A Sightseeing Tour of mm Interferometry

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Towards Higher Resolution:
I. Problem

Telescope resolution:

- $\sim \lambda/D$;
- IRAM-30m: $\sim 11''$ @ 1 mm.

Needs to:

- increase D;
- increase precision of telescope positionning;
- keep high surface accuracy.

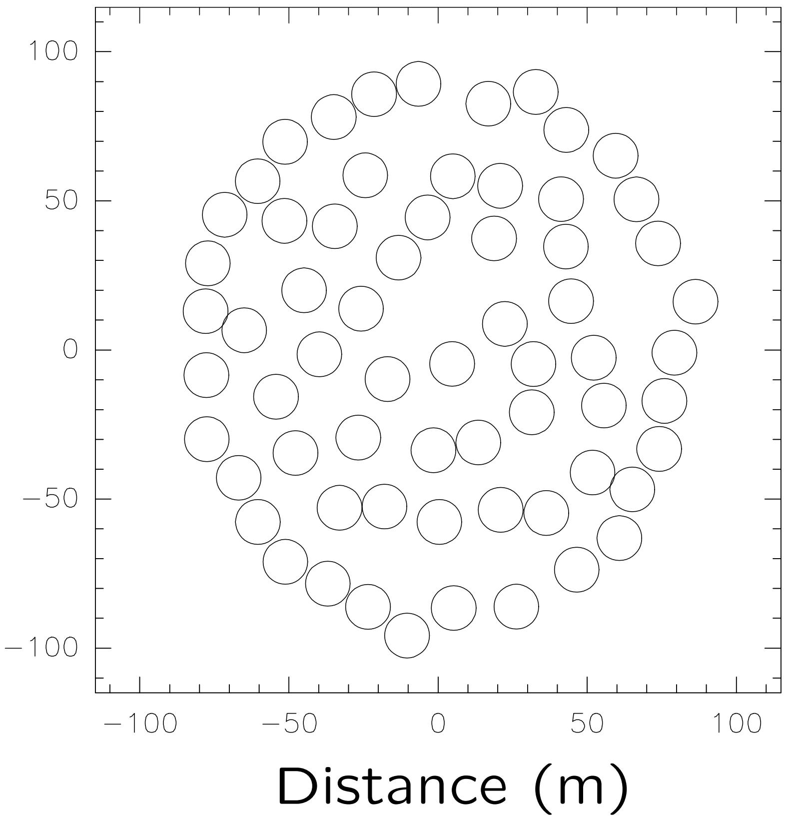
$\Rightarrow$ Technically difficult (perhaps impossible?).

Towards Higher Resolution: II. Solution

Aperture Synthesis: Replacing a single large telescope by a collection of small telescope “filling” the large one.

$\Rightarrow$ Technically difficult but feasible.

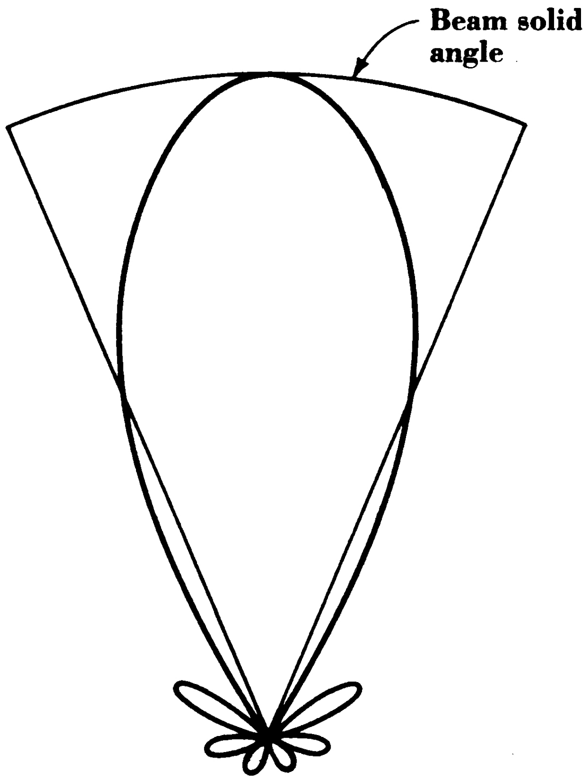
ALMA



Vocabulary and notations:

- **Baseline** Line segment between two antenna.
- b_{ij} Baseline length between antenna i and j.
- **Configuration** Antenna layout (*e.g.* compact configuration).
- D configuration size (*e.g.* 150 m).
- **Primary beam** resolution of one antenna (*e.g.* 27″ @ 1 mm).
- **Synthesized beam** resolution of the array (*e.g.* 2″ @ 1 mm).

Parenthesis: PSF = Diffraction Pattern = Beam Pattern



Single-Dish sensitivity
in polar coordinates.

Combination of:

- Antenna properties;
- Optical system (*i.e.* how the waves are feeding the receiver).

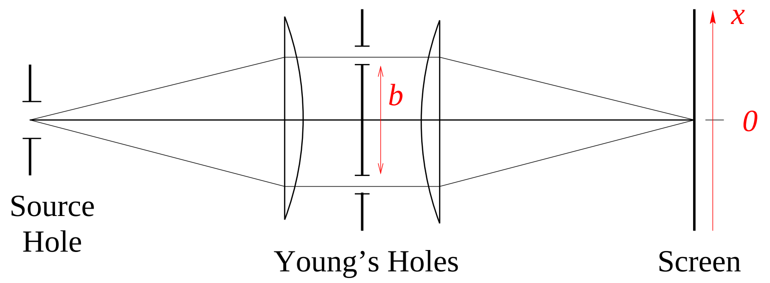
Typical kind:

Optic/IR Airy function;
Radio Gaussian function.

(Lecture by P. Hily-Blant)

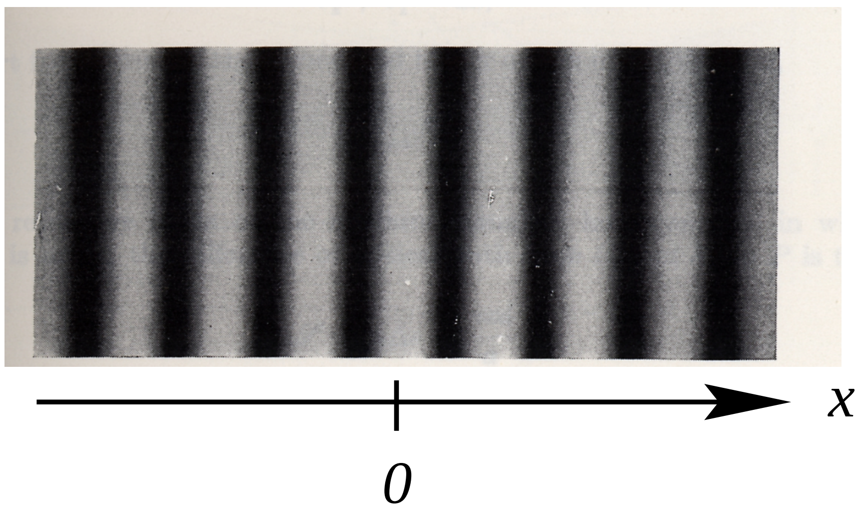
Young's Experiment

Setup



Lens ⇒ Fraunhofer conditions (*i.e.* Plane waves as if the source were placed at infinity).

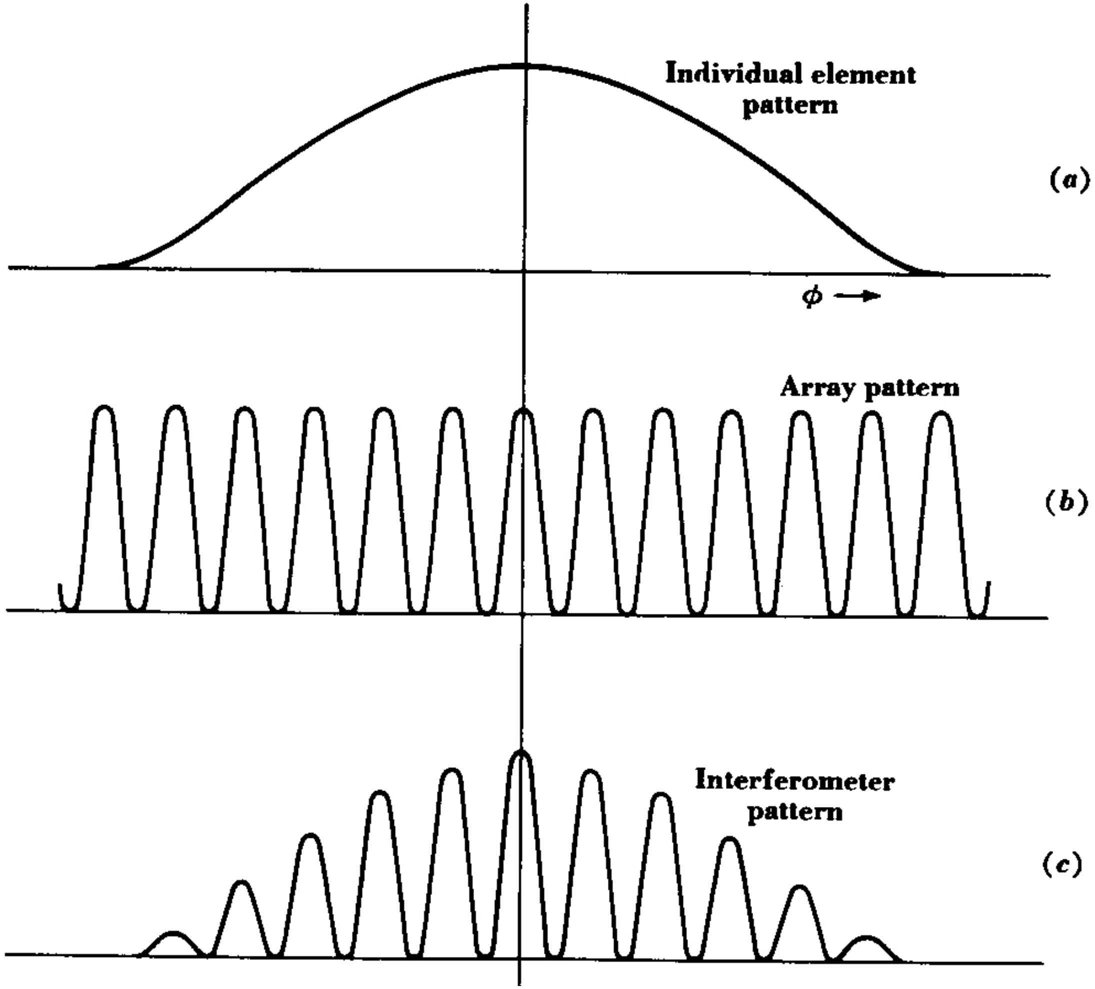
Obtained image of interference: fringes



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{bx}{\lambda}\right)$$

with
- λ Source wavelength;
- b Distance between the two Young's holes;
- x Distance from the optical center on the screen.

