

# Imaging, Deconvolution & Image Analysis

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# Scientific Analysis of a mm Interferometer Output

mm interferometer output:

Calibrated visibilities in the  $uv$  plane ( $\simeq$  the Fourier plane).

2 possibilities:

- $uv$  plane analysis (cf. Lecture by F. Gueth):  
Always better ... when possible!  
(in practice for “simple” sources as point sources or disks)
- Image plane analysis:  
 $\Rightarrow$  Mathematical transforms to go from  $uv$  to image plane!

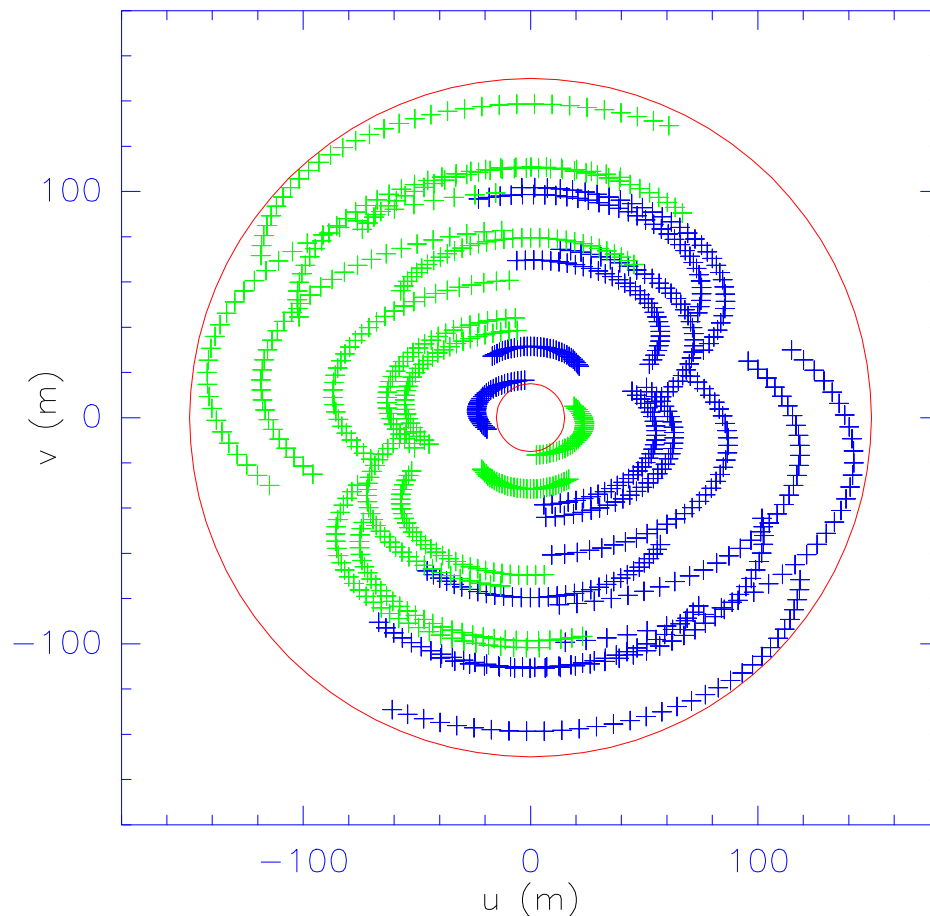
Goal: Understand effects of the imaging process on

- The resolution;
- The field of view (single pointing or mosaicing, cf. Lecture by F. Gueth);
- The reliability of the image;
- The noise level and repartition (cf. lecture by S. Guilloteau).

# From Calibrated Visibilities to Images:

## I. Comparison Visibilities/Source Fourier Transform

$$V_{ij}(b_{ij}) = 2D \text{ FT} \left\{ B_{\text{primary}} \cdot I_{\text{source}} \right\} (b_{ij}) + N$$



- Primary Beam  
⇒ Distorted source information.
- Noise ⇒ Sensitivity problems.
- Irregular, limited sampling  
⇒ incomplete source information:
  - Support limited at:
    - \* High spatial frequency  
⇒ limited resolution;
    - \* Low spatial frequency ⇒ problem of wide field imaging;
  - Inside the support, incomplete (*i.e.* Nyquist's criterion not respected) sampling ⇒ lost of information.

## From Calibrated Visibilities to Images: II. Effect of Irregular, Limited Sampling

Definitions:

- $V = 2D \text{ FT} \{B_{\text{primary}} \cdot I_{\text{source}}\};$
- Irregular, limited sampling function:
  - $S(u, v) = 1$  at  $(u, v)$  points where visibilities are measured;
  - $S(u, v) = 0$  elsewhere;
- $B_{\text{dirty}} = 2D \text{ FT}^{-1} \{S\};$
- $I_{\text{meas}} = 2D \text{ FT}^{-1} \{S \cdot V\}.$

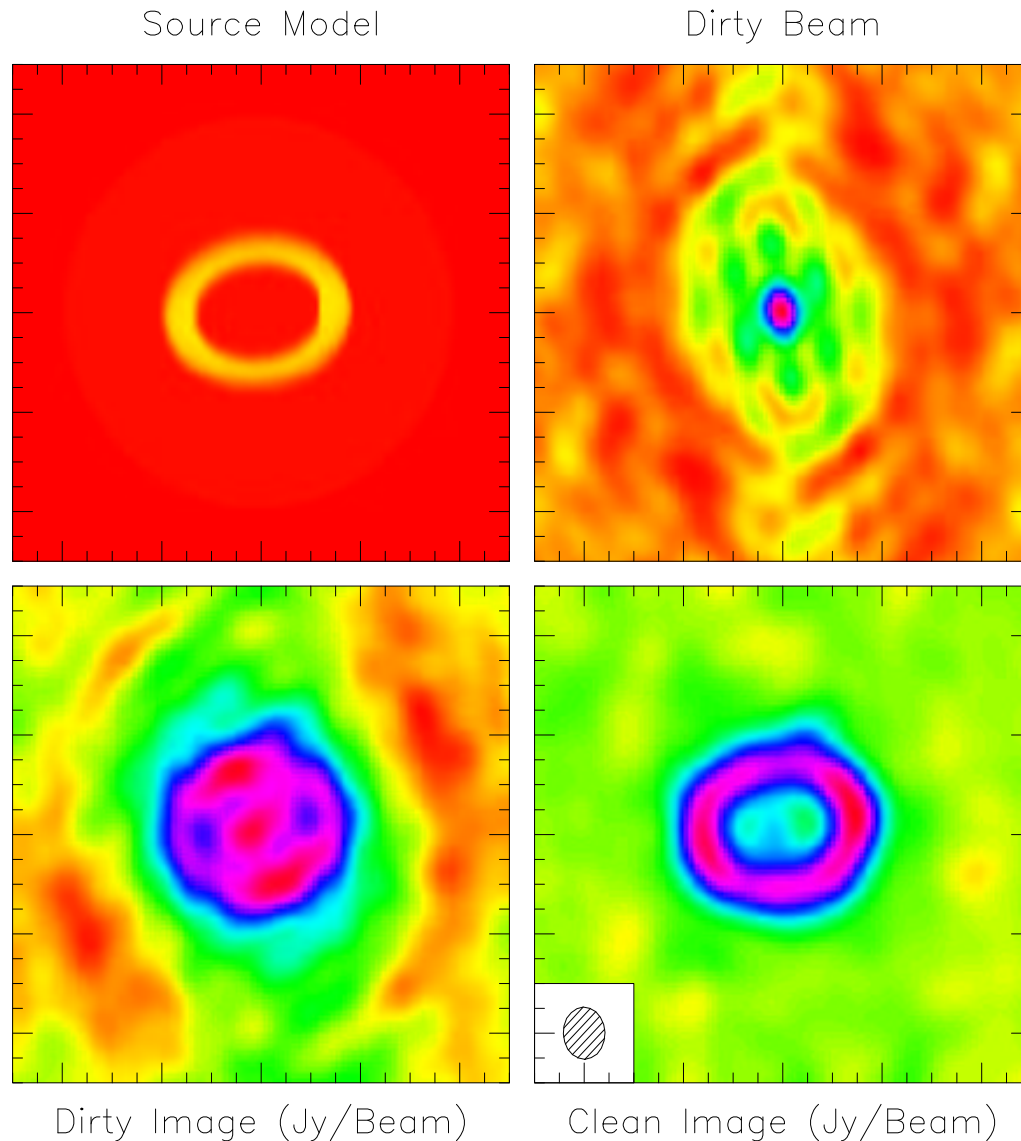
Fourier Transform Property #1:

$$I_{\text{meas}} = B_{\text{dirty}} * \{B_{\text{primary}} \cdot I_{\text{source}}\}.$$

$B_{\text{dirty}}$ : Point Spread Function (PSF) of the interferometer  
(i.e. if the source is a point, then  $I_{\text{meas}} = I_{\text{tot}} \cdot B_{\text{dirty}}$ ).

# From Calibrated Visibilities to Images:

## III. Why Deconvolving?



- Difficult to do science on dirty image.
- Deconvolution  $\Rightarrow$  a clean image compatible with the sky intensity distribution.

# From Calibrated Visibilities to Images:

## IV. Summary

Fourier Transform and Deconvolution:  
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
↓ Fourier Transform	UV_STAT, UV_MAP
Dirty beam & image	
↓ Deconvolution	HOGBOM, CLARK
Clean beam & image	
↓ Visualization	GO_BIT, GO_VIEW
↓ Image analysis	GO_NOISE, GO_FLUX, GO_MOMENTS
Physical information on your source	

# Direct vs. Fast Fourier Transform

Direct FT:

- Advantage: Direct use of the irregular sampling;
- Inconvenient: Slow.

Fast FT:

- Inconvenient: Needs a regular sampling  $\Rightarrow$  Gridding;
- Advantage: Quick for images of size  $2^M \times 2^N$ .

$\Rightarrow$  In practice, everybody use FFT.

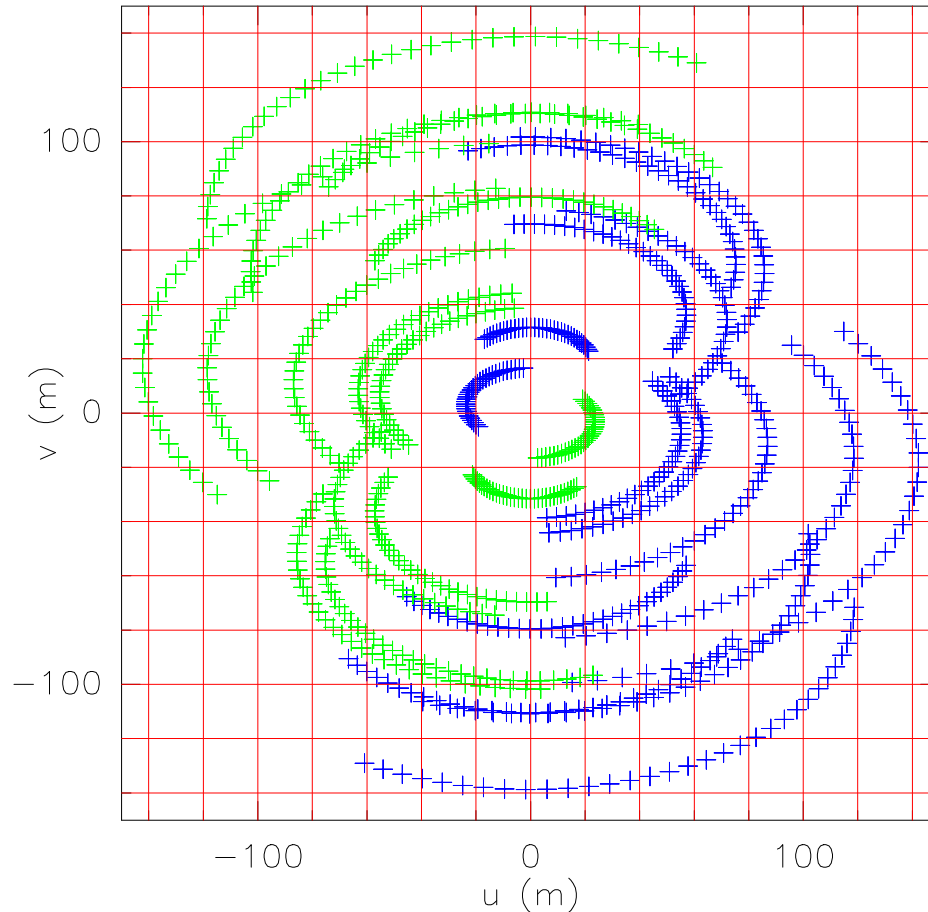
# From Calibrated Visibilities to Images: Summary

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Physical information on your source	



# Gridding: I. Interpolation Scheme



Convolution because:

- Visibilities = noisy samples of a smooth function.  
⇒ Some smoothing is desirable.
- Nearby visibilities are not independent.
  - $V = 2D \text{ FT} \{ B_{\text{primary}} \cdot I_{\text{source}} \}$   
 $= \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}};$
  - $\text{FWHM}(\text{convolution kernel}) < \text{FWHM}(\tilde{B}_{\text{primary}})$   
⇒ No real information lost.

## Gridding: II. Convolution Equation is Kept Through Gridding

Demonstration:

- $I_{\text{meas}}^{\text{grid}} \xLeftrightarrow[2\text{D FT}] G * (S.V) \Leftrightarrow I_{\text{meas}}^{\text{grid}} = \tilde{G} . (\widetilde{S.V});$
- $B_{\text{dirty}}^{\text{grid}} \xLeftrightarrow[2\text{D FT}] G * S \Leftrightarrow B_{\text{dirty}}^{\text{grid}} = \tilde{G} . \tilde{S};$

$$\Rightarrow I_{\text{meas}} = B_{\text{dirty}} * \{B_{\text{primary}} . I_{\text{source}}\}$$

with  $I_{\text{meas}} = I_{\text{meas}}^{\text{grid}} / \tilde{G}$   
and  $B_{\text{dirty}} = B_{\text{dirty}}^{\text{grid}} / \tilde{G}.$

Remark: Gridding may be hidden in equations but it is still there.

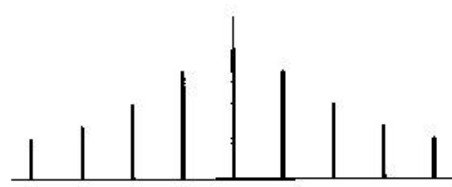
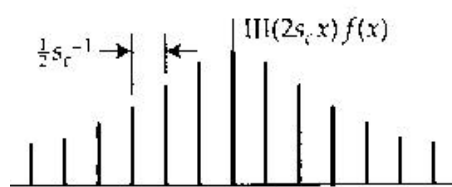
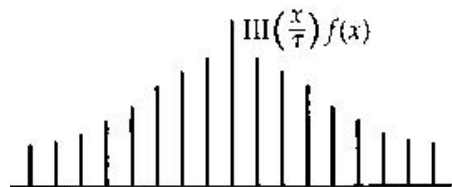
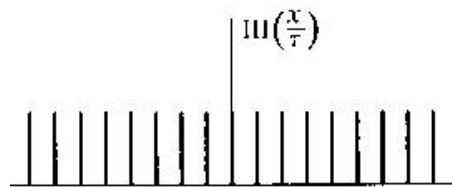
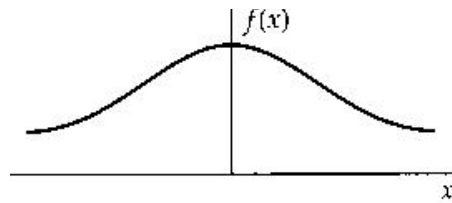
$\Rightarrow$  Artifacts due to gridding! (cf. next transparencies)

# Gridding:

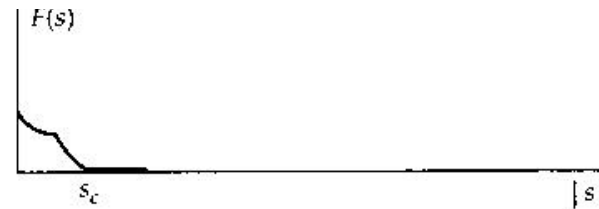
## III. Effect of a Regular Sampling (Periodic Replication)

$uv$  Plane

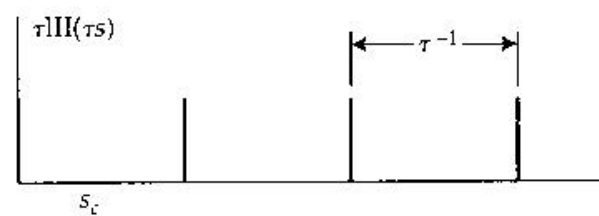
Image Plane



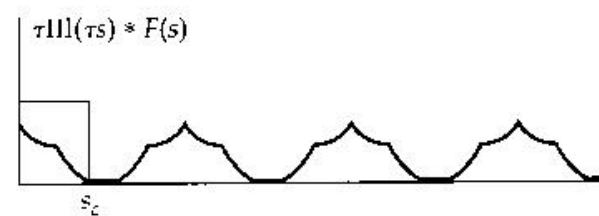
(a)



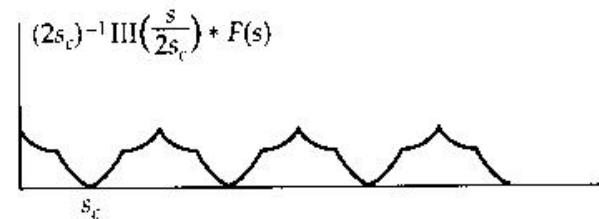
(b)



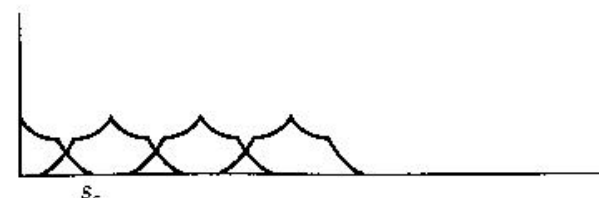
(c)



(d)



(e)



$B_{\text{primary}} \cdot I_{\text{source}}$

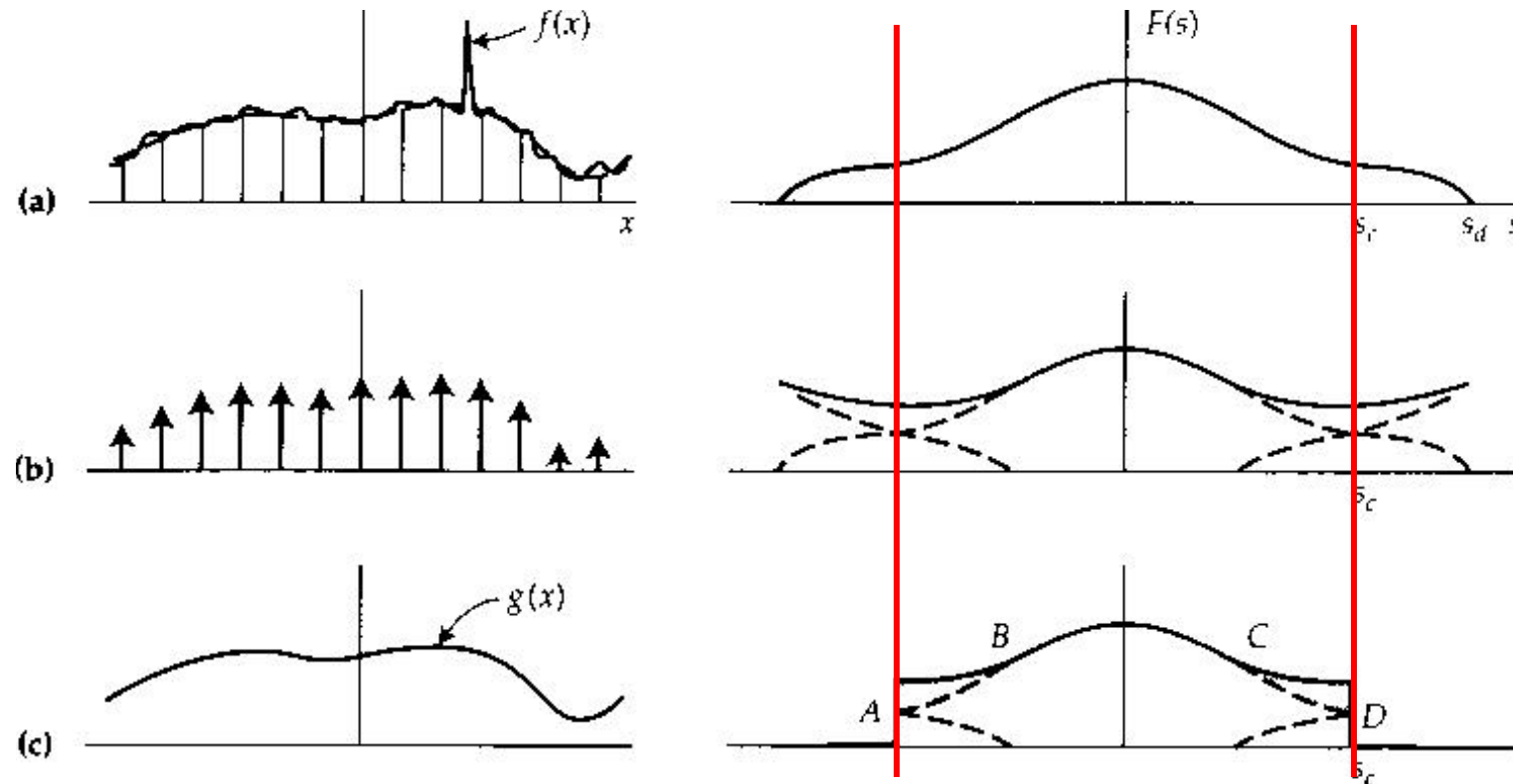
Regular Sampling function

Result for a fine sampling

Result for critical sampling  
(Nyquist's criterion)

Result for a coarse sampling

### Gridding: III. Effect of a Regular Sampling (Aliasing)



Aliasing = Folding of intensity outside the image size into the image.

⇒ Image size must be large enough.

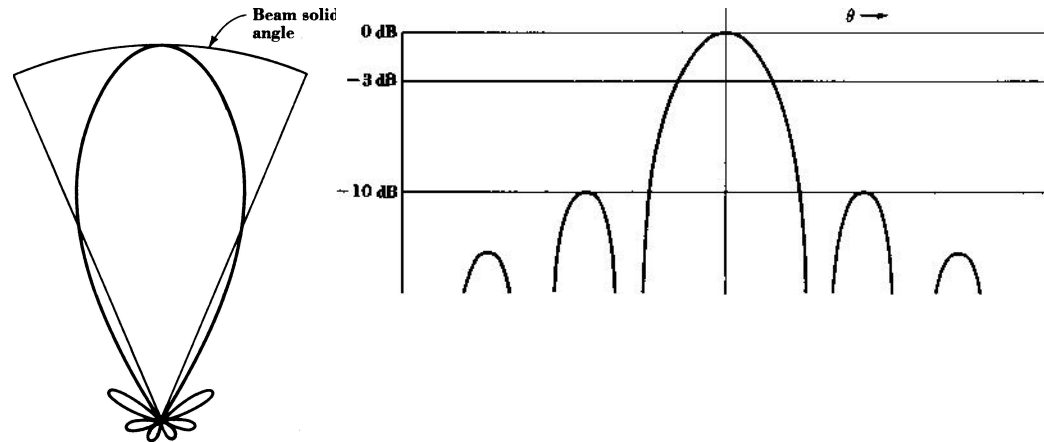
## Gridding: IV. Pixel and Image Sizes

Pixel size: Between  $1/3$  and  $1/4$  of the synthesized beam size (*i.e.* more than the Nyquist's criterion in image plane to ease deconvolution).

Image size:

- =  $uv$  plane sampling rate (FT property # 2);
  - Natural resolution in the  $uv$  plane:  $\tilde{B}_{\text{primary}}$  size;
- ⇒ At least twice the  $B_{\text{primary}}$  size (*i.e.* Nyquist's criterion in  $uv$  plane).

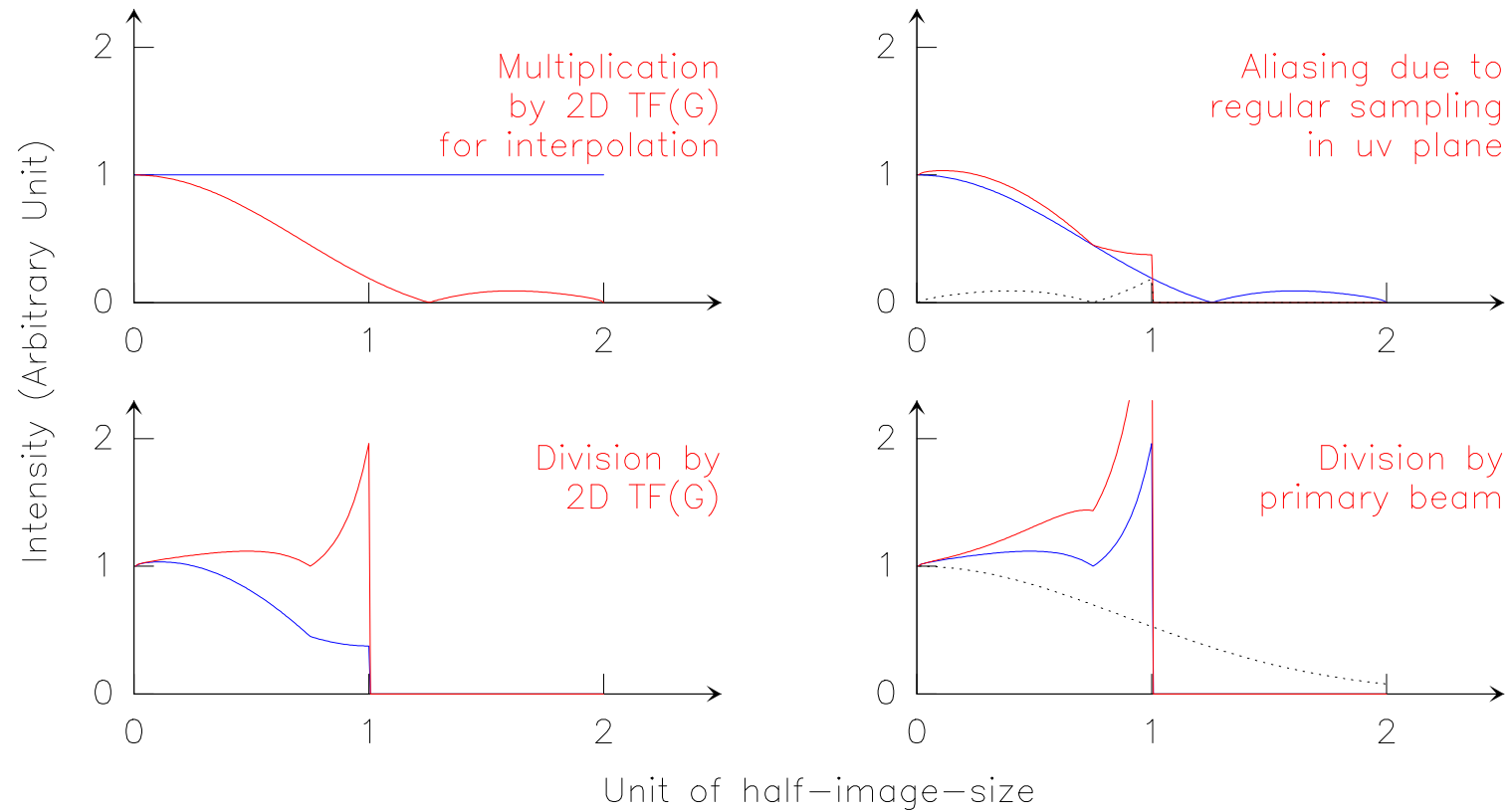
## Gridding: V. Bright Sources in $B_{\text{primary}}$ Sidelobes



Bright Sources in  $B_{\text{primary}}$  sidelobes  
outside image size will be aliased into image.  
 $\Rightarrow$  Spurious source in your image!

Solution: Increase the image size.  
(Be careful: only when needed for efficiency reasons!)

## Gridding: VI. Noise Distribution



## Gridding: VII. Choice of Gridding function

Gridding function must:

- Fall off quickly in image plane (to avoid noise aliasing);
- Fall off quickly in  $uv$  plane (to avoid too much smoothing).

⇒ Define a mathematical class of functions: **Spheroidal functions**.

GILDAS implementation: In `UV_MAP`

- Spheroidal functions = Default gridding function;
- Tabulated values are used for speed reasons.



# Dirty Beam Shape and Image Quality

$$B_{\text{dirty}} = 2\text{D FT}^{-1} \{S\}.$$

Importance of the Dirty Beam Shape:

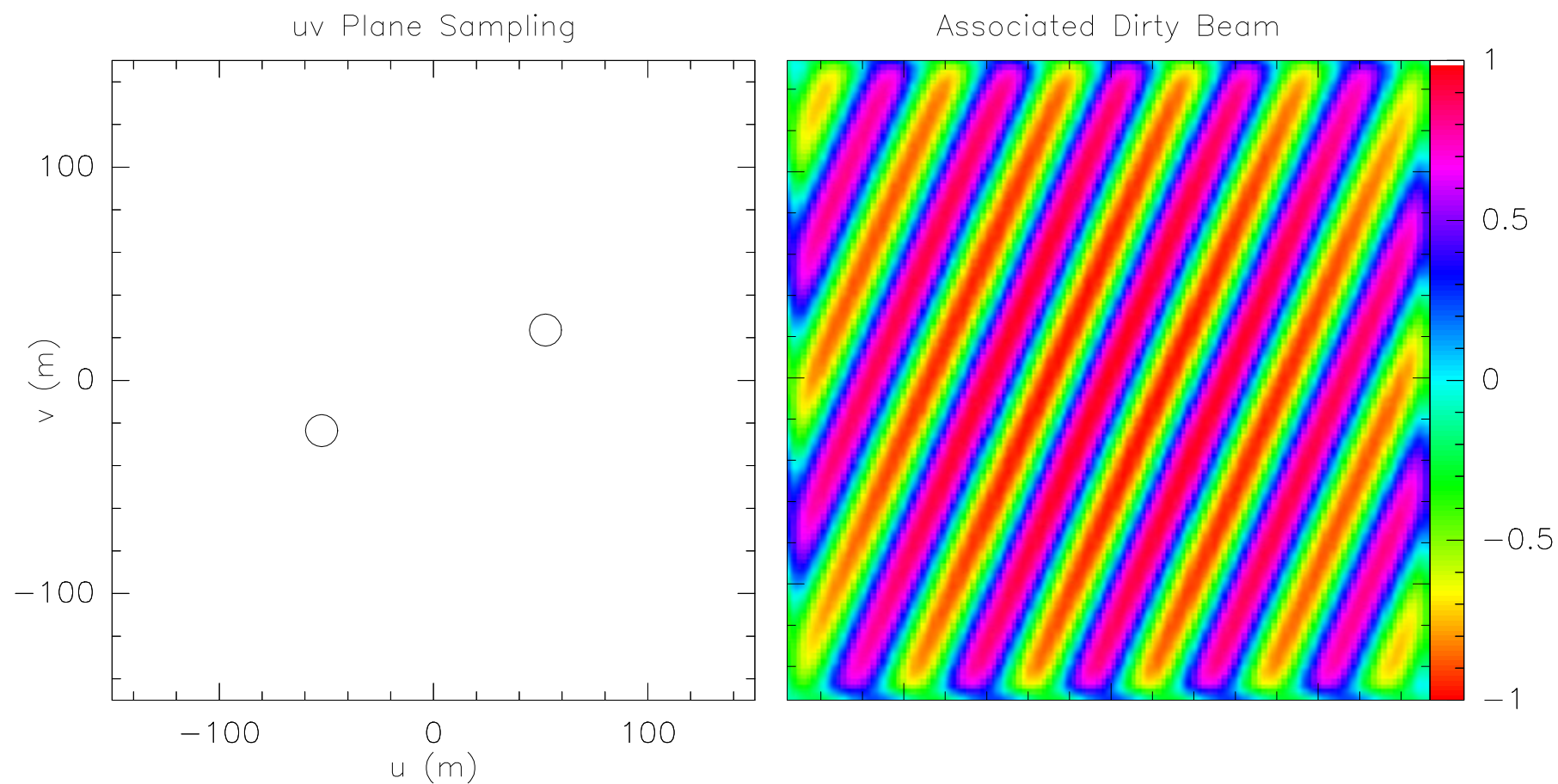
- Deconvolving a dirty image is a delicate stage;
- The closest to a Gaussian  $B_{\text{dirty}}$  is, the easier the deconvolution;
- Extreme case:  
 $B_{\text{dirty}} = \text{Gaussian} \Rightarrow$  No deconvolution needed at all!

Ways to improve (at least change)  $B_{\text{dirty}}$  shape:

- Increase the number of antenna (costly).
- Change the antenna layout (technically difficult).
- Weight the irregular, limited sampling function  $S$  (the only thing you can do in practice).

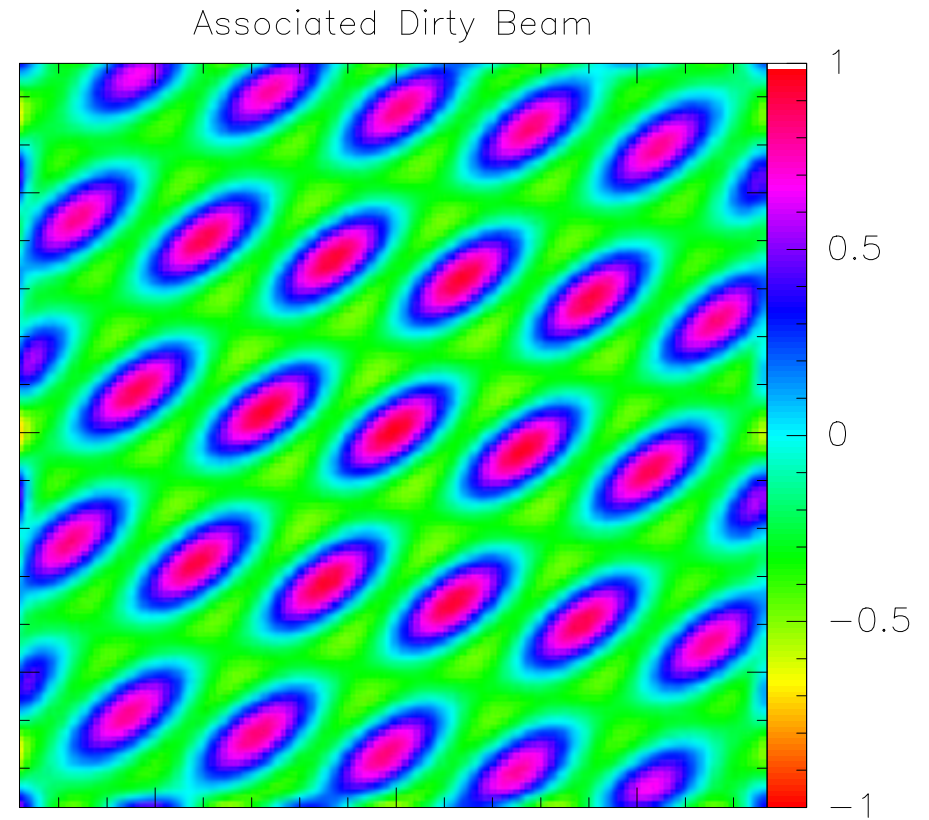
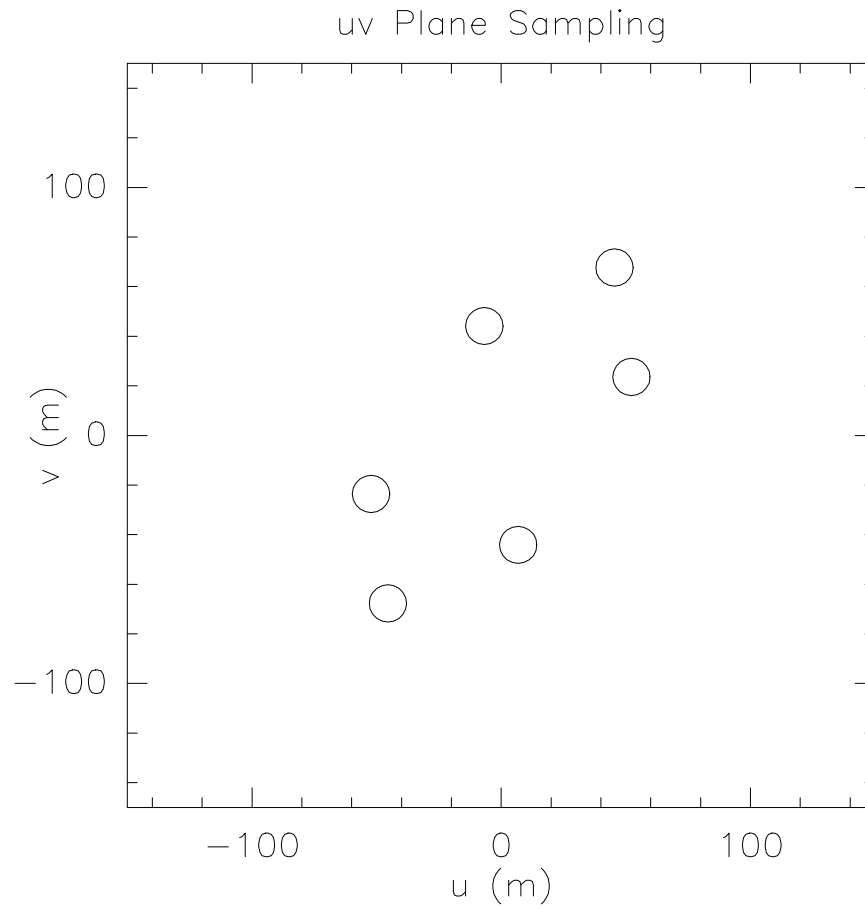
# Dirty Beam Shape and Number of Antenna:

## 2 Antenna



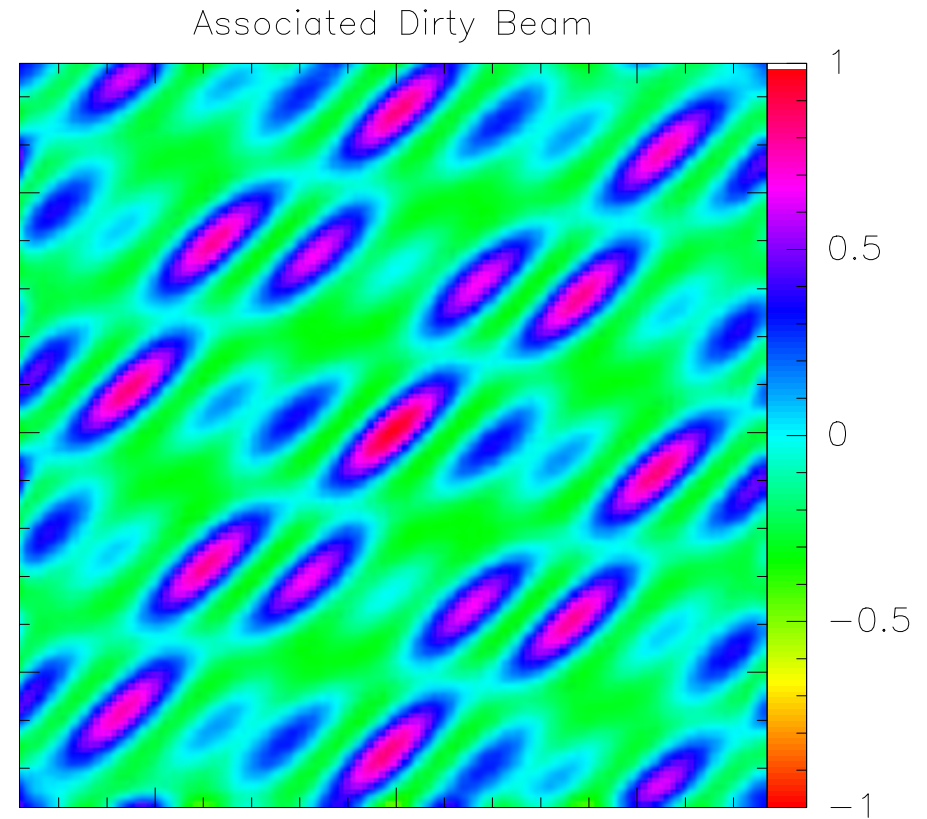
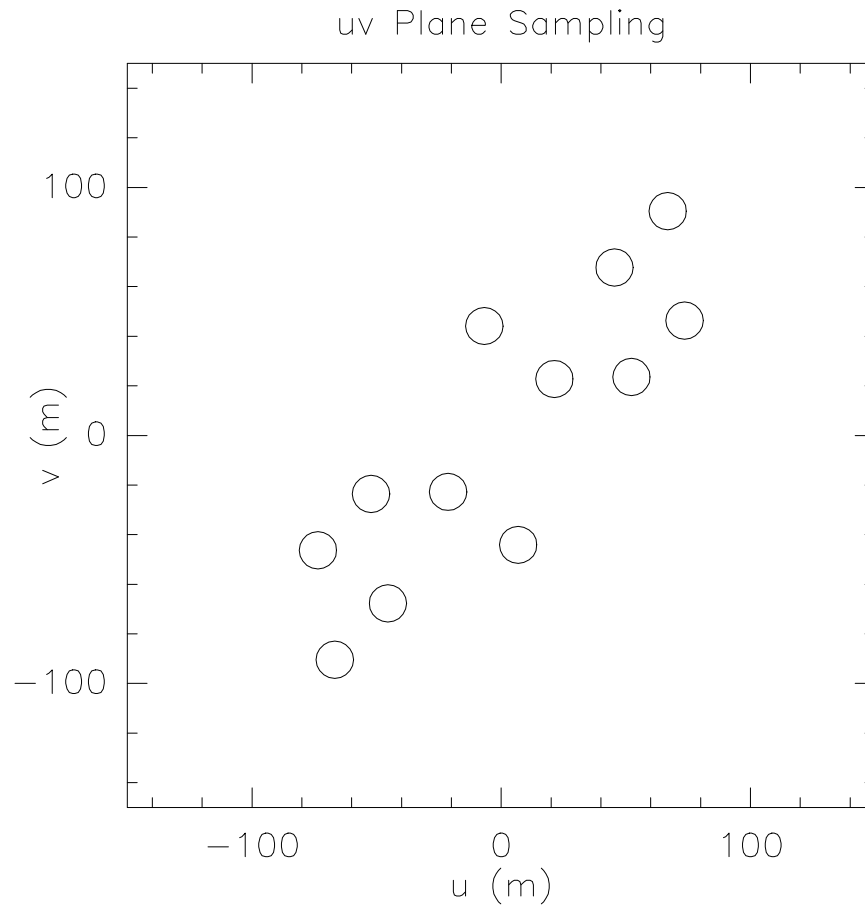
# Dirty Beam Shape and Number of Antenna:

## 3 Antenna



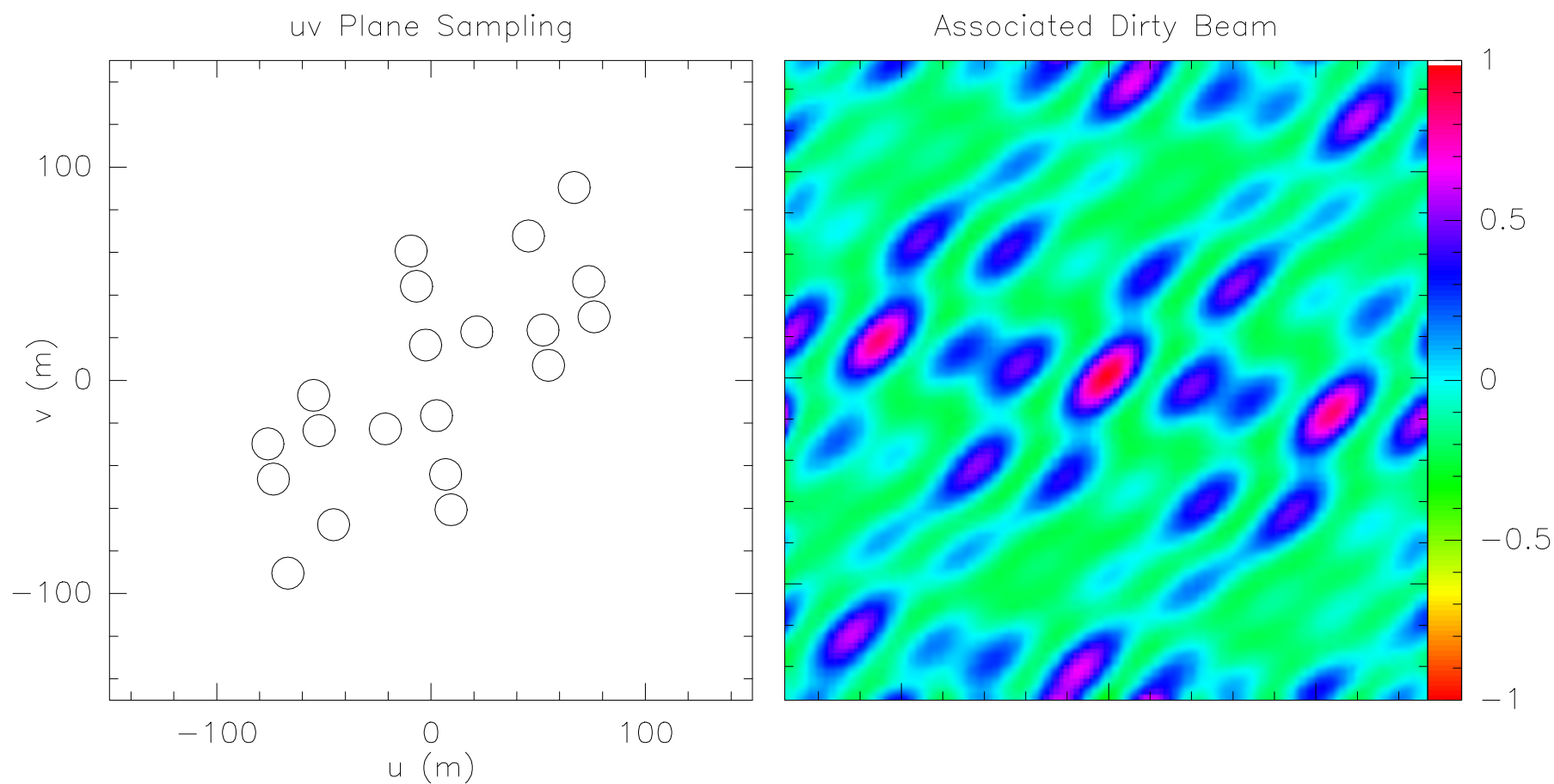
# Dirty Beam Shape and Number of Antenna:

## 4 Antenna



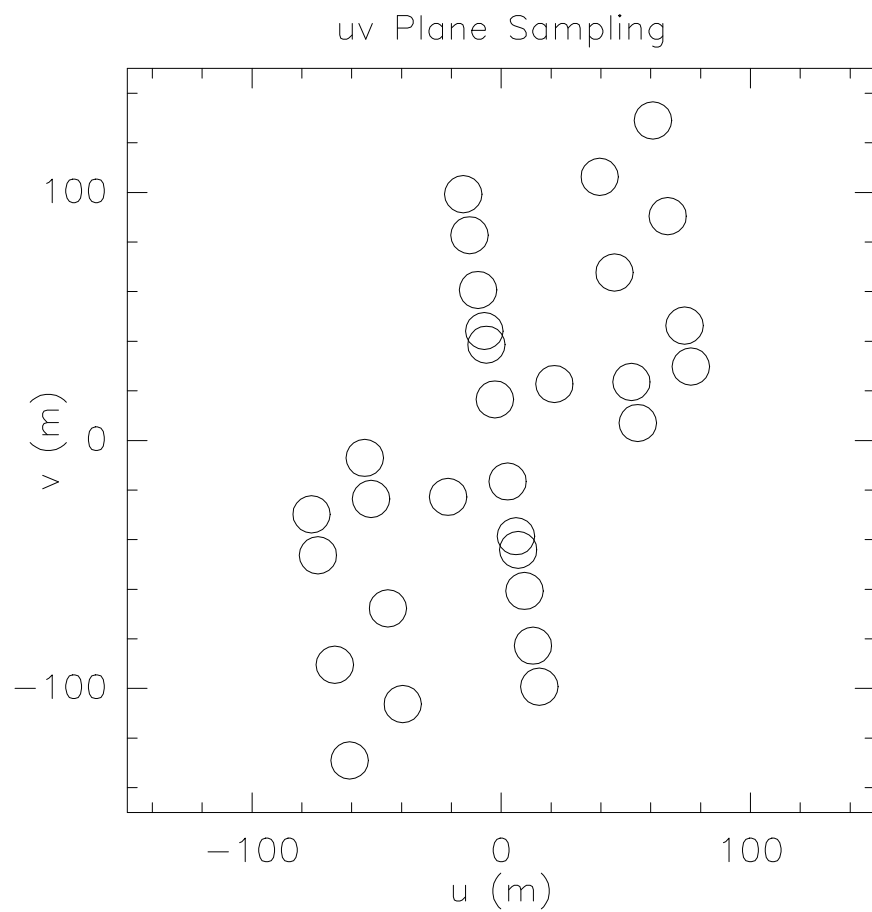
# Dirty Beam Shape and Number of Antenna:

## 5 Antenna

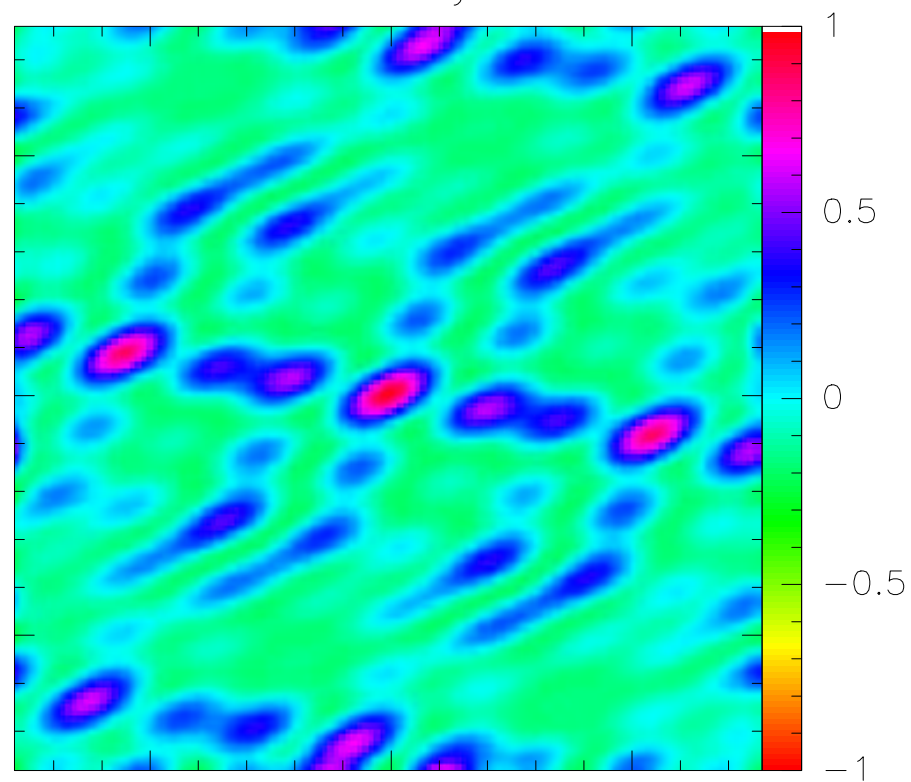


# Dirty Beam Shape and Number of Antenna:

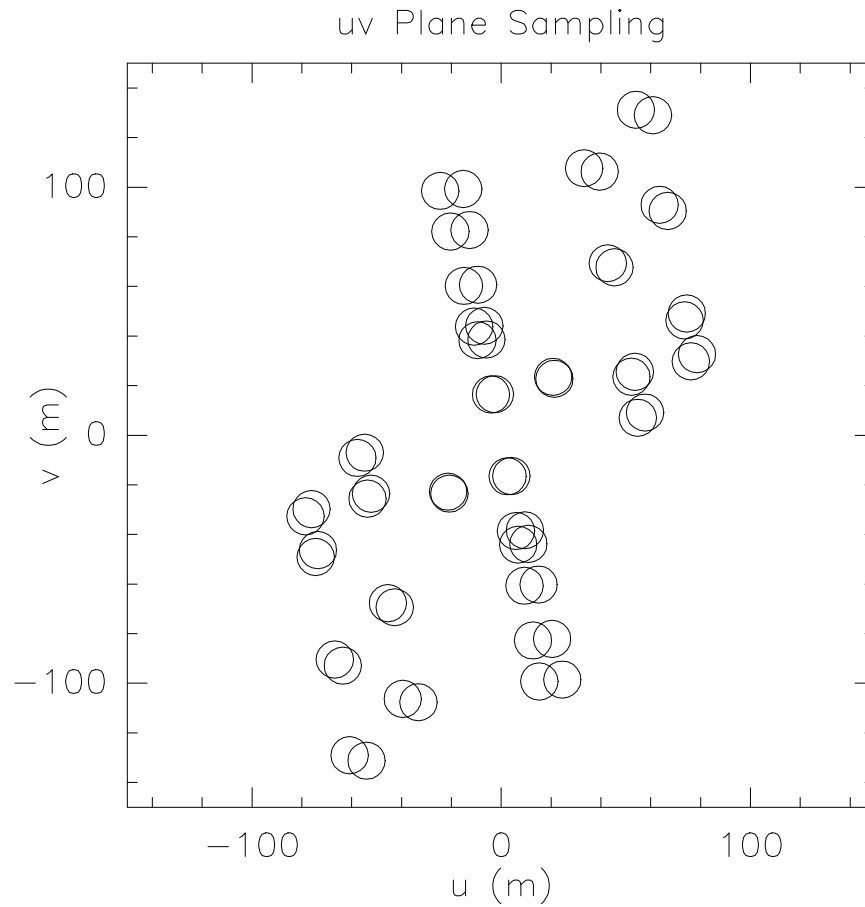
## 6 Antenna



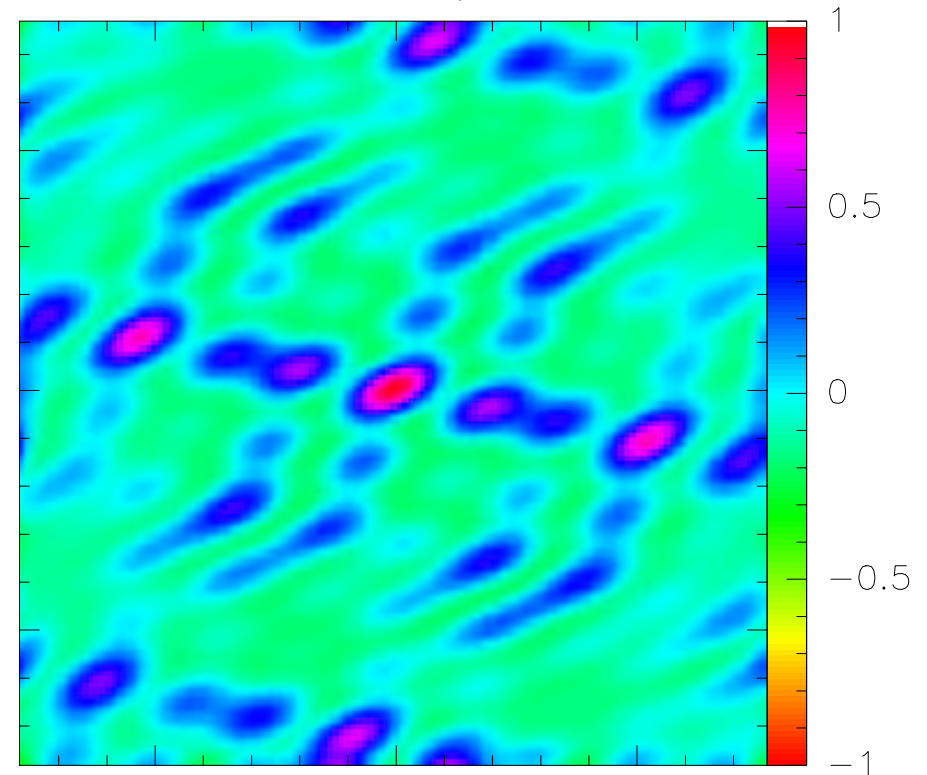
Associated Dirty Beam



# Dirty Beam Shape and Super Synthesis

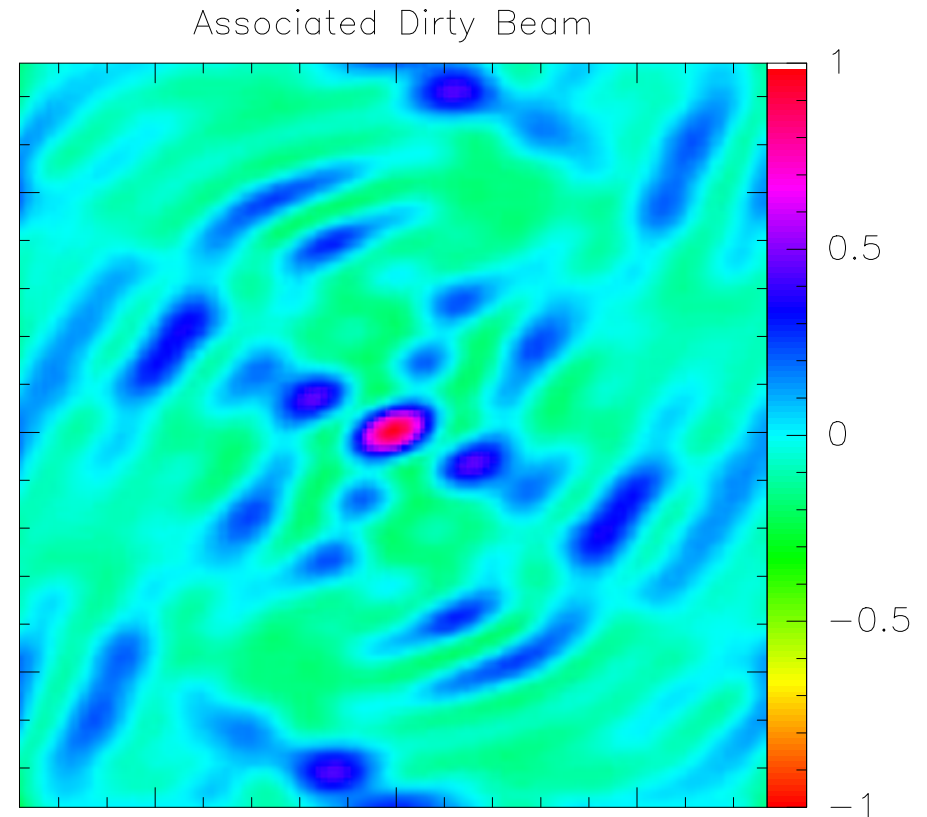
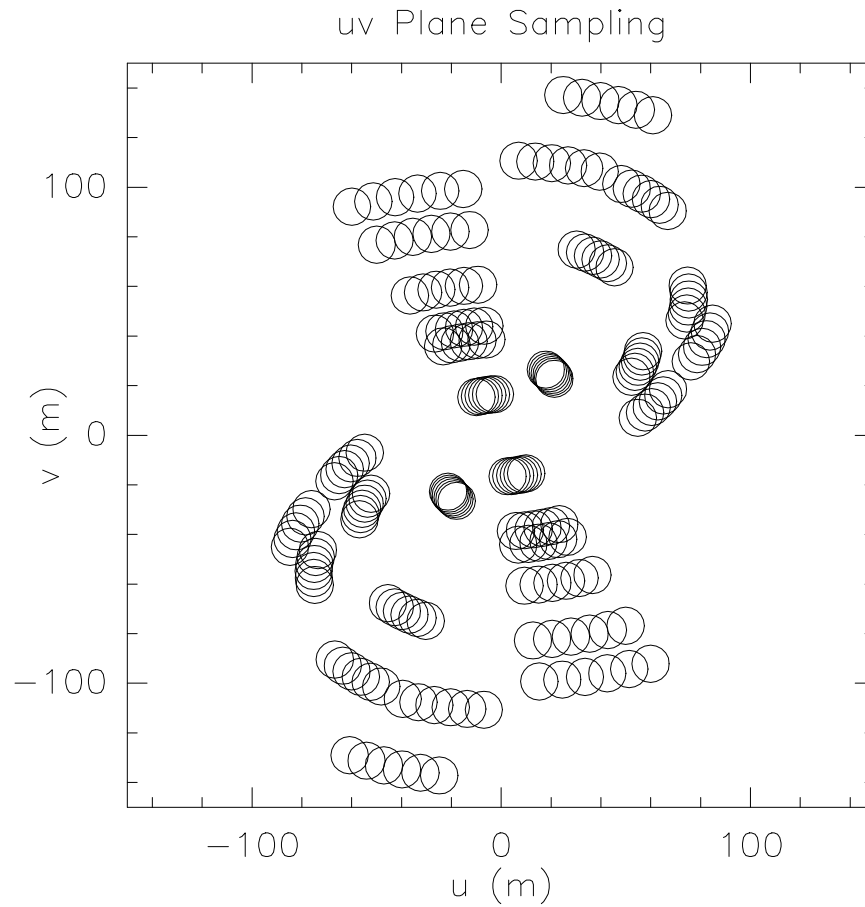


Associated Dirty Beam



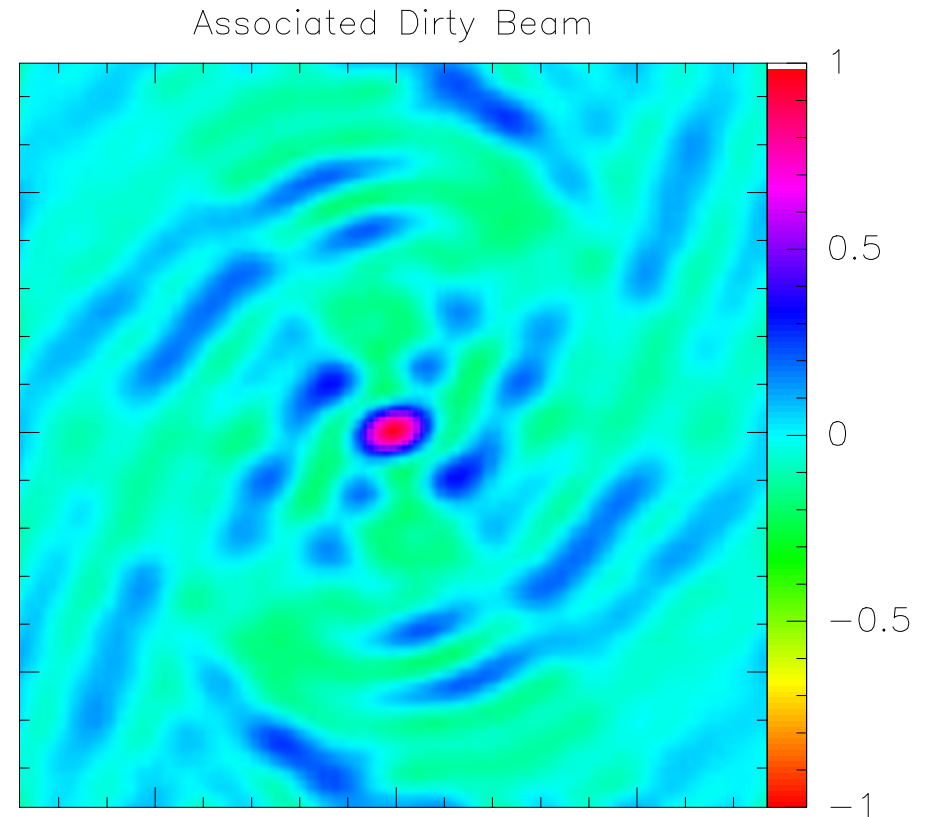
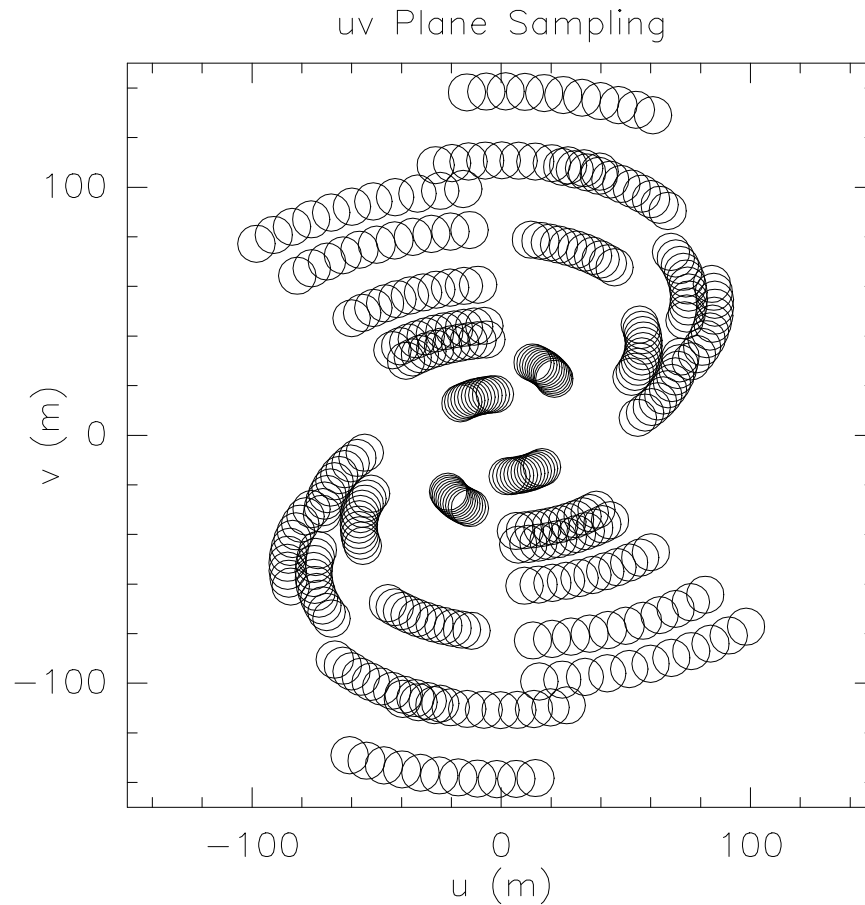


# Dirty Beam Shape and Super Synthesis

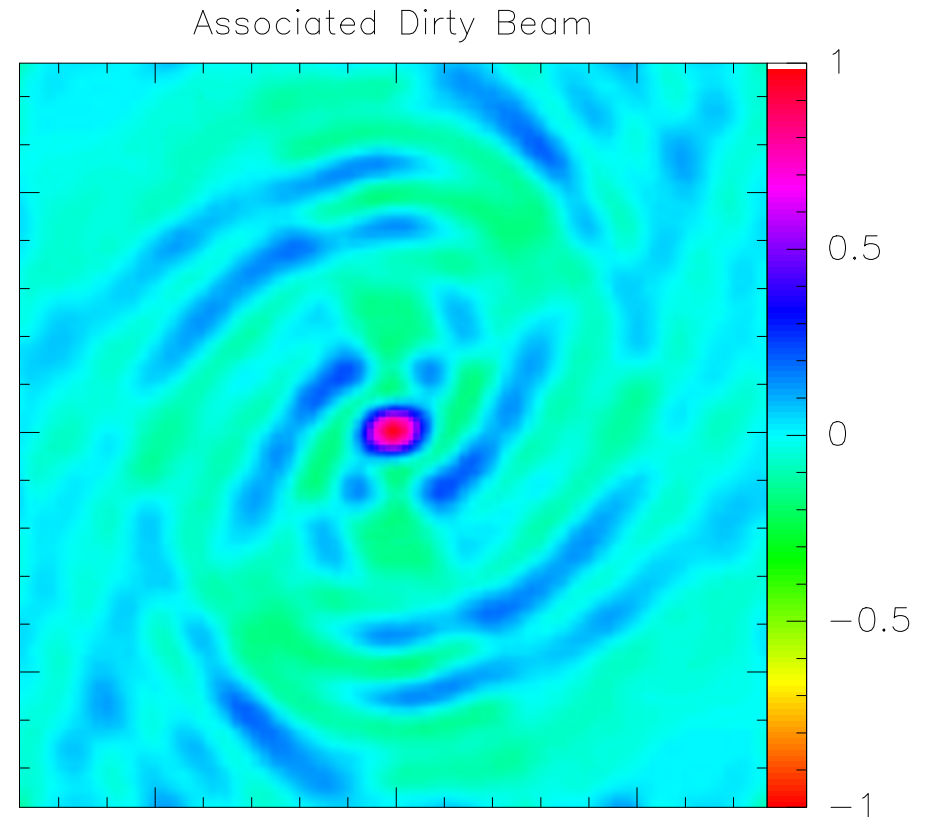
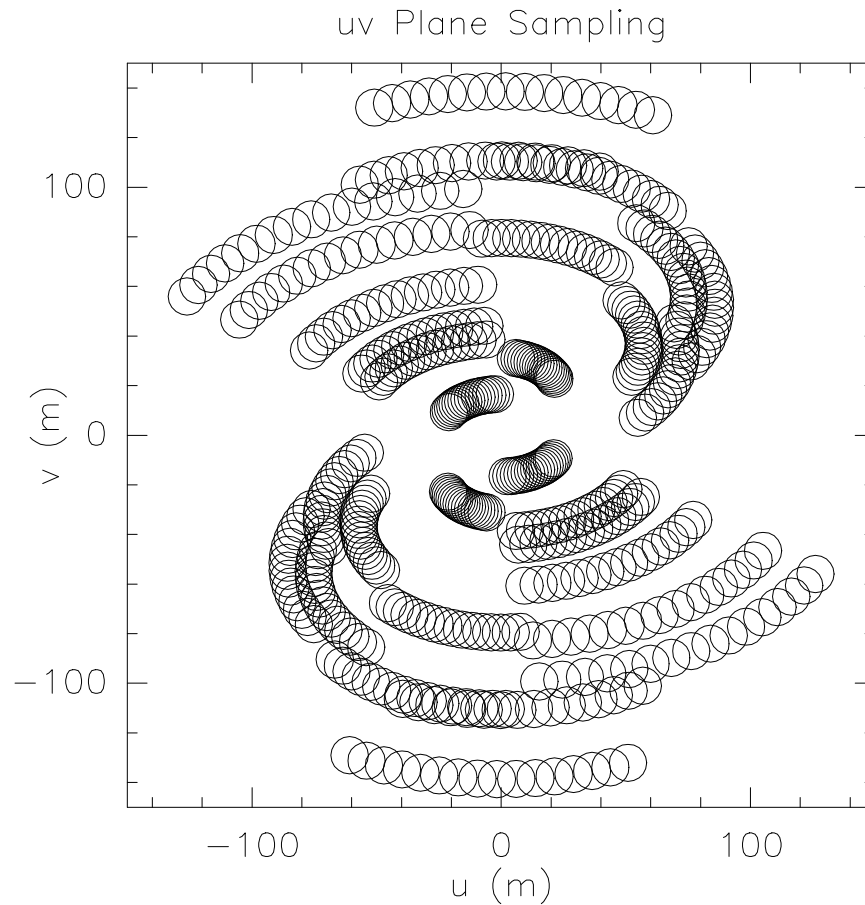




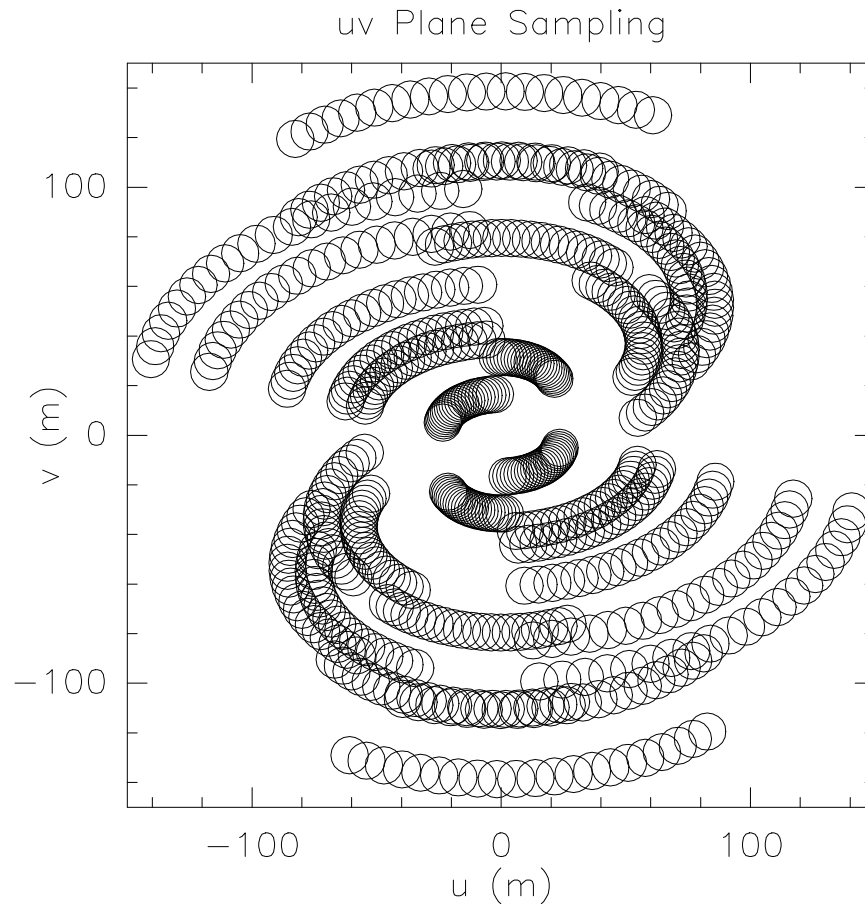
# Dirty Beam Shape and Super Synthesis



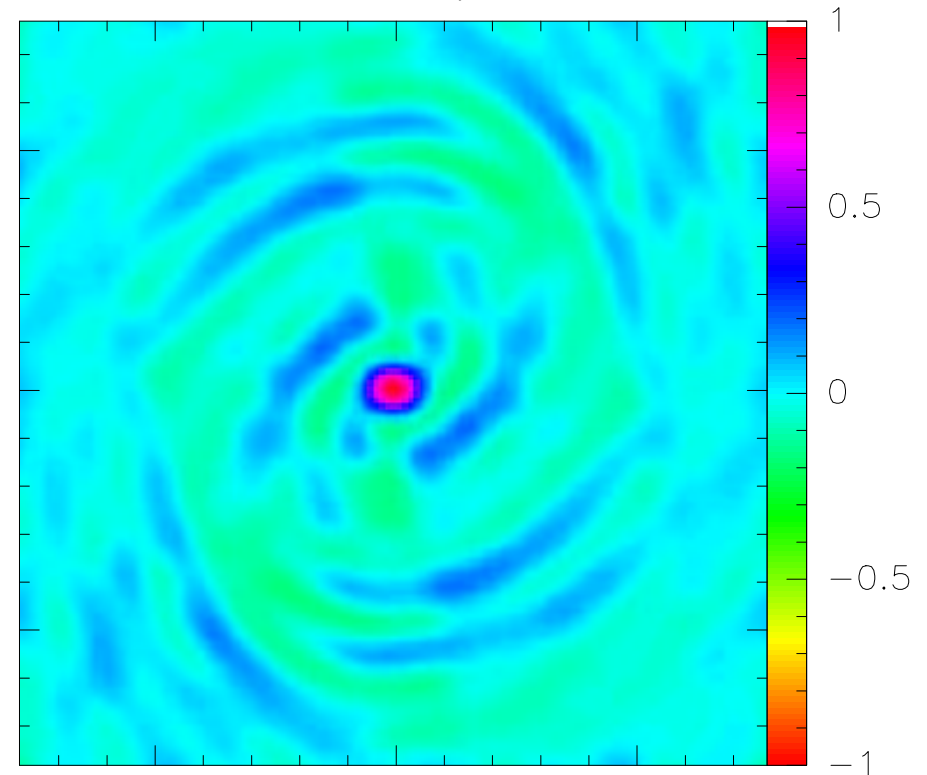
# Dirty Beam Shape and Super Synthesis



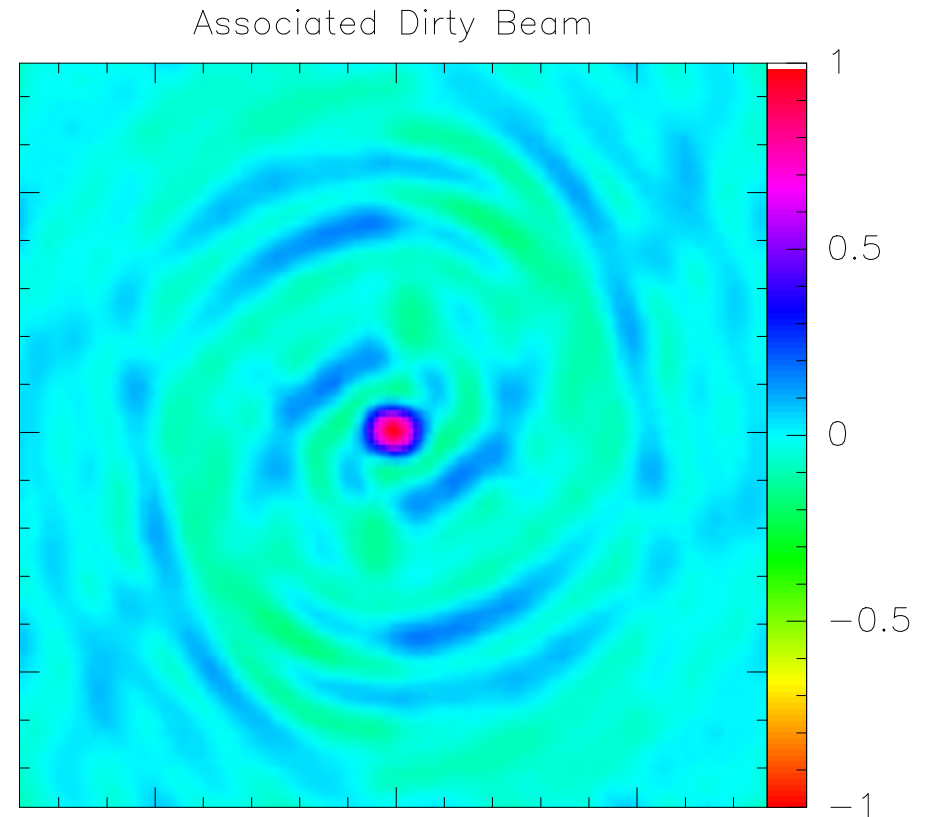
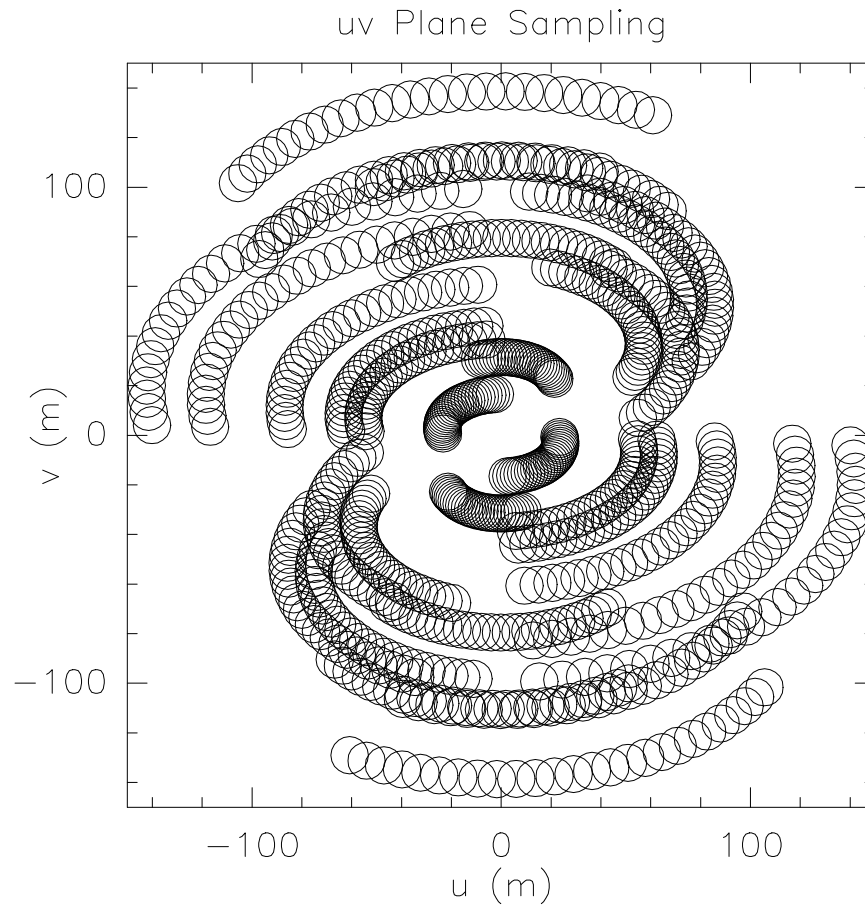
# Dirty Beam Shape and Super Synthesis



