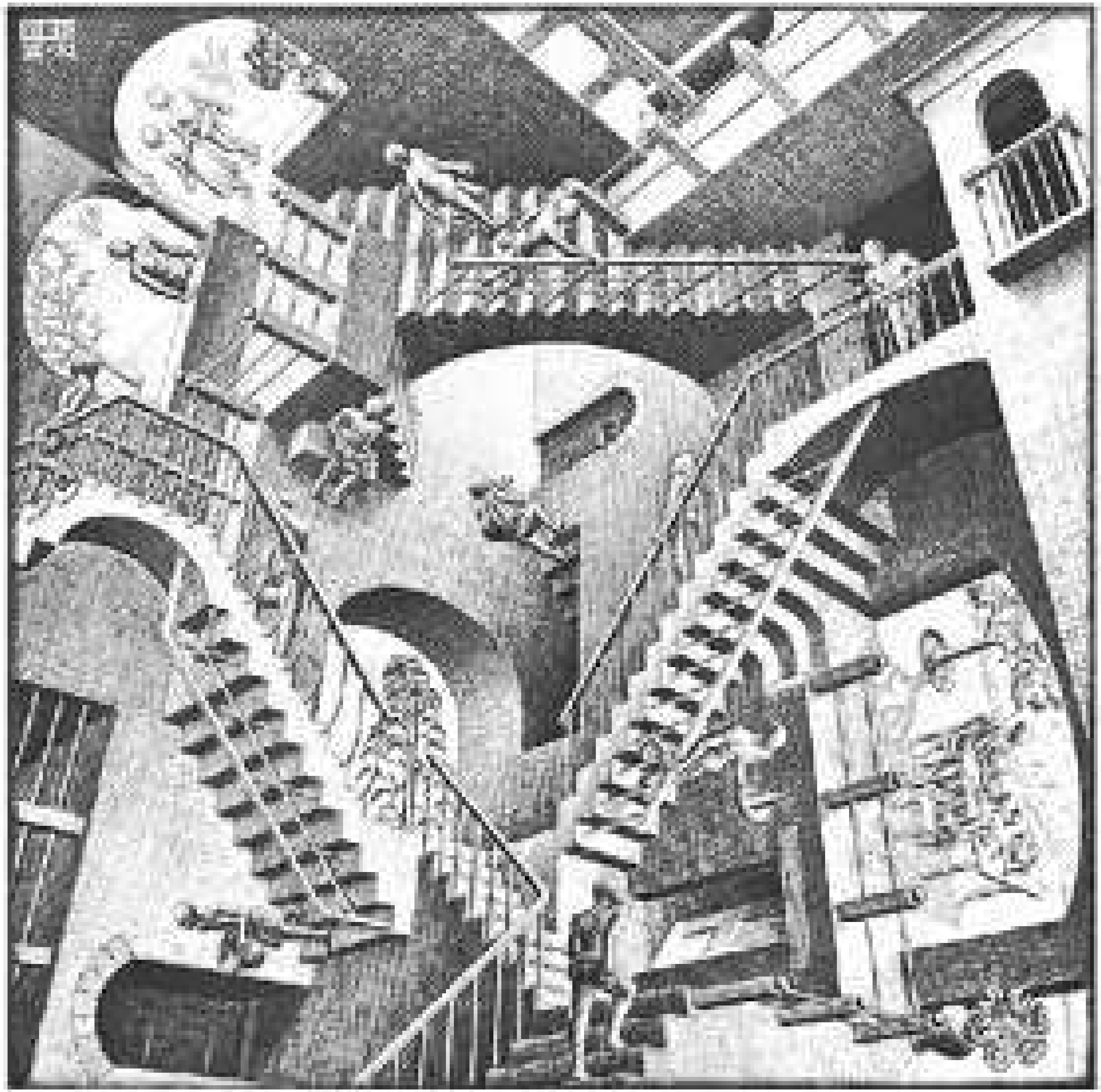
Disadvantage: every other time lag missing.