Effect of the Antenna Diffraction Pattern



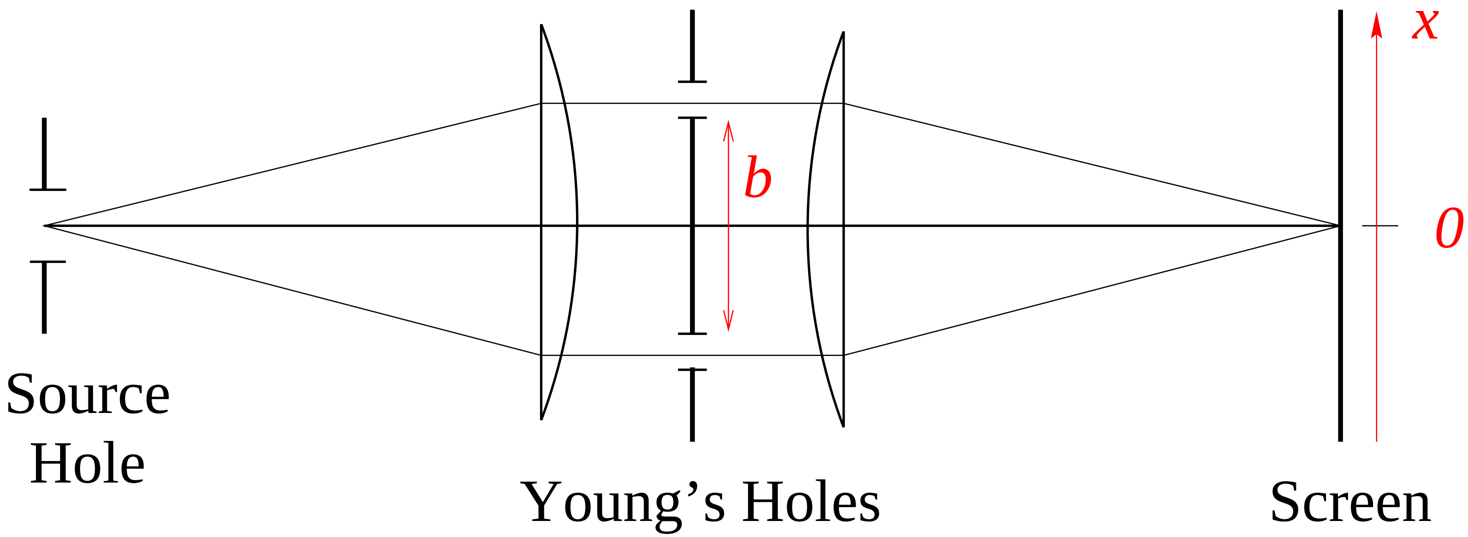
$$I(x) = B(x).\left\{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{bx}{\lambda}\right)\right\}$$

Effect of the Source Hole Size:
I. Description

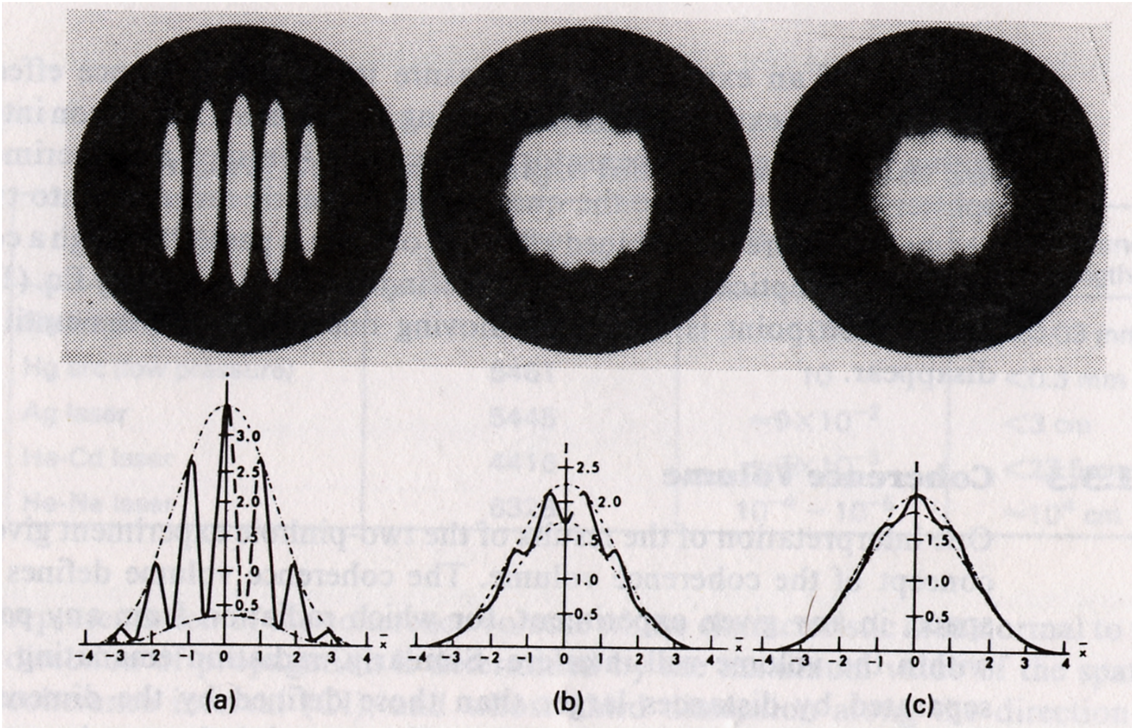
Hypothesis: Monochromatic source (but not a laser).

Description:

- The Source Hole Size is increased.
- Everything else is kept equal.



Effect of the Source Hole Size:
II. Results



Fringes disappear! $\Rightarrow$ $\begin{cases}\text{Fringe contrast is linked to the} \\ \text{spatial properties of the source.}\end{cases}$

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

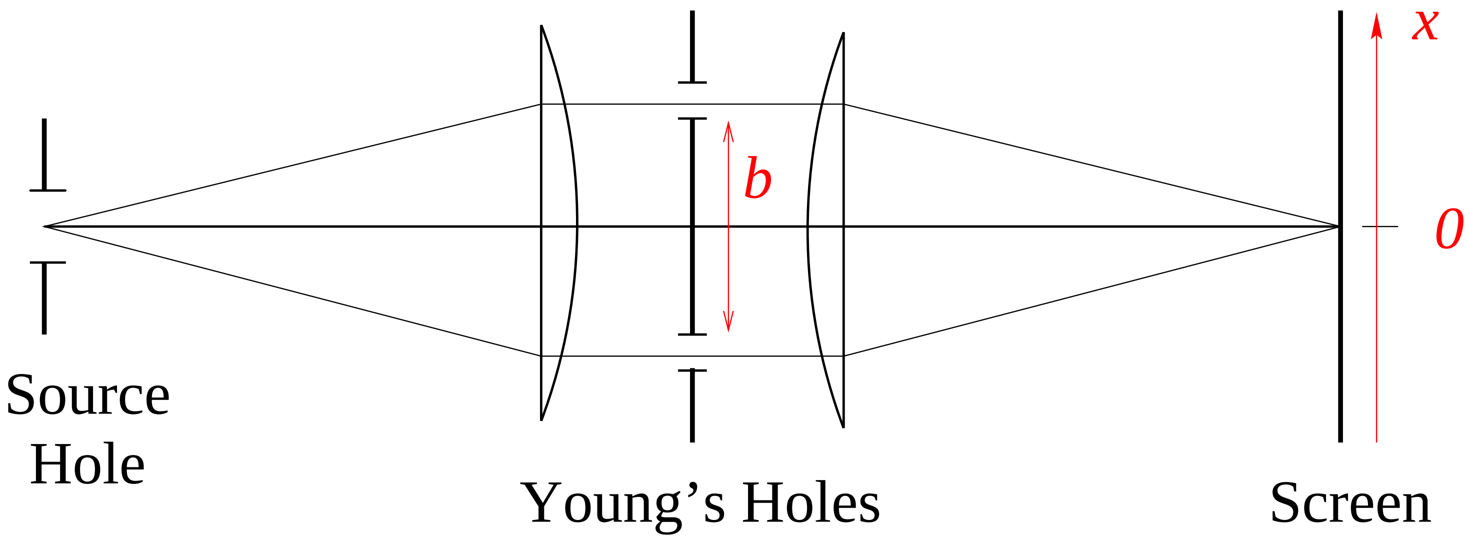
Effect of the Distance Between Young's Holes:
I. Description

Hypothesis:

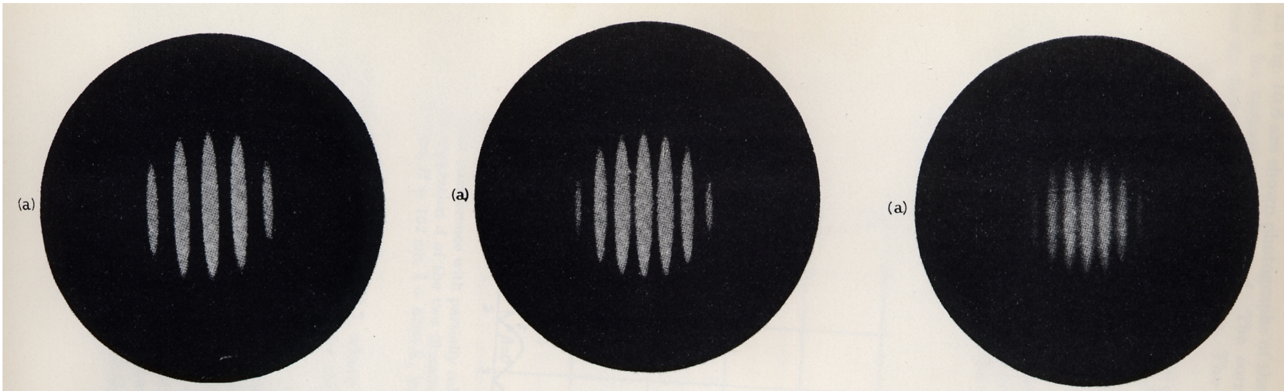
- Monochromatic source (but not a laser).
- The source hole is a circular disk.

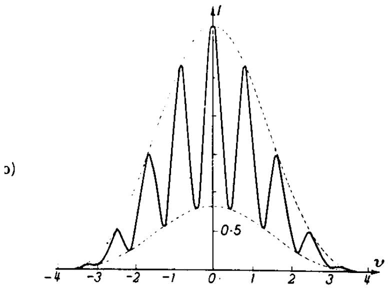
Description:

- The distance between the two Young's holes is increased.
- Everything else is kept equal (in particular the hole size).