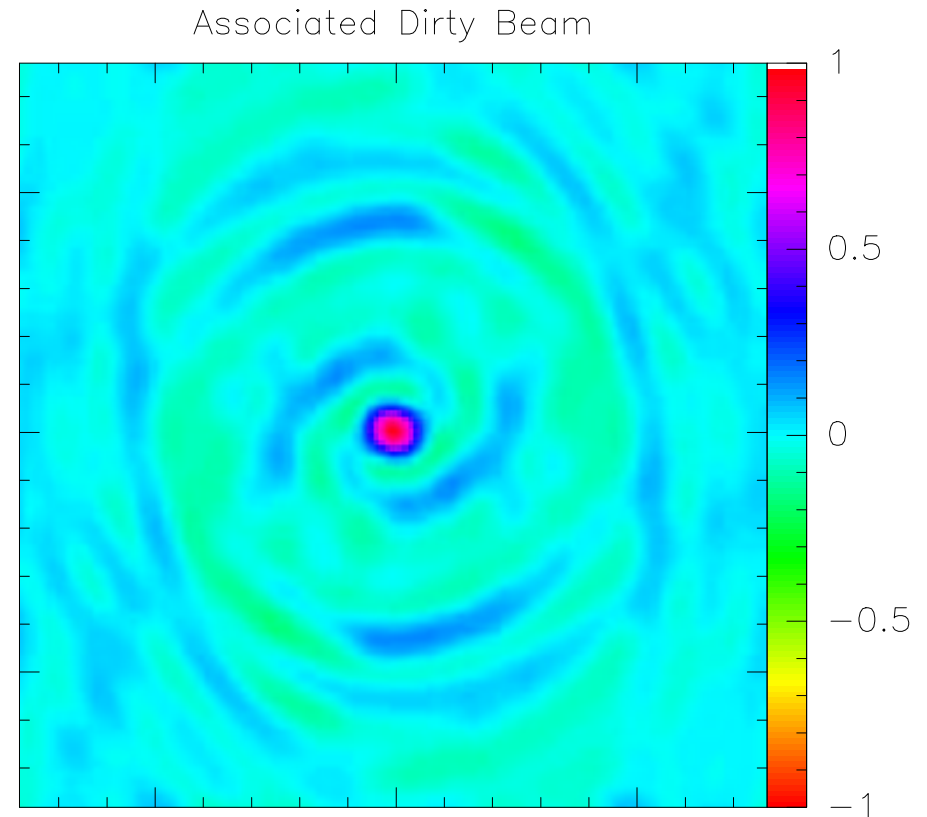
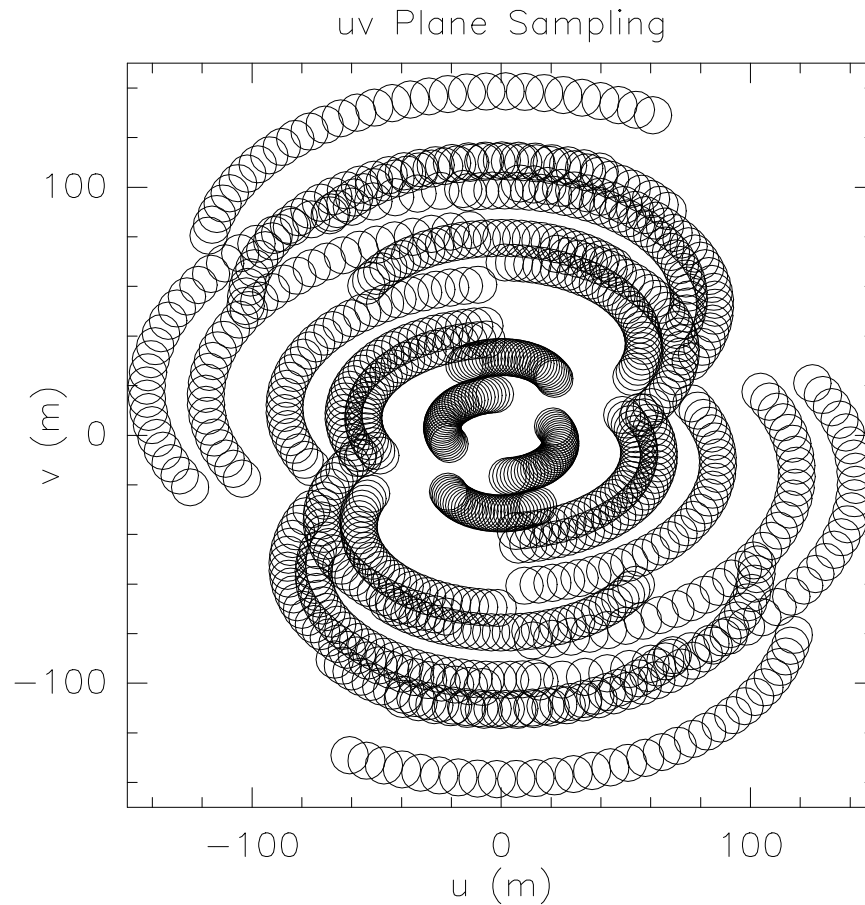
Associated Dirty Beam



# Dirty Beam Shape and Super Synthesis

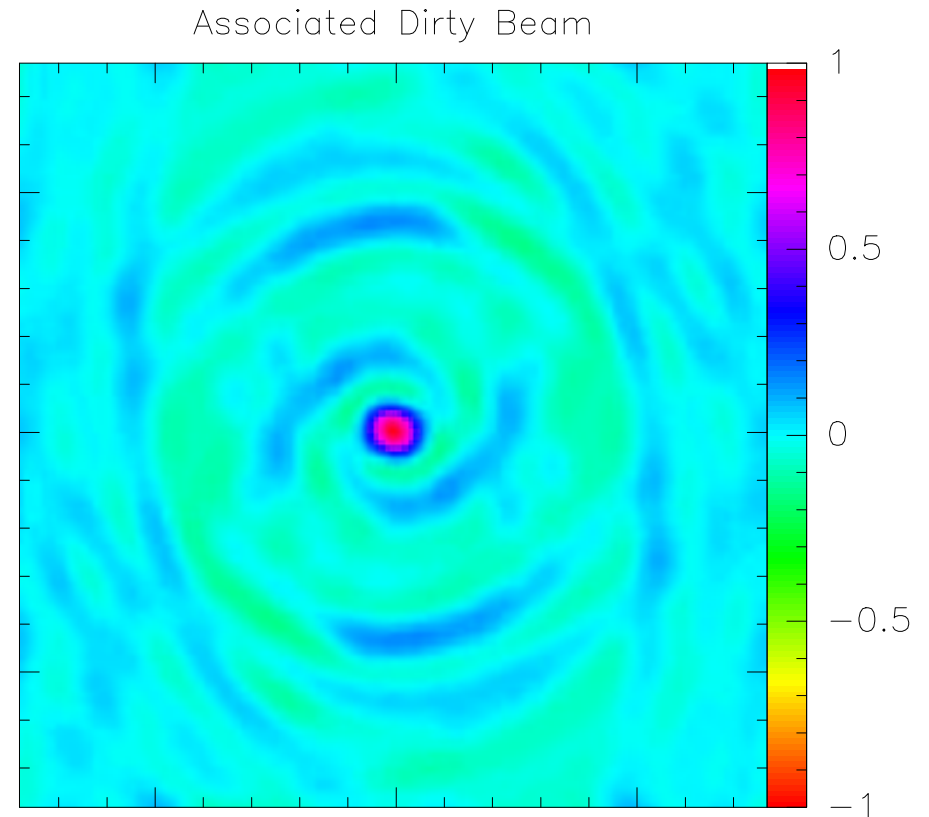
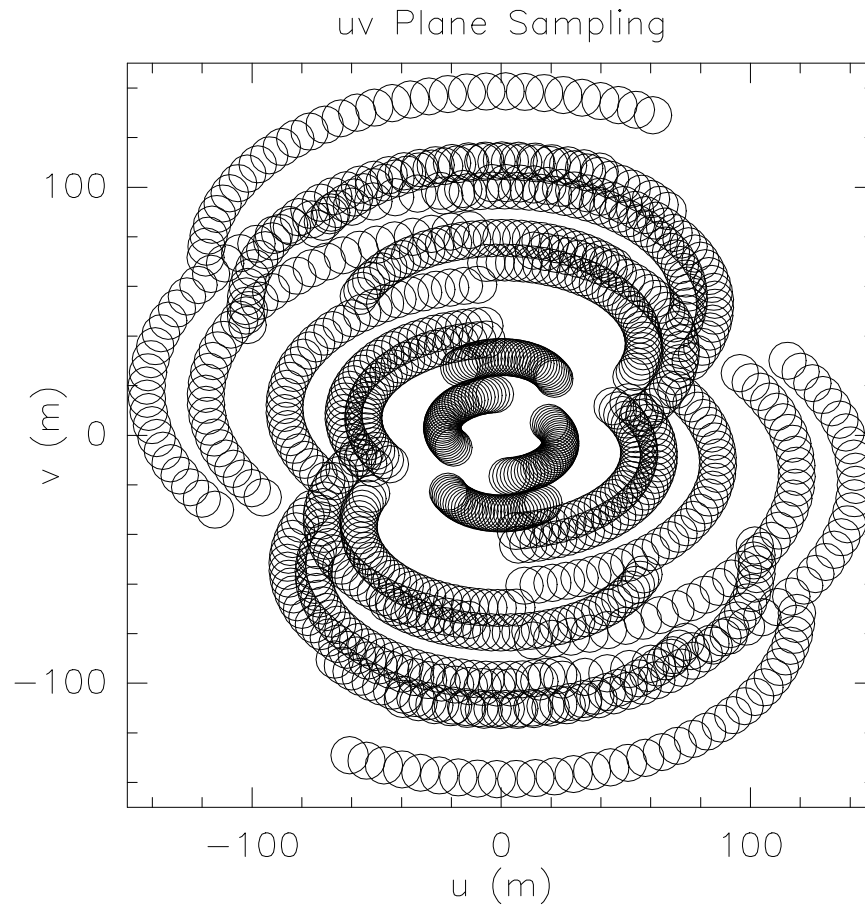


# Dirty Beam Shape and Super Synthesis





# Dirty Beam Shape and Super Synthesis



# Dirty Beam Shape and Weighting

**Natural Weighting:** Default definition of the irregular sampling function at  $uv$  table creation.

- $S(u, v) = 1/\sigma^2$  at  $(u, v)$  points where visibilities are measured;
- $S(u, v) = 0$  elsewhere;

with  $\sigma^2(u, v)$  the noise variance of the visibility.

Introduction of a weighting function  $W(u, v)$ :

- $B_{\text{dirty}} = 2\text{D FT}^{-1} \{W.S\}$ ;
- **Robust weighting:**  $W$  enhance the **large** baseline contribution;
- **Tapering:**  $W$  enhance the **small** baseline contribution.

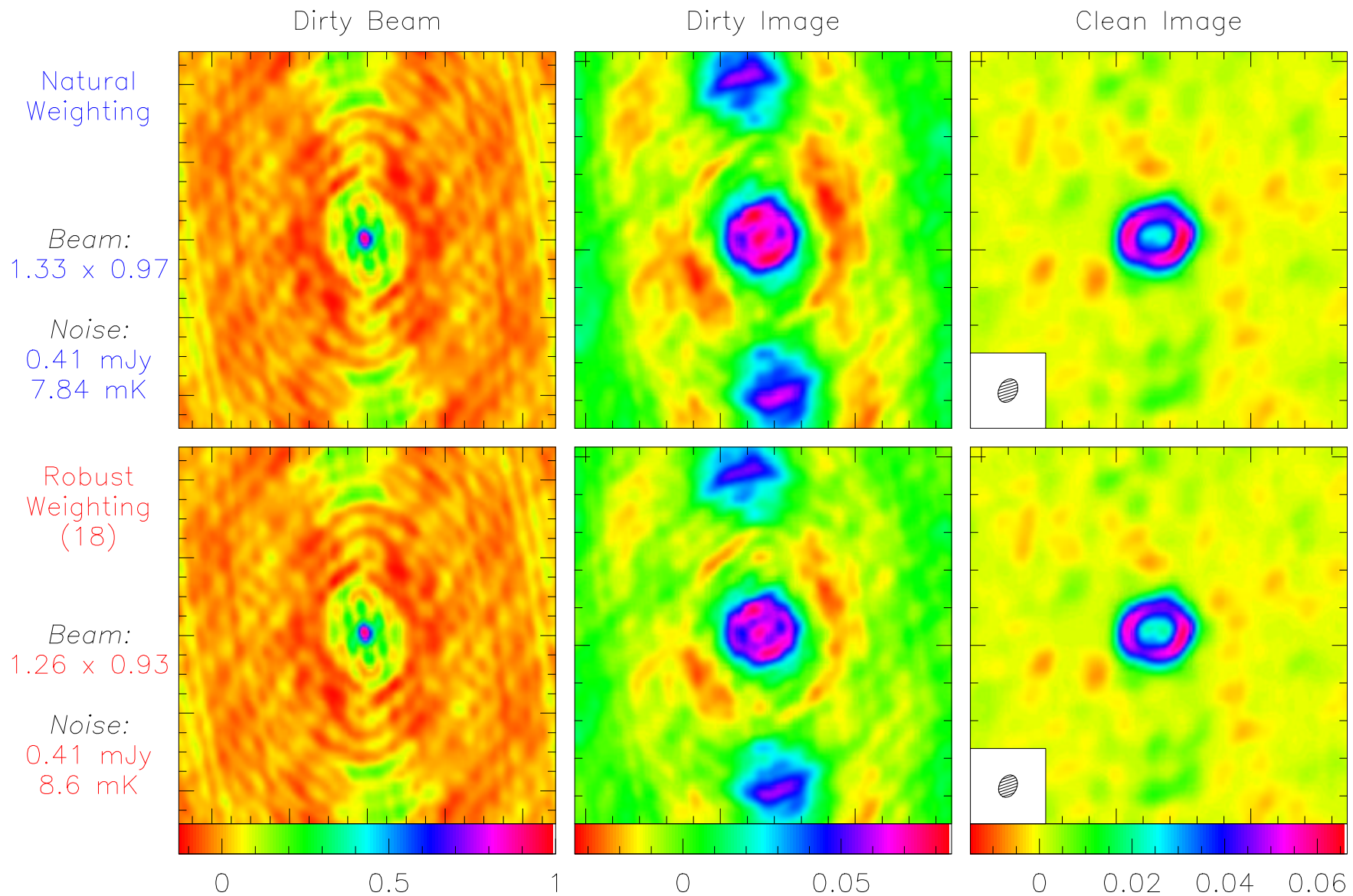
# Robust Weighting: I. Definition

Definitions:

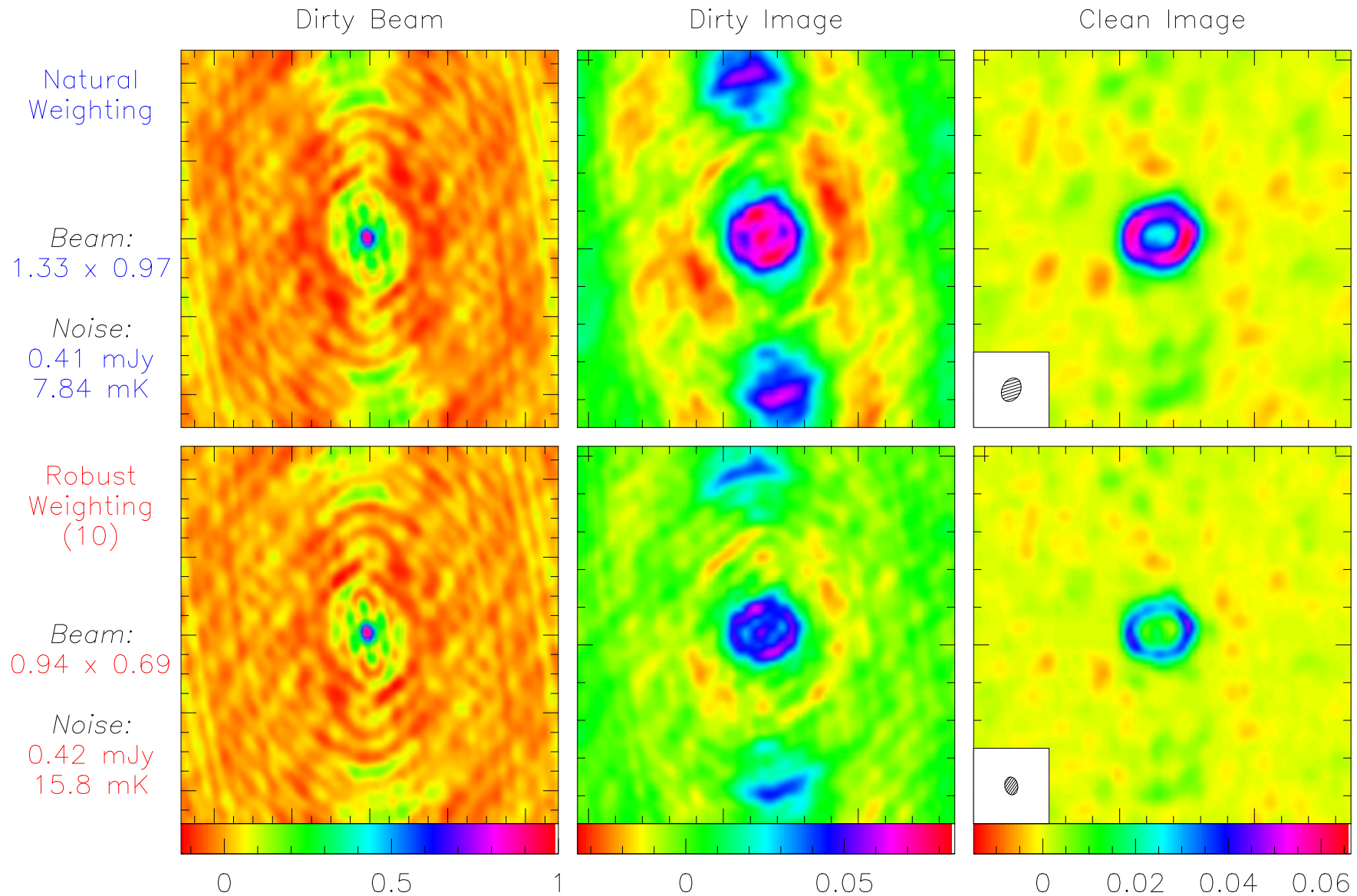
- $\text{Natural} = \sum_{(u,v) \in \text{Cell}} S;$
- $\sum_{(u,v) \in \text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if } (\text{Natural} \geq \text{Threshold}); \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is  $0.5D$ .



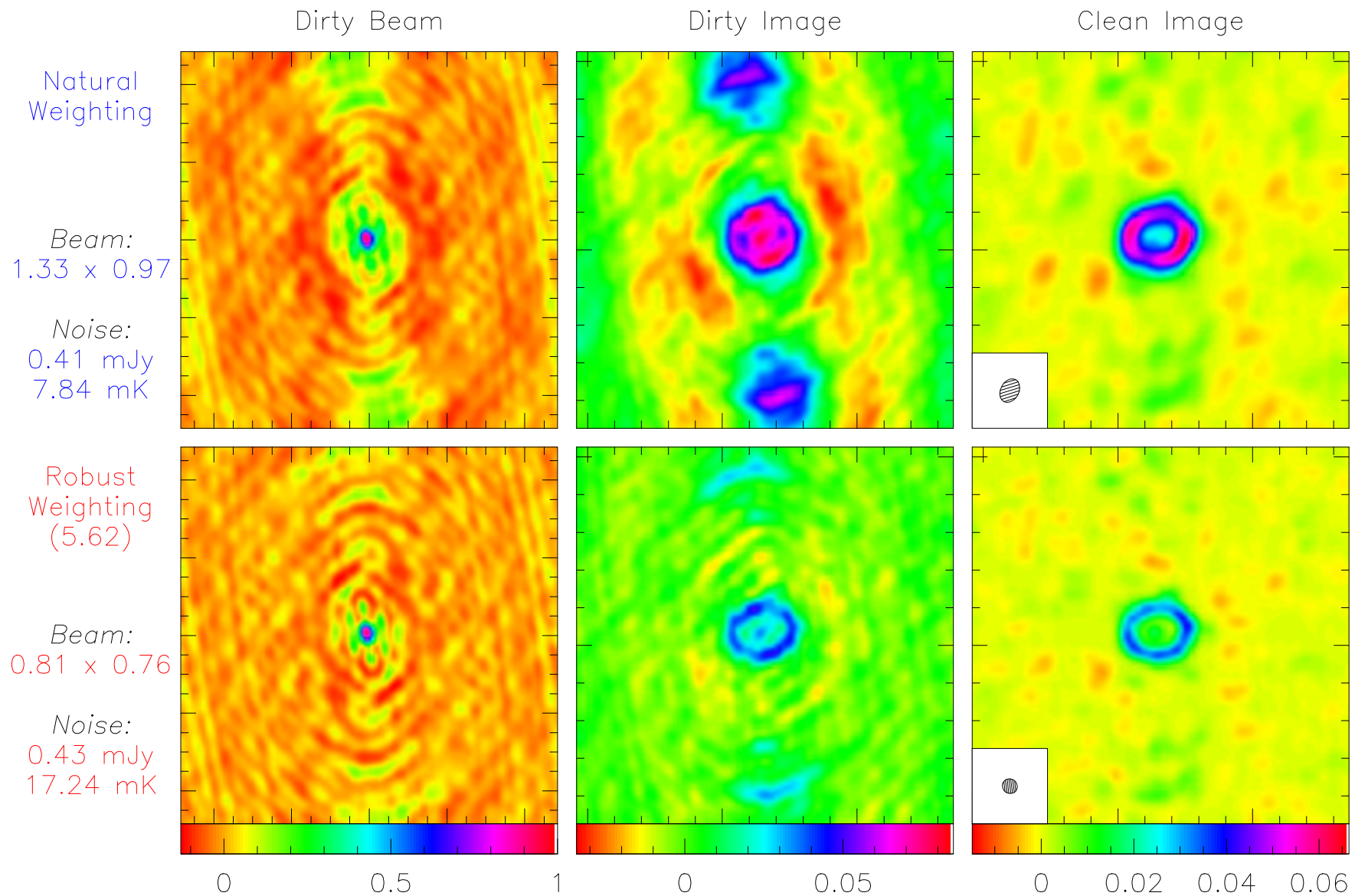
## Robust Weighting: II. Examples



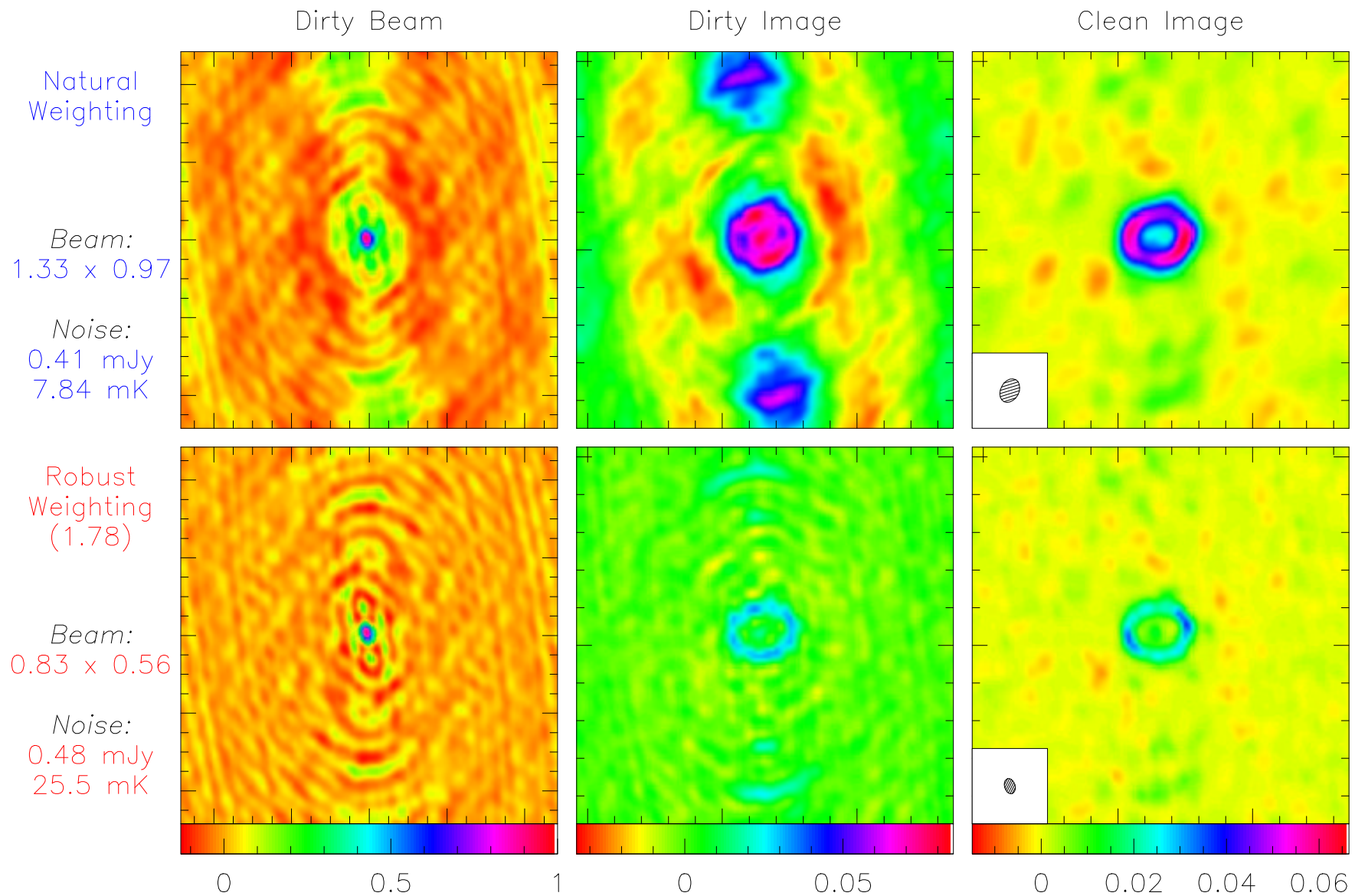
## Robust Weighting: II. Examples



## Robust Weighting: II. Examples



## Robust Weighting: II. Examples





## Robust Weighting: III. Definition and Properties

Definitions:

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- $\sum_{(u,v) \in \text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if } (\text{Natural} \leq \text{Threshold}); \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is  $0.5D$ .

Properties:

- Increase the resolution;
- Lower the sidelobes;
- Degrade point source sensitivity.

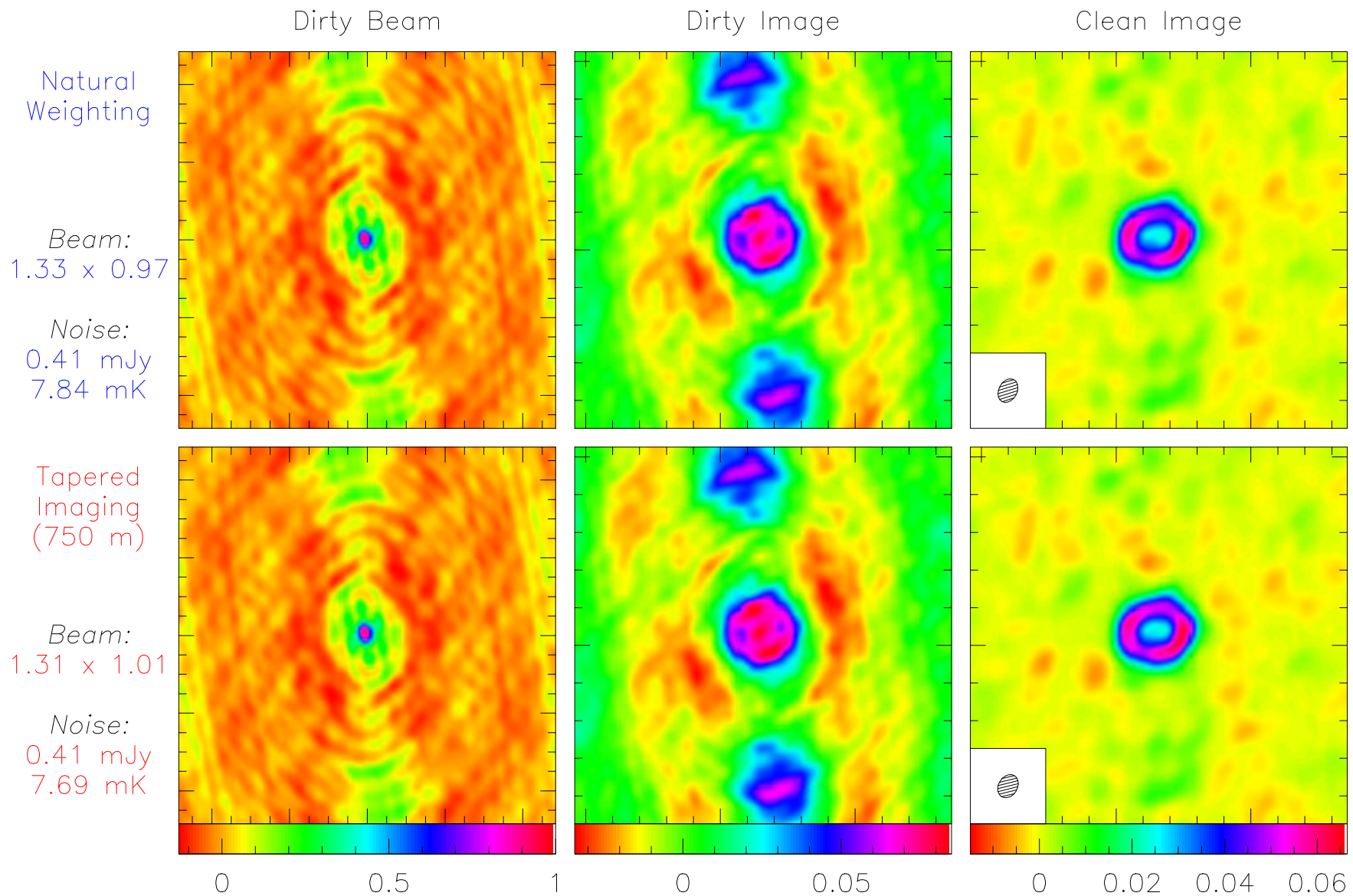
Unfortunately: GILDAS implementation gives it the name of “uniform” weighting!

# Tapering: I Definition

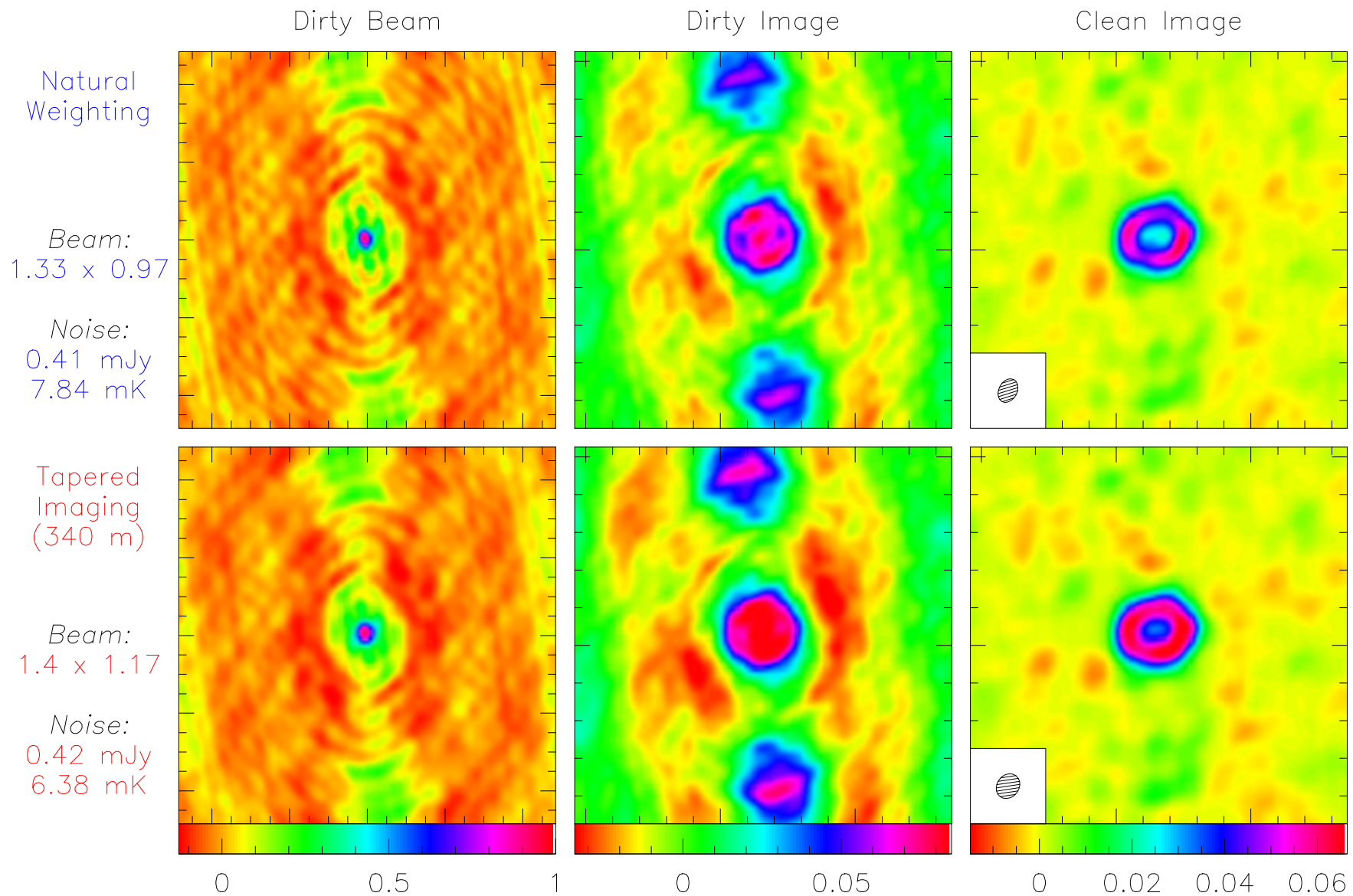
Definition:

- Apodization of the  $uv$  coverage in general by a Gaussian;
  - $W = \exp \left\{ -\frac{(u^2 + v^2)}{t^2} \right\}$  where  $t$  = tapering distance.
- ⇒ Convolution (*i.e.* smoothing) of the image by a Gaussian.