Signal 1	1	2	3	4	5	6	7	8
Product ?T	-7	-5	-3	-1	1	3	5	7
Signal 2	8	7	6	5	4	3	2	1

The correlator in practice (continued)

Technical implementation: both signals are in shift registers.





M. C. E S C H E R

The correlator in practice (continued)

The products are evaluated in the following way

	S(x)	0	0	1	1
	M(x)	1	0	0	1
S(y)	M(y)				
0	1	n^2	n	$-n$	$-n^2$
0	0	n	1	-1	$-n$
1	0	$-n$	-1	1	n
1	1	$-n^2$	$-n$	n	n^2

The correlator in practice (continued)

The correlator's output then amounts to:

$$\rho_4 = \frac{2n^2\left(N_{01,01} - 2N_{01,11}\right) + 4n\left(N_{00,01} - N_{00,11}\right) + 2\left(N_{00,00} - N_{00,10}\right)}{2\left(n^2 N_{01,01} + N_{00,00}\right)_{\rho=1}}$$

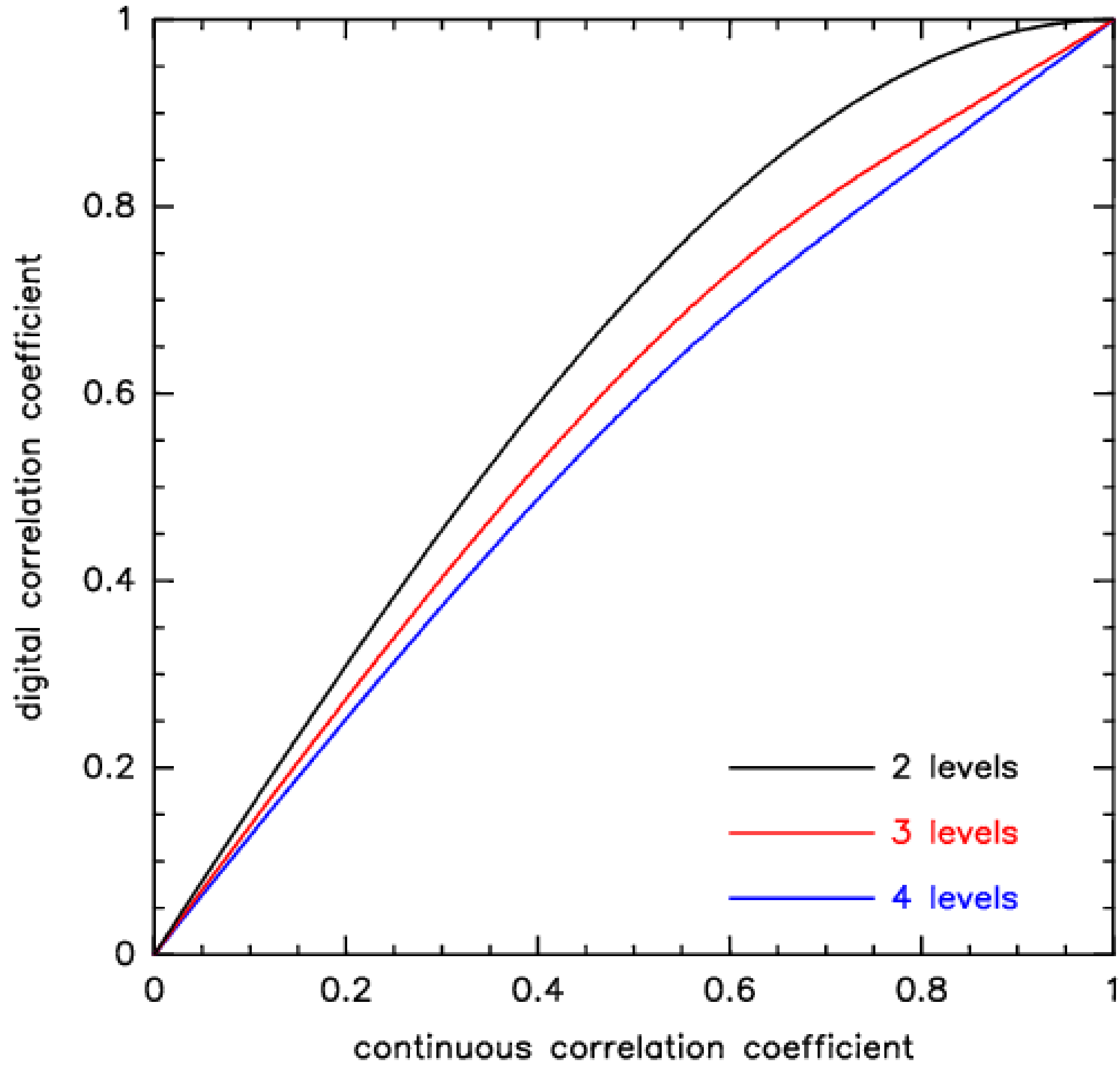
Clipping correction:

calculate the jointly Gaussian probability distributions as a function of the continuous correlation coefficient, such as

$$N_{01,01} = NP_{01,01} = \frac{N}{2\pi\sigma^2\sqrt{1-\rho^2}} \int_{v_s}^{\infty}\int_{v_s}^{\infty} \exp\left(-\frac{x^2+y^2-2\rho xy}{2\sigma^2\left(1-\rho^2\right)}\right) dx\, dy$$

and tabulate the result in a look-up table, or fit a polynomial function.

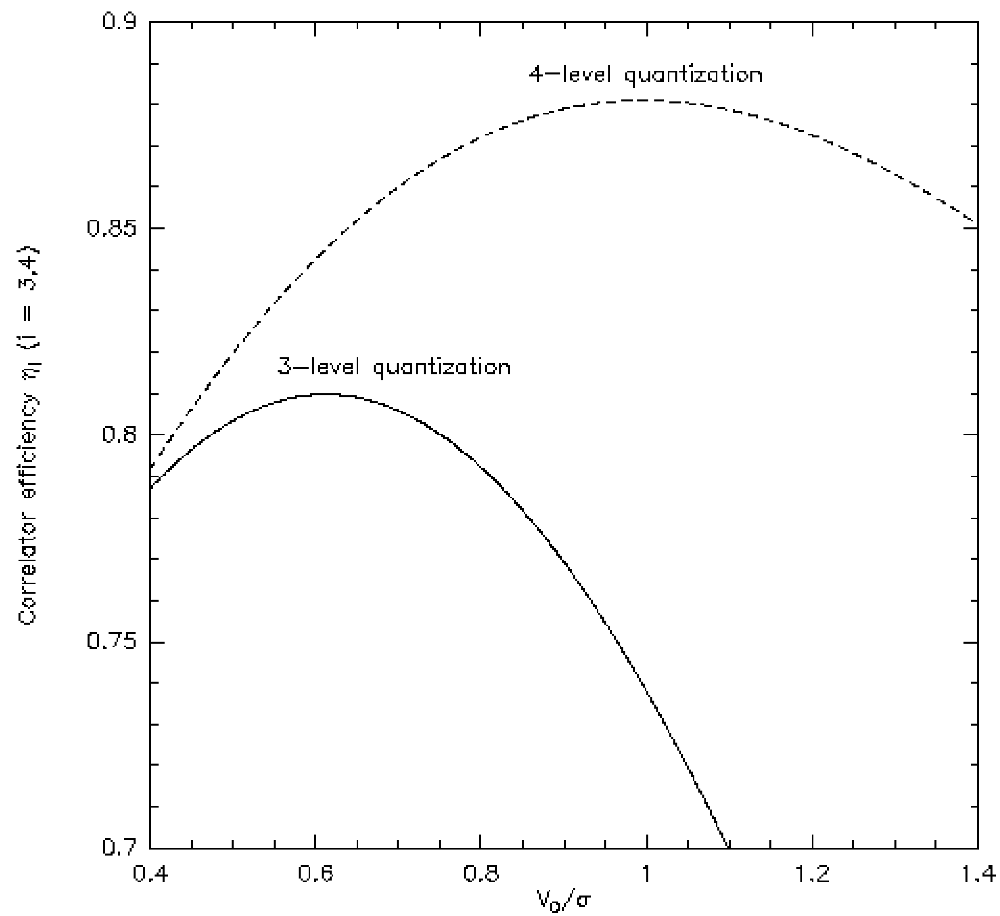
The correlator in practice (continued)