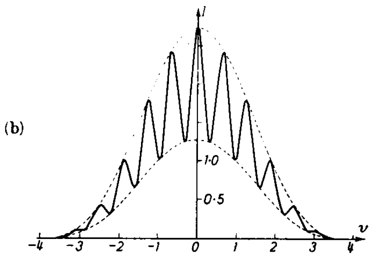


Effect of the Distance Between Young's Holes: II. Results

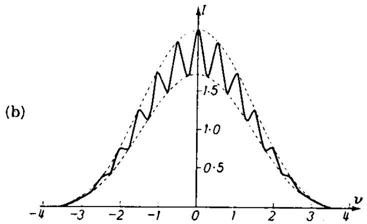




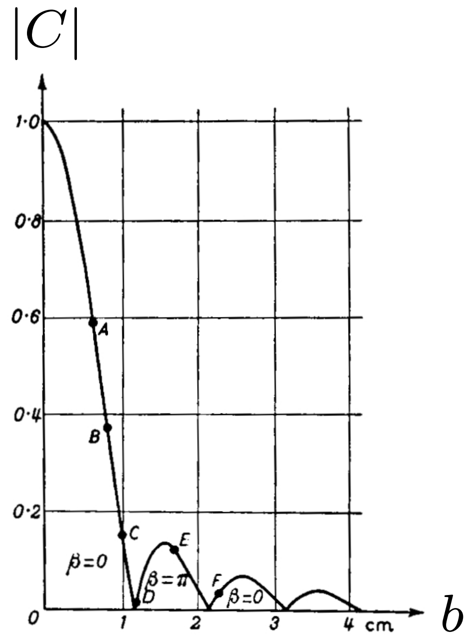
(A): $d = 0{\cdot}6$ cm
$|\mu_{12}| = 0{\cdot}593$, $\beta_{12} = 0$



(B): $d = 0{\cdot}8$ cm
$|\mu_{12}| = 0{\cdot}361$, $\beta_{12} = 0$

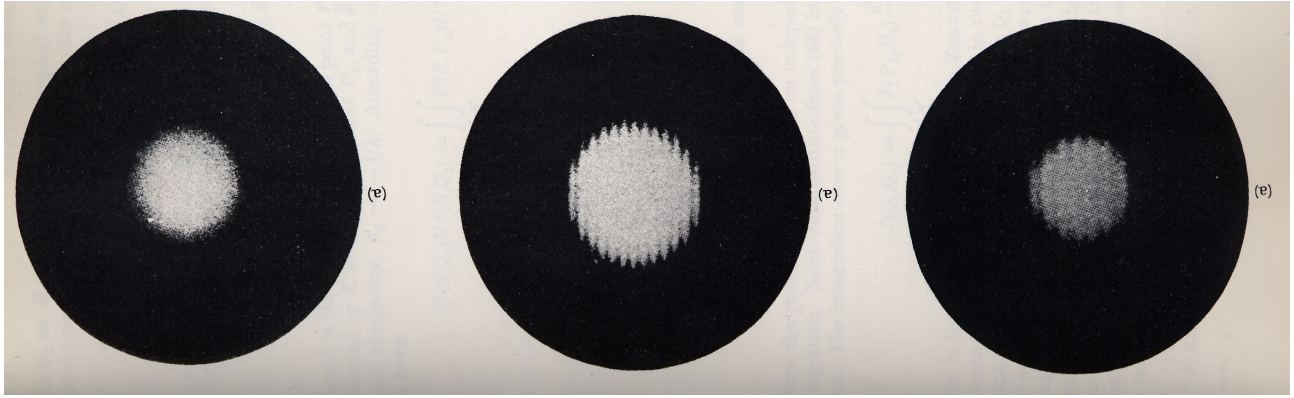


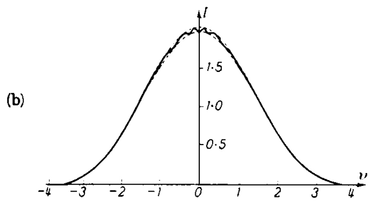
(C): $d = 1$ cm
$|\mu_{12}| = 0{\cdot}146$, $\beta_{12} = 0$



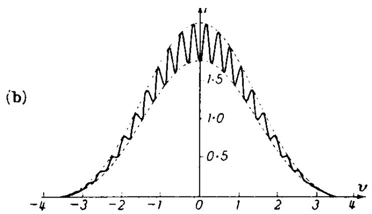
$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Effect of the Distance Between Young's Holes:
II. Results (Continued)

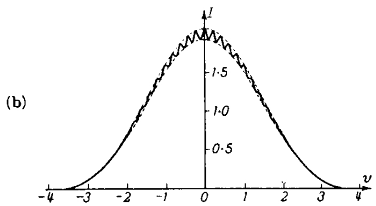




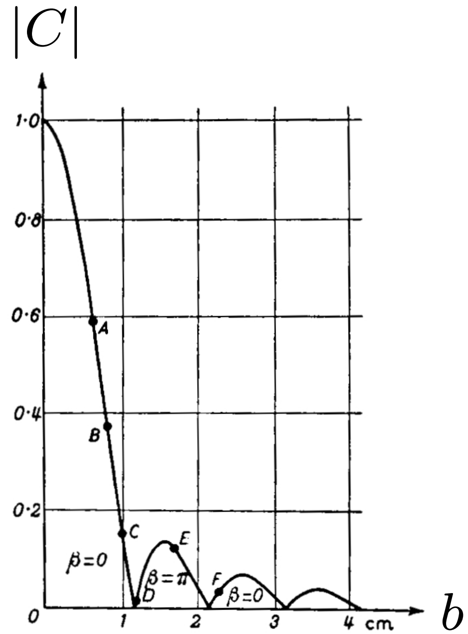
(D): d = 1·2 cm
$|\mu_{12}| = 0{\cdot}015,\ \beta_{12} = \pi$



(E): d = 1·7 cm
$|\mu_{12}| = 0{\cdot}123,\ \beta_{12} = \pi$

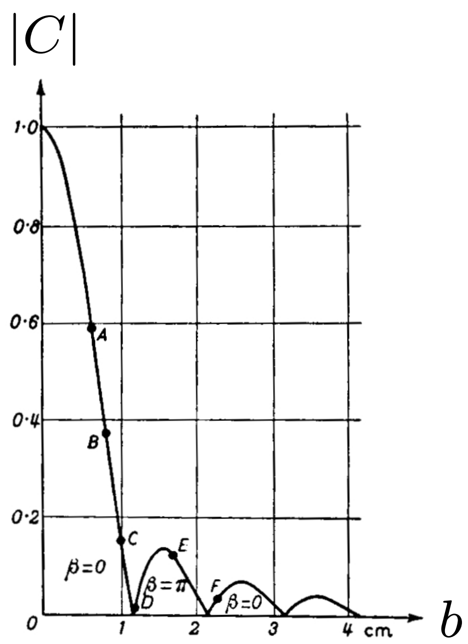


(F): d = 2·3 cm
$|\mu_{12}| = 0{\cdot}035,\ \beta_{12} = 0$

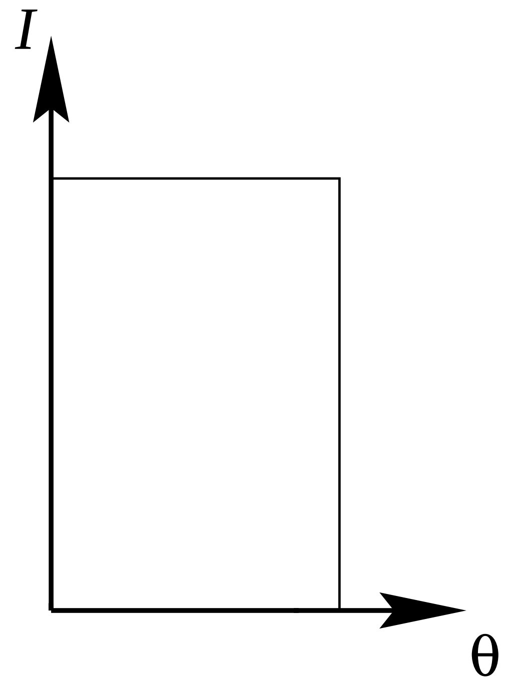


$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

Measured Curve = 2D Fourier Transform of the Source



$$\frac{J_1(b)}{b} \overset{\text{2D FT}}{\rightleftharpoons} \text{Heaviside}(\theta)$$



Source = Uniformly illuminated disk.

Theoretical Basis of the Aperture Synthesis

The van Citter-Zernike theorem

$$V_{ij}(b_{ij}) = C_{ij}(b_{ij}).I_{\text{tot}} \overset{\text{2D FT}}{\rightleftharpoons} B_{\text{primary}}.I_{\text{source}}$$

- Young's holes = Telescopes;
- Signal received by telescopes are combined by pairs;
- Fringe visibilities are measured.

⇒ One Fourier component of the source (*i.e.* one visibility) is measured by baseline (or antenna pair).

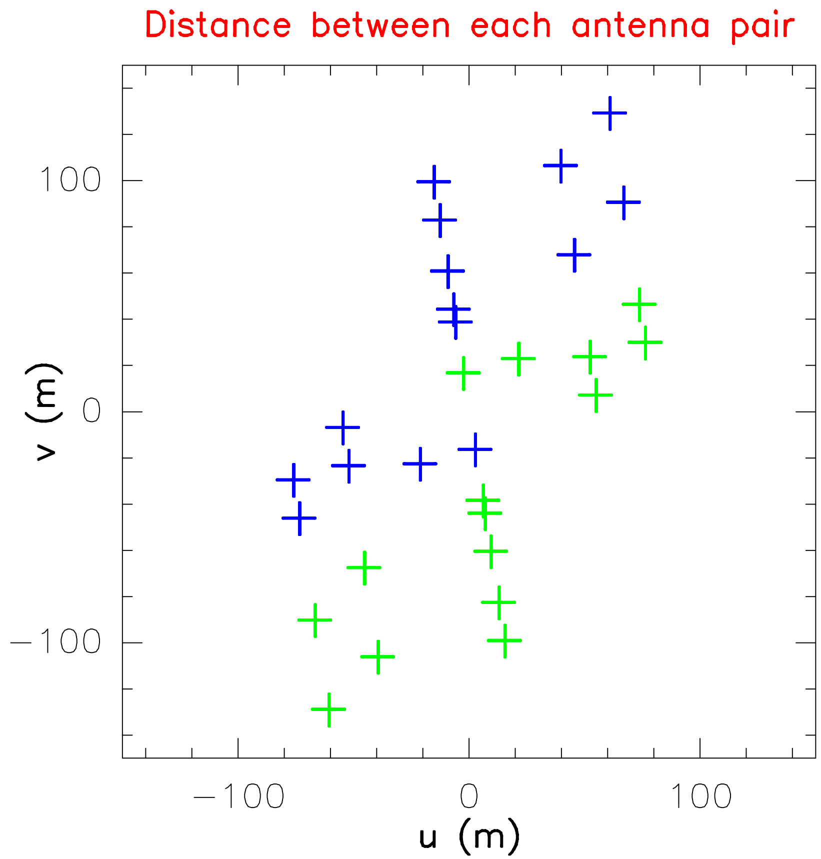
⇒ Convention: Spatial frequencies are measured in meter.

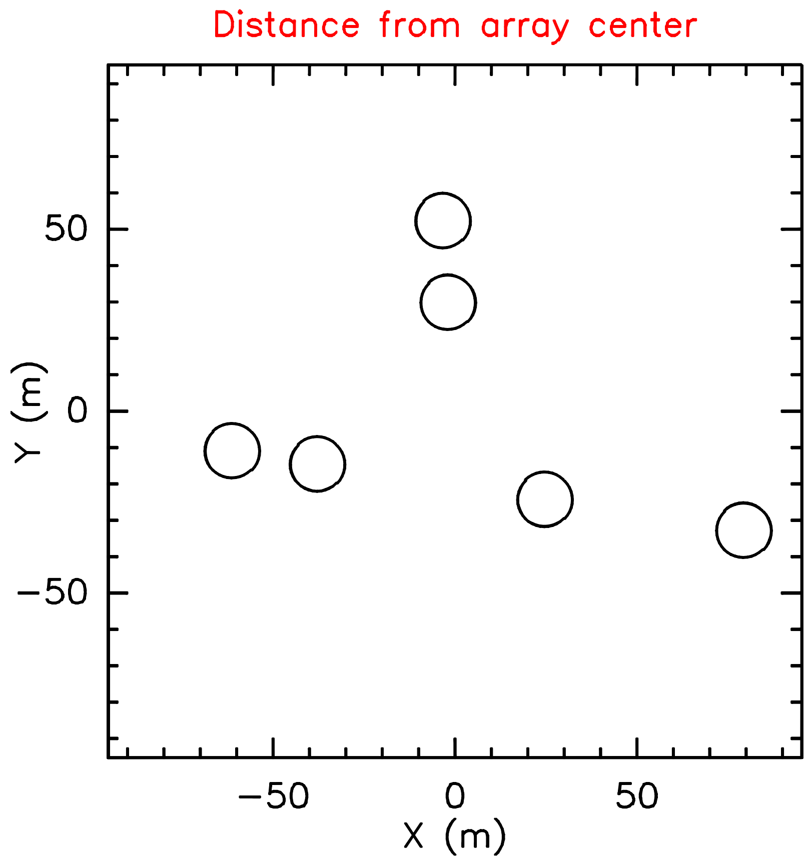
⇒ Each baseline lenght b_{ij} = a spatial frequency.