## Tapering: II. Examples

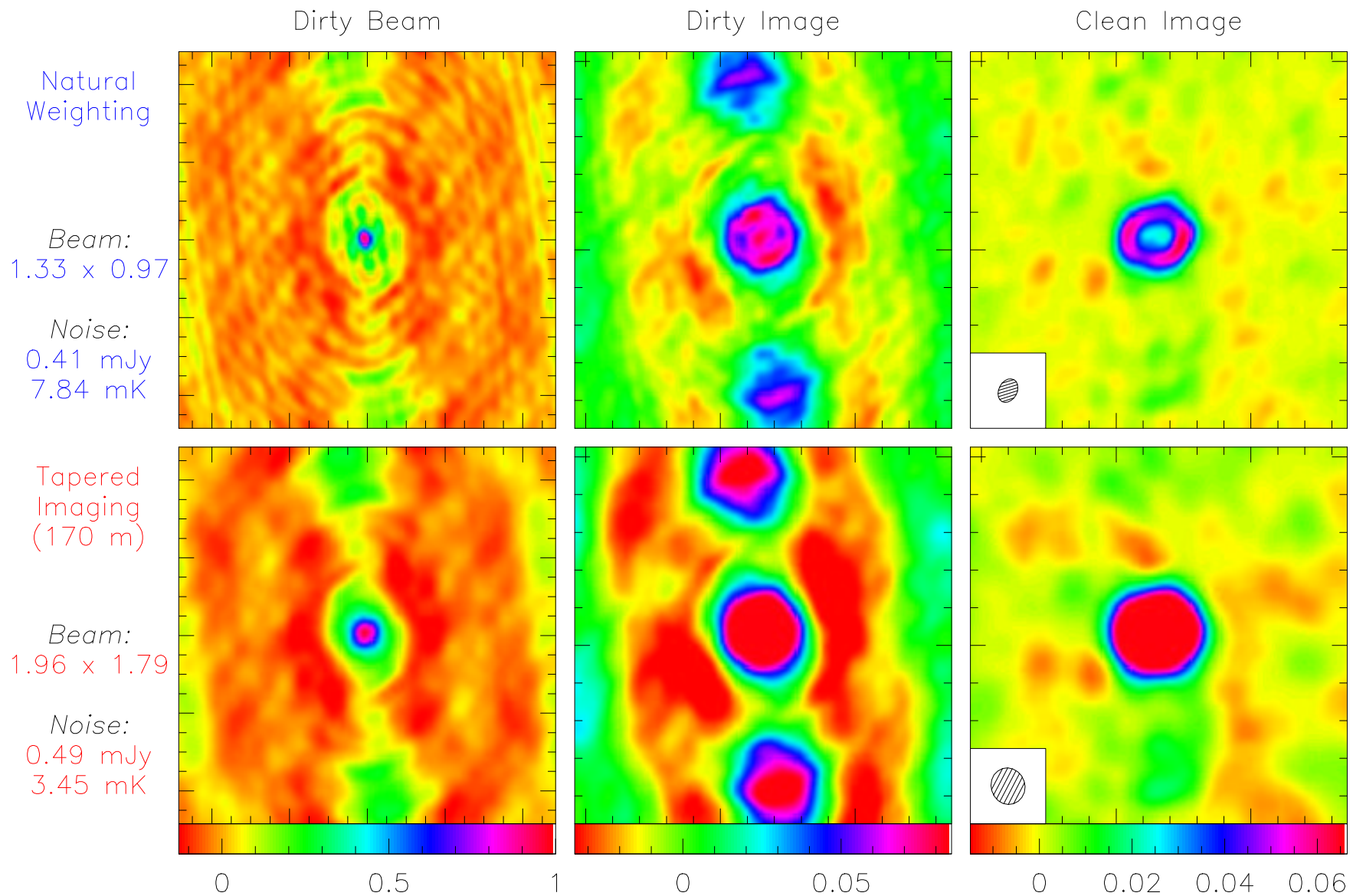


## Tapering: II. Examples





## Tapering: II. Examples



## Tapering: III. Definition and Properties

Definition:

- Apodization of the  $uv$  coverage in general by a Gaussian;
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- ⇒ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Properties:

- Decrease the resolution;
- Degrade point source sensitivity;
- Increase sensitivity to “medium size” structures.

Inconvenient: Throw out some information.

⇒ To increase sensitivity to extended sources, use compact arrays not tapering.

## Weighting and Tapering: Summary

	Robust	Natural	Tapering
Resolution	High	Medium	Low
Side Lobes	↘	Medium	?
Point Source Sensitivity	↘	Maximum	↘
Extended Source Sensitivity	↘	Medium	↗

Non-circular tapering + Robust weighting:  
Sometimes  $\Rightarrow$  Better (*i.e.* smaller and more circular) beams.

## Weighting and Tapering: GILDAS implementation

Resolution, point/extended source sensitivity  
as a function of  
robust threshold or tapering distance.

MAPPING> UV\_STAT WEIGHT

Robust	Major (")	Minor (")	PA (deg)	Noise (mJy)	Noise (K)	Sidelobe %	
0.10	0.840	0.527	18.5	0.791	0.044	-16.1	13.9
None	1.271	0.940	155.1	0.406	0.008	-2.7	25.9

I-UV\_STAT, Recommended pixel size is 0.080 - 0.264"

MAPPING> UV\_STAT TAPER

Taper (m)	Major (")	Minor (")	PA (deg)	Noise (mJy)	Noise (K)	Sidelobe %	
30.00	4.844	3.817	11.9	1.019	0.001	-44.7	14.5
None	1.323	0.967	151.5	0.406	0.008	-12.9	28.7

# From Calibrated Visibilities to Images: GILDAS implementation

MAPPING> INPUT UV\_MAP: display UV\_MAP control parameters.

Default = 0  $\Rightarrow$  Try something clever.

- Gridding kernel: **CONVOLUTION** (default: 5, *i.e.* spheroidals).
- Data cube setup:
  - First and last data channels to map: **MCOL** (default: 0, *i.e.* all channels).
  - Pixel size: **MAP\_CELL** (in arcsec, default: 0, *i.e.* 1/3 to 1/4 synthesized beam).
  - Map size: **MAP\_SIZE** (in pixels, default: 0, *i.e.*  $\sim$  twice primary beam size).
- Weighting:
  - kind: **WEIGHT\_MODE** (Default: NATURAL).
  - Size of *uv* plane cell and robust threshold: **UV\_CELL** (Defaults: 7.5 m, 1).
- Tapering: Gaussian taper parameters **UV\_TAPER** (in m and degree, default: 0, *i.e.* no tapering).

# From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:  
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
↓ Fourier Transform	UV_STAT, UV_MAP
Dirty beam & image	
↓ Deconvolution	HOGBOM, CLARK
Clean beam & image	
↓ Visualization	GO_BIT, GO_VIEW
↓ Image analysis	GO_NOISE, GO_FLUX, GO_MOMENTS
Physical information on your source	

## Deconvolution: I. Philosophy

$$I_{\text{meas}} = B_{\text{dirty}} * \{B_{\text{primary}} \cdot I_{\text{source}}\} + N.$$

Information lost:

- Irregular, incomplete sampling  $\Rightarrow$  convolution by  $B_{\text{dirty}}$ ;
- Noise  $\Rightarrow$  Low signal structures undetected.

$\Rightarrow$  1. Impossible to recover the intrinsic source structure!

$\Rightarrow$  2. Infinite number of solution!

$$\left\{ \begin{array}{l} S \text{ solution (i.e. } I_{\text{meas}} = B_{\text{dirty}} * S + N) \\ B_{\text{dirty}} * R = 0 \end{array} \right\} \Rightarrow (S + R) \text{ solution.}$$

## Deconvolution: I. Philosophy (continued)

$$I_{\text{meas}} = B_{\text{dirty}} * \{B_{\text{primary}} \cdot I_{\text{source}}\} + N.$$

Information lost:

- ⇒ 1. Impossible to recover the intrinsic source structure!
- ⇒ 2. Infinite number of solution!

Deconvolution goal: Finding a **sensible** intensity distribution **compatible** with the intrinsic source one.

Deconvolution needs:

- Some *a priori* assumptions about the source intensity distribution;
- As much as possible knowledge of
  - $B_{\text{dirty}}$  (OK in radioastronomy);
  - Noise properties.

The best solution: A Gaussian  $B_{\text{dirty}} \Rightarrow$  No deconvolution needed!



## Deconvolution: II. MEM principle

*a priori* assumptions: Smoothed and positive intensity.

Idea:

“Select from the images that agree with the measured visibilities to within the noise level the one that maximizes entropy.”

Algorithm:

- Entropy:

$$\mathcal{S} = - \sum_{ij} I_{ij} \log(I_{ij}/M_{ij}) \text{ with } M = \text{first guess image.}$$

- Constraint:

$$\sum_k \frac{|V(u_k, v_k) - \tilde{I}(u_k, v_k)|^2}{\sigma_k^2} = \text{number of visibilities}$$

with  $\tilde{I} = 2D \text{ FT}(I)$ .

## Deconvolution: II. MEM properties

### Advantages:

- Fast:  
Computational load  $\propto N \ln(N)$  with  $N =$  number of pixels.
- Easier to generalize (Arrays with different antenna diameters).
- Flatten low-level extended emission.
- Resolve peaks.

### Inconvenients:

- Angular resolution increases with peak height.
- Unable to clean ripples (e.g. point source sidelobes) in extended emission.
- Biased residuals:  
 $\Rightarrow$  Noise increase and spurious emission at low signal.
- Impossibility to deal with absorption features.
- Poor performance with limited  $uv$  coverage  
 $\Rightarrow$  **Not** used at PdBI.

## Deconvolution: III. The Basic CLEAN Algorithm

*a priori* assumption: Source = Collection of point sources.

Idea: “Matching pursuit”.

Algorithm:

### 1 Initialize

- the residual map to the dirty map;
- the Clean component list to an empty (NULL) value;

2 Identify pixel of  $|I_{\max}|$  in residual map as a point source;

3 Add  $\gamma \cdot I_{\max}$  to clean component list;

4 Subtract  $\gamma \cdot I_{\max}$  from residual map;

5 Go back to point 2 while stopping criterion is not matched;

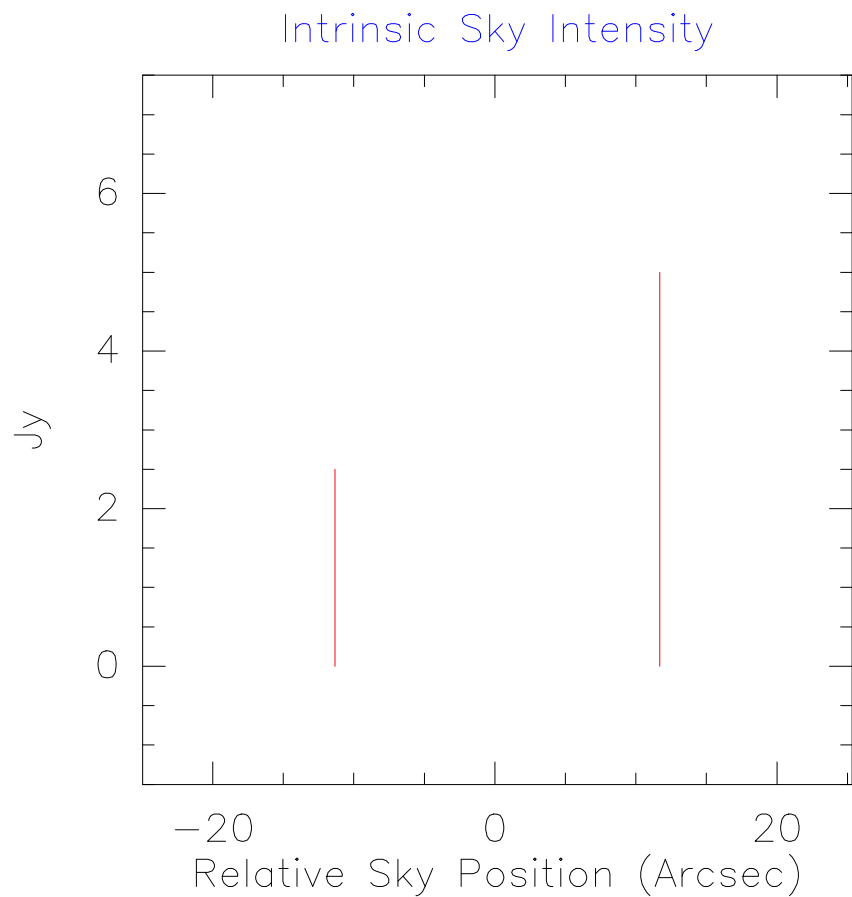
6 Convolution by Clean beam (*a posteriori* regularization);

5 Addition of residual map to enable:

- Correction when cleaning is too superficial;
- Noise estimation.

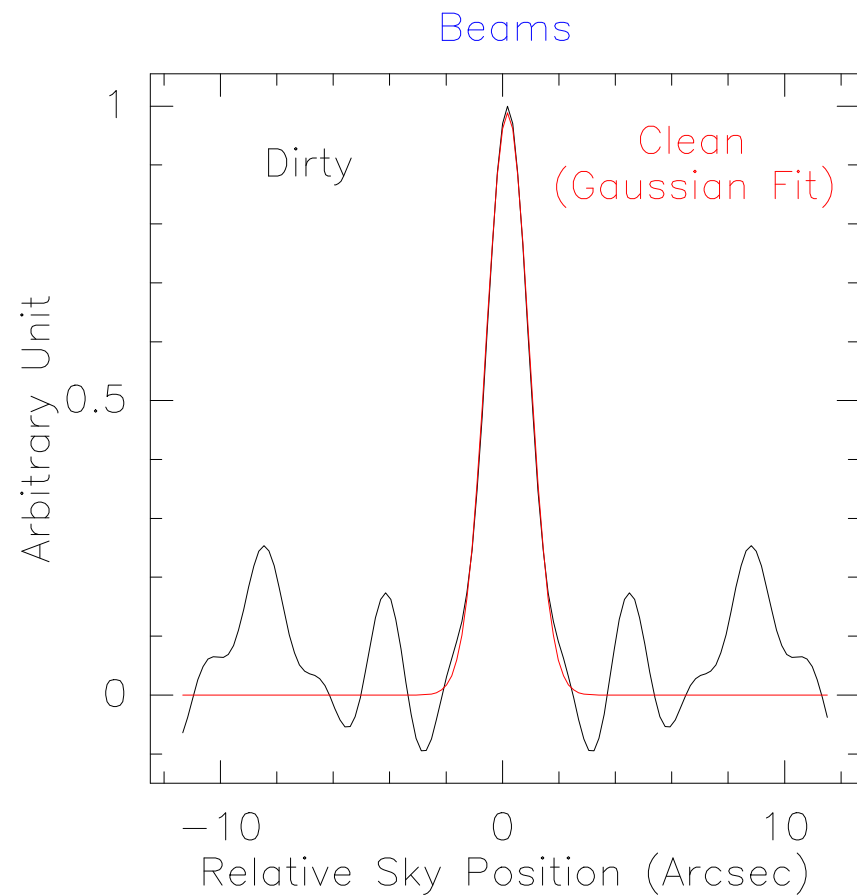
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



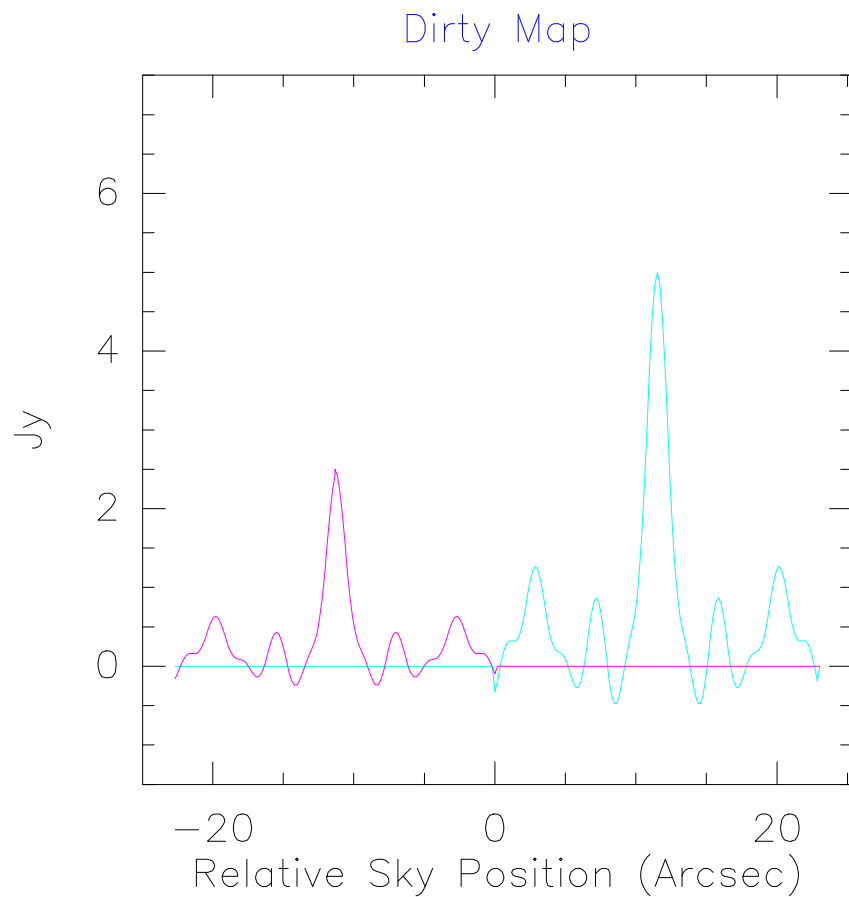
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



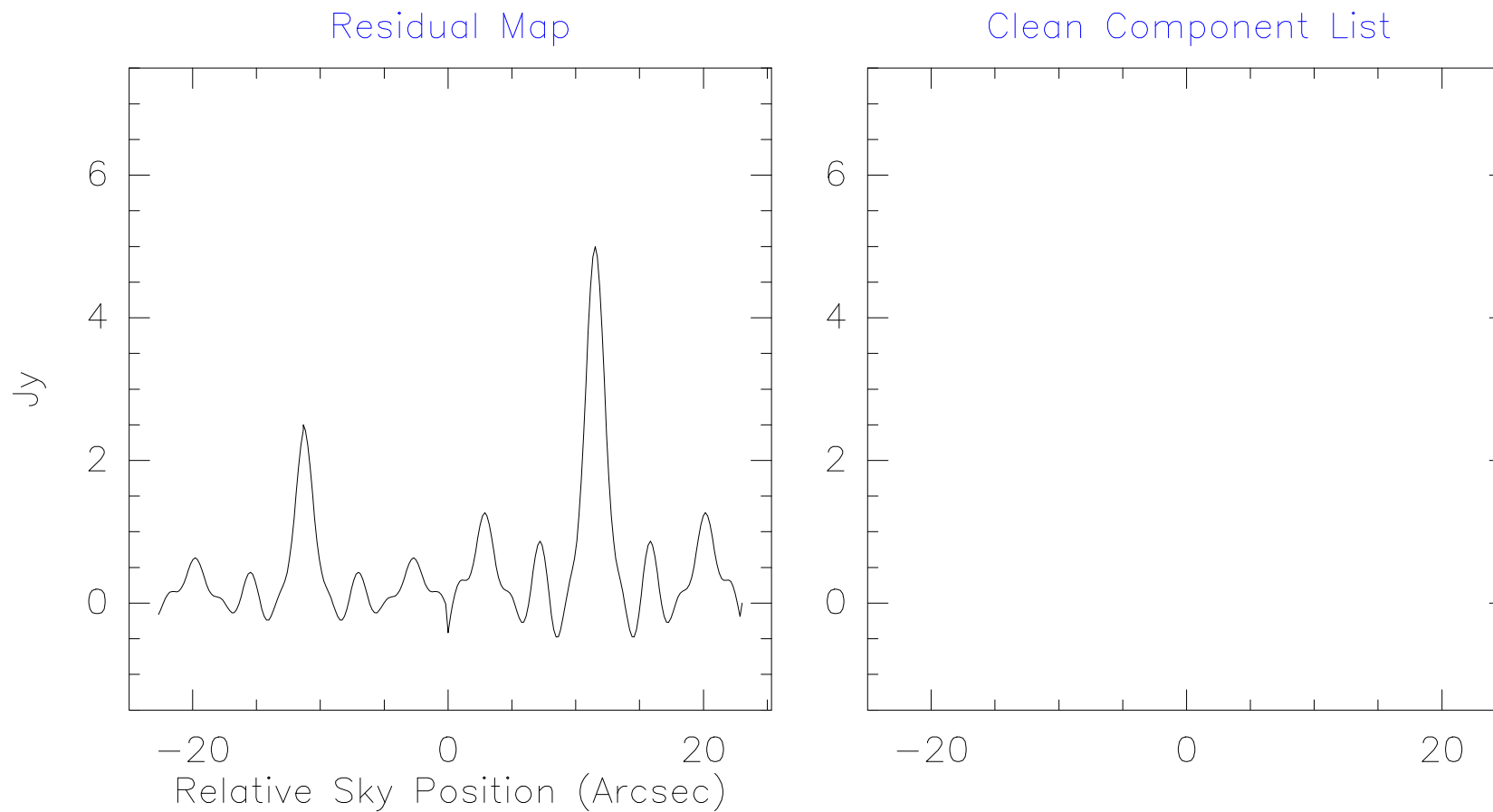
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



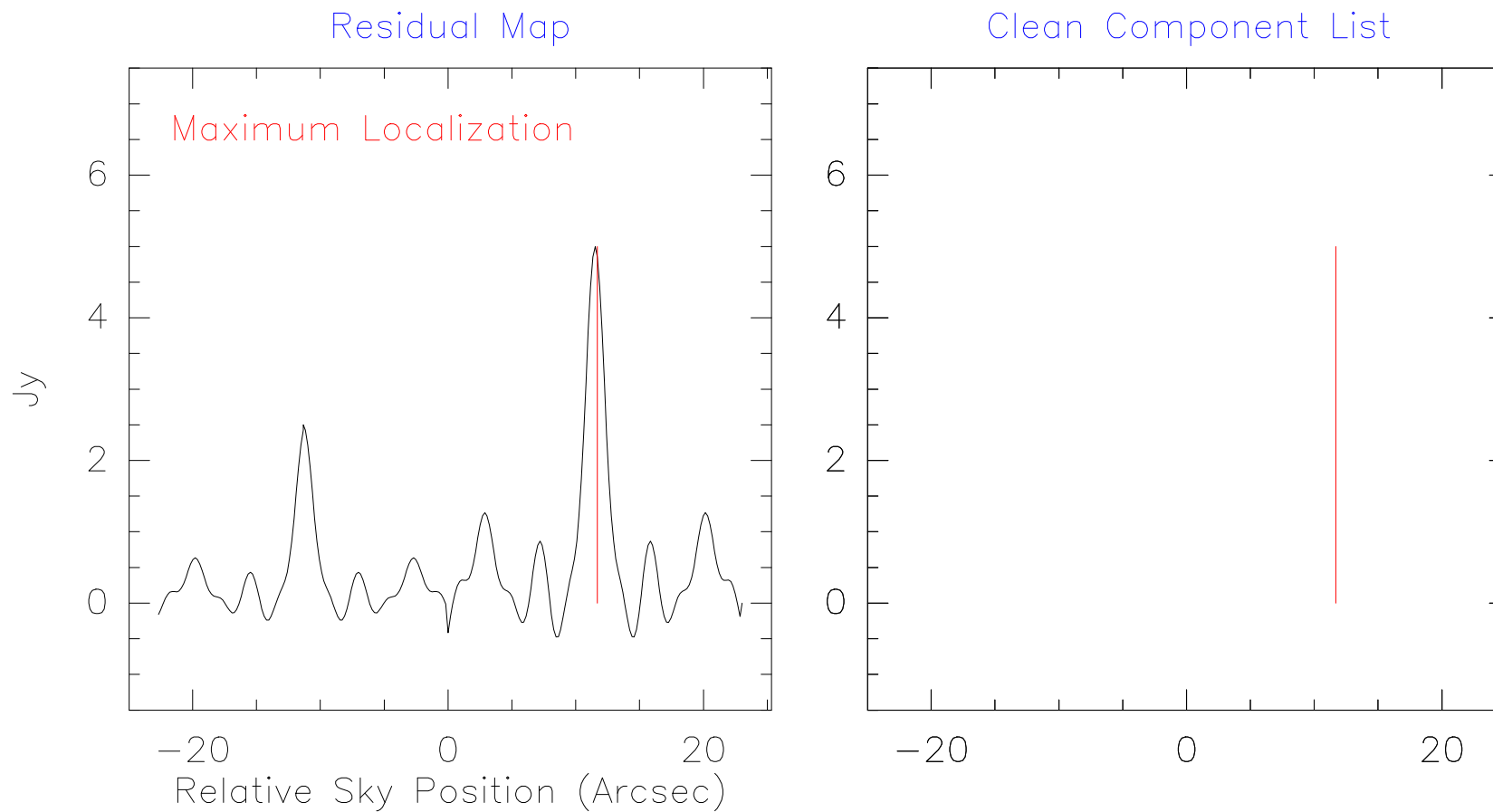
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



# Deconvolution: III. The Basic Clean Algorithm

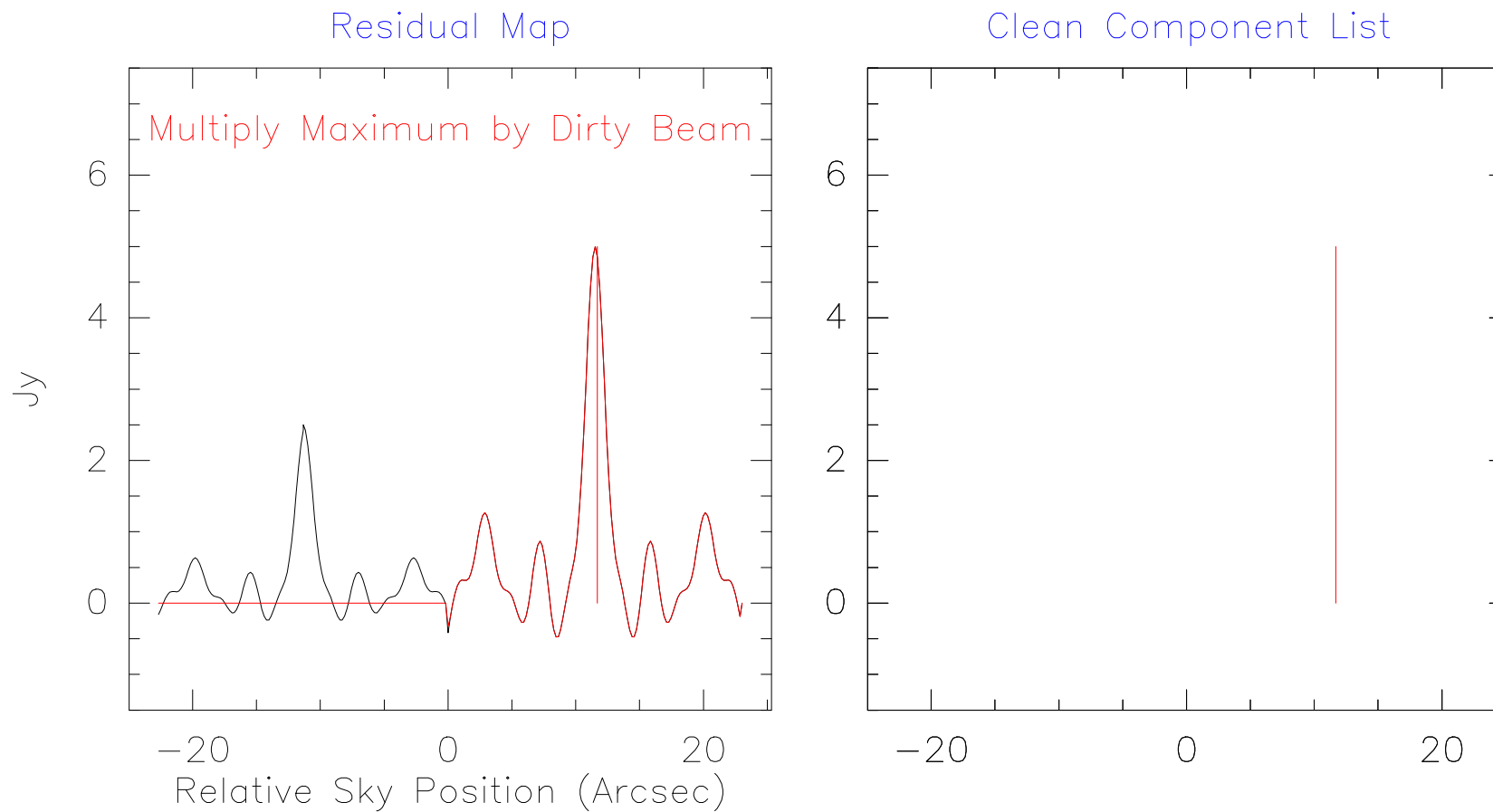
## 1. First Illustration





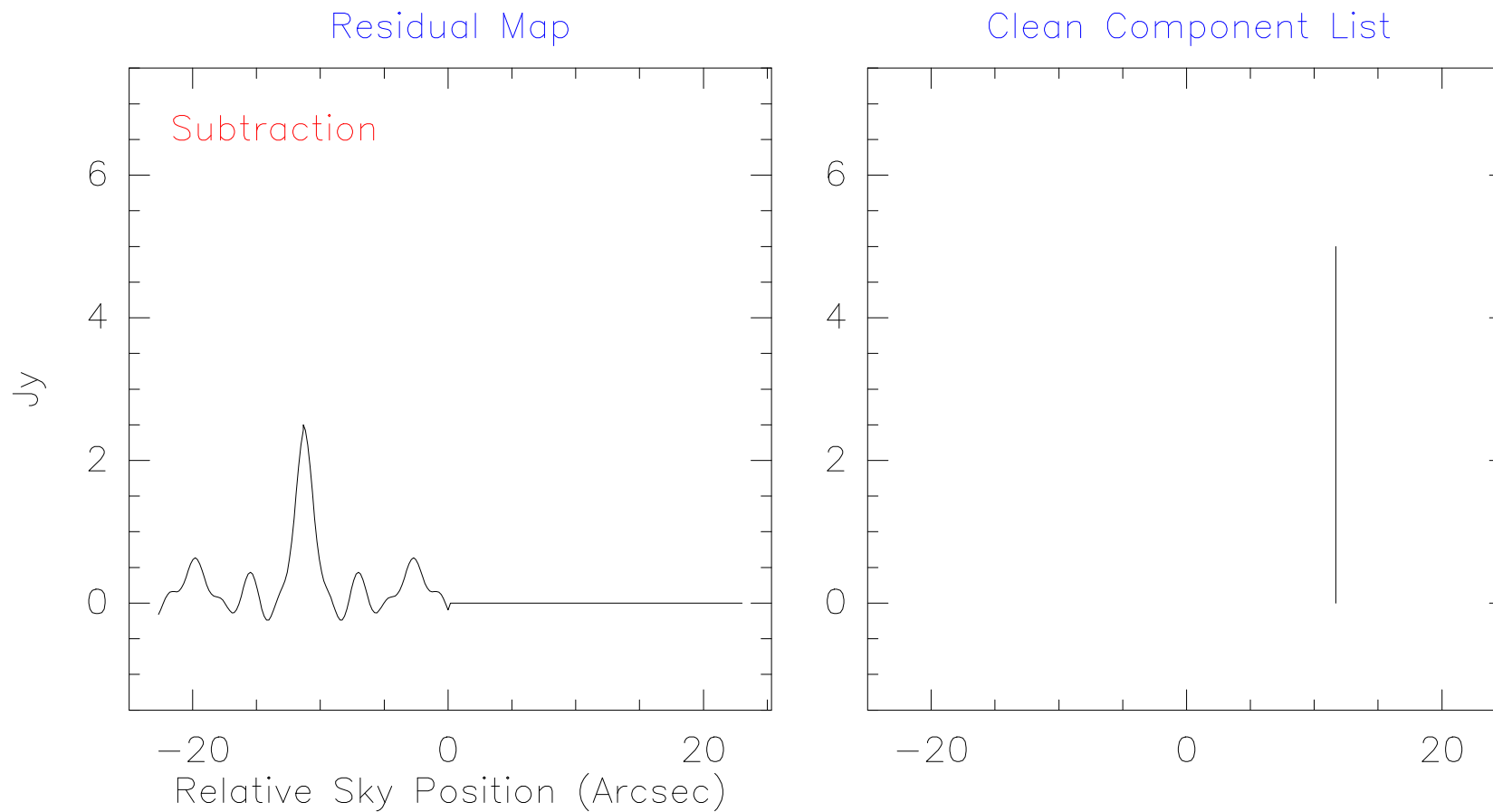
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



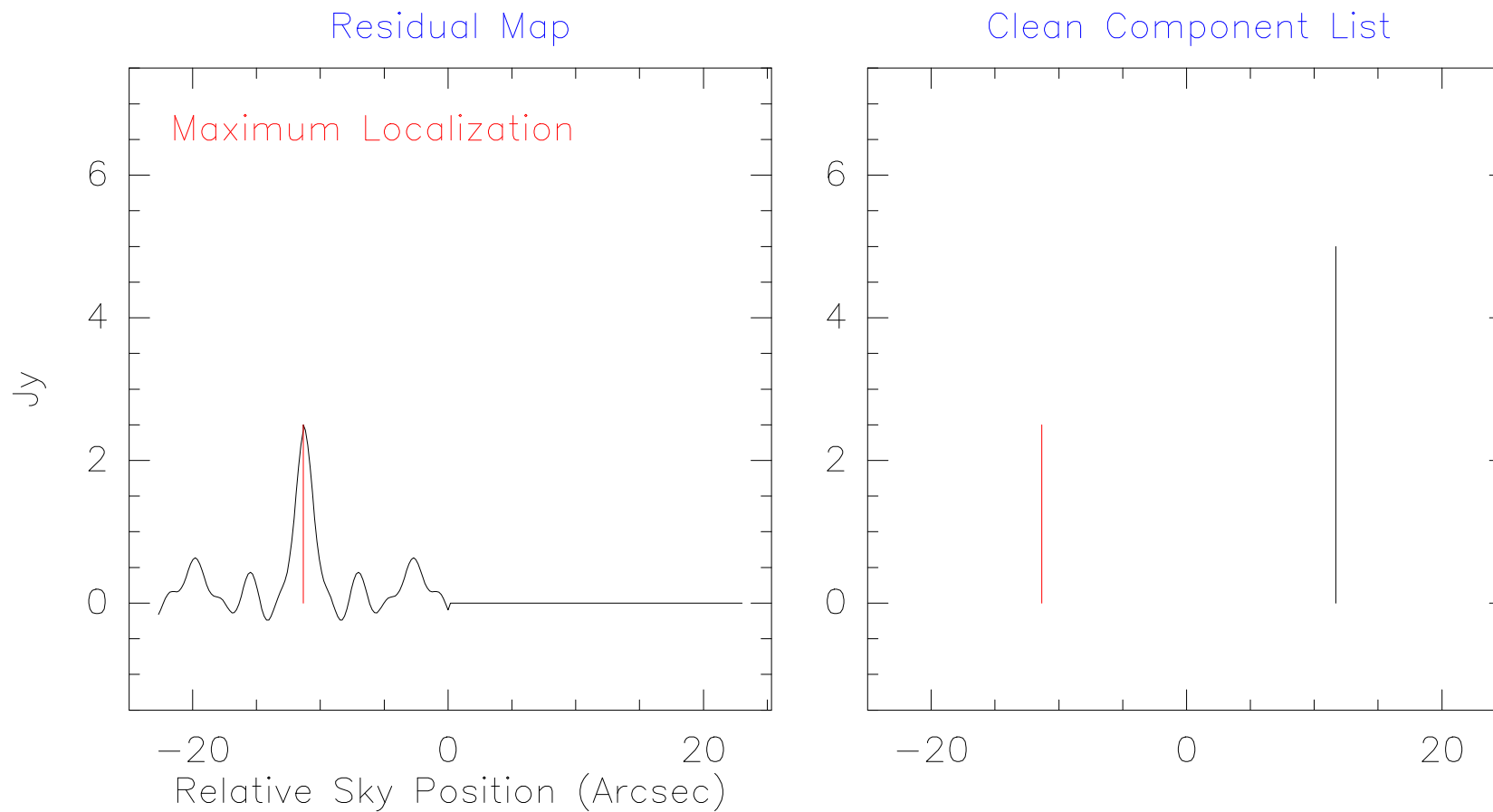
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



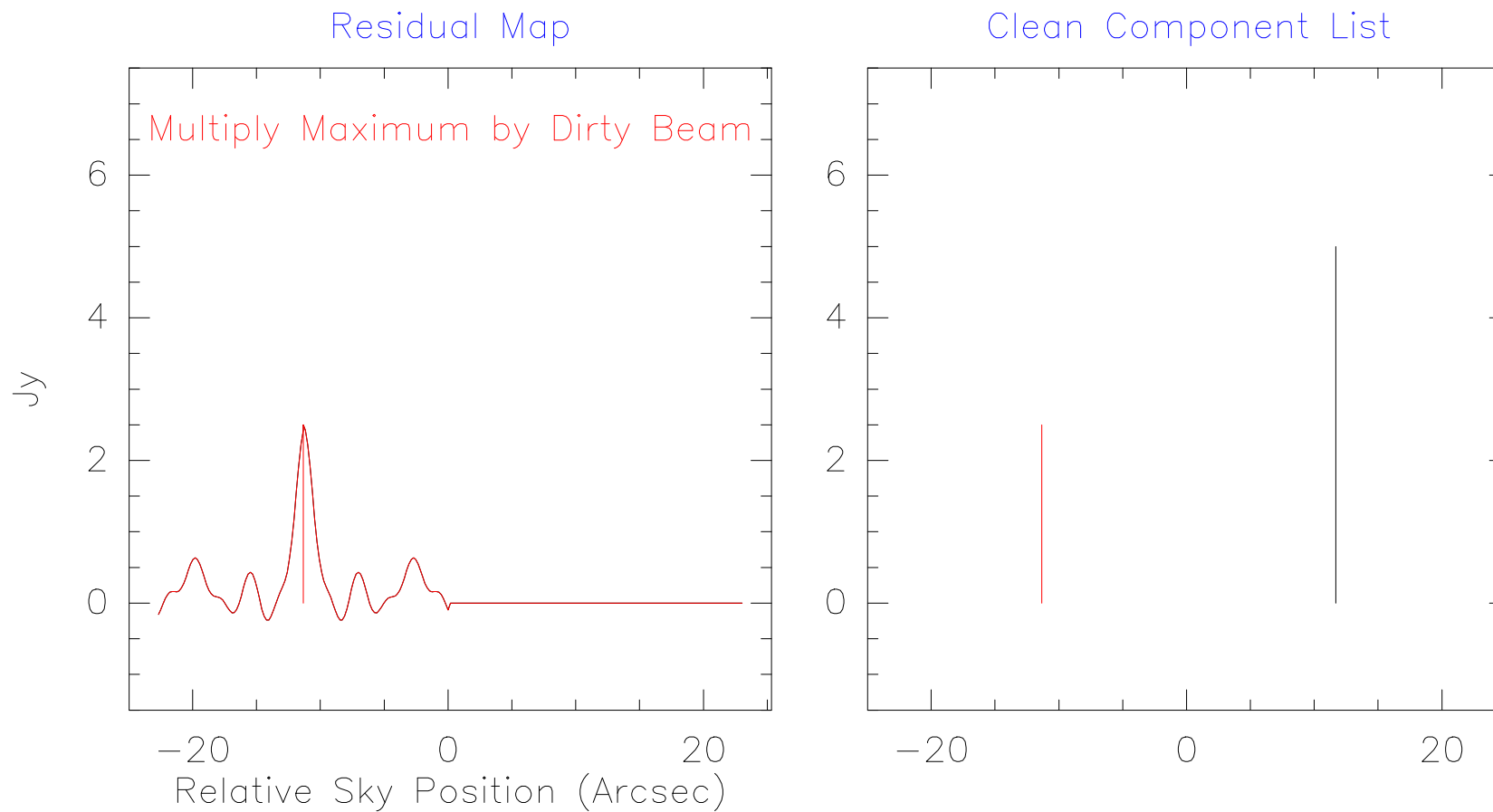
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