The correlator efficiency

is defined as the signal-to-noise ratio of the digital cross correlation, normalized by that of the continuous one:

$$\eta_k = \frac{R_k}{\sqrt{\langle R^2 \rangle - \langle R \rangle^2}} \cdot \frac{1}{\rho \sqrt{N}}$$

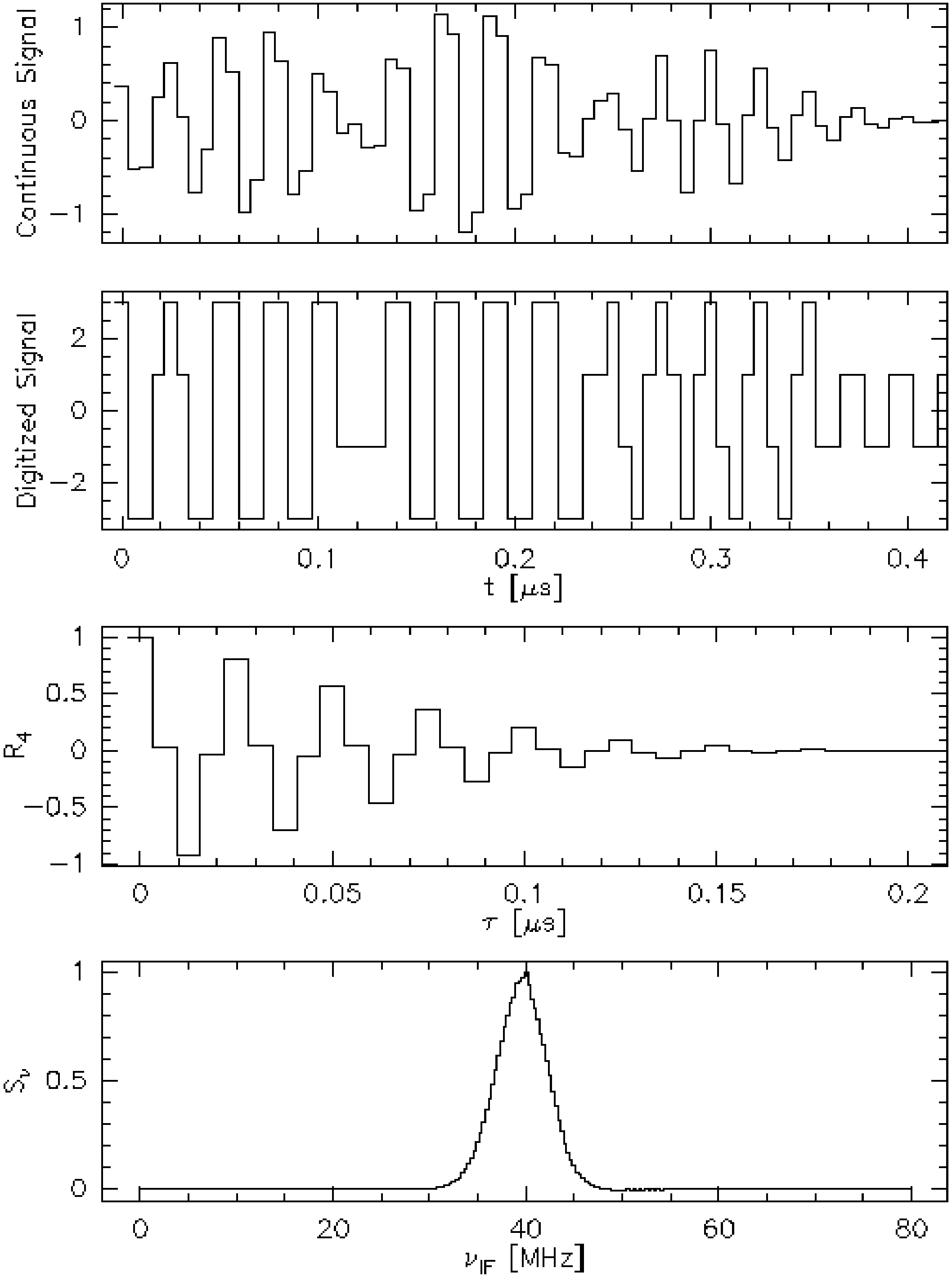


The correlator in practice

Signal processing steps in a 3 level - 2 bit correlator

From top to bottom:

- original time series
- digitized time series
- digital correlation
- output spectral line