An Example: The PdBI

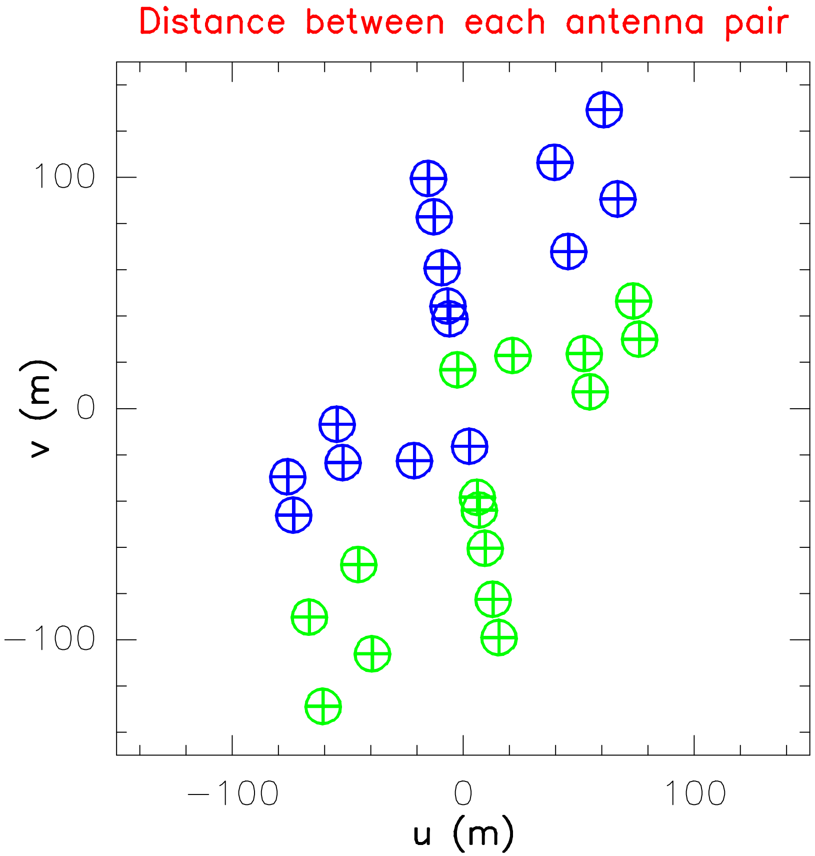
Number of baselines: $N(N-1) = 30$ for $N = 6$ antennas.

Convention: Fourier plane = uv plane.





Each Visibility is a Weighted Sum of the Fourier Components of the Source



$V_{ij}(b_{ij}) \overset{\text{2D FT}}{\rightleftharpoons} B_{\text{primary}}.I_{\text{source}}$

i.e. $V_{ij}(b_{ij}) = \left\{\tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}}\right\}(b_{ij})$

with $\tilde{B}_{\text{primary}}$ a Gaussian of FWHM=15 m.

$\Rightarrow$ Indirect information on the source (important for mosaicing).

Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_1 \text{and} F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_2), \text{ then } F_1.F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_1 * \tilde{F}_2.$$

2 Sampling size $\overset{\text{FT}}{\rightleftharpoons}$ Image size.

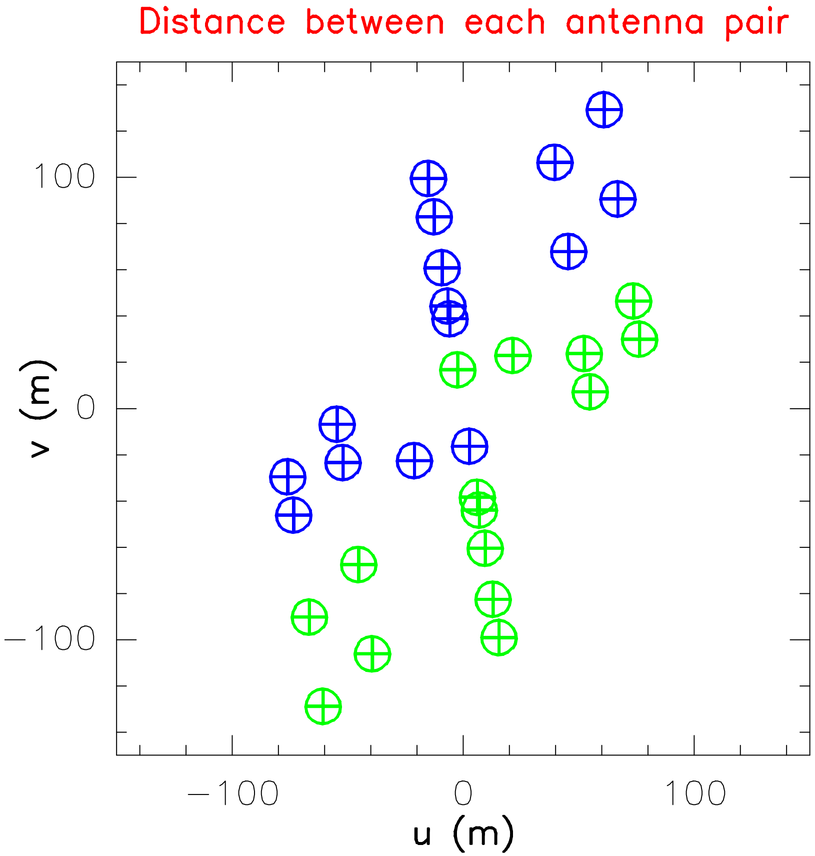
3 Bandwidth size $\overset{\text{FT}}{\rightleftharpoons}$ Pixel size.

4 Finite support $\overset{\text{FT}}{\rightleftharpoons}$ Infinite support.

5 Fourier transform evaluated at zero spacial frequency
= Integral of your function.

$$V(u=0, v=0) \overset{\text{FT}}{\rightleftharpoons} \sum_{ij \,\in\, \text{image}} I_{ij}.$$

Each Visibility is a Weighted Sum of the Fourier Components of the Source



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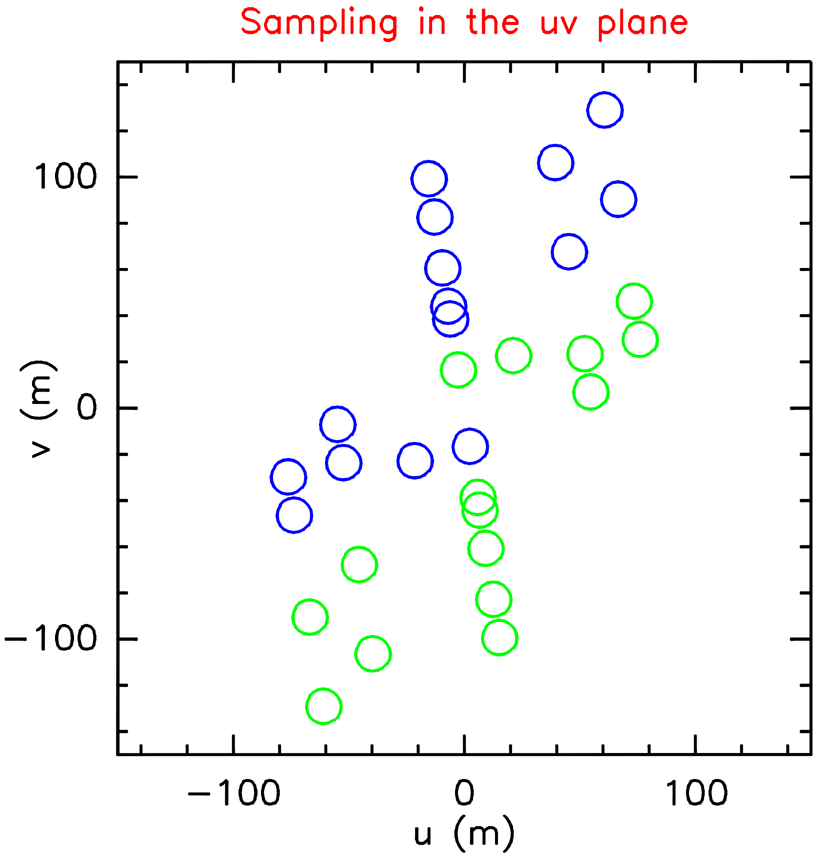
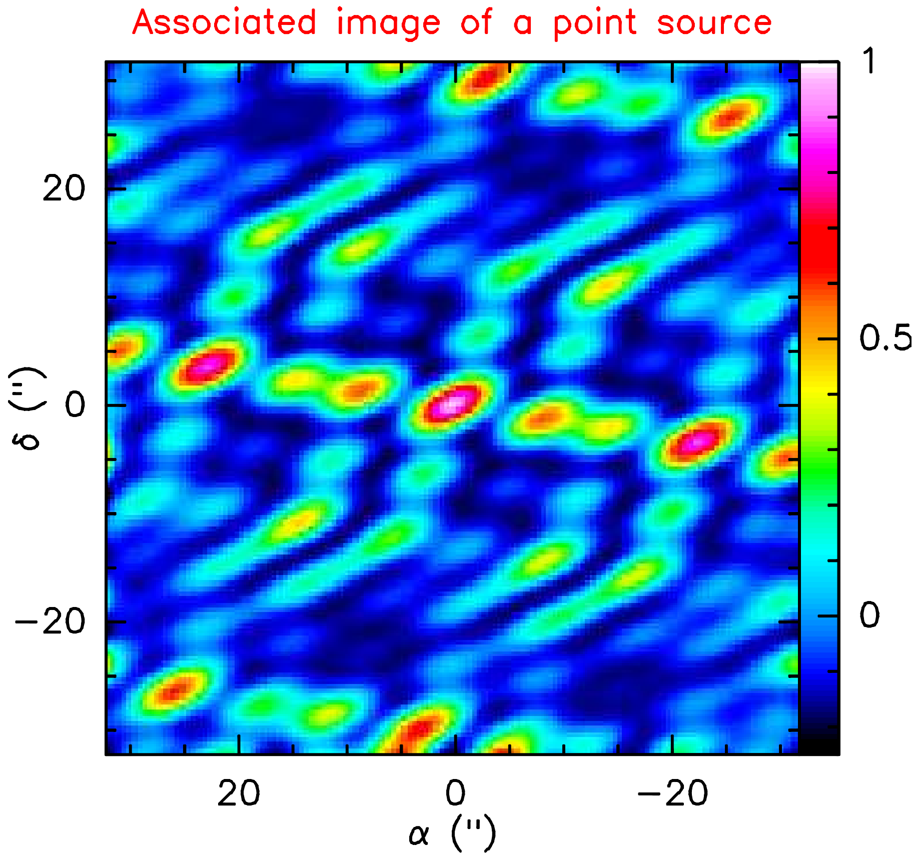
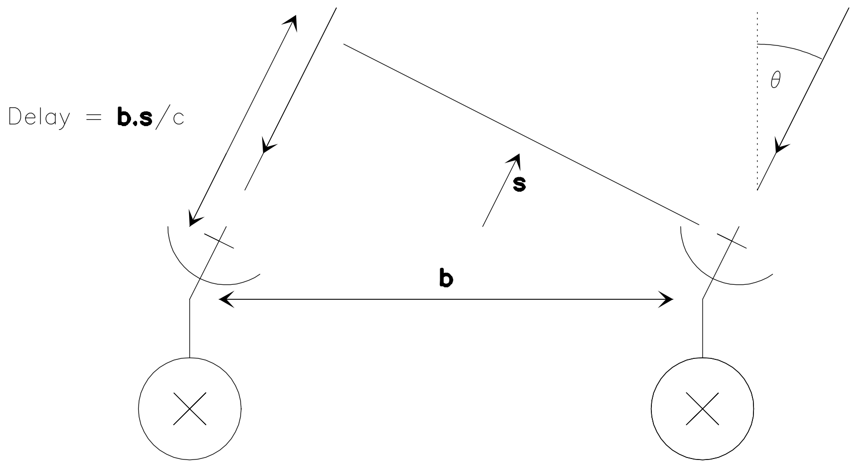




Incomplete uv plane coverage $\Rightarrow$ difficult to make a reliable image (Lectures by A. Castro–Carrizo, J. Pety and F. Gueth).