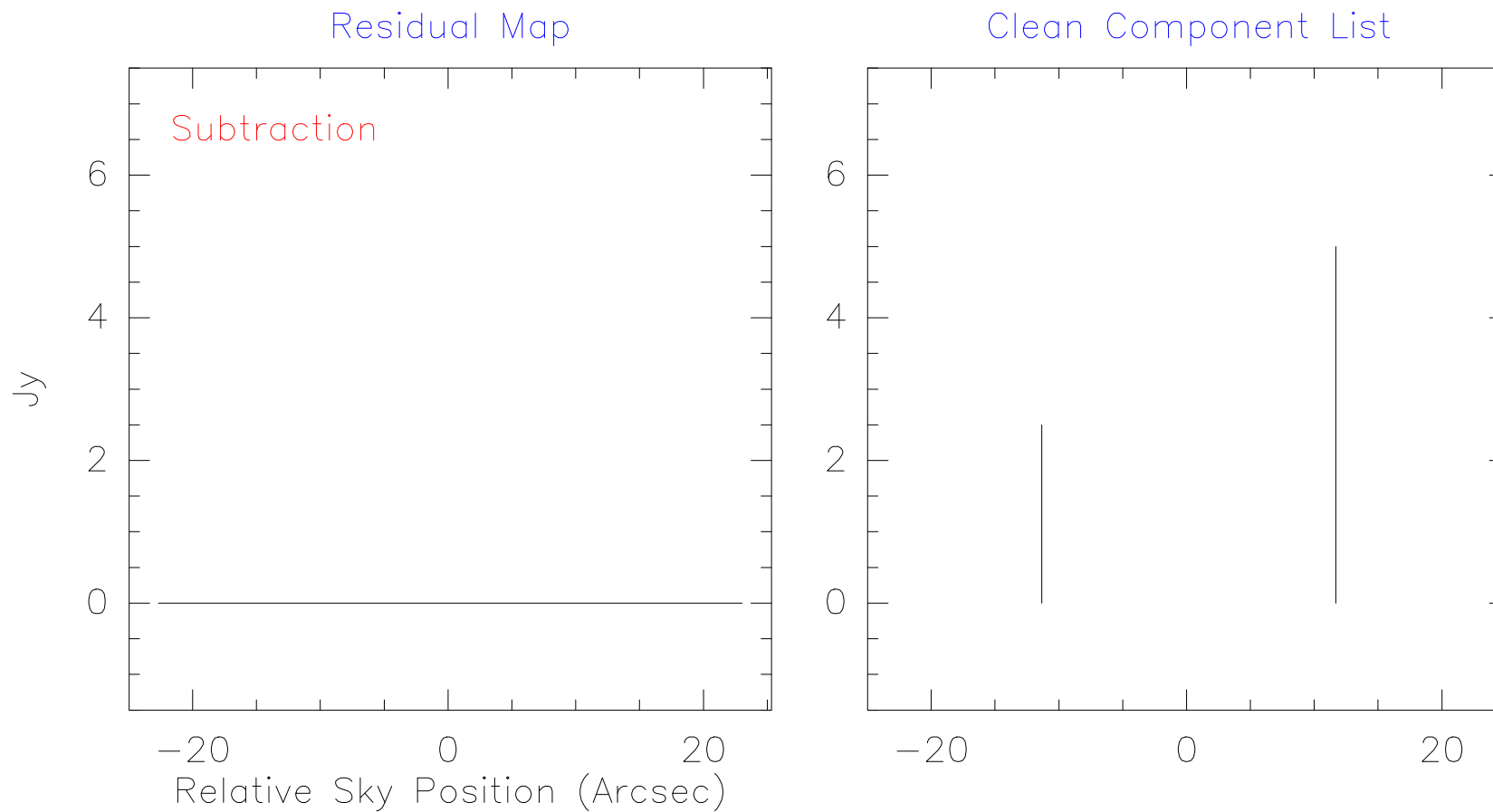
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



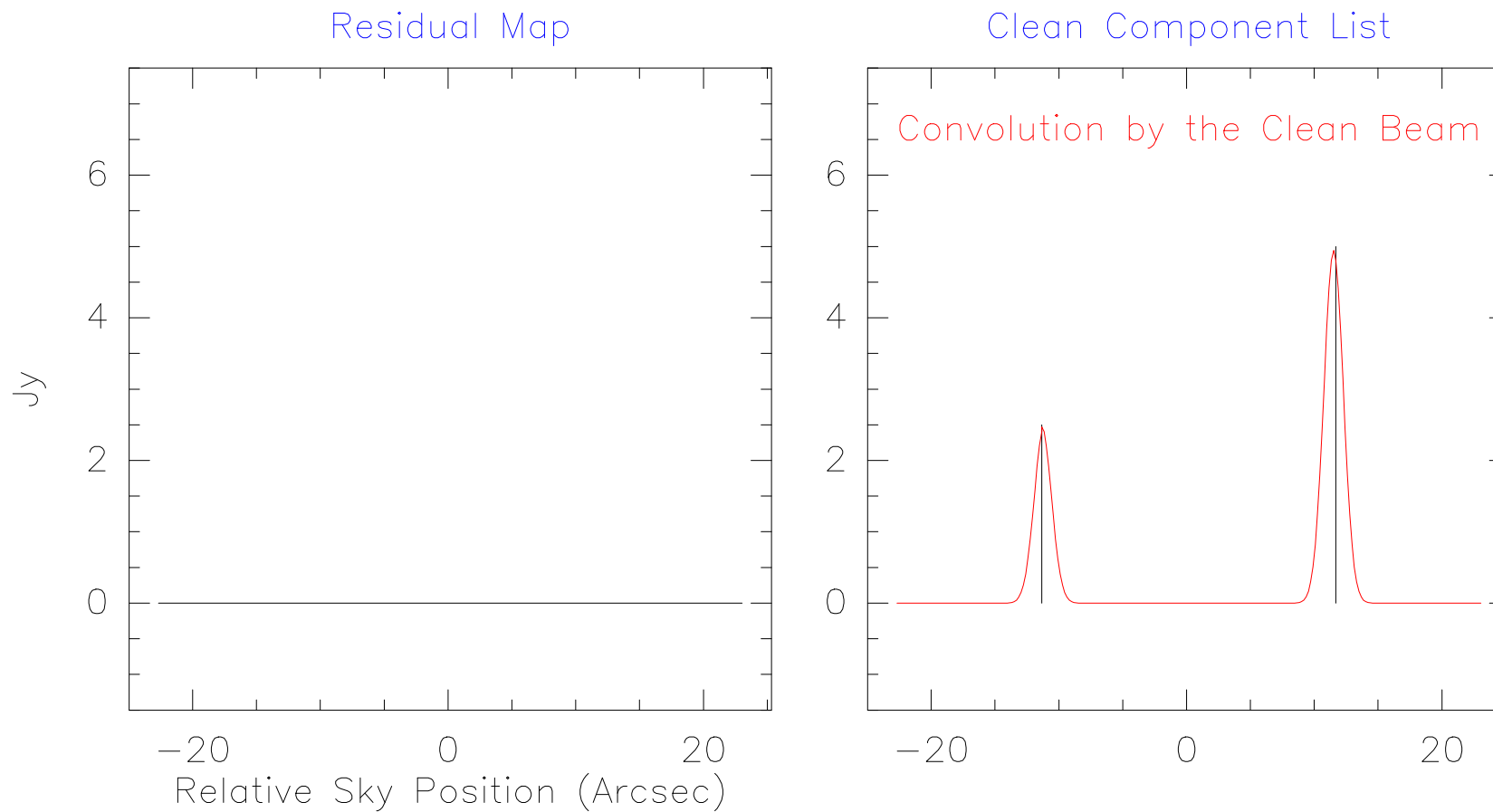
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



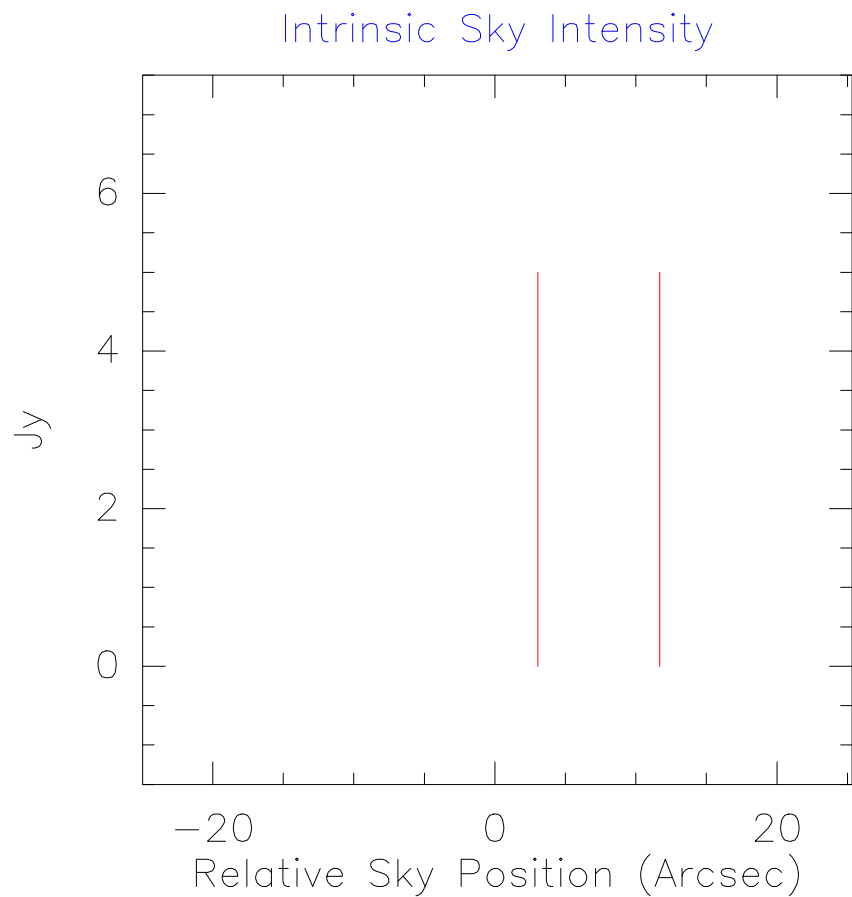
# Deconvolution: III. The Basic Clean Algorithm

## 1. First Illustration



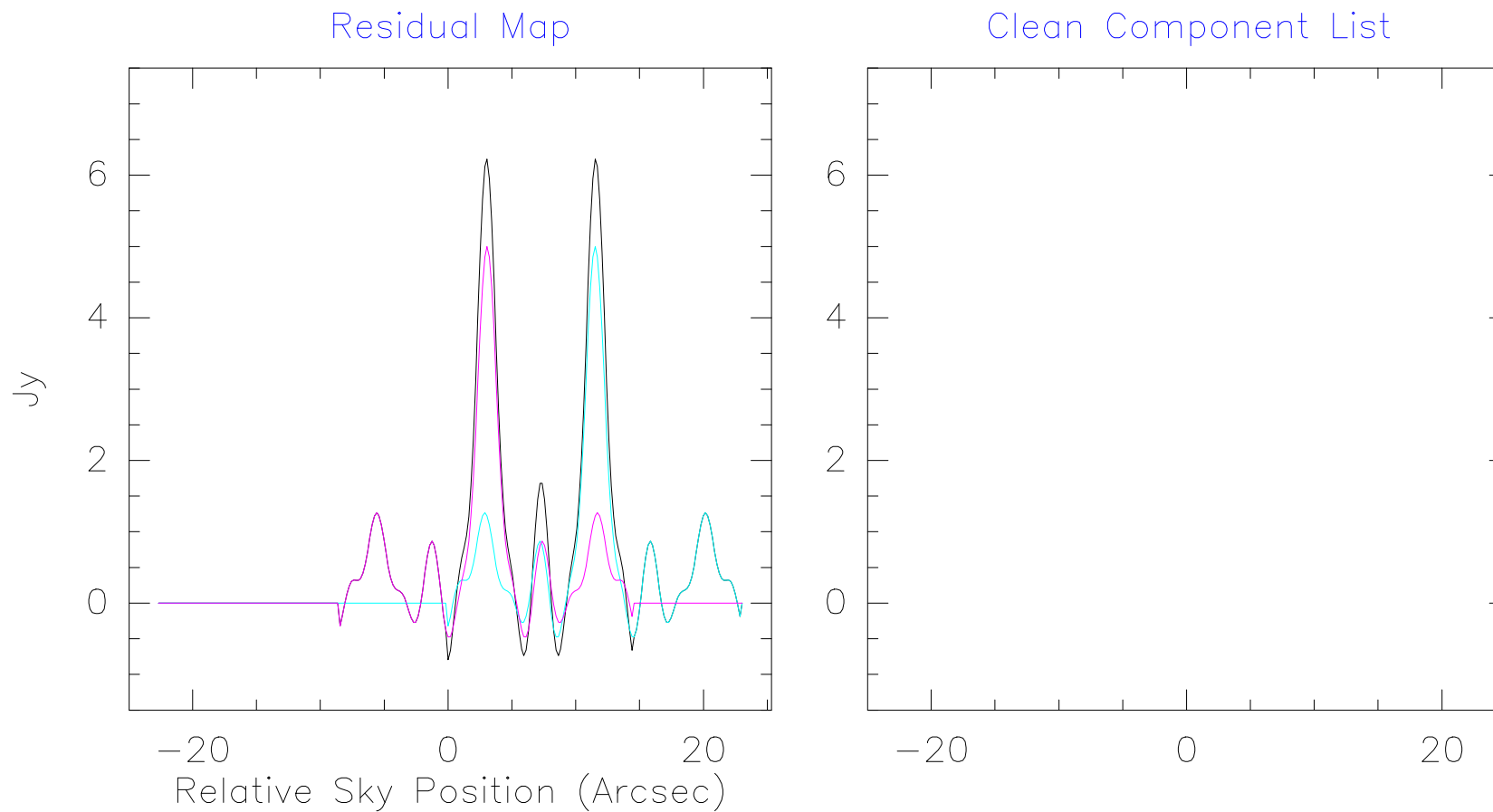
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



# Deconvolution: III. The Basic Clean Algorithm

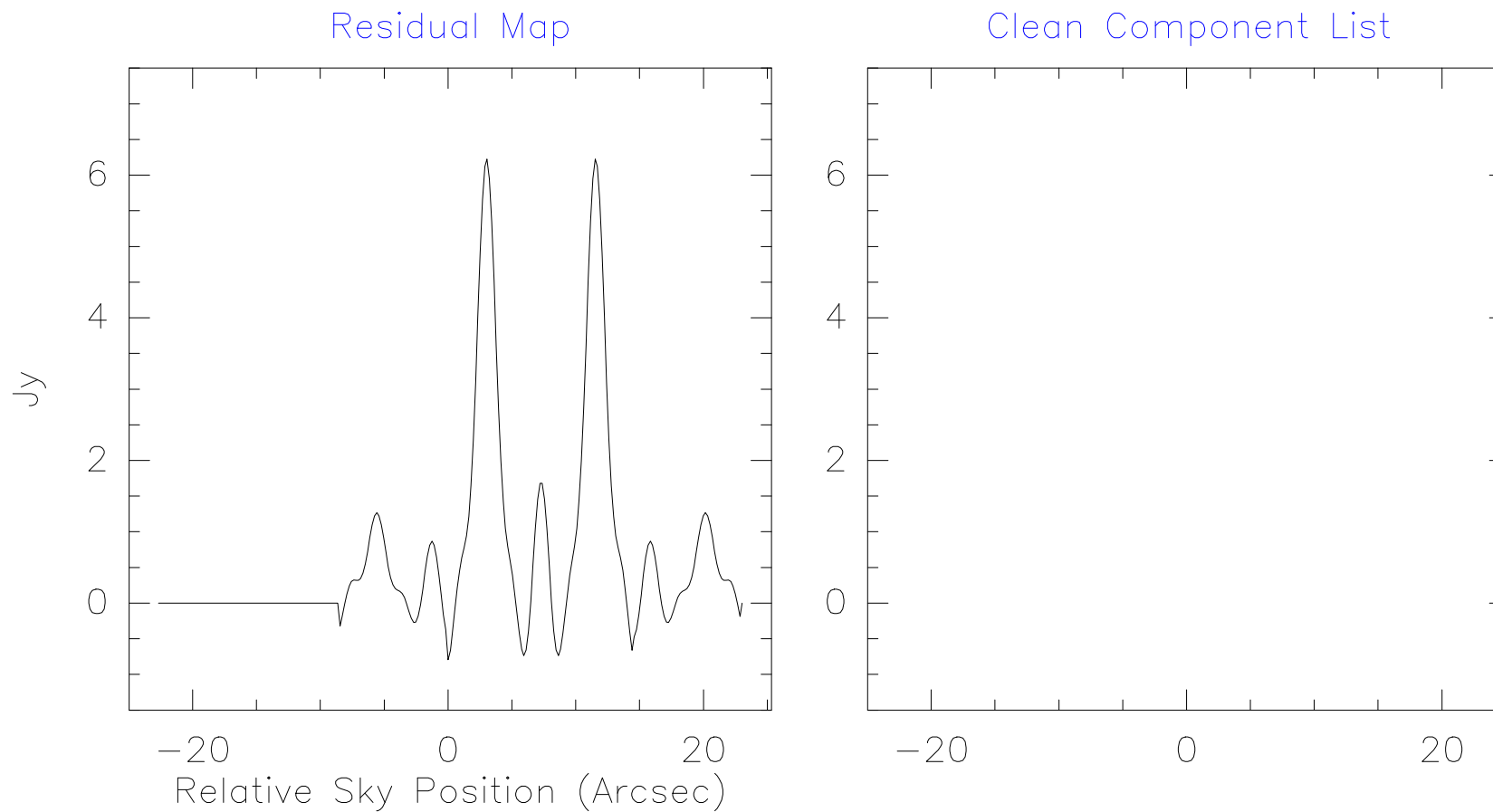
## 2. Second Illustration





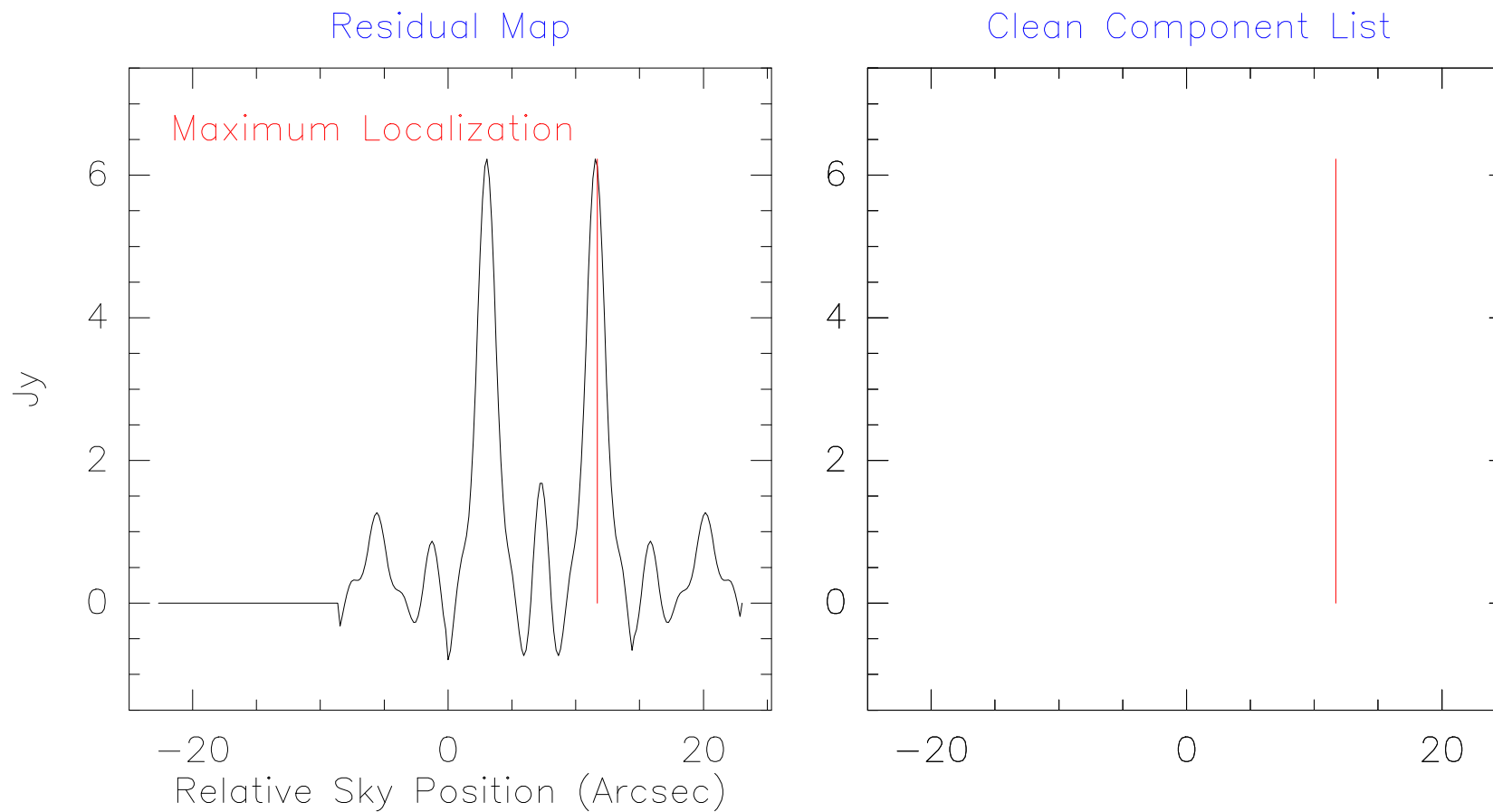
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



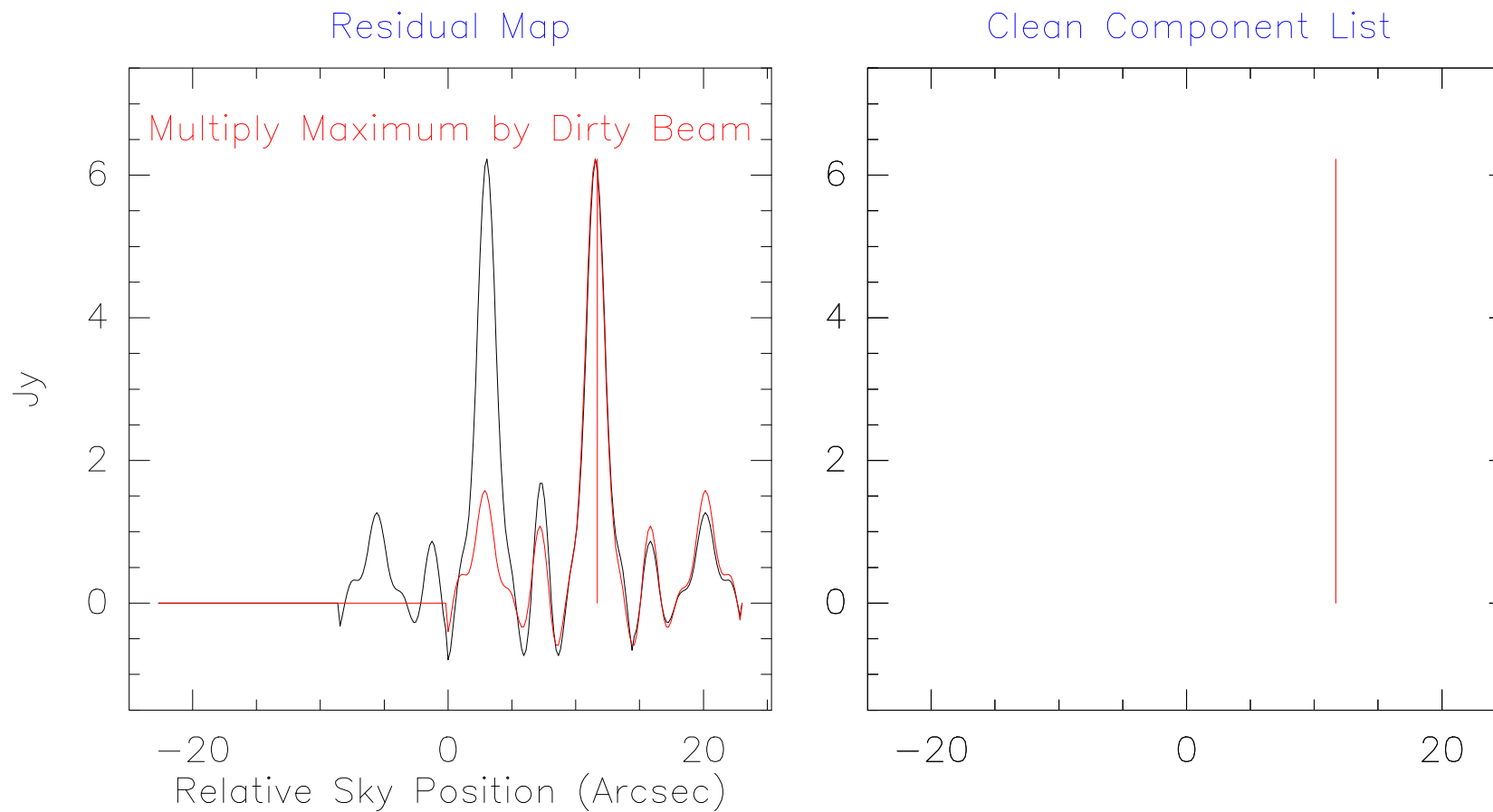
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



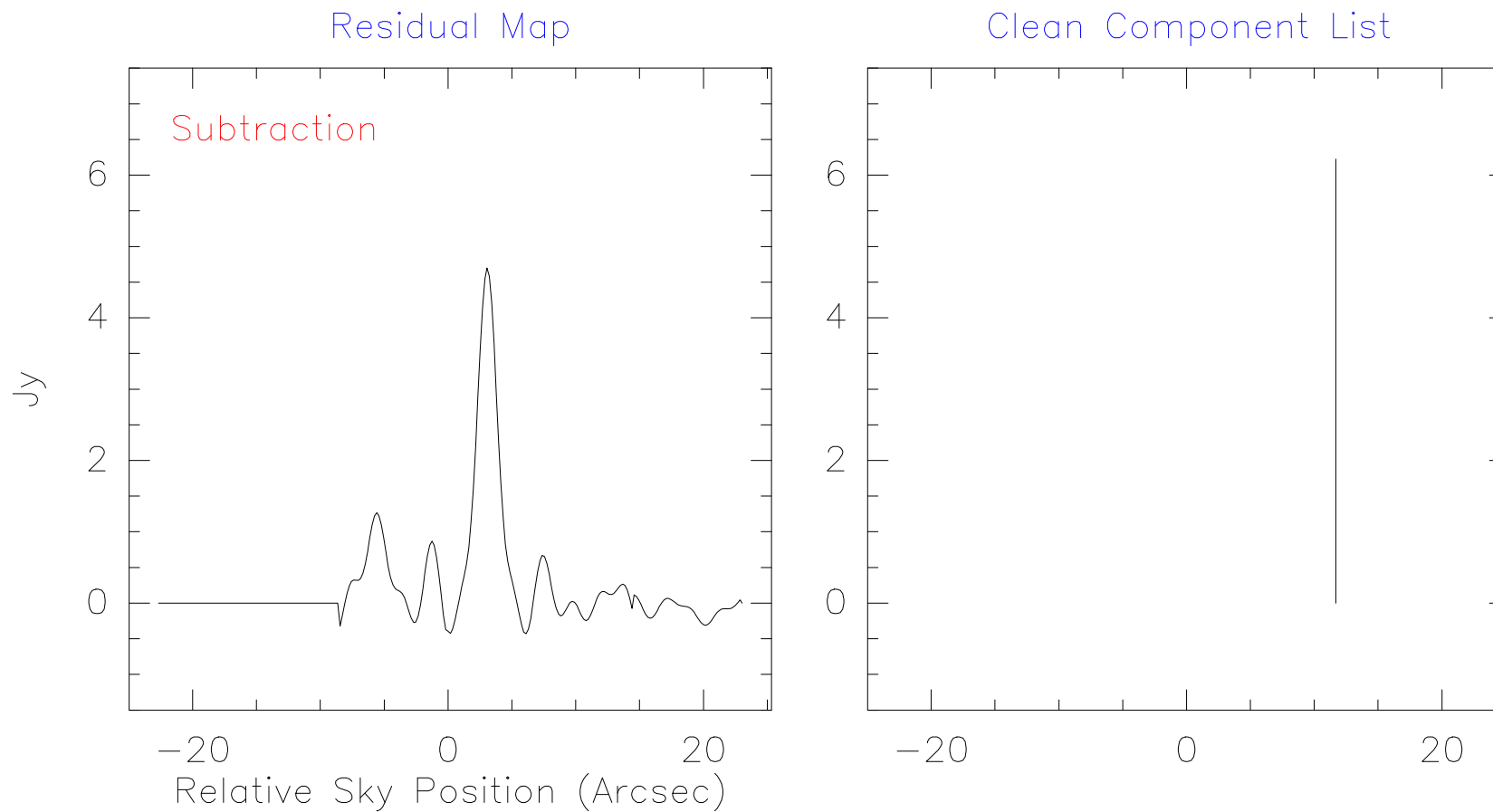
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



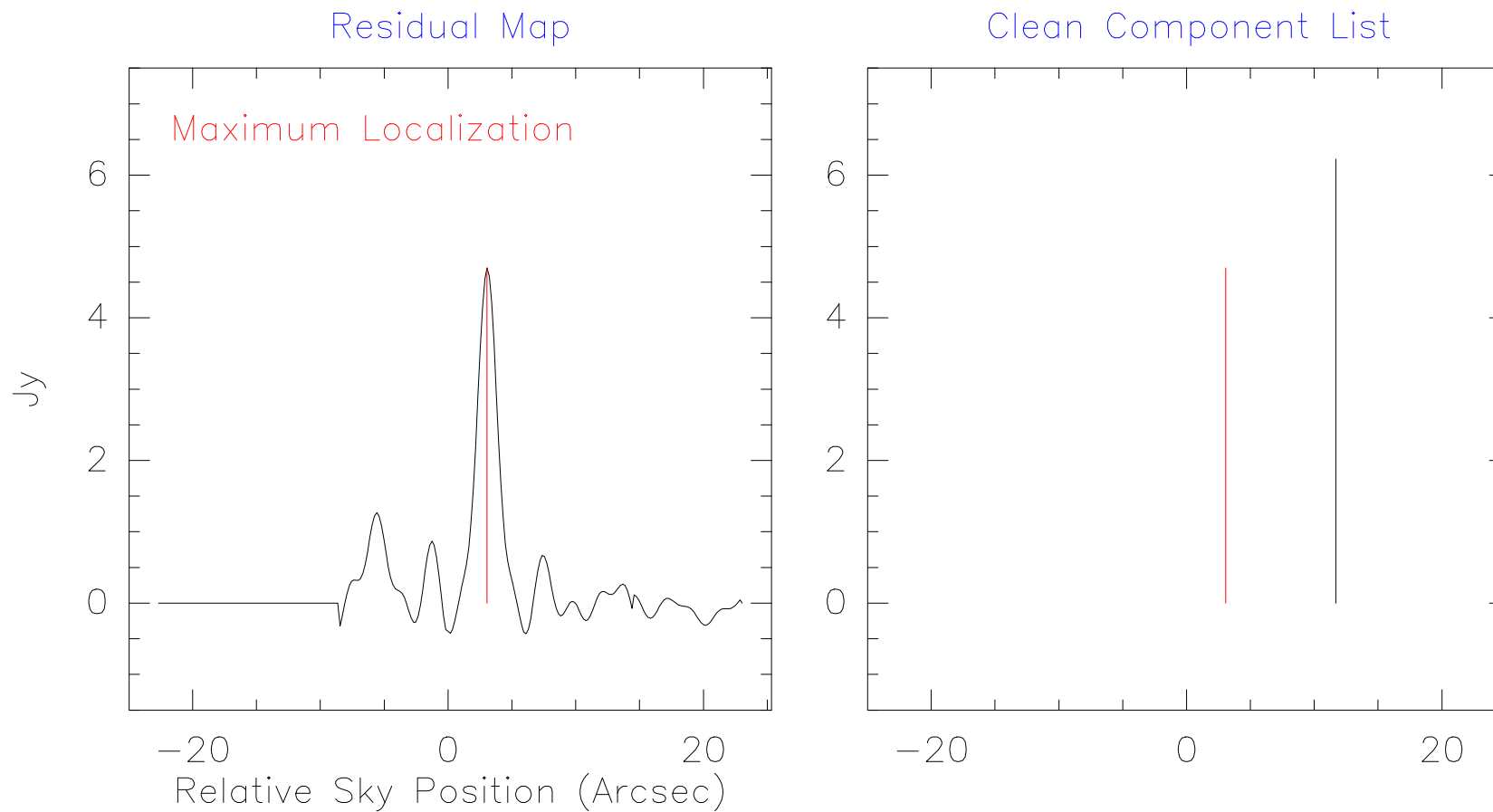
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



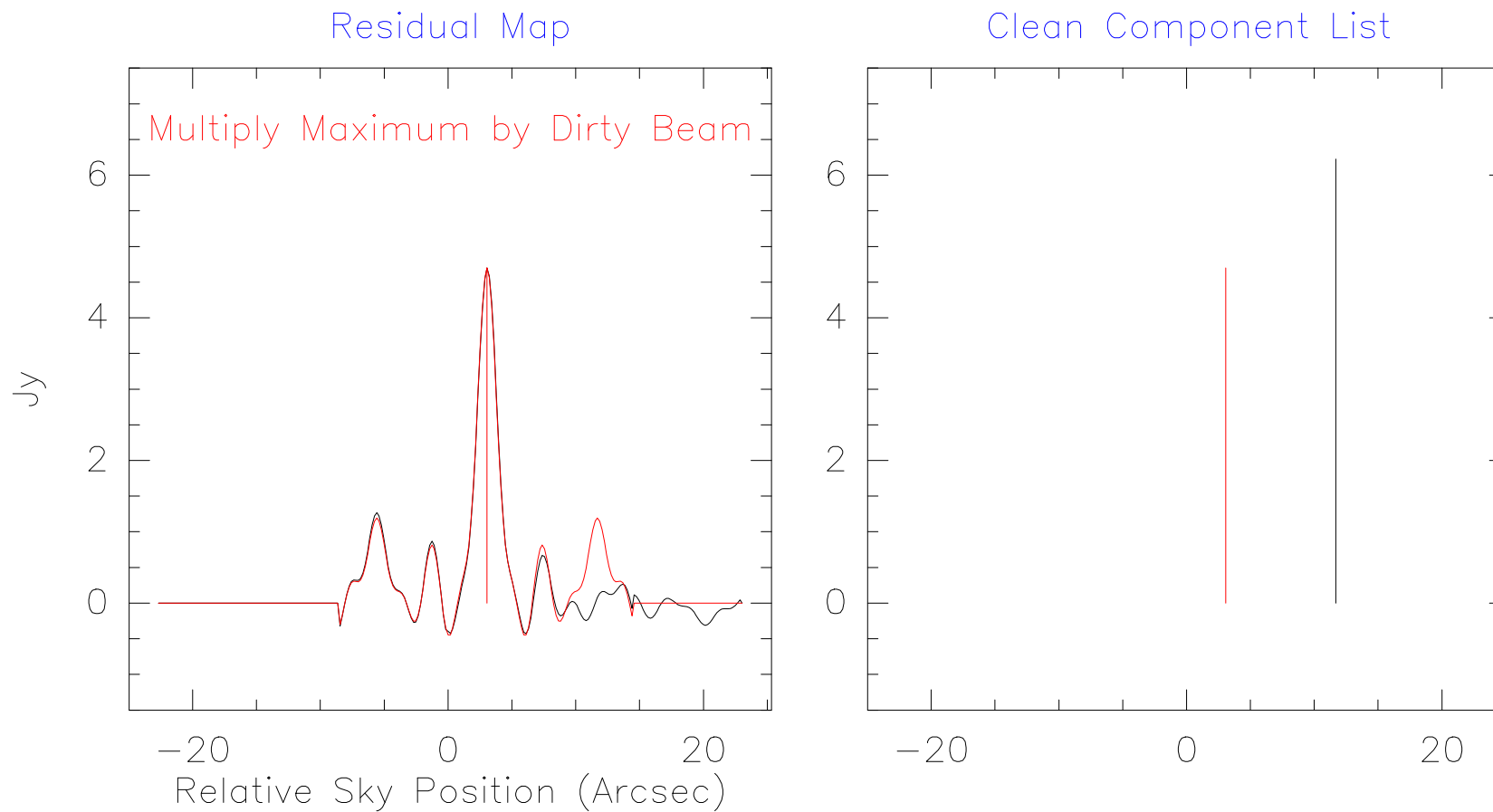
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