The frequency resolution of a spectral correlator...

- The sampling theorem yields:

$$\Delta t = \frac{1}{2\Delta \nu_{IF}}$$

- The resulting channels spacing is given by the largest time lag:

$$\delta \nu = \frac{1}{2N\Delta t}$$

- In practice, the channel separation does not equal the spectral resolution.

The correlator in practice

Consequence of the finite time lag

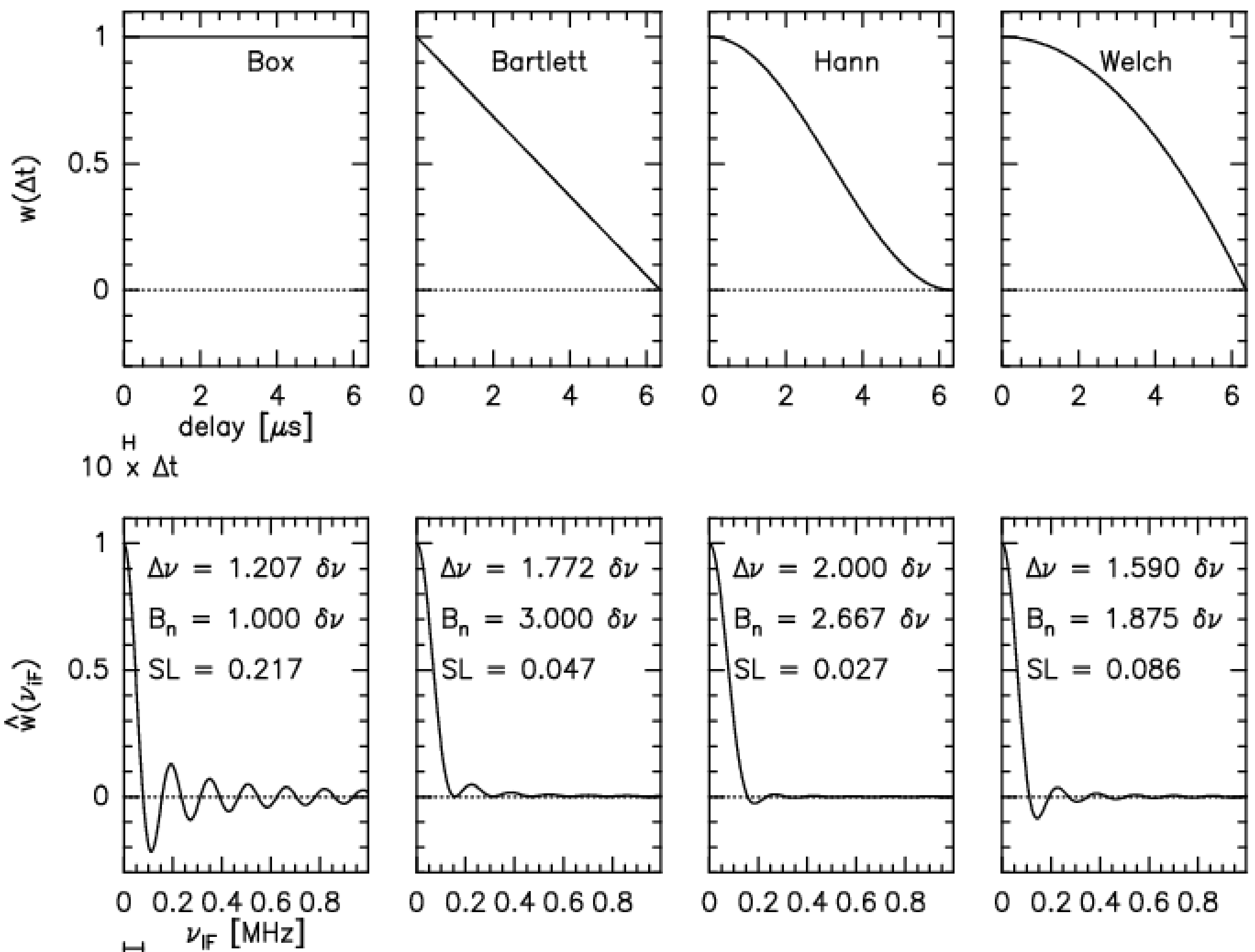
delay space: multiplication with a rectangular time lag window