Earth Rotation and Super Synthesis

Precision: Spatial frequencies = baseline lengths projected in a plane perpendicular to the source mean direction.



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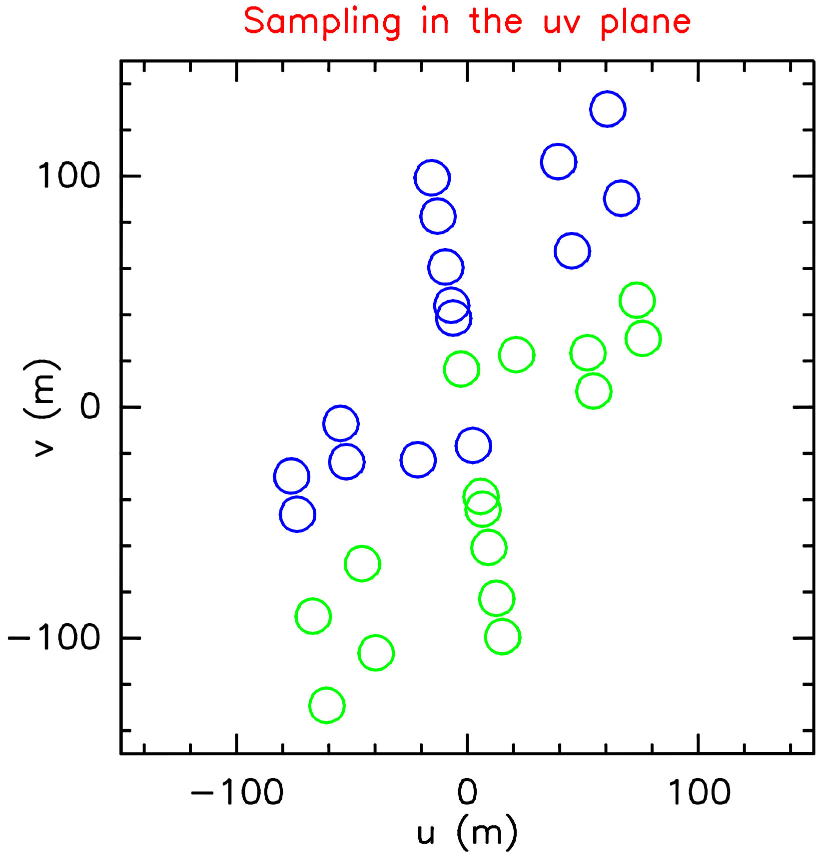
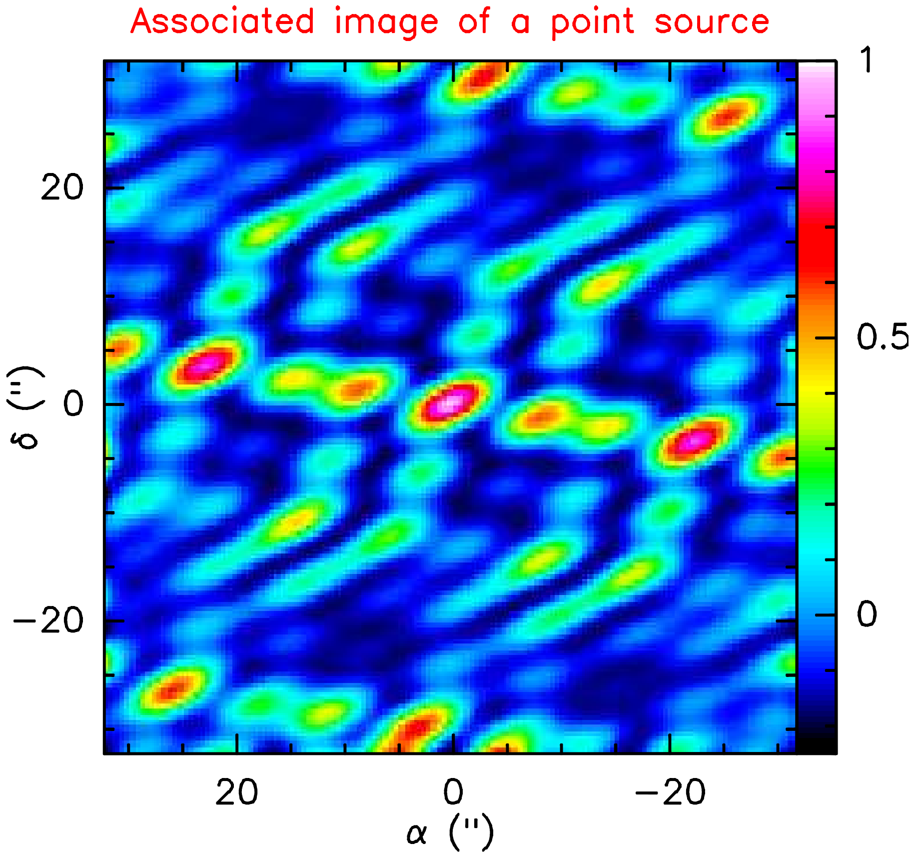
Precision: Spatial frequencies = baseline lengths projected in a plane perpendicular to the source mean direction.

Advantage: Possibility to measure different Fourier components without moving antennas!





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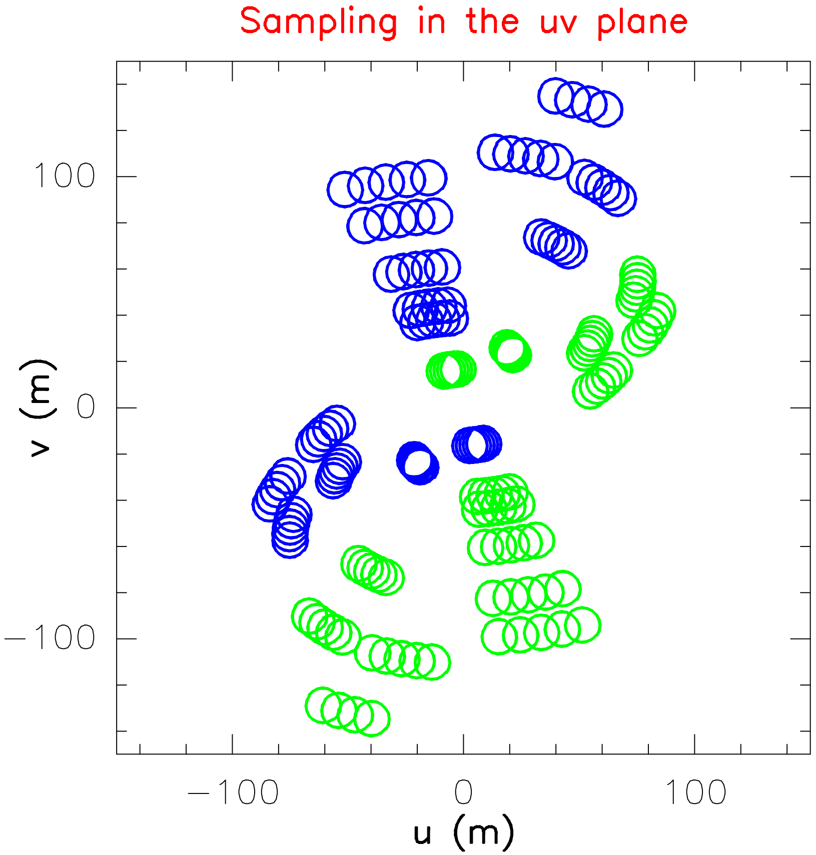