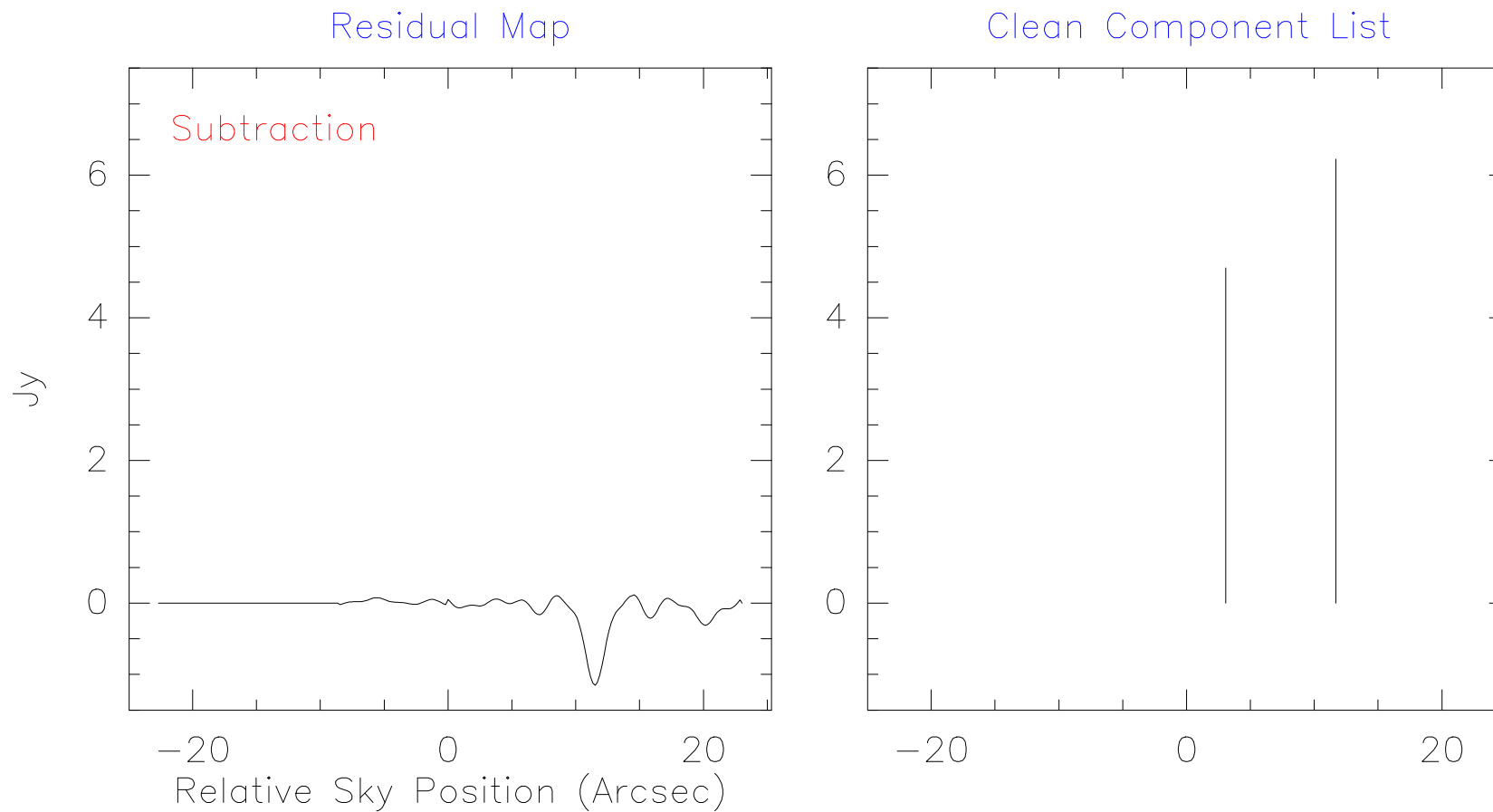
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



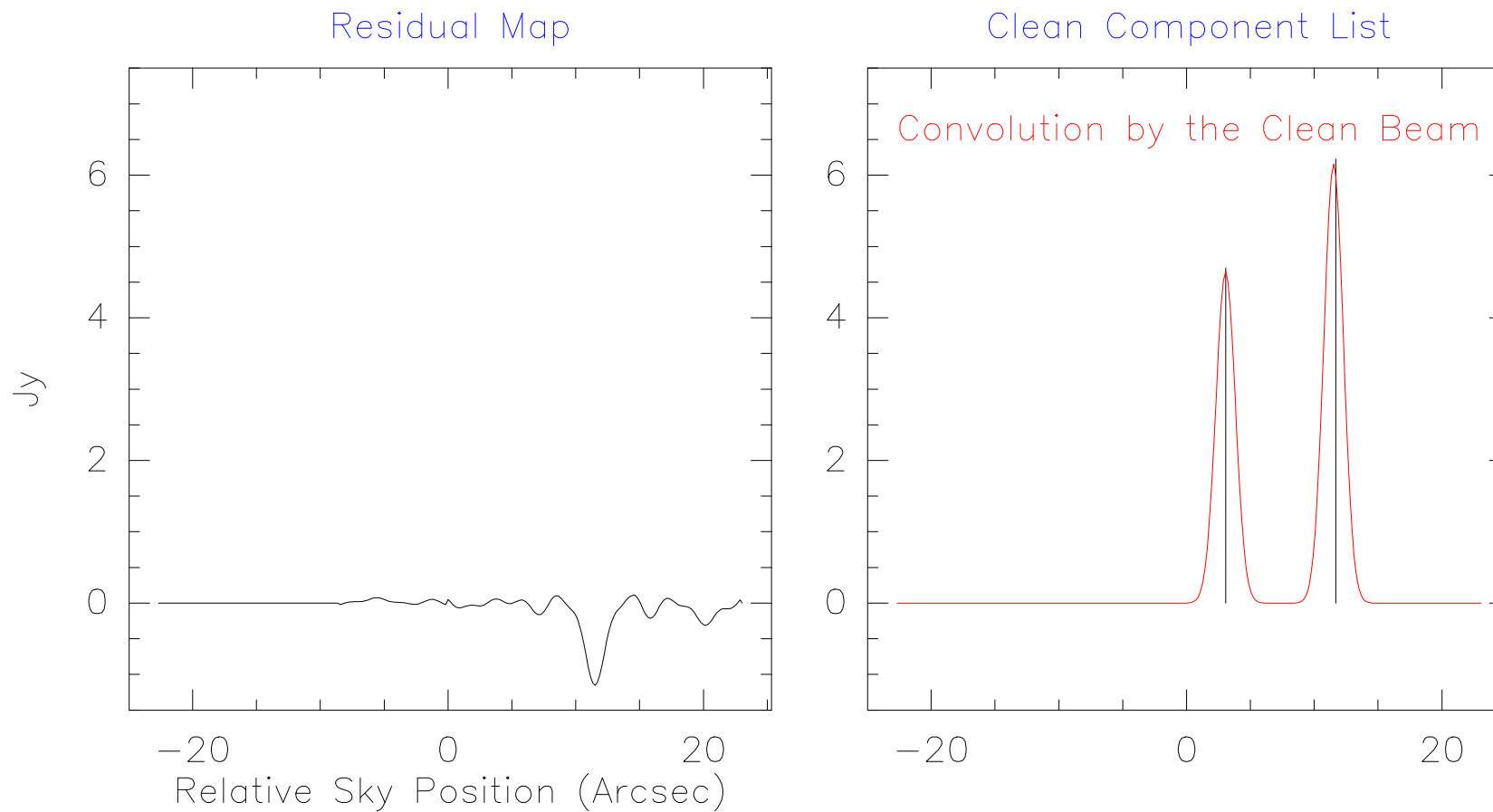
# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration



# Deconvolution: III. The Basic Clean Algorithm

## 2. Second Illustration





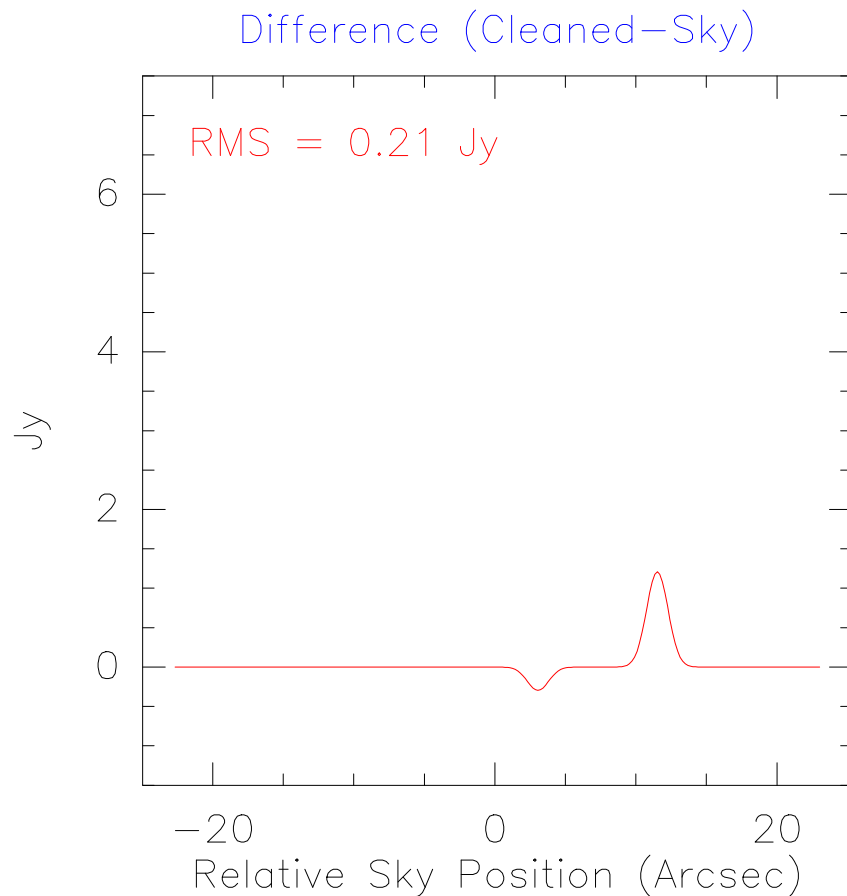
# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

Convergence:

Too superficial cleaning  $\Rightarrow$  Approximate results.

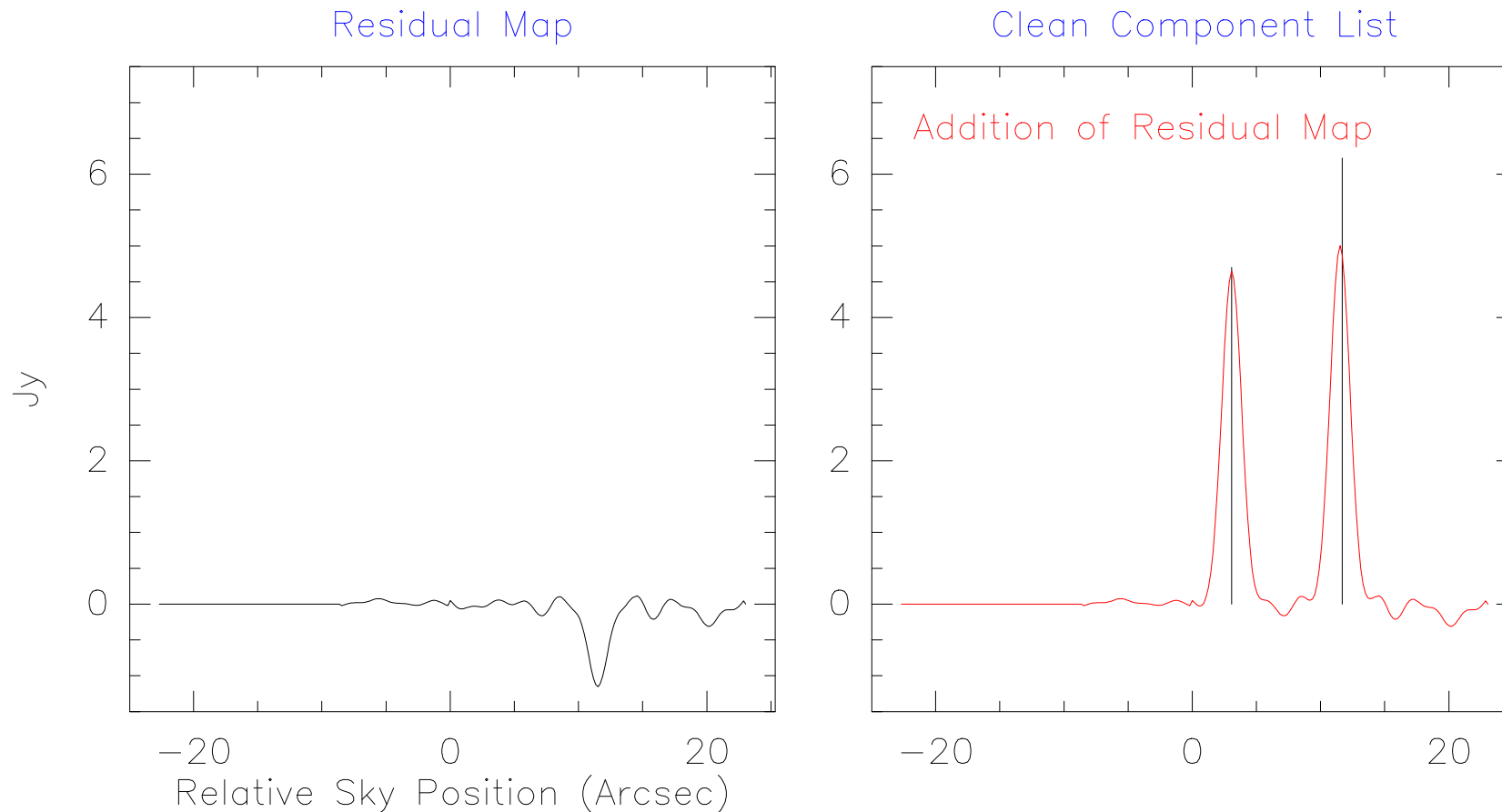
Too deep cleaning  $\Rightarrow$  Divergence.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

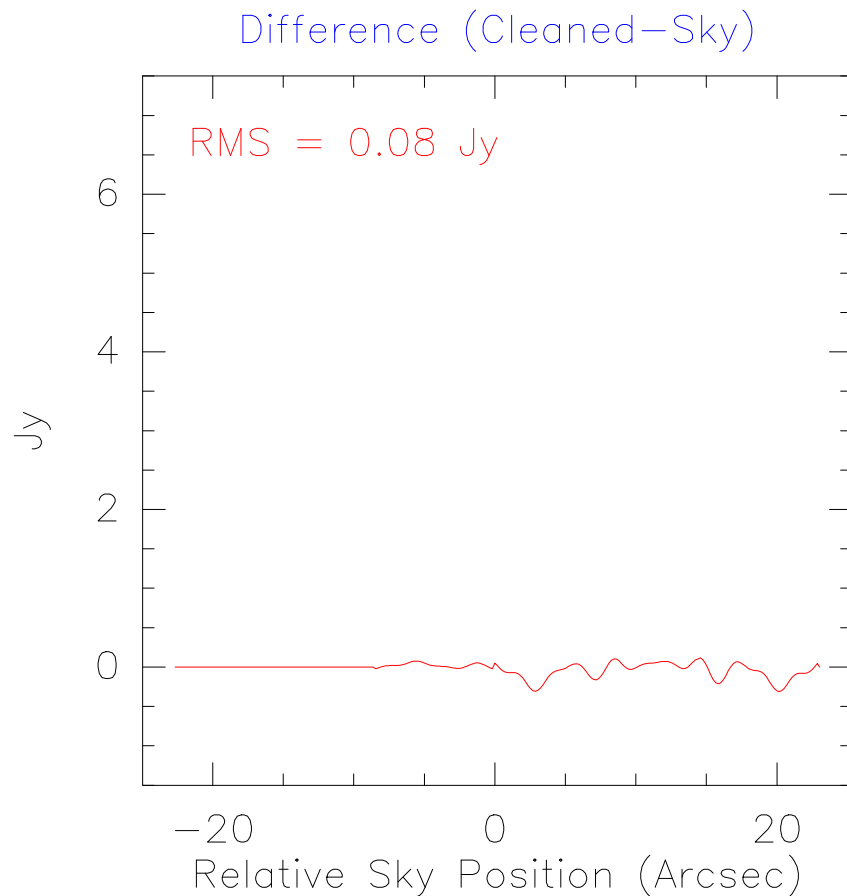
Addition of residual map:  
Improvement when convergence **not** reached;  
Noise estimation.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

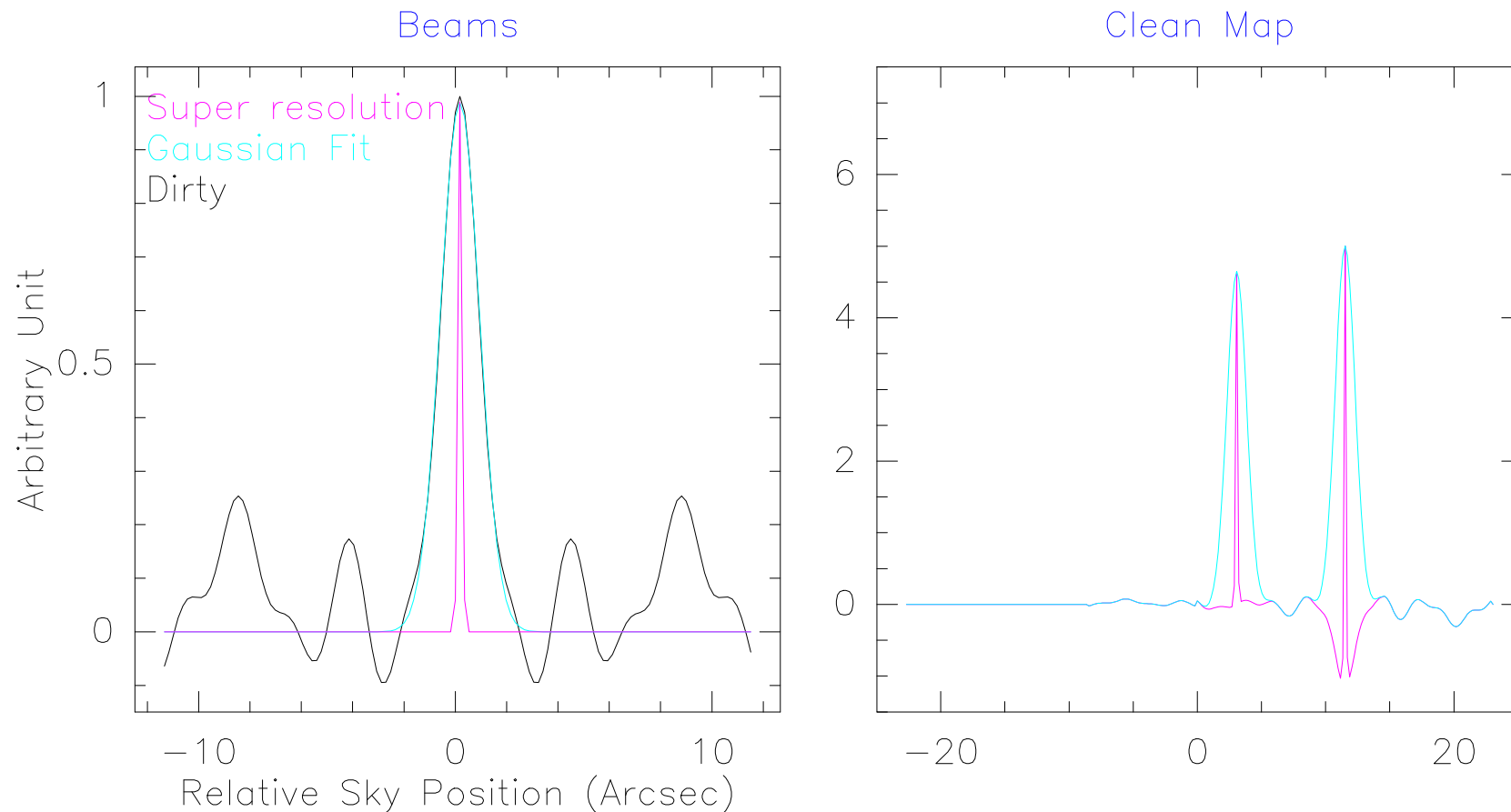
Addition of residual map:  
Improvement when convergence **not** reached;  
Noise estimation.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

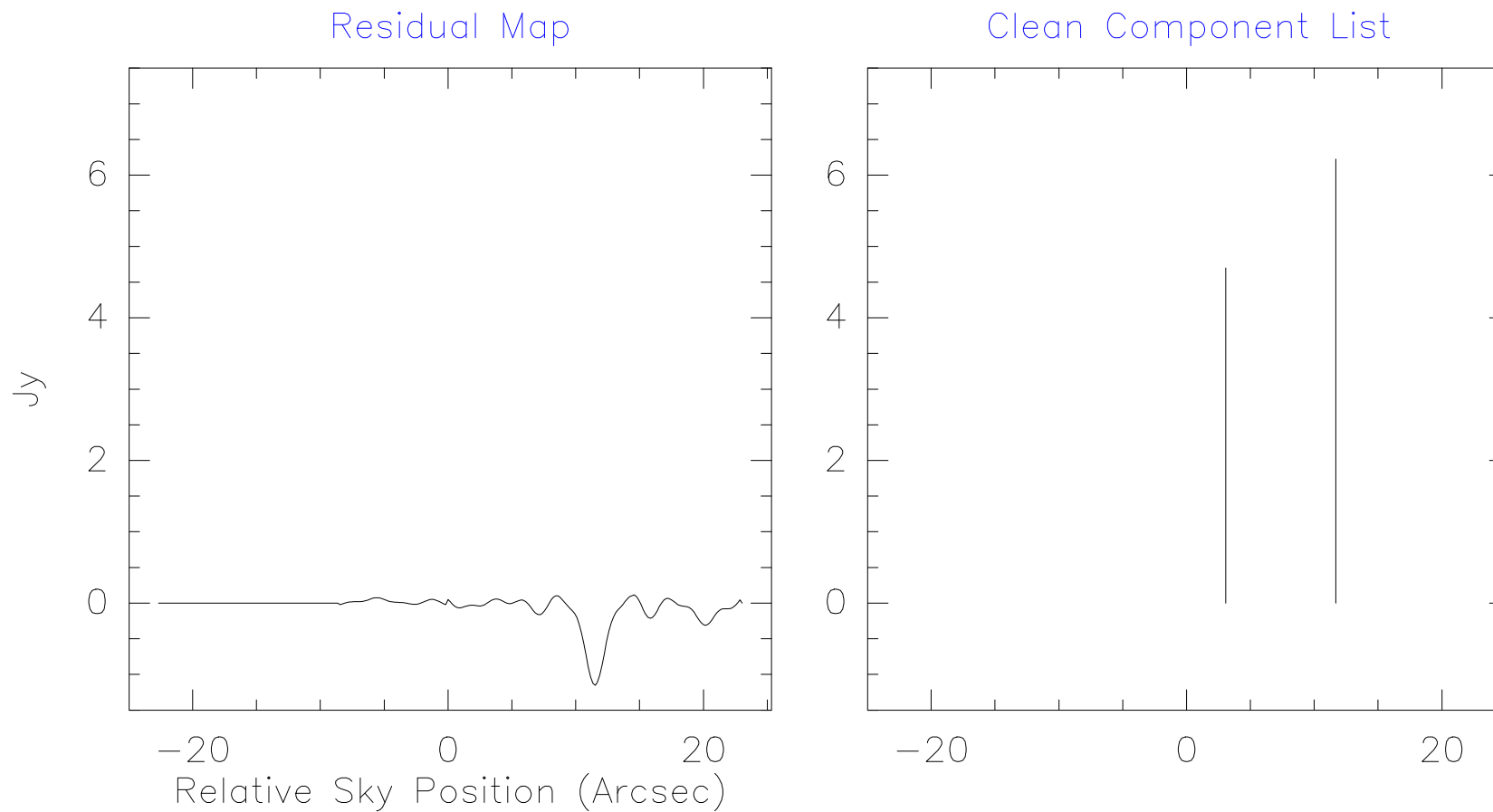
Choice of clean beam:  
Gaussian of FWHM matching the synthesized beam size.  
⇒ Super resolution **strongly** discouraged.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

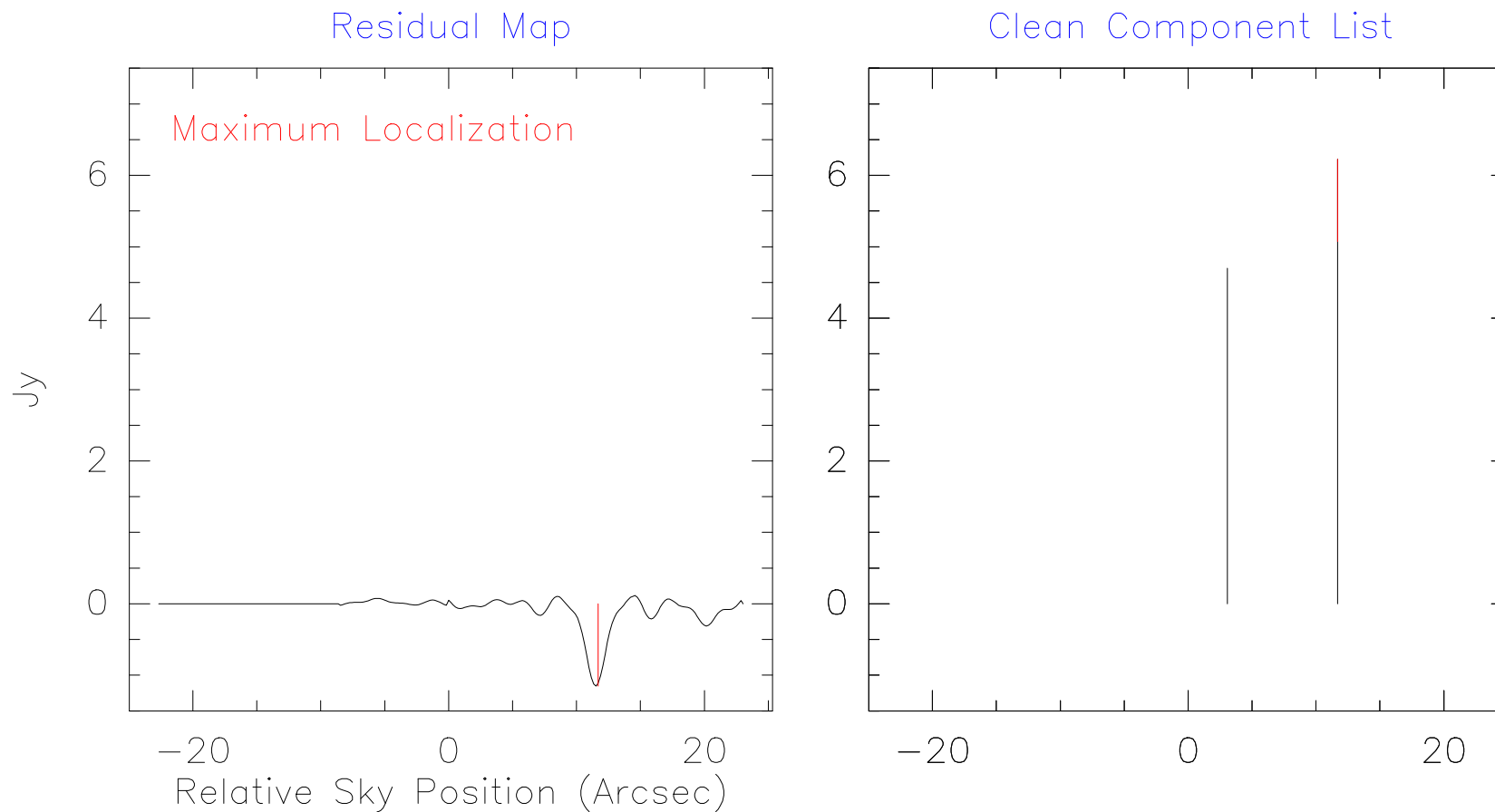
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

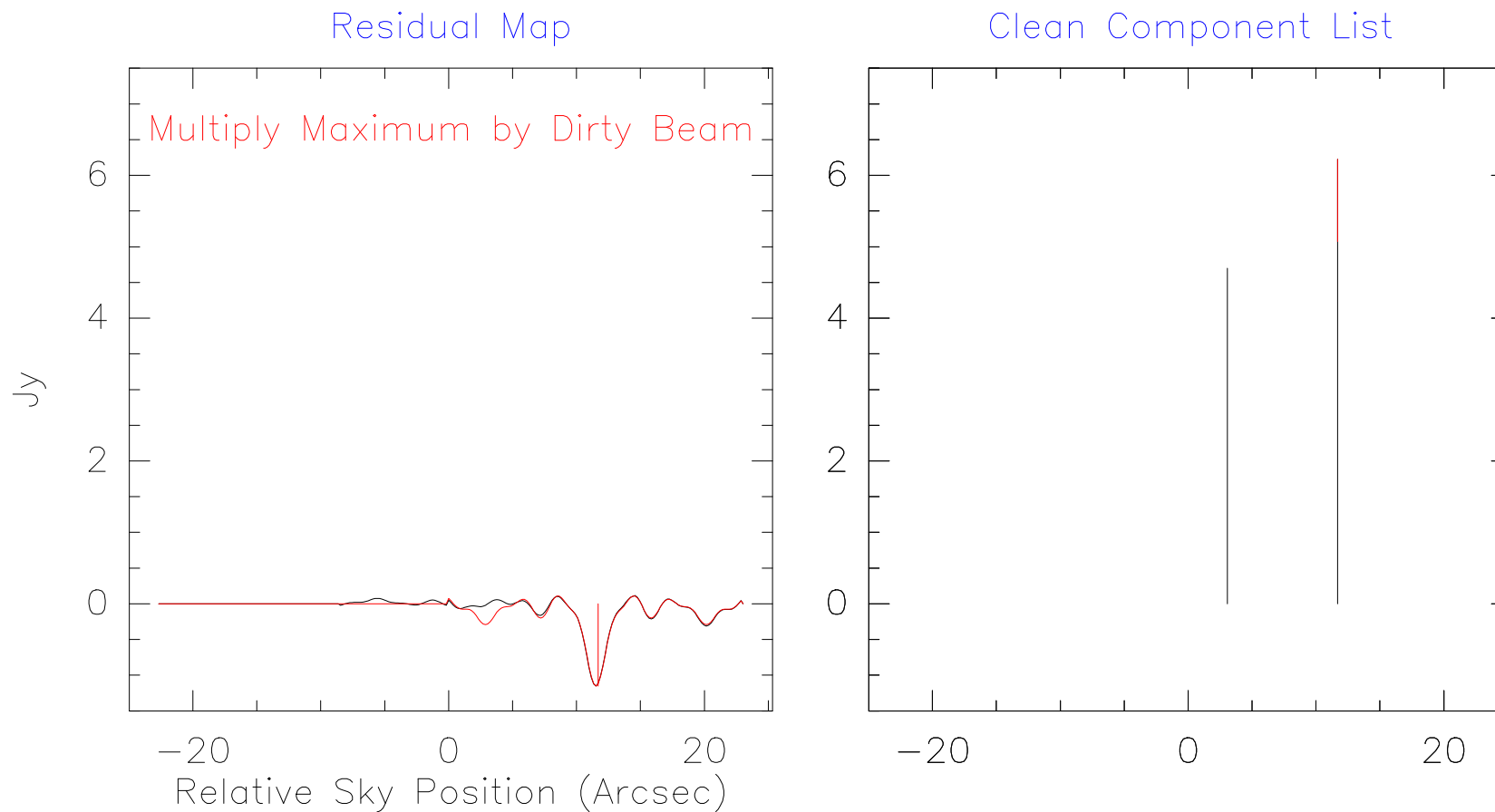
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

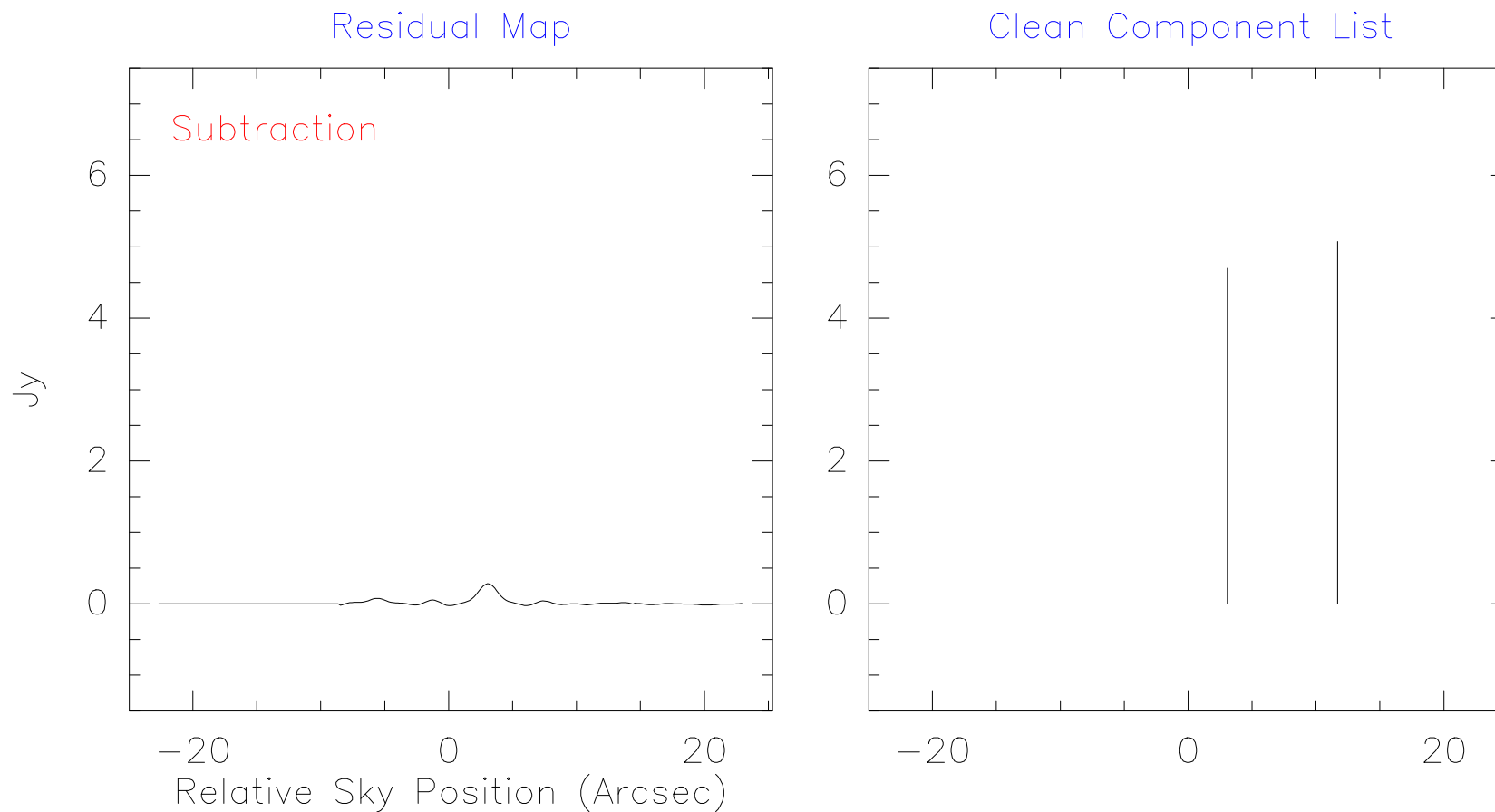
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

Negative clean components are mandatory.