? ? Fourier transform

frequency space: convolution with a sinc function

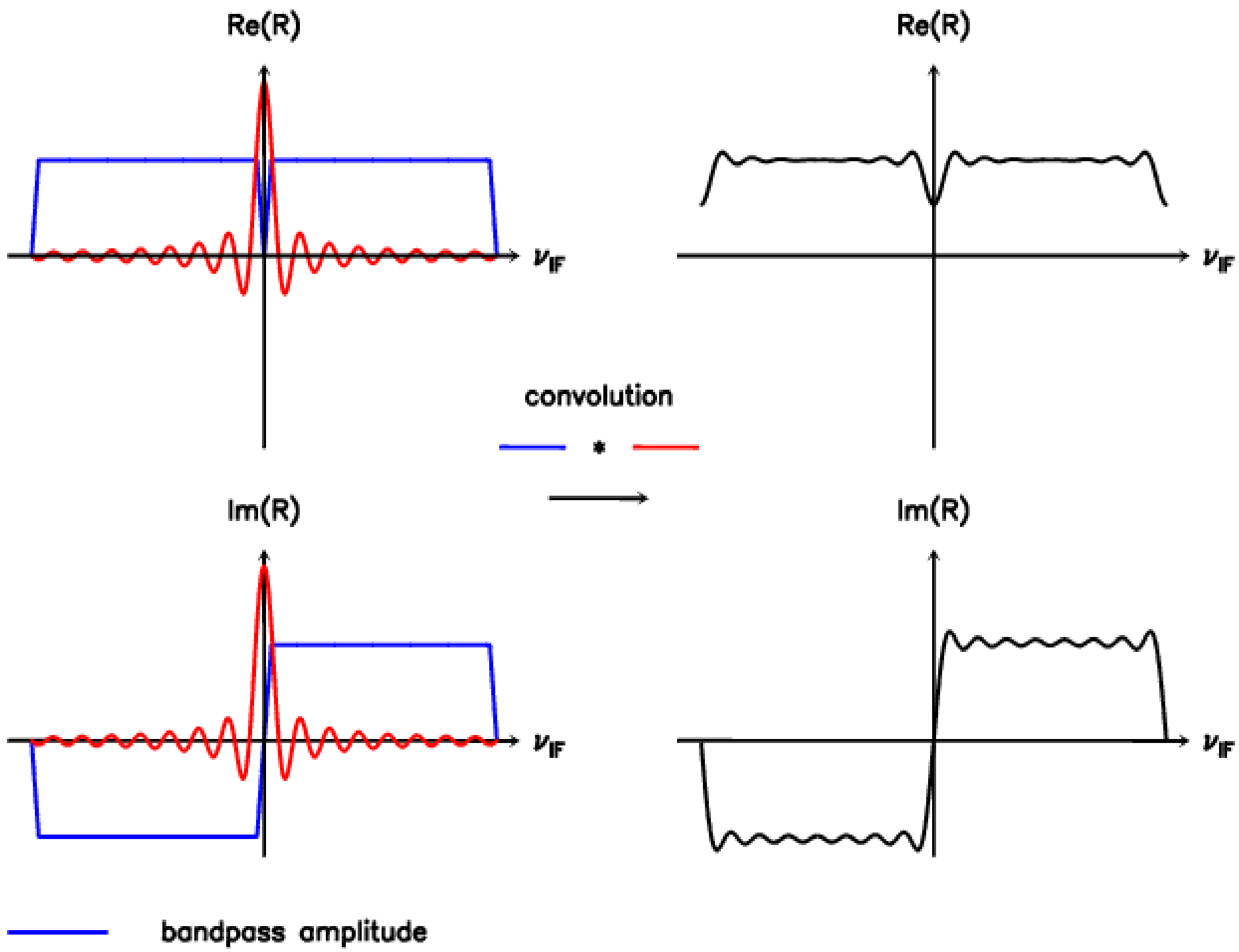
→ The effect appears in the cross power spectrum.