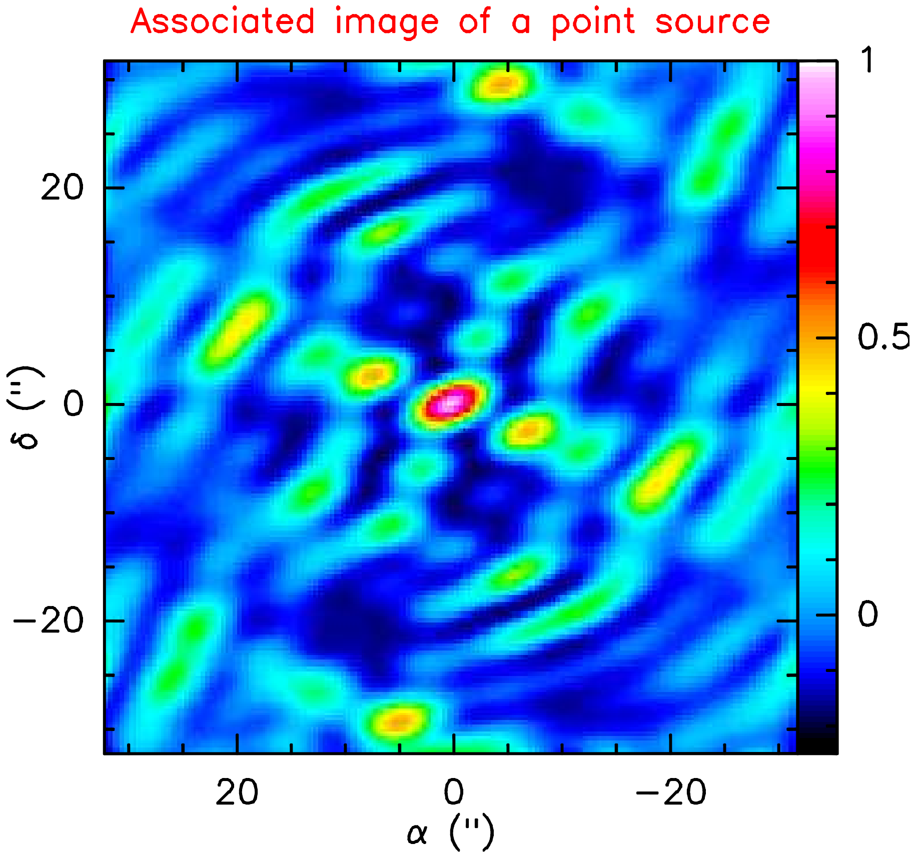

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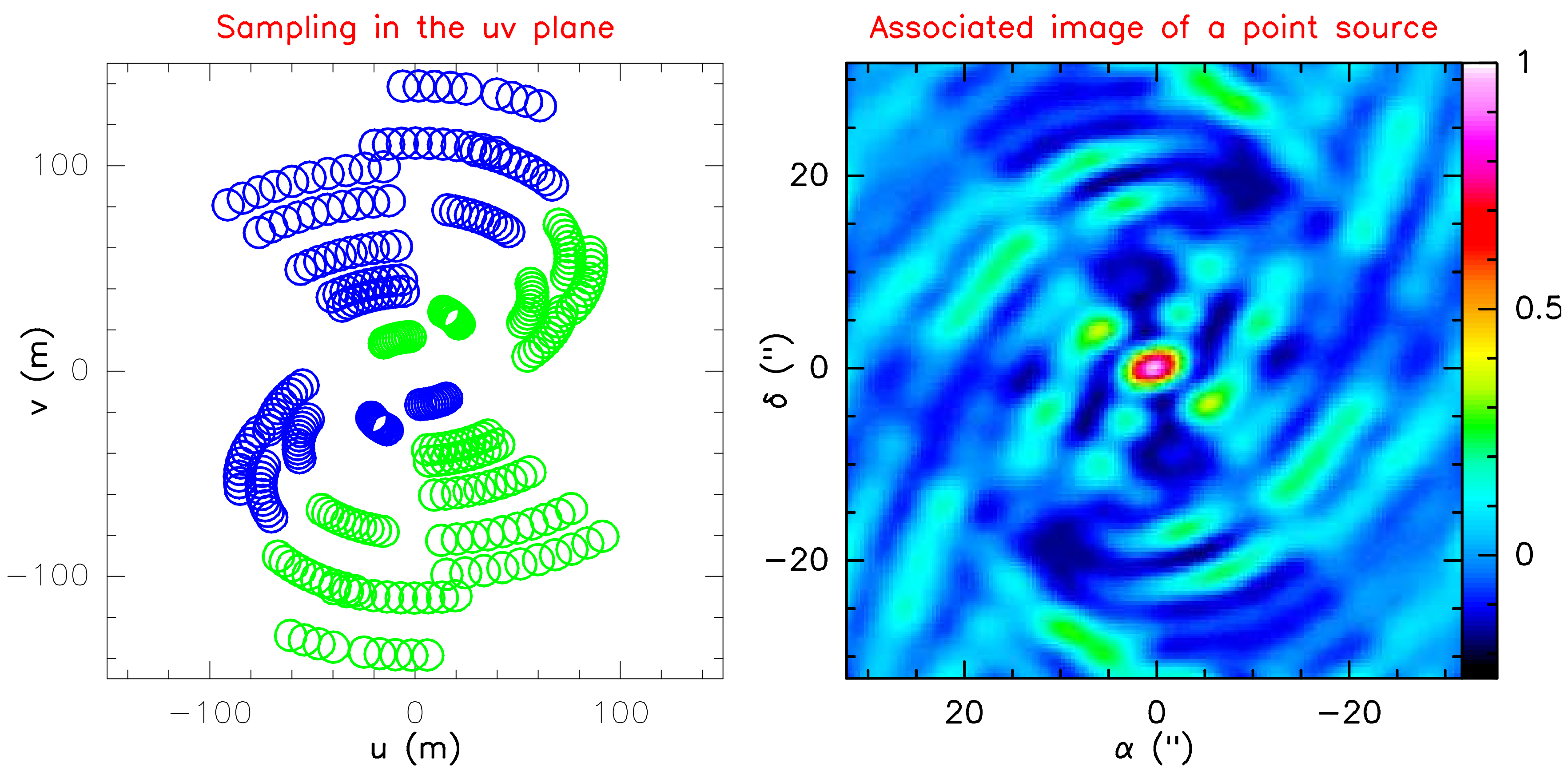

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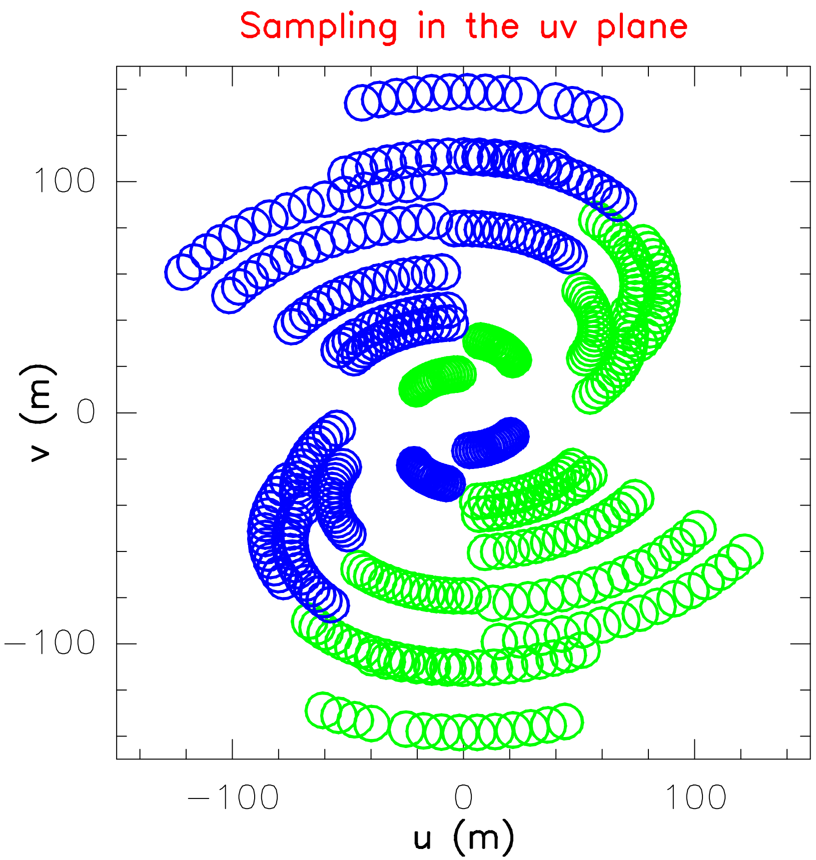

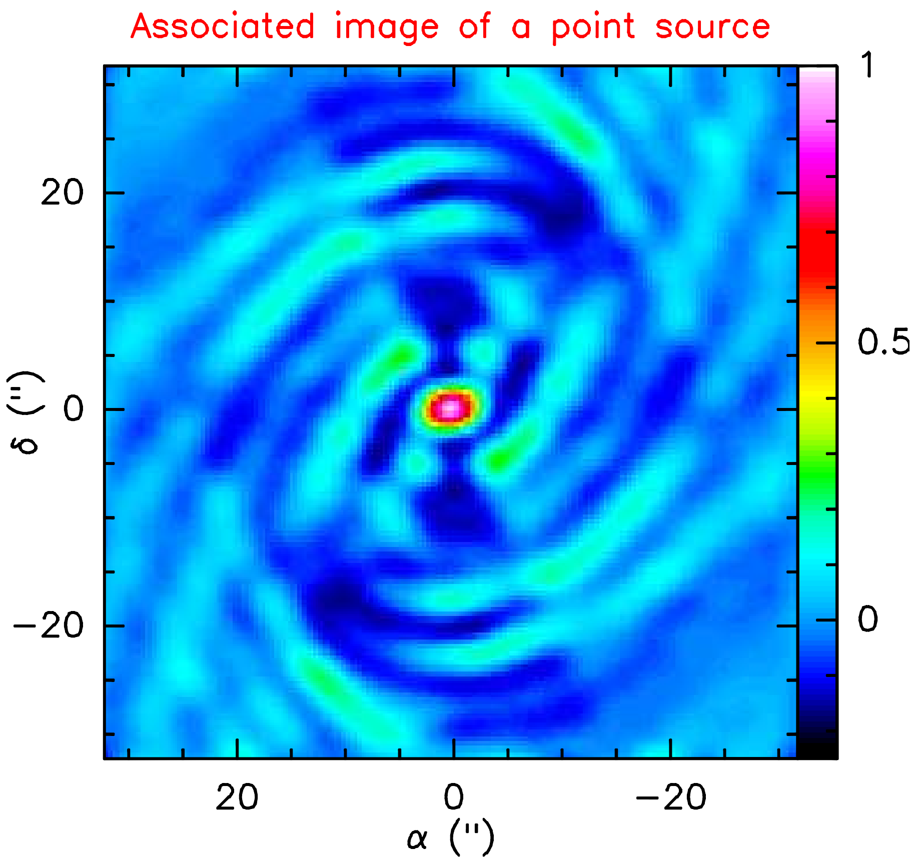

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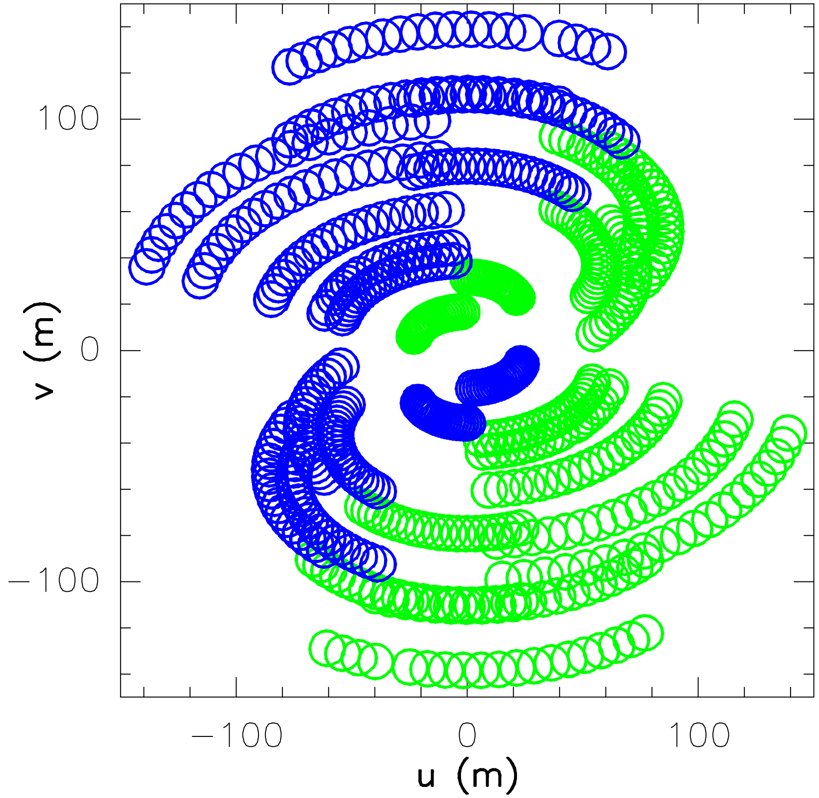
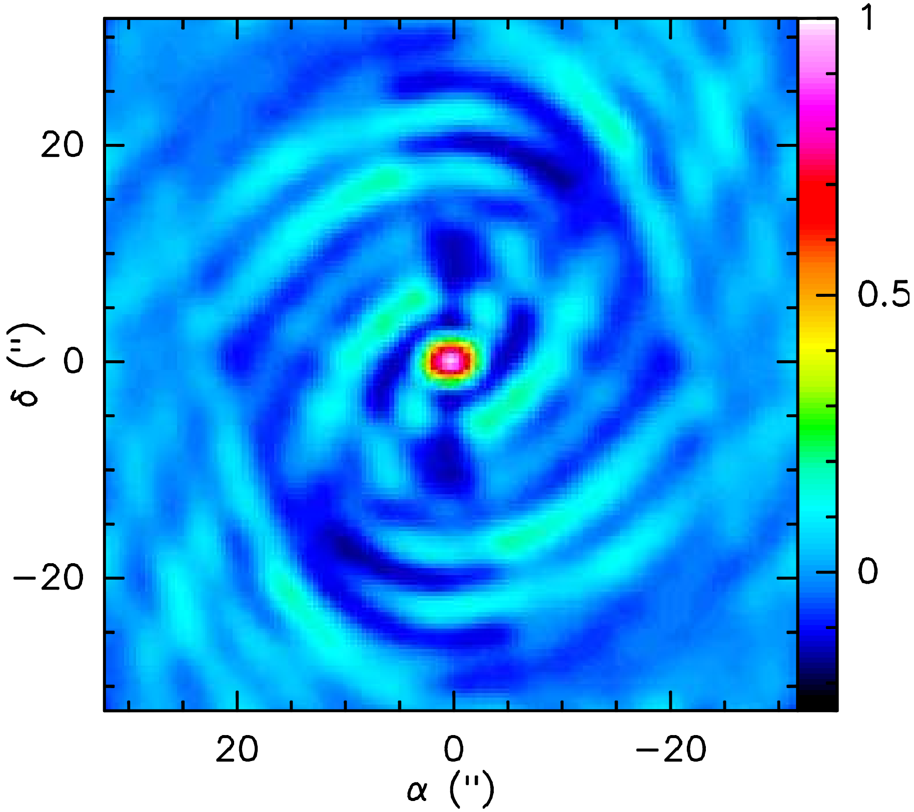
Sampling in the uv plane