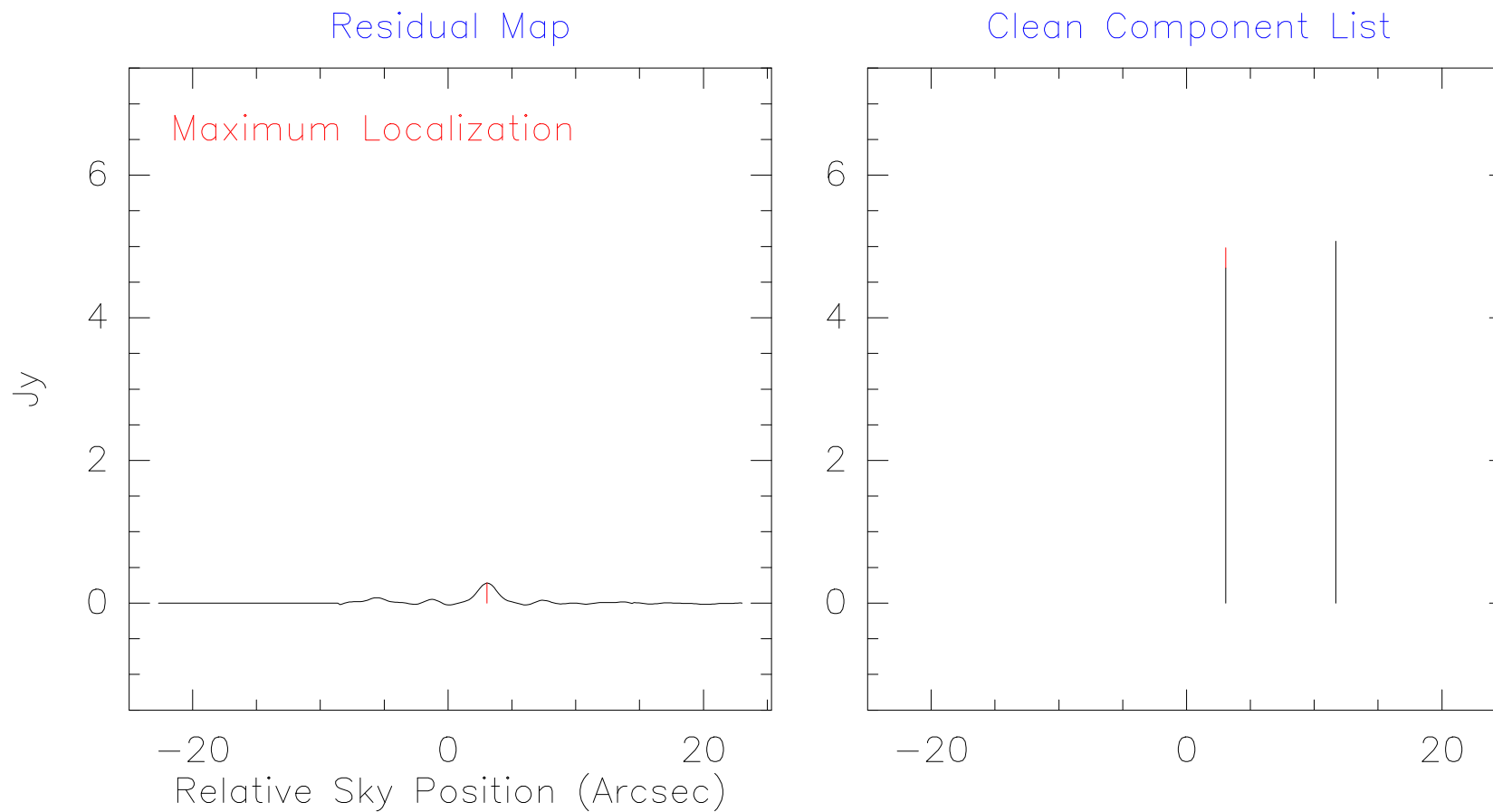




# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

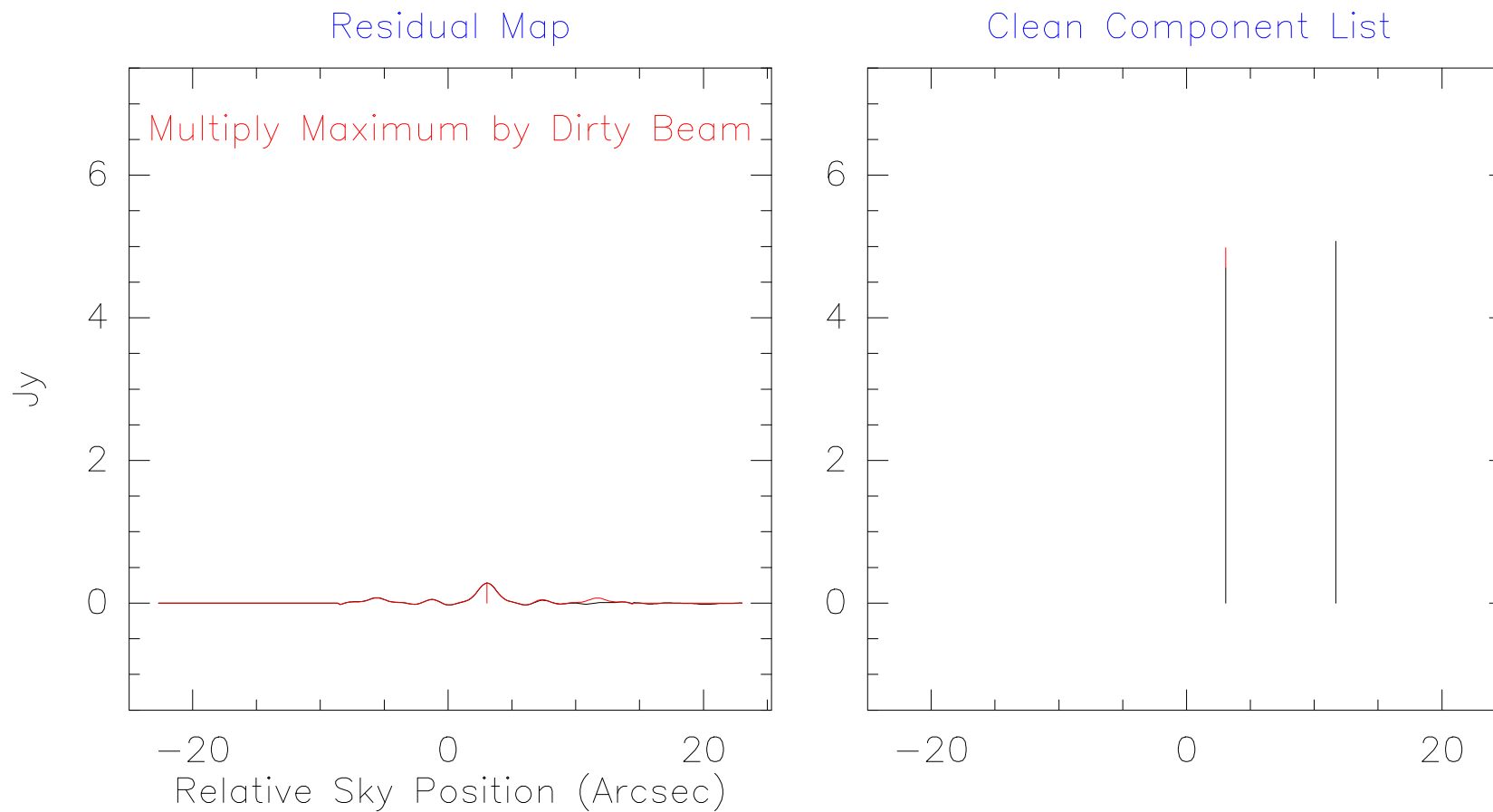
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

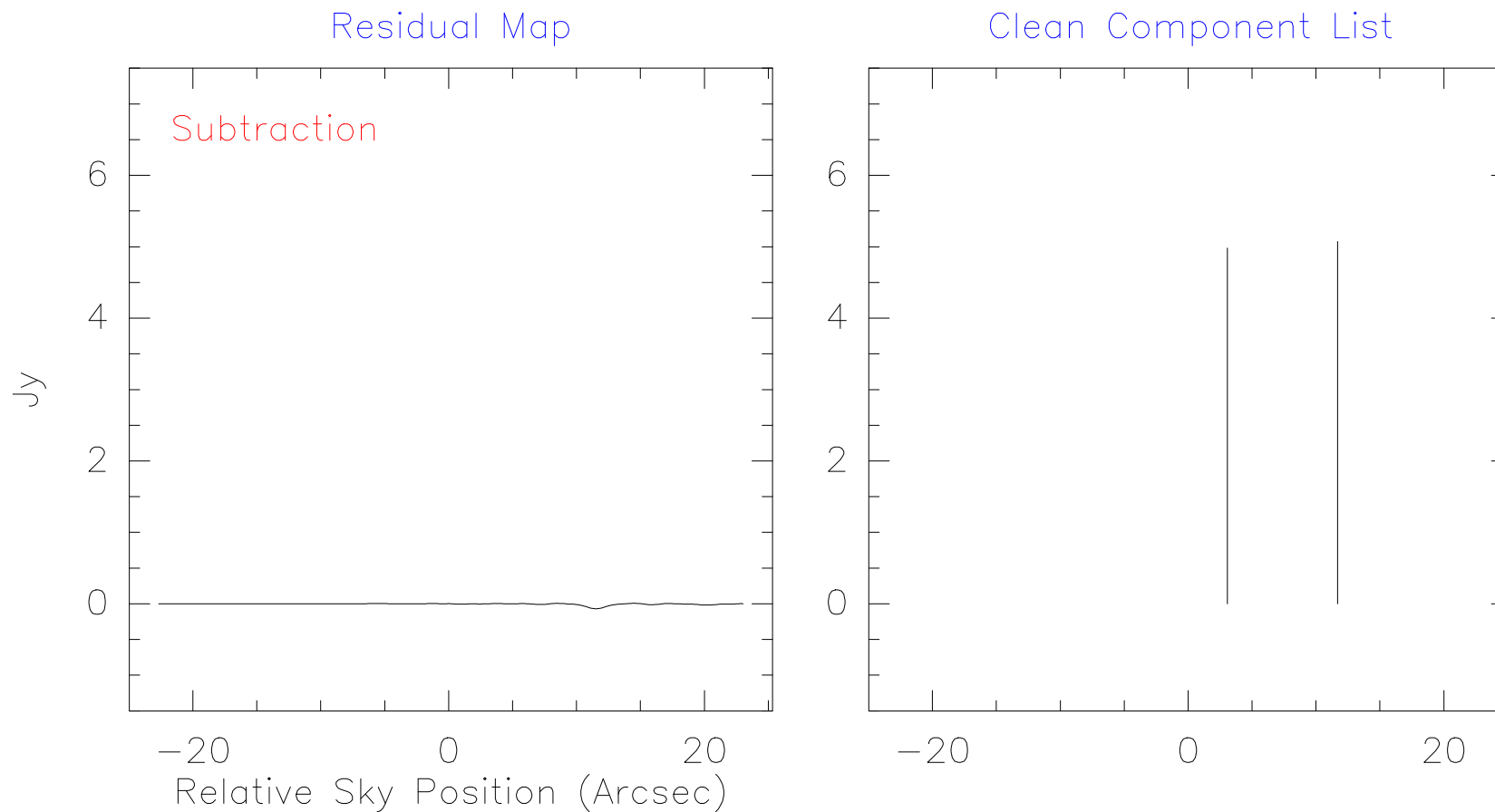
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

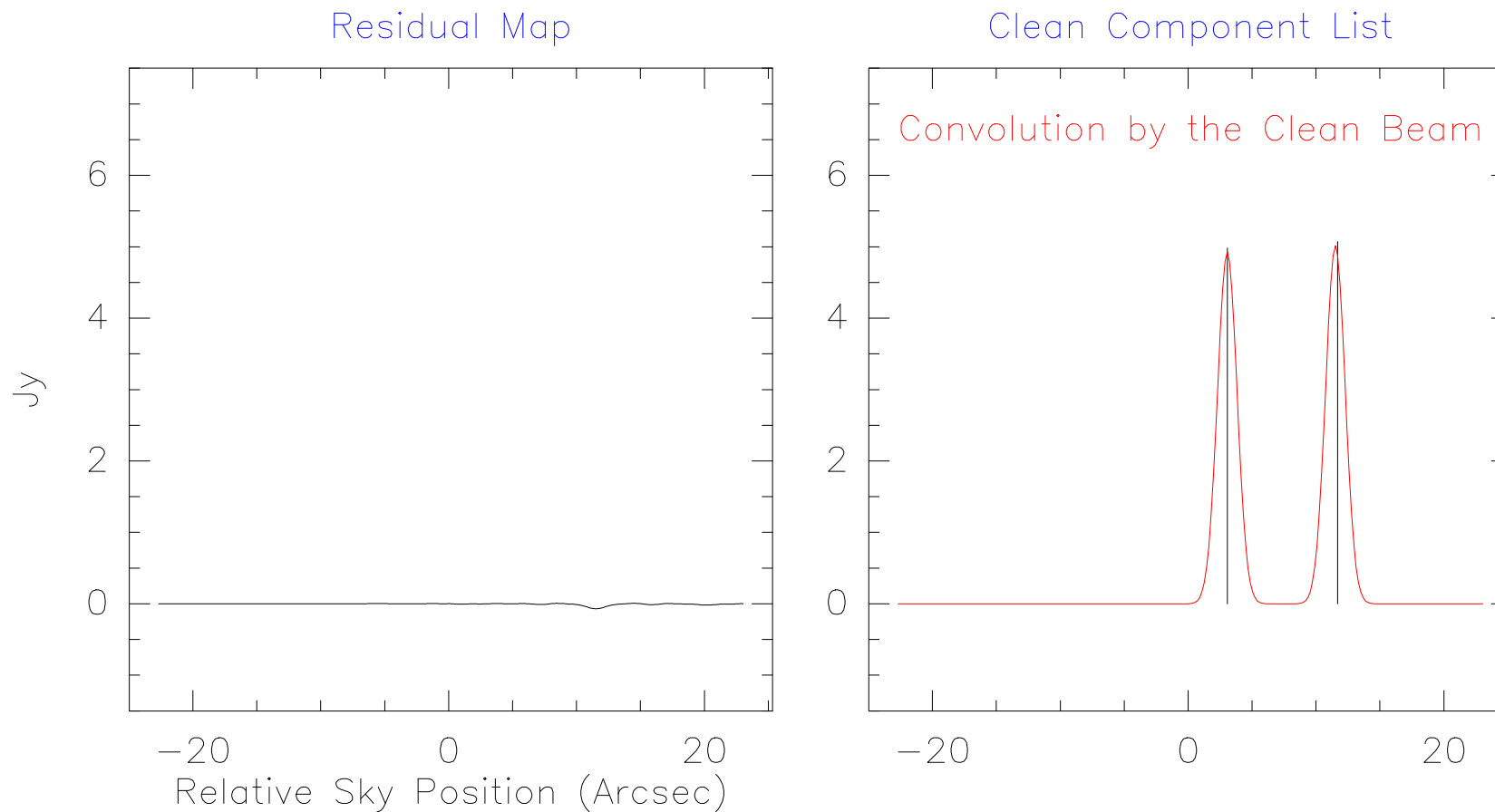
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

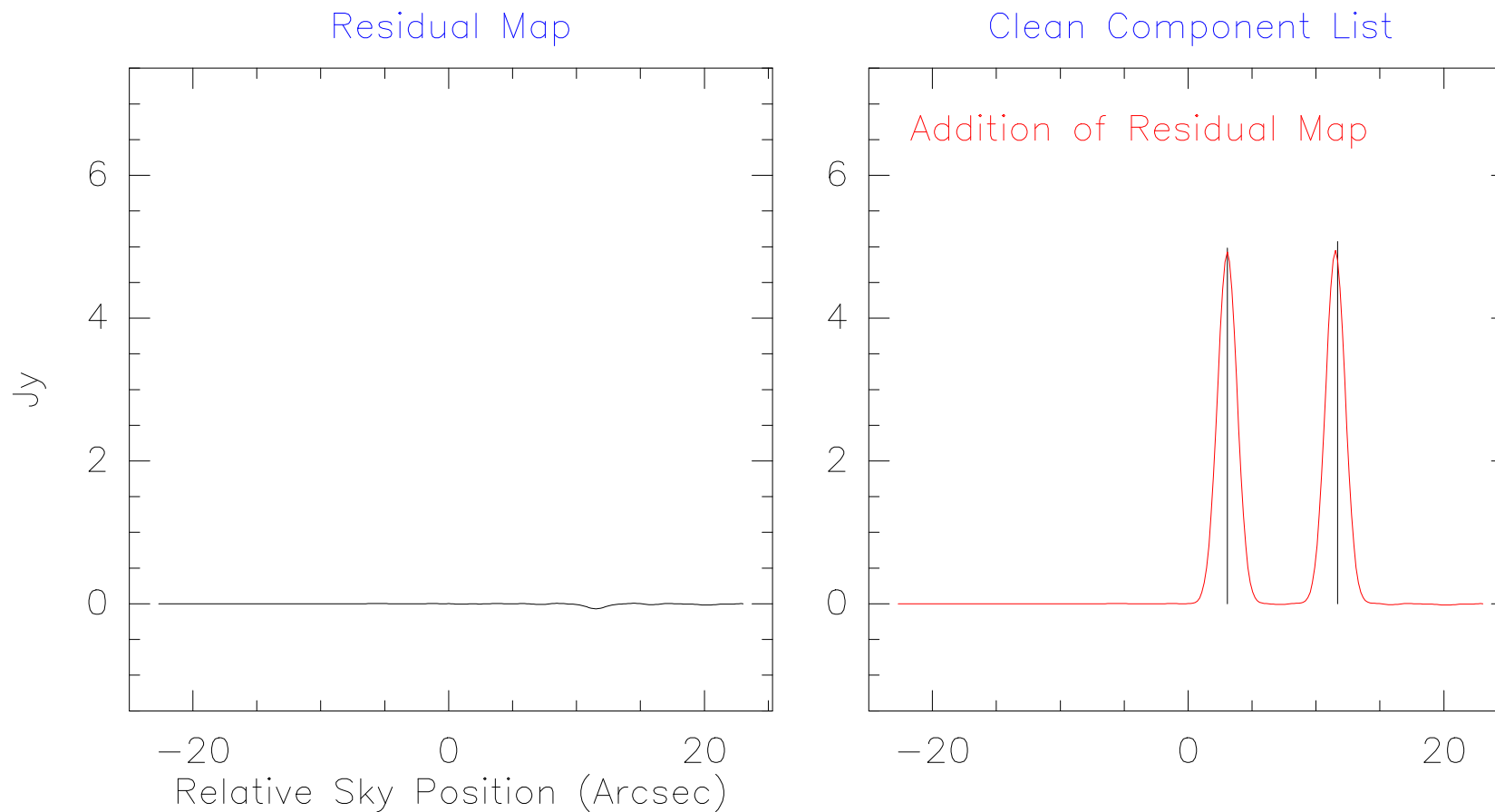
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

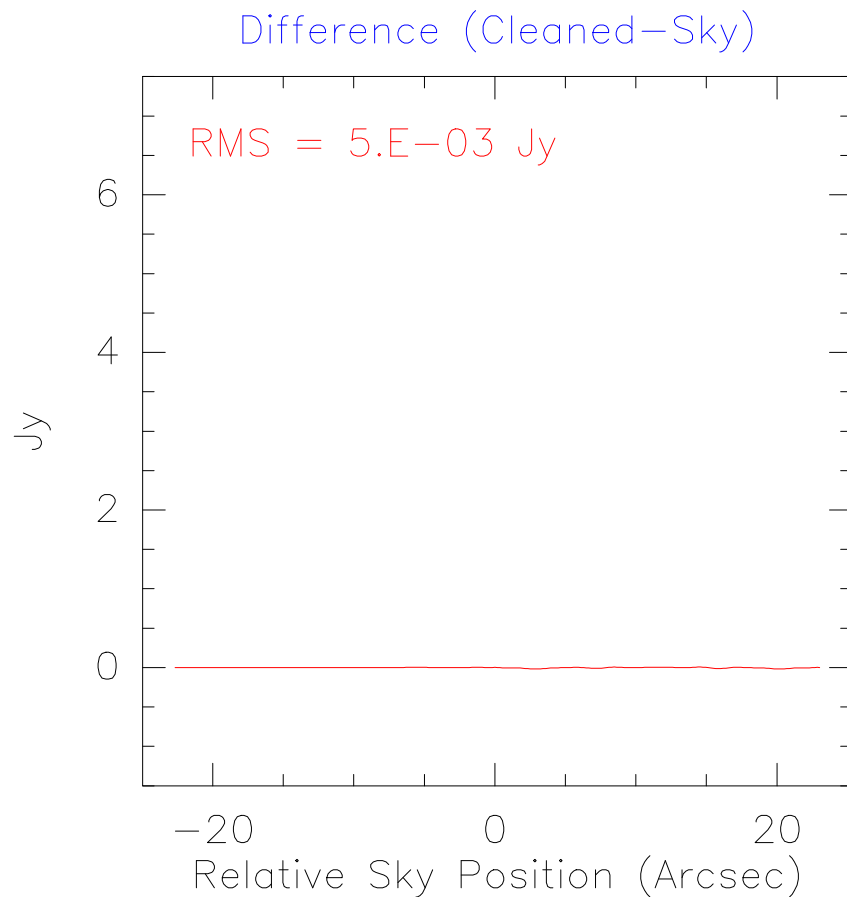
Negative clean components are mandatory.



# Deconvolution: III. The Basic Clean Algorithm

## 3. Little Secrets

Negative clean components are mandatory.



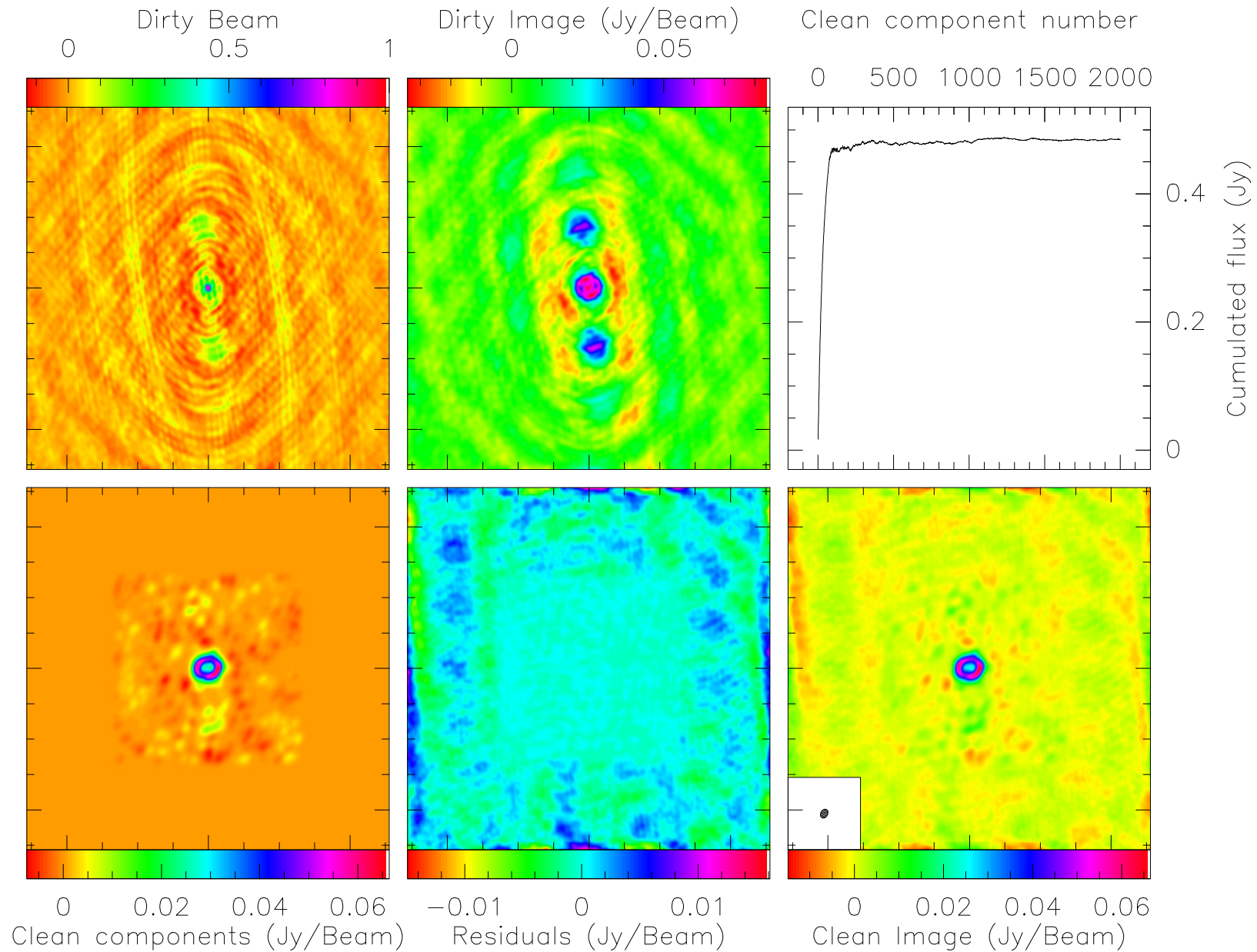
## Deconvolution: III. The Basic Clean Algorithm

### 4. Other Little Secrets

- Stopping criteria:
  - Total number of Clean components;
  - $|I_{\max}| < \text{fraction of noise (when noise limited)}$ ;
  - $|I_{\max}| < \text{fraction of dirty map max (when dynamic limited)}$ .
- Loop gain: Good results when  $\gamma \sim 0.1 - 0.3$ .
- Cleaned region: Only the inner quarter of the dirty image.
- Support: Definition of a region where CLEAN components are searched.
  - *A priori* information  $\Rightarrow$  Help CLEAN convergence.
  - But *bias* if support excludes signal regions  
 $\Rightarrow$  Be wise!

# Deconvolution: III. The Basic Clean Algorithm

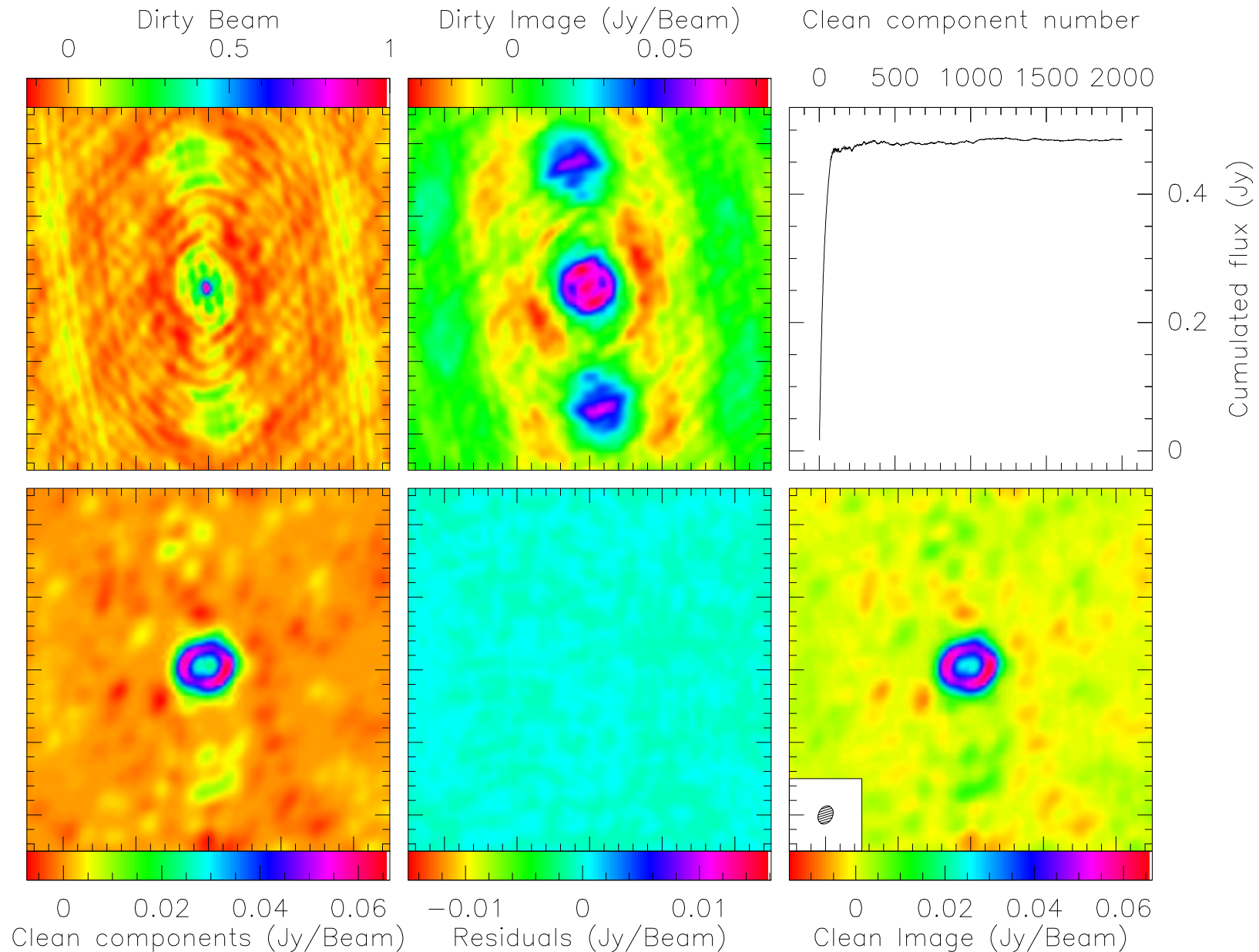
## 5. A True Example **without** support





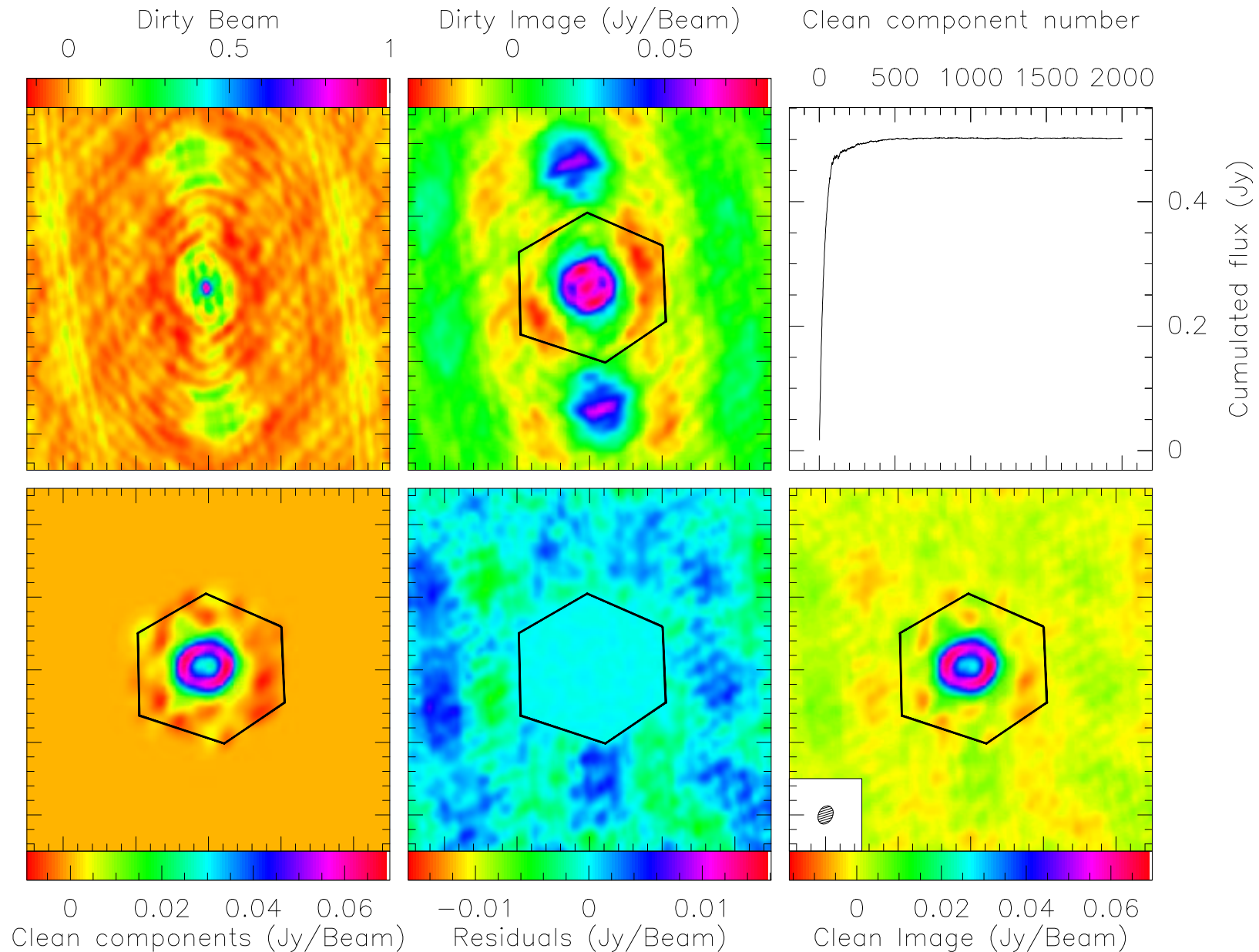
# Deconvolution: III. The Basic Clean Algorithm

## 5. A True Example without support (zoom)



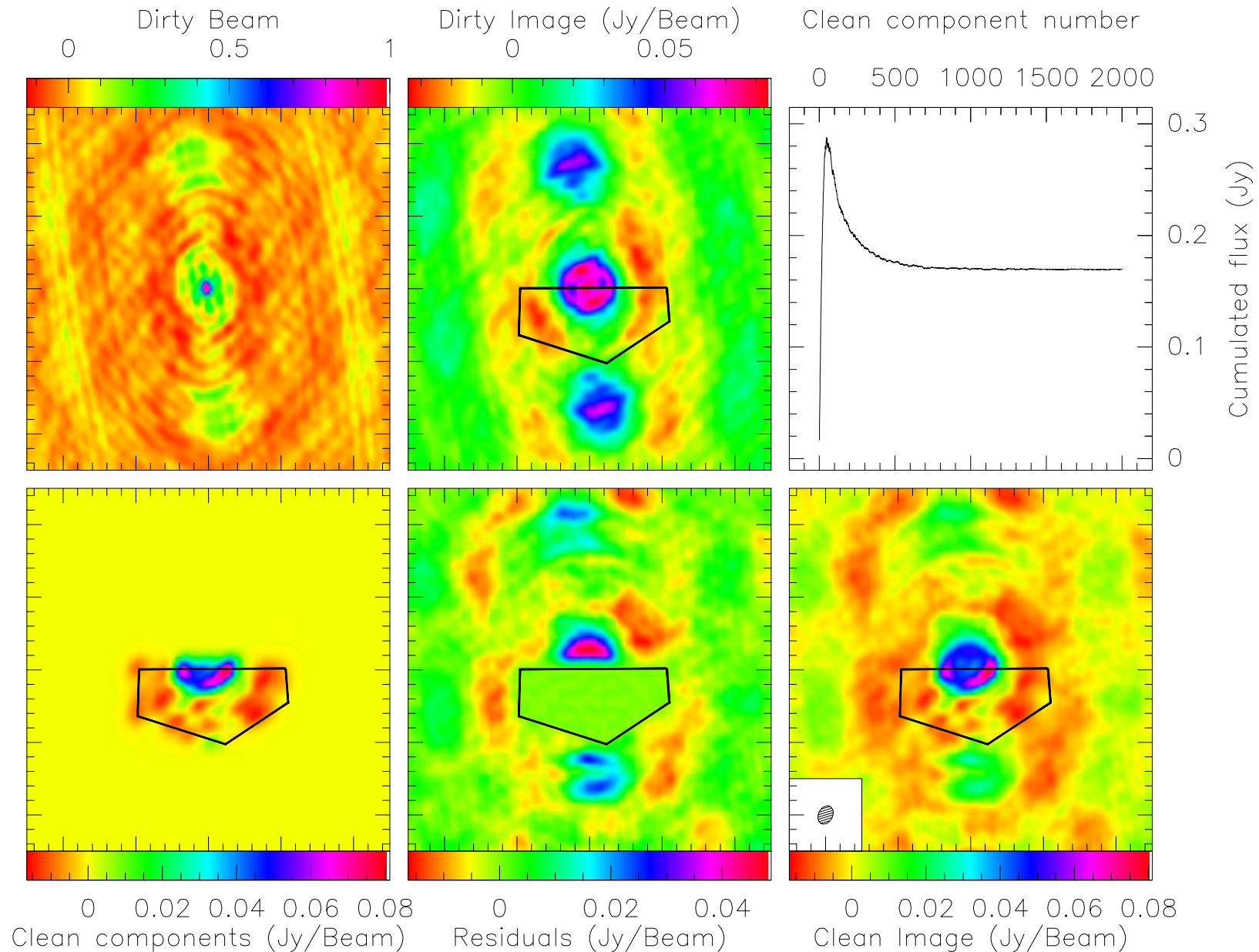
# Deconvolution: III. The Basic Clean Algorithm

## 5. A True Example with **right** support



# Deconvolution: III. The Basic Clean Algorithm

## 5. A True Example with **wrong** support



## Deconvolution: IV. CLEAN Variants

Basic:

- HOGBOM (Hogböm 1974)  
Robust but slow.

Faster Search Algorithms:

- CLARK (Clark 1980)  
Fast but unstable (when sidelobes are high).
- MX (Cotton & Schwab 1984)  
Better accuracy (Source removal in the  $uv$  plane), but slower (gridding steps repeated).

## Deconvolution: IV. CLEAN Variants (continued)

Better Handling of Extended Sources:

- SDI (Steer, Dewdney, Ito 1984)  
Created to minimize stripes.
- MRC (Multi-Resolution Clean by Wakker & Schwarz 1988)  
Simple multi-resolution approach.
- MULTI (Multi-Scale Clean by Cornwell 1998)  
Complex multi-resolution approach.

In practice: Exotic use at PdBI!

# Deconvolution: IV. GILDAS Implementation

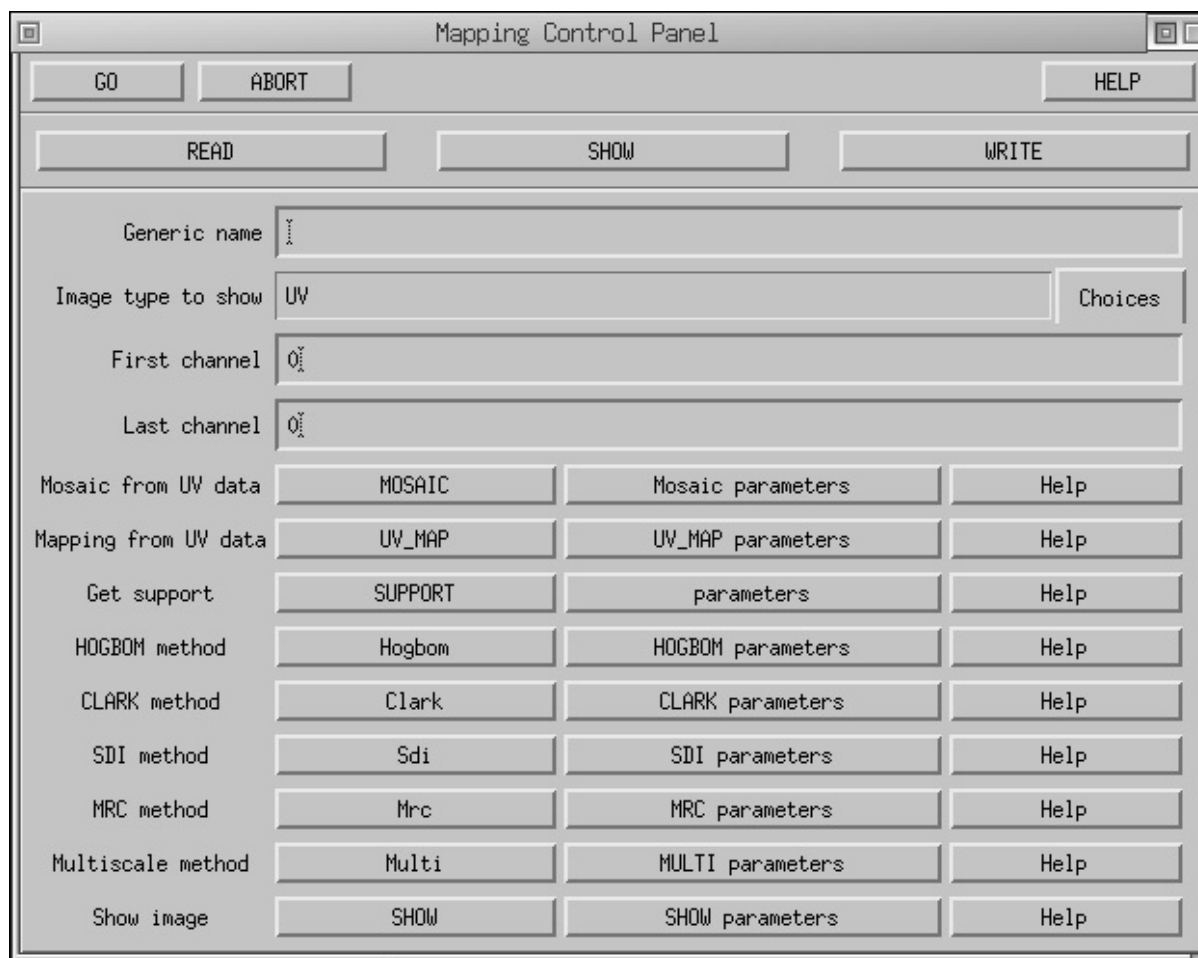
## 1. MAPPING Command Line

MAPPING> INPUT CLEAN: display CLEAN control parameters.

- Loop gain: **GAIN** (default: 0.2).
- Stopping criteria:
  - Maximum number of iterations: **NITER** (default: 100).
  - Maximum residual amplitude, expressed as fraction of peak intensity: **FRES** (default: 0.05).
  - Maximum residual amplitude, in map units (Jy/Beam): **ARES** (default: 0).
- Support: **BLC** and **TRC** are the Bottom Left Corner and Top Right Corner of a square support in pixel units (default: 0, *i.e.* no support).
- Gaussian clean beam parameters: **MAJOR**, **MINOR**, **ANGLE** (Default: 0, *i.e.* Fit dirty beam).