Time lag windows and their Fourier Transform



The Gibbs phenomenon

Convolution of the bandpass with the spectral window for continuum signals:



Main Limitations

... of the analog part:

- The Gibbs phenomenon is a convolution. The instrument-dependent factors do not cancel out anymore in a calibration. Channels concerned have to be flagged.
- The same holds for spectral lines: do not place the most important part of your line in those channels.

... of the digital part (samplers):

- The noise level may change drastically and needs to be adjusted by injecting a noise source.
- Threshold variations can be decomposed in an even and an odd part. The even part is equivalent to a gain variation and can be calibrated out. The odd part cancels out by periodic half-wave changes of both the local oscillator phase and the digitized signals (such that the output is not concerned). The treshold errors cancel out with high precision.

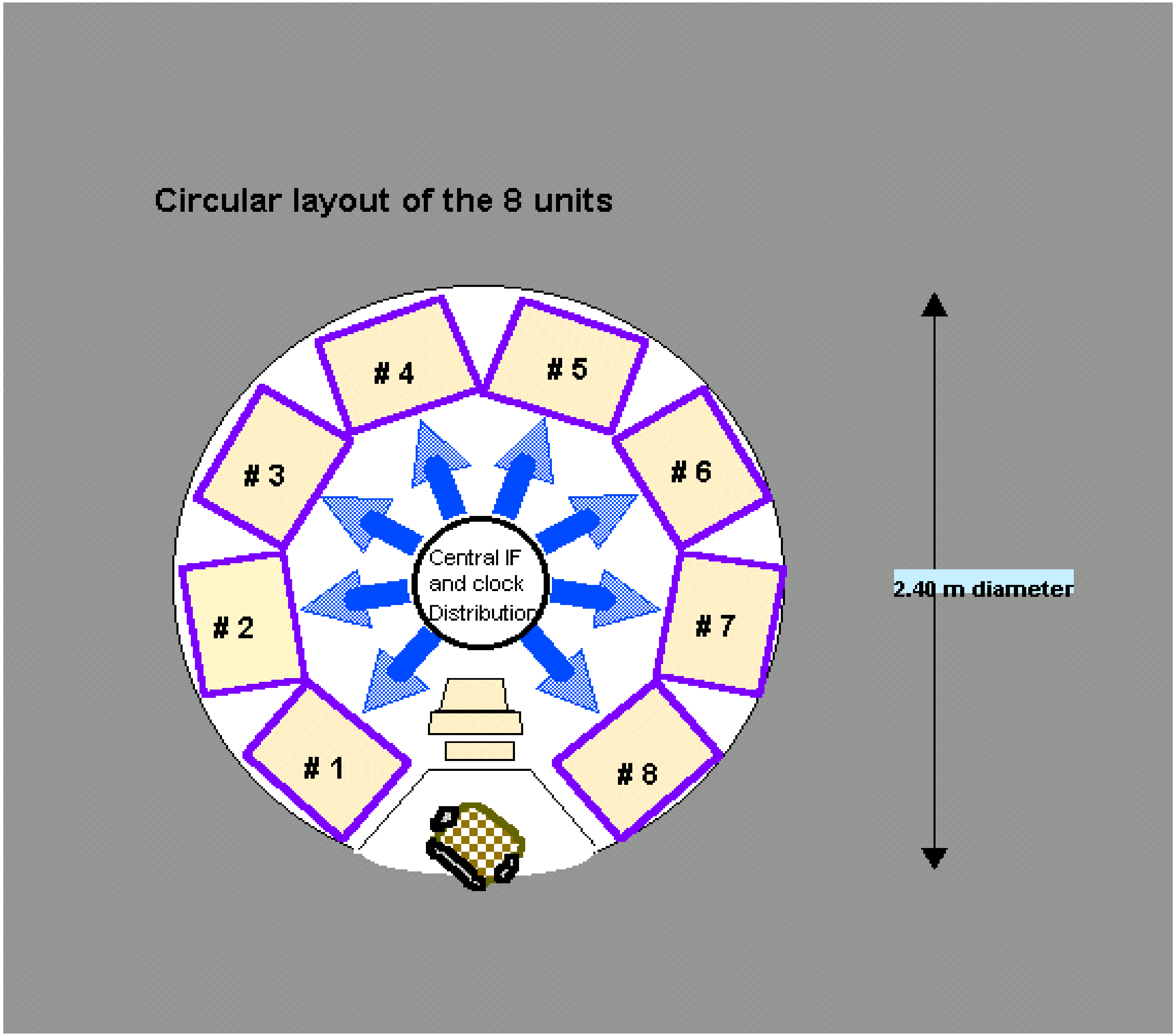
The correlator on Plateau de Bure

The third-generation correlator has 2.56 GHz global bandwidth, and performs 9.8 TeraMultiplications per second. It has a flexible architecture comprising 8 independent units. Each unit consists of 3 parts :

- an IF processor for the analog functions (frequency setting, low-pass filter selection, oscillator phase control)