Associated image of a point source



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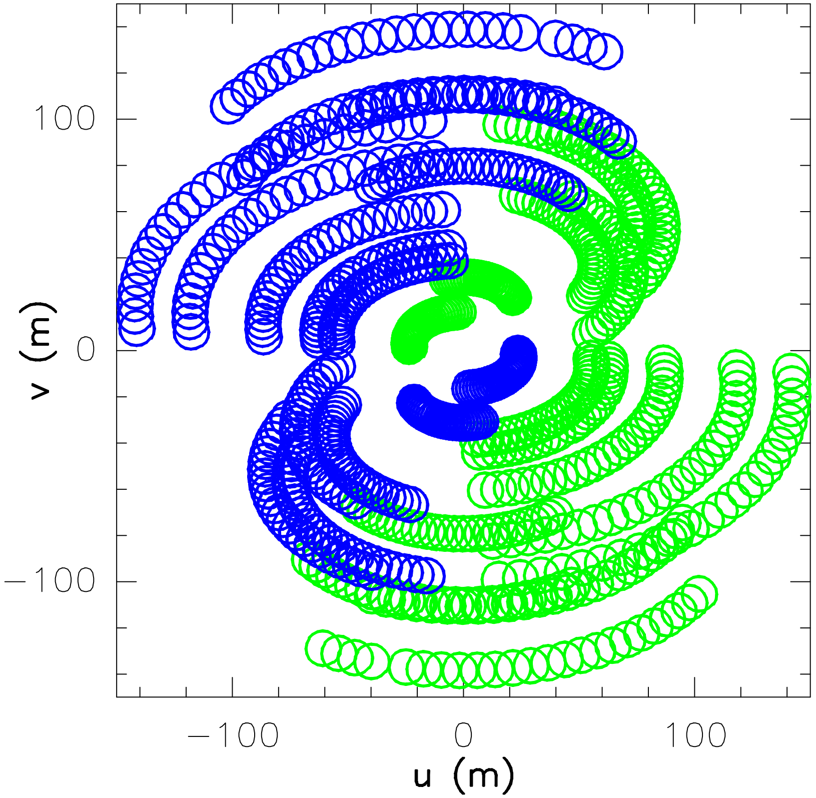

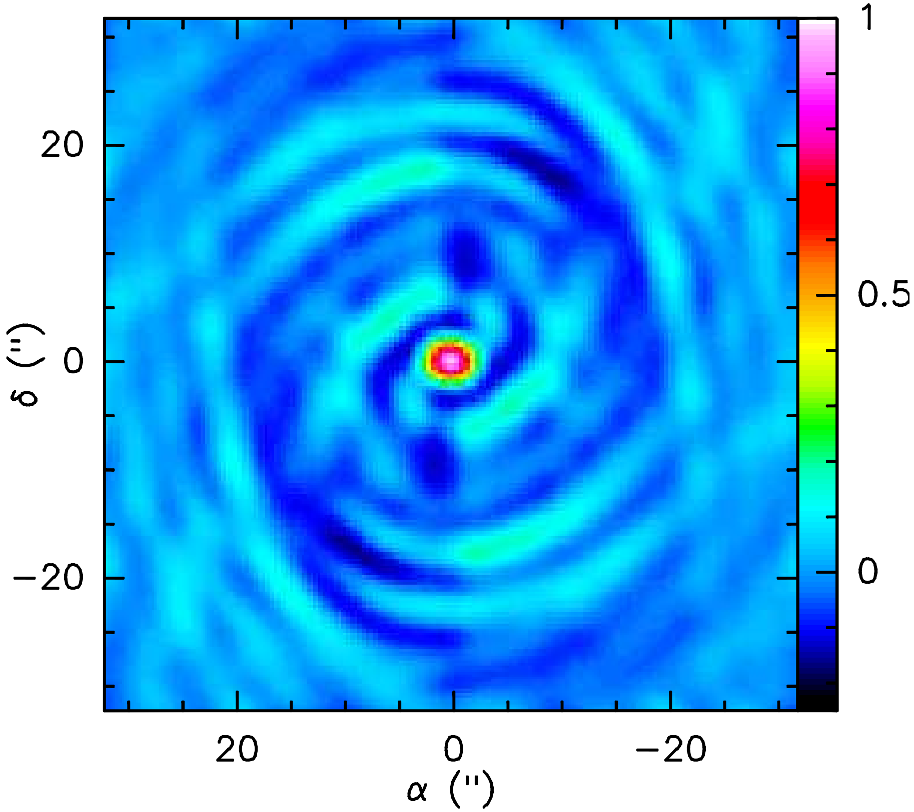

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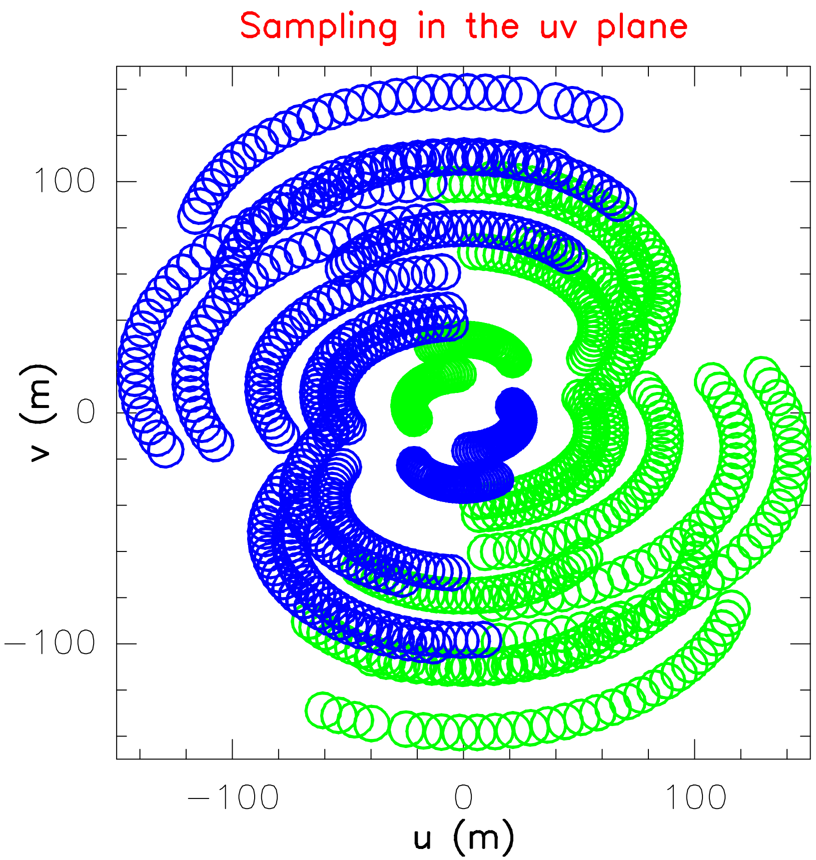