# Deconvolution: IV. GILDAS Implementation

## 2. MAPPING Window Interface



## Deconvolution: IV. Recommended Practices

- Method: Start with CLARK and turn to HOGBOOM in case of problems.
- Support:
  - Start without one.
  - Define one on your first clean image if really needed (*i.e.* difficulties of convergence).
- Stopping criterion:
  - Use a large enough number of iterations to ensure convergence.
  - Clean down to the noise level unless a very strong source is present.
- Misc: Consult an expert until you become one.



## Typical MAPPING session

Look at the `mapping-tutorial.map` procedure for details.

```
MAPPING> read uv 1mm
MAPPING> show uv
MAPPING> input uv_map
MAPPING> uv_map
MAPPING> show beam
MAPPING> show dirty 1mm
MAPPING> write beam 1mm
MAPPING> write dirty
MAPPING> input clean
MAPPING> hogbom /flux 0 1
MAPPING> show clean
MAPPING> show residual
MAPPING> write clean 1mm
MAPPING> write residual 1mm
MAPPING> exit
```

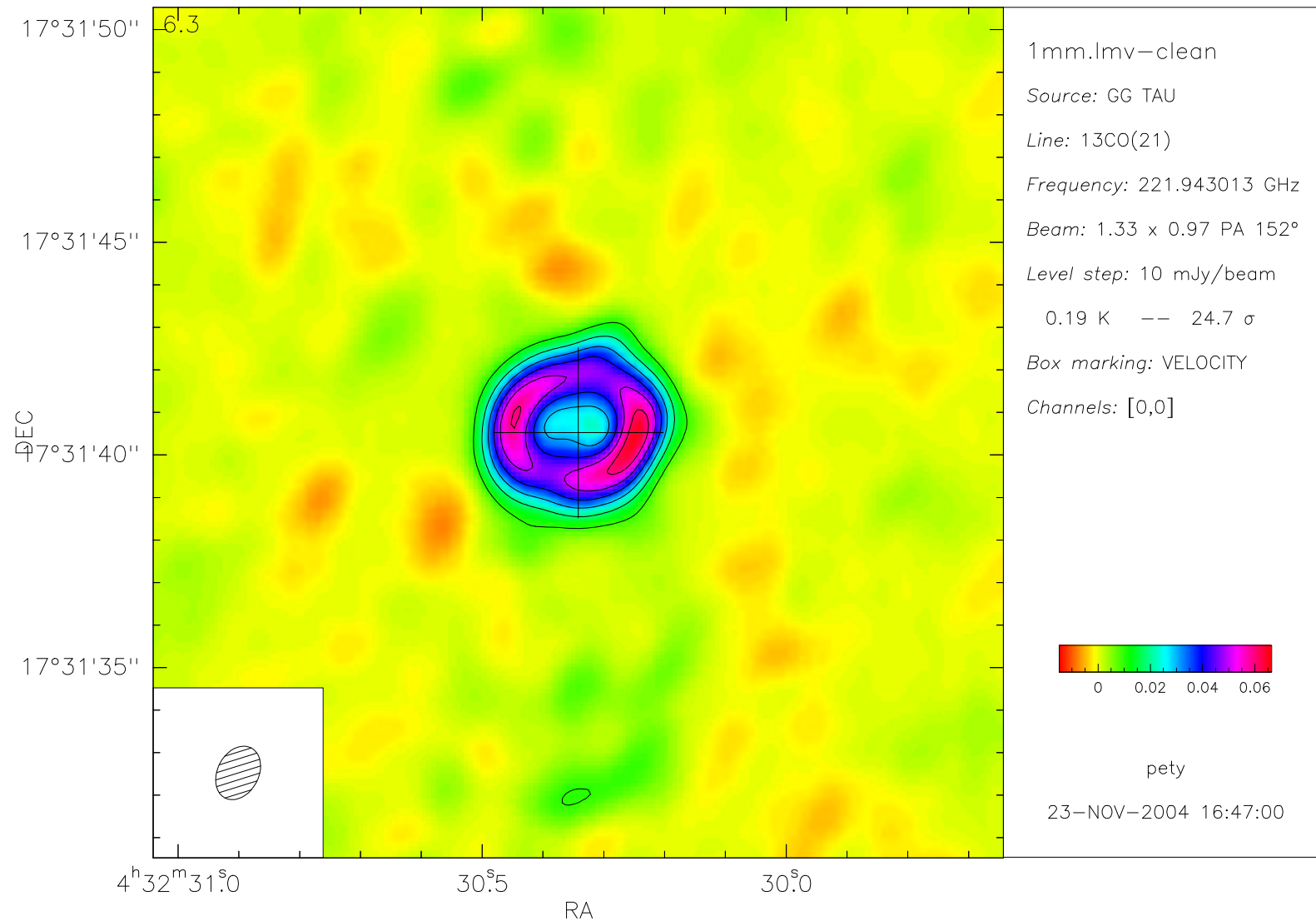
# Visualization and Image Analysis

Fourier Transform and Deconvolution:  
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
↓ Fourier Transform	UV_STAT, UV_MAP
Dirty beam & image	
↓ Deconvolution	HOGBOM, CLARK
Clean beam & image	
↓ Visualization	GO_BIT, GO_VIEW
↓ Image analysis	GO_NOISE, GO_FLUX, GO_MOMENTS
Physical information on your source	

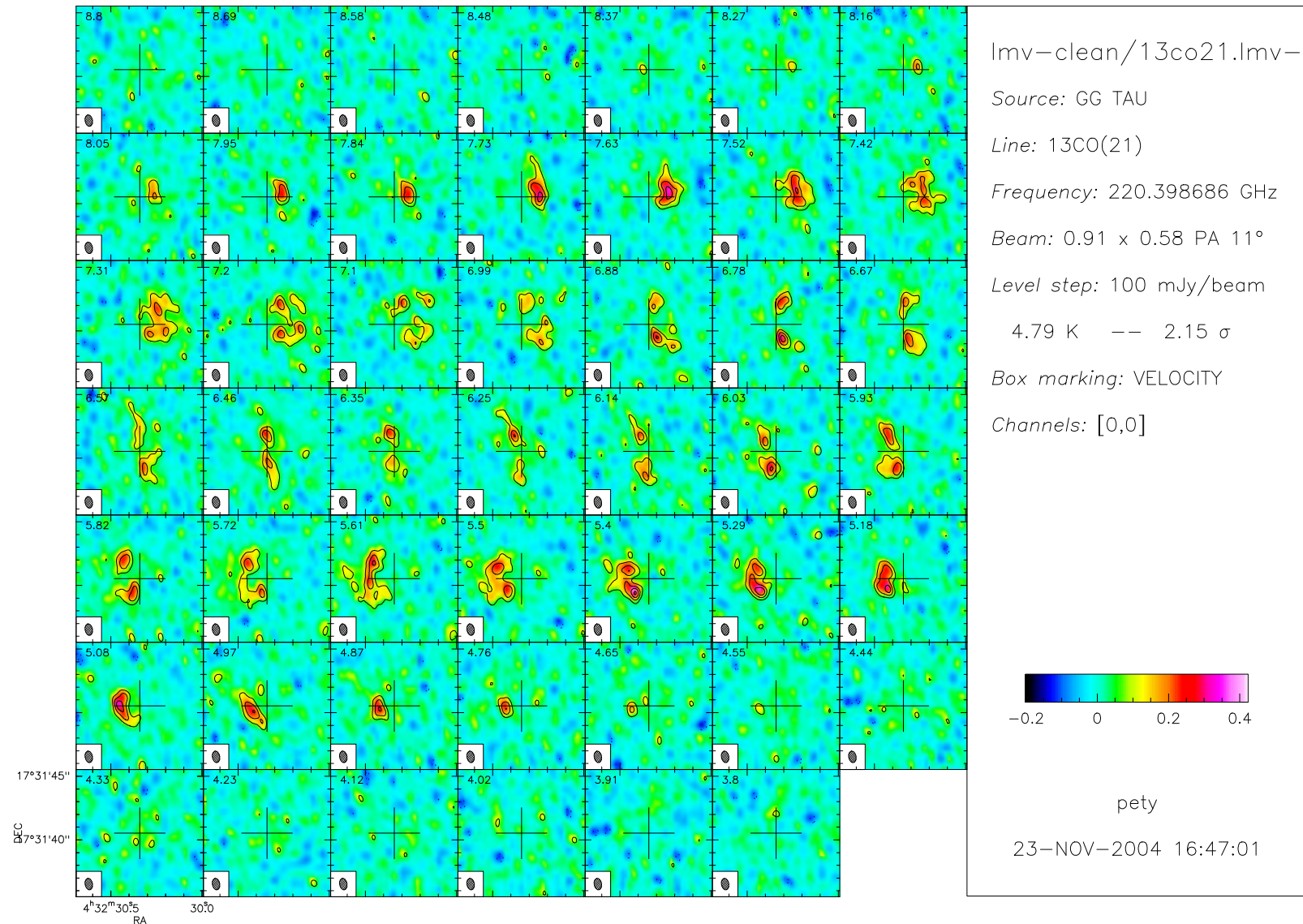
## Visualization: I. GO BIT for continuum

```
MAPPING> let name 1mm  
MAPPING> let type lmv-clean  
MAPPING> go bit  
MAPPING> input bit
```



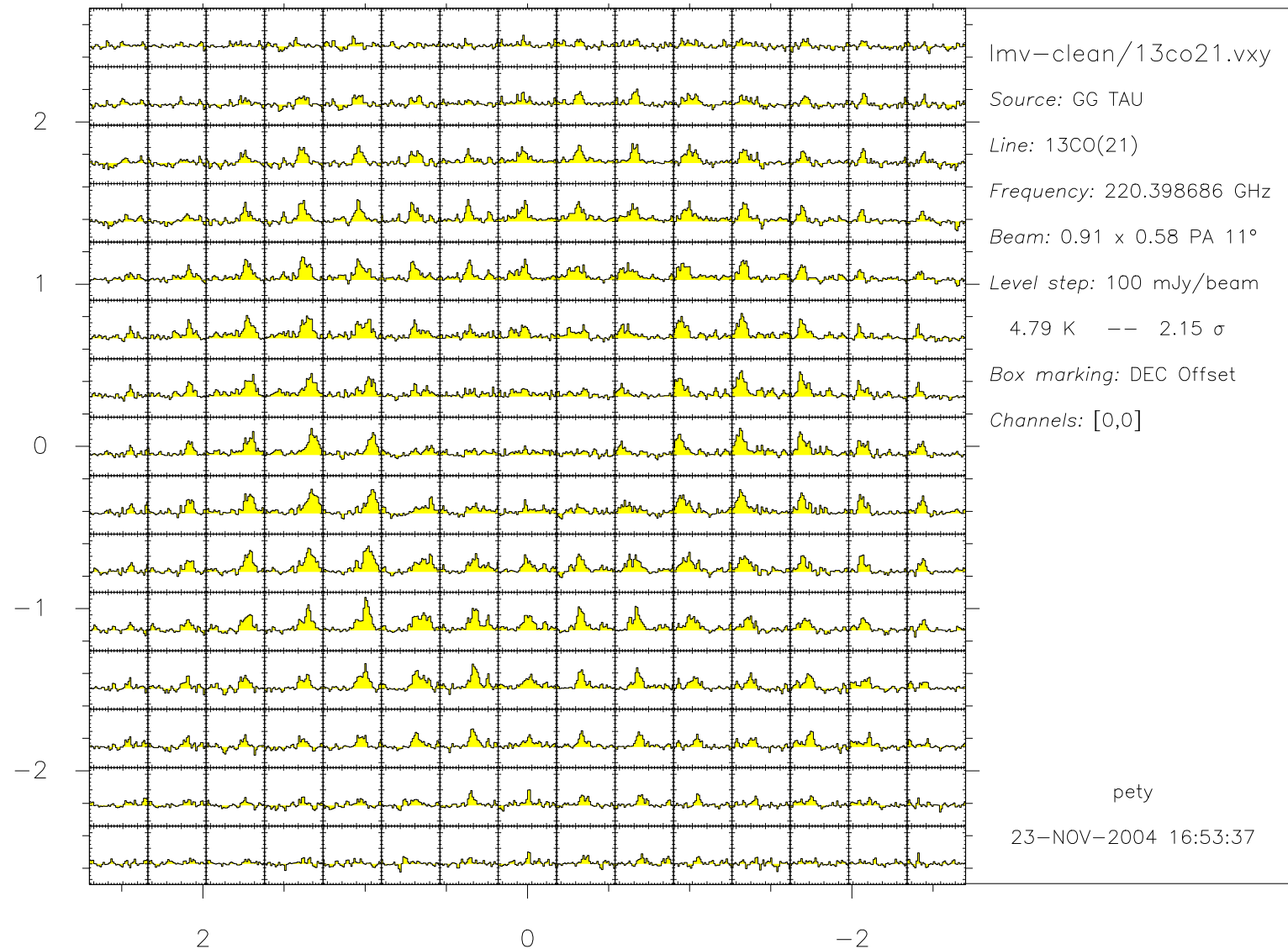
## Visualization: II. GO BIT for spectra

```
MAPPING> let name 13co21  
MAPPING> let type lmv-clean  
MAPPING> go bit
```



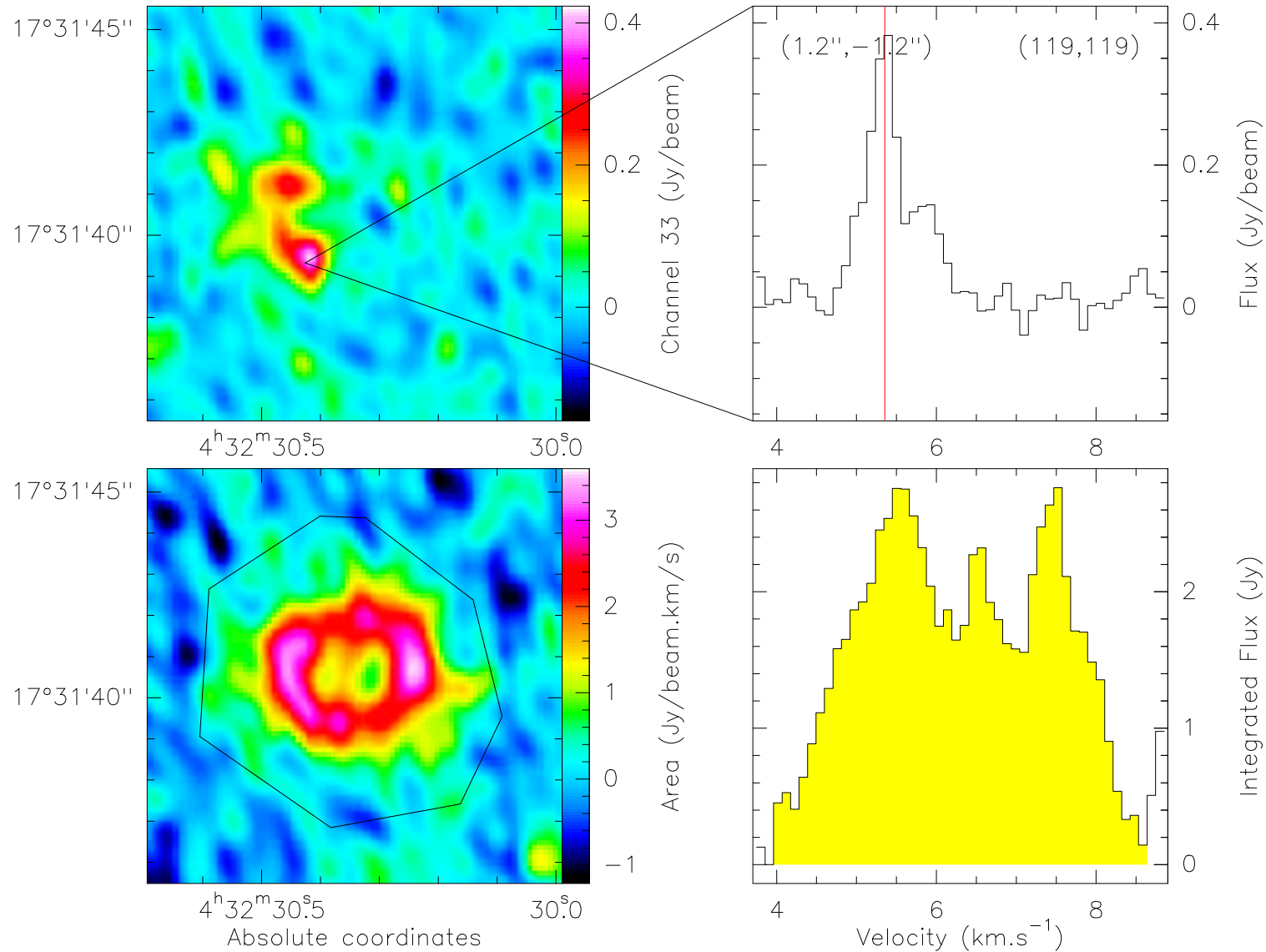
## Visualization: III. GO SPECTRUM for spectra

```
MAPPING> let name 13co21  
MAPPING> let type lmv-clean  
MAPPING> go spectrum
```



## Visualization: IV. GO VIEW for spectra

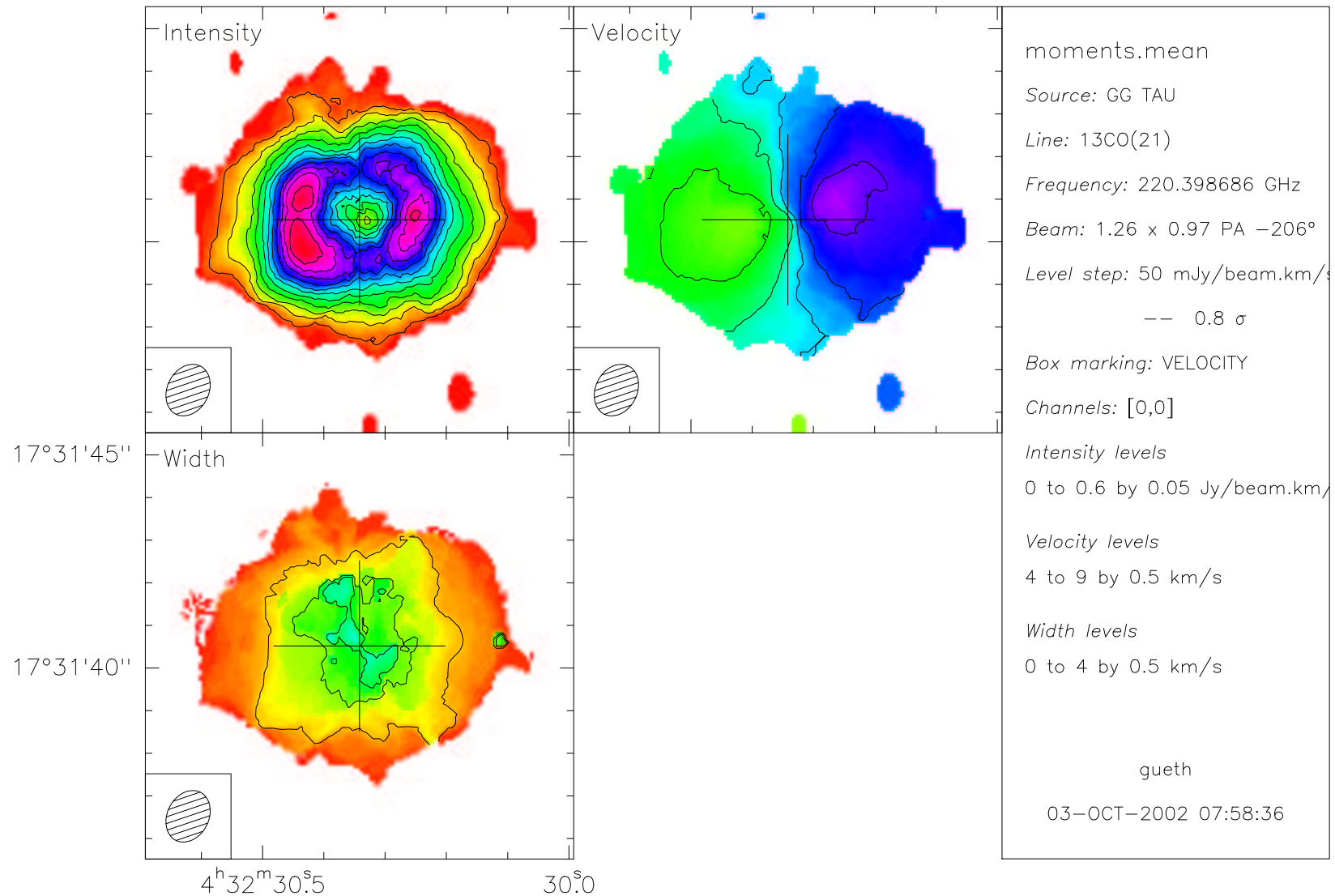
```
MAPPING> let name 13co21  
MAPPING> let type lmv-clean  
MAPPING> go view
```





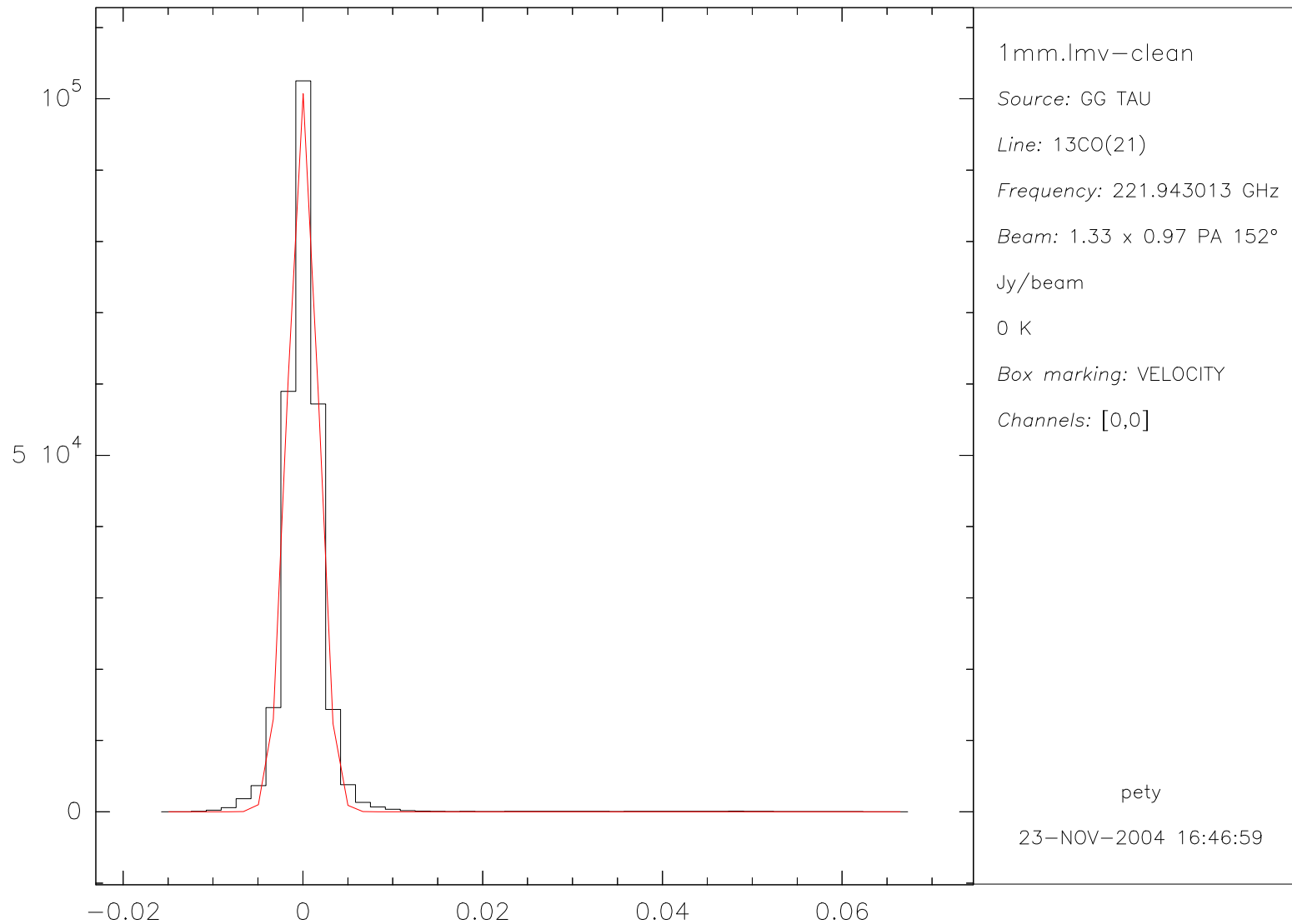
# Analysis: I. Moment Estimation with GO MOMENT + GO VELOCITY

```
MAPPING> let name 13co21  
MAPPING> let type lmv-clean  
MAPPING> go moments  
MAPPING> go velocity
```



## Analysis: II. Noise Estimation with GO NOISE

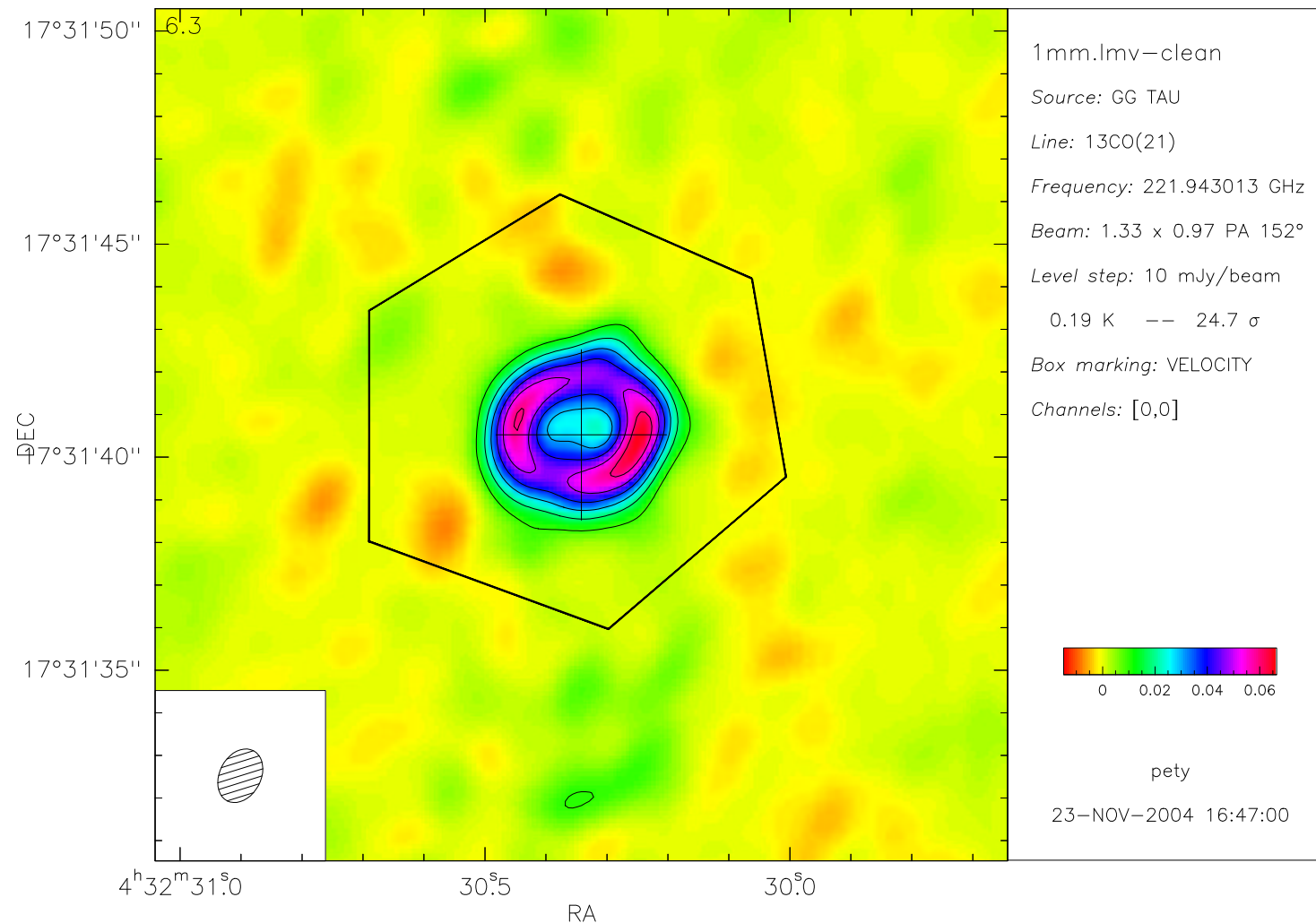
```
MAPPING> let name 1mm  
MAPPING> let type lmv-clean  
MAPPING> go noise  
NOISE = 1.6312926E-03 ! Real GLOBAL
```





## Analysis: III. Flux Estimation with GO FLUX

```
MAPPING> let name 1mm  
MAPPING> let type lmv-clean  
MAPPING> go bit  
MAPPING> go flux  
Total Flux 461 mJy
```



## Photometry: I Generalities

- Brightness = Intensity (e.g.  $\text{Power} = I_\nu(\alpha, \beta) dA d\Omega d\nu$ )
- Flux unit:  $1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ .
- Source flux measured by a single-dish antenna:  
 $F_\nu = B * I_\nu$  with  $B$  the antenna beam.
- Relationship between measured flux and temperature scales:  
 $T_A = \frac{\lambda^2}{2k\Omega_A} F_\nu$ ,  $T_A^* = \frac{\lambda^2}{2k \cdot (2\pi)} F_\nu$  and  $T_{mb} = \frac{\lambda^2}{2k\Omega_{mb}} F_\nu$  because
  - $P_\nu = \frac{1}{2} A_e F_\nu$  Power detected by the single-dish antenna.
  - $P'_\nu = kT$  Power emitted by a resistor at temperature  $T$ .
  - $P_\nu = P'_\nu \Rightarrow T_A = \frac{A_e}{2k} F_\nu$ .
  - $\lambda^2 = A_e \Omega_A$  (diffraction).
  - $2\pi = F_{\text{eff}} \Omega_A$  or  $F_{\text{eff}} = \frac{\text{Forward beam}}{\text{Total beam}}$ .
  - $\Omega_{mb} = B_{\text{eff}} \Omega_A$  or  $B_{\text{eff}} = \frac{\text{Main beam}}{\text{Total beam}}$ .

## Photometry: II Visibilities

Visibility unit: **Jy** because:

$$\begin{aligned} V &= 2D \text{ FT } \{ B_{\text{primary}} \cdot I_{\text{source}} \} \\ &= \iint B_{\text{primary}}(\sigma) \cdot I_{\text{source}}(\sigma) \exp(-i2\pi \mathbf{b} \cdot \sigma / c) d\Omega. \end{aligned}$$

Effect of flux calibration errors on your image:

- Multiplicative factor if uniform in  $uv$  plane.
- Convolution (*i.e.* distortion) else.

## Photometry: III Dirty map

III-defined because:

- $S(u = 0, v = 0) = 0 \Rightarrow$  Area of the dirty beam is 0!
- $V(u = 0, v = 0) = 0 \Rightarrow$  Total flux of the dirty image is 0!  
 $\Rightarrow$  A source of constant intensity will be fully filtered out.
- A single point source of 1 Jy appears with peak intensity of 1.
- Several close-by point sources of 1 Jy appears with peak intensities different of 1.

## Photometry: IV Clean map

(my dream: Don't take it seriously)

$I_{\text{clean}} = \frac{1}{\Omega_{\text{clean}}} (B_{\text{clean}} * I_{\text{point}})$ : *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* clean beam).

Unit: Jy/sr

Consequences:

- Source flux computation by integration inside a support:

$$\text{Flux} = \sum_{ij \in \mathcal{S}} I_{\text{clean}} d\Omega$$

[Jy]                      [Jy/sr] [sr]

with  $d\Omega$  the image pixel surface.

- From Brightness to temperature:  $T_{\text{clean}} = \frac{\lambda^2}{2k} I_{\text{clean}}$

## Photometry: IV Clean map (**reality**)

$I_{\text{clean}} = B_{\text{clean}} * I_{\text{point}}$ : *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* clean beam).

Unit: Jy/**beam** with 1 beam =  $\Omega_{\text{clean}}$  sr.

Consequences:

- Source flux computation by integration inside a support:

$$\begin{array}{ccccc} \text{Flux} = & \sum_{ij \in \mathcal{S}} & I_{\text{clean}} & \cdot & \frac{d\Omega}{\Omega_{\text{clean}}} \\ & & [\text{Jy}] & & [\text{Jy/beam}] \quad [\text{beam}] \end{array}$$

with  $\frac{d\Omega}{\Omega_{\text{clean}}}$  the nb of beams in the surface of an image pixel.

- From Brightness to temperature:  $T_{\text{clean}} = \frac{\lambda^2}{2k\Omega_{\text{clean}}} I_{\text{clean}}$

## Photometry: IV Clean map

Consequences of a **Gaussian** clean beam shape:

- No error beams, no secondary beams.
- $T_{\text{clean}}$  is a main beam temperature.

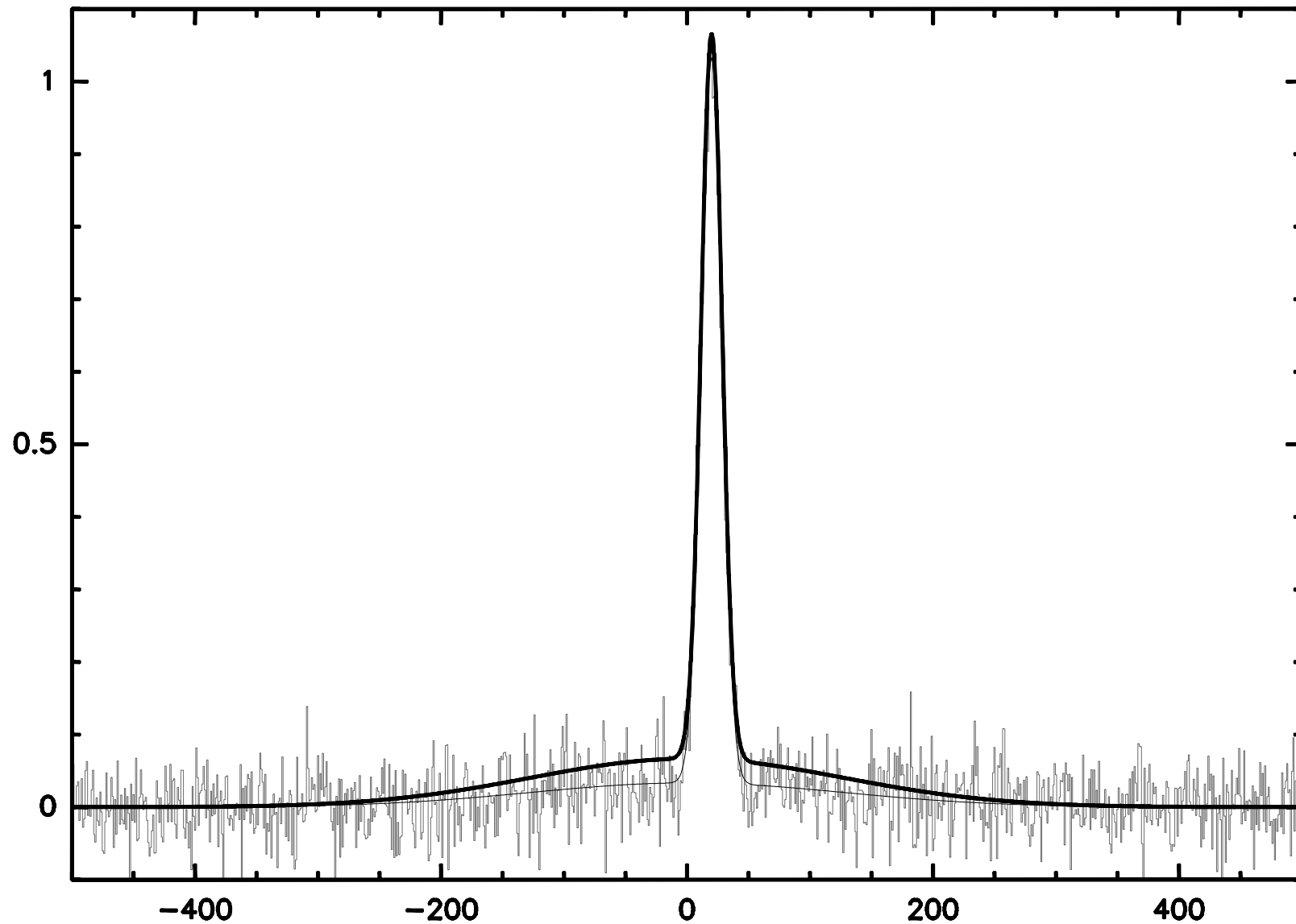
Natural choice of clean beam size: Synthesized beam size  
(i.e. fit of the central peak of the dirty beam).

⇒ Minimize unit problems when adding the dirty map residuals.

Caveats of flux measurements:

- **CLEAN does not conserve flux**  
(i.e. CLEAN extrapolates unmeasured short spacings).
- **Large scales are filtered out** (source size  $> 1/3$  primary beam size ⇒ need of short spacings, cf. lecture by F. Gueth).
- $I_{\text{clean}} = B_{\text{primary}} \cdot I_{\text{source}} + N$   
⇒ **Primary beam correction** may be needed:  
$$I_{\text{clean}}/B_{\text{primary}} = I_{\text{source}} + N/B_{\text{primary}} \Rightarrow \text{Varying noise!}$$
- **Seeing scatters flux.**

## Photometry: $V$ Importance of Extended, Low Level Intensity





**Writing the Paper: Your job!**

# Mathematical Properties of Fourier Transform

- 1 Fourier Transform of a product of two functions  
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \xLeftrightarrow{\text{FT}} \tilde{F}_1 \text{ and } F_2 \xLeftrightarrow{\text{FT}} \tilde{F}_2), \text{ then } F_1 \cdot F_2 \xLeftrightarrow{\text{FT}} \tilde{F}_1 * \tilde{F}_2.$$

- 2 Sampling size  $\xLeftrightarrow{\text{FT}}$  Image size.

- 3 Bandwidth size  $\xLeftrightarrow{\text{FT}}$  Pixel size.

- 4 Finite support  $\xLeftrightarrow{\text{FT}}$  Infinite support.

- 5 Fourier transform evaluated at zero spacial frequency  
= Integral of your function.

$$V(u=0, v=0) \xLeftrightarrow{\text{FT}} \sum_{ij \in \text{image}} I_{ij}.$$

## Photographic Credits and References

- R. N. Bracewell, “The Fourier Transform and its Applications”.
- J. D. Kraus, “Radio Astronomy”.
- R. Narayan and R. Nityananda, Ann. Rev. Astron. Astrophys., 1986, 24, 127–170, “Maximum Entropy Image Restoration in Astronomy”