- a digital part, controlled by a master processor (delay steps, clipping correction, FFT, small delay corrections, bandpass correction)
- a satellite micro processor, reading out and further processing the correlations.

Each unit can be placed in the [100,1100] MHz IF band, in steps of 0.625 MHz (using a third frequency conversion). Note that the receivers actually limit the bandwidth to [100, 680] MHz.

The correlator on Plateau de Bure



The correlator on Plateau de Bure (continued)

The data are sampled at 320 Ms/ sec => 160 MHz bandwidth (SSB mode).

Time multiplexing means:

instead of a time series	1	2	3	4	5	6	7	8	(one shift register)							
generate 2 time series	1		3		5		7		9		11		13		15	
		2		4		6		8		10		12		14		
									(two shift registers)							

at 160 Ms/sec each => 80 MHz bandwidth, but 2 x higher spectral resolution.

The correlator on Plateau de Bure (continued)

Bandwidth [MHz]	*Subband of IRM*	*Clock rate [MHz]*	*Time Multiplex Factor*	*Number of time lags*	*Number of complex channels*	*Channel spacing [MHz]*	*Spectral Resolution [MHz] for apodization according to*	
							Box	*Welch*
2 x 160	DSB	80	4	2 x 128	2 x 64	2.500	3.018	3.975
1 x 160	SSB	80	4	1 x 256	1 x 128	1.250	1.509	1.988
2 x 80	DSB	80	2	2 x 256	2 x 128	0.625	0.754	0.994
1 x 80	SSB	80	2	1 x 512	1 x 256	0.312	0.377	0.497
2 x 40	DSB	80	1	2 x 512	2 x 256	0.156	0.189	0.248
1 x 40	SSB	80	1	1 x 1024	1 x 512	0.078	0.094	0.124
1 x 20	SSB	40	1	1 x 1024	1 x 512	0.039	0.047	0.062