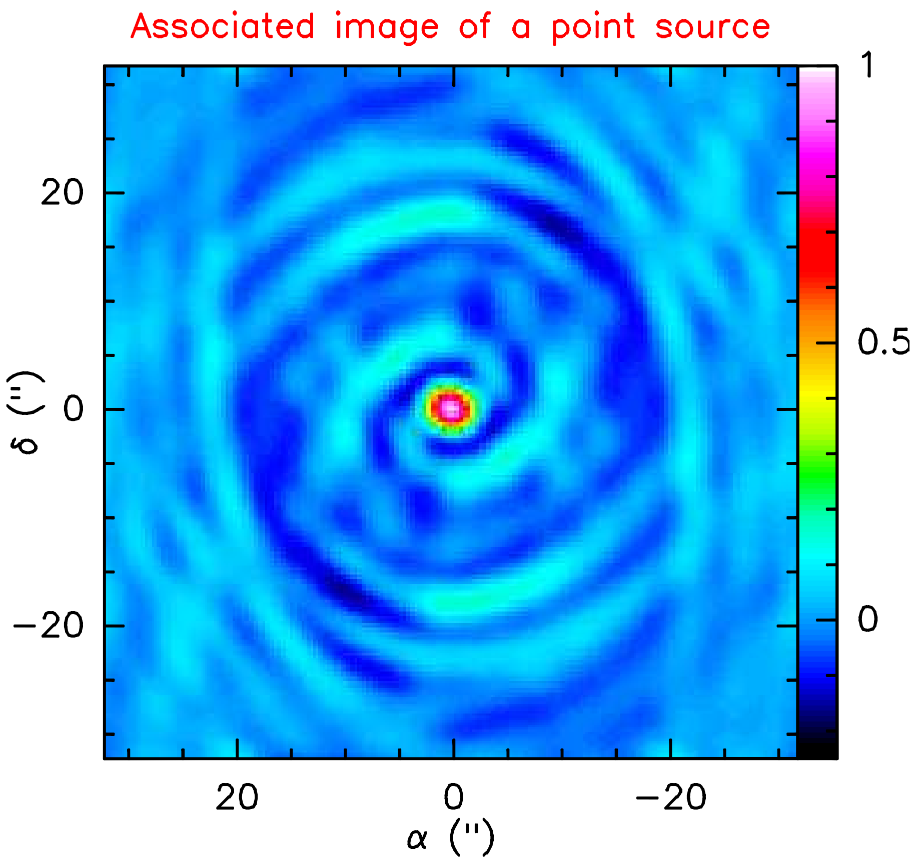

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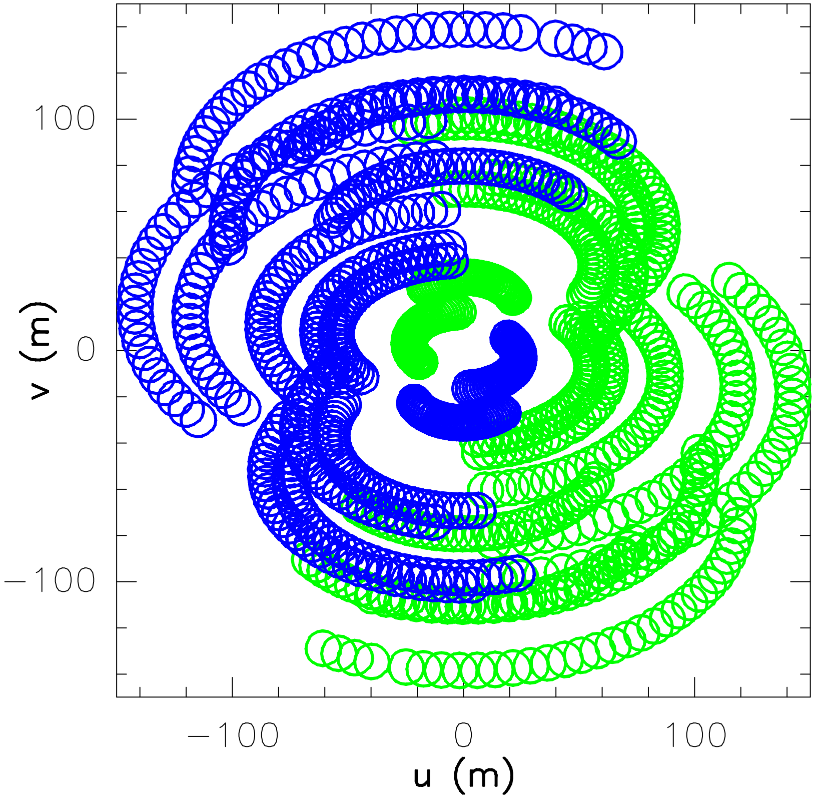
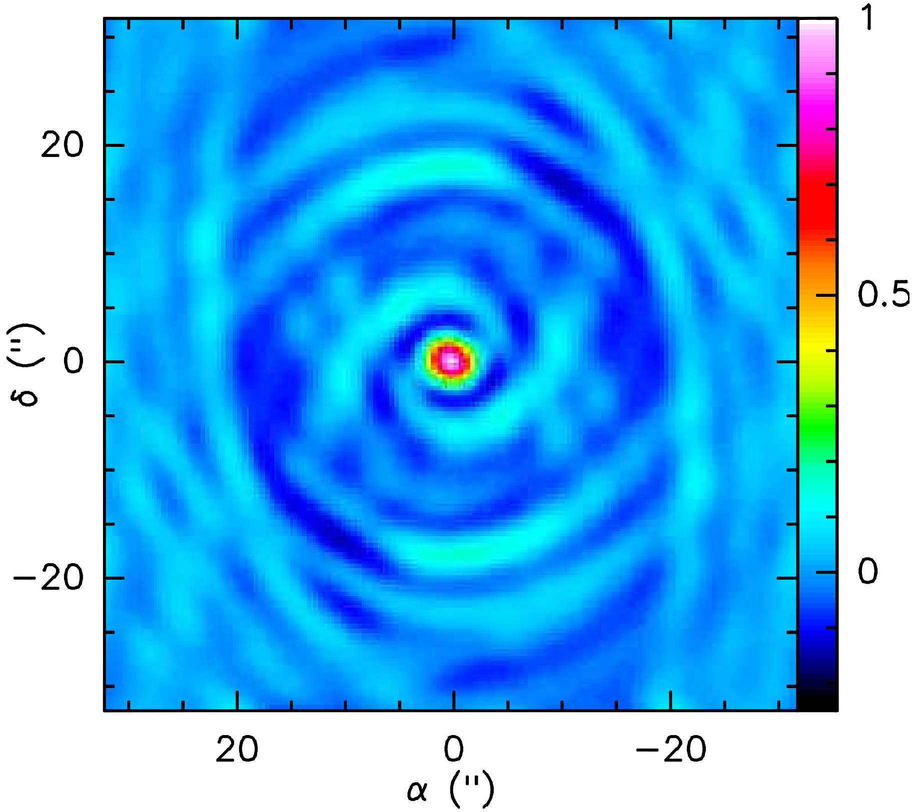
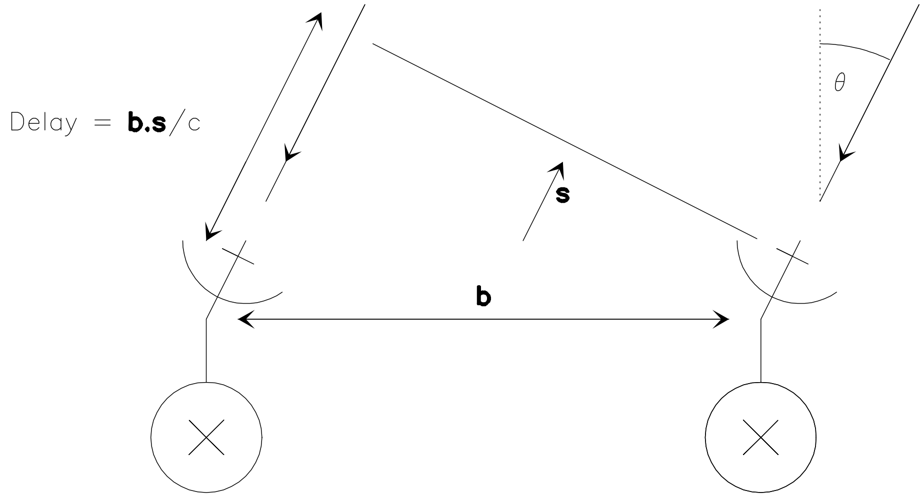




Delay Correction: I. Why?

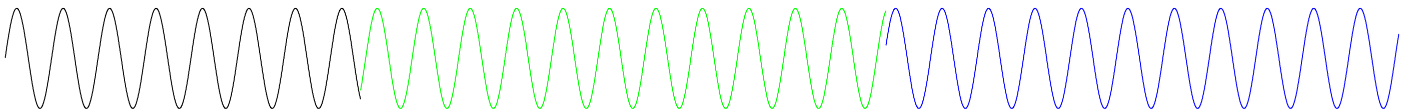
Real life: Source not at zenith.

$\Rightarrow$ Wave plane arrives at different moment on each antenna.



Temporal coherence:

- $E(t) = E_0 \cos(\omega t + \psi)$
- Temporally Incoherent Source = random phase changes.
- Coherence time: mean time over which wave phase = constant.



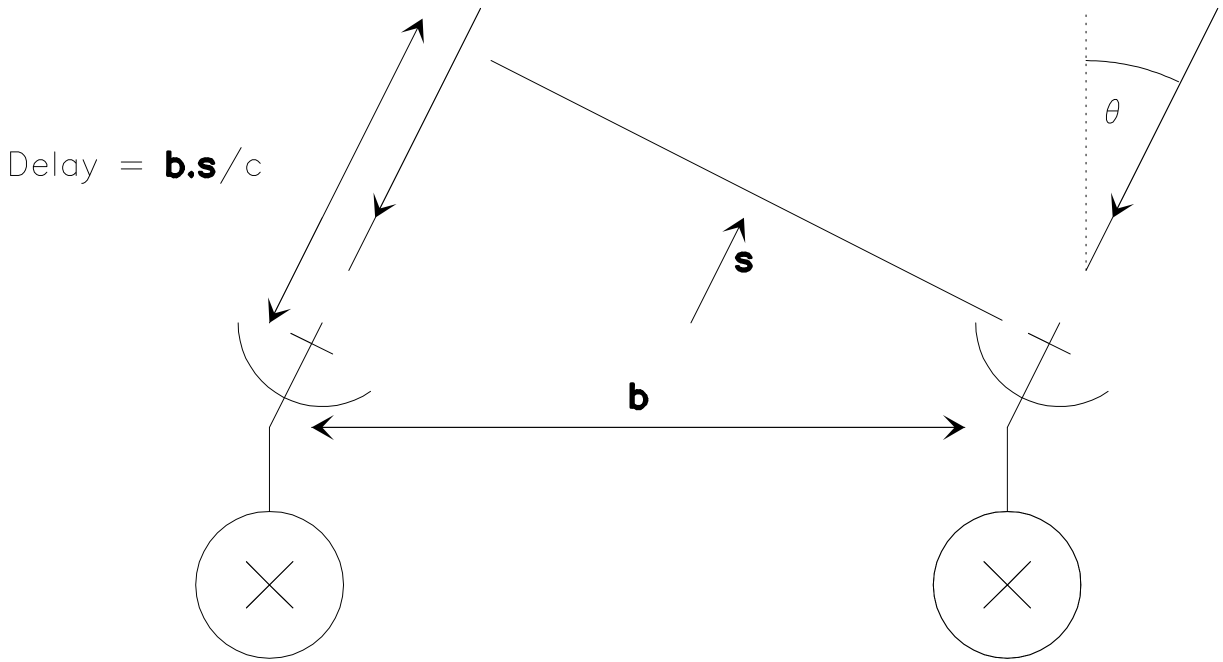
$\psi = 0$ $\psi = 1.5$ $\psi = 0.5$

Problem: (Coherence time $\lesssim$ delay) $\Rightarrow$ fringes disappear!

Delay Correction: II. Earth rotation

Earth rotation:

- Advantage: Super synthesis;
- Inconvenient: Delay correction varies with time!



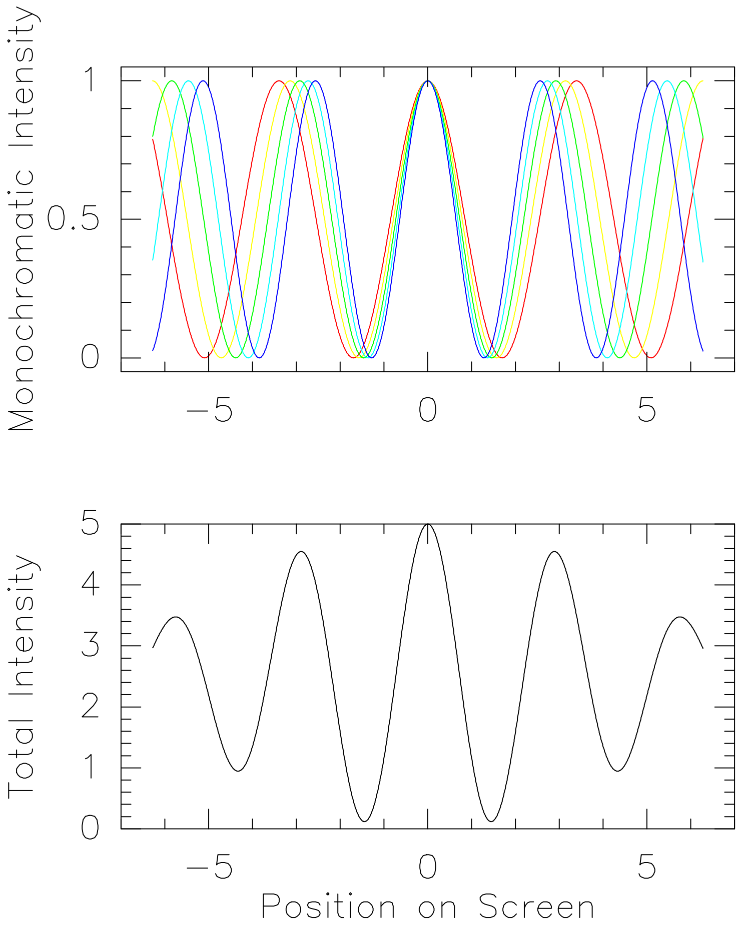
Delay Correction: III. Finite Bandwidth

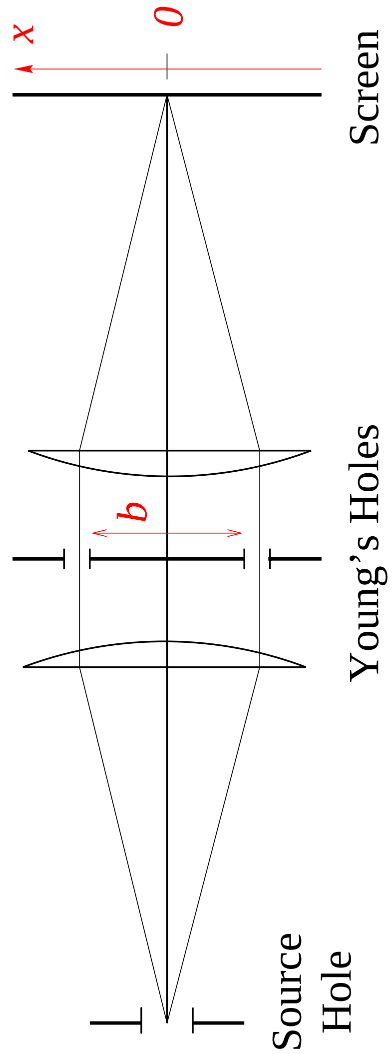
Real life: Observation of finite bandwidth.

$\Rightarrow$ polychromatic light.

Perfect delay correction

$\Rightarrow$ White fringes in 0.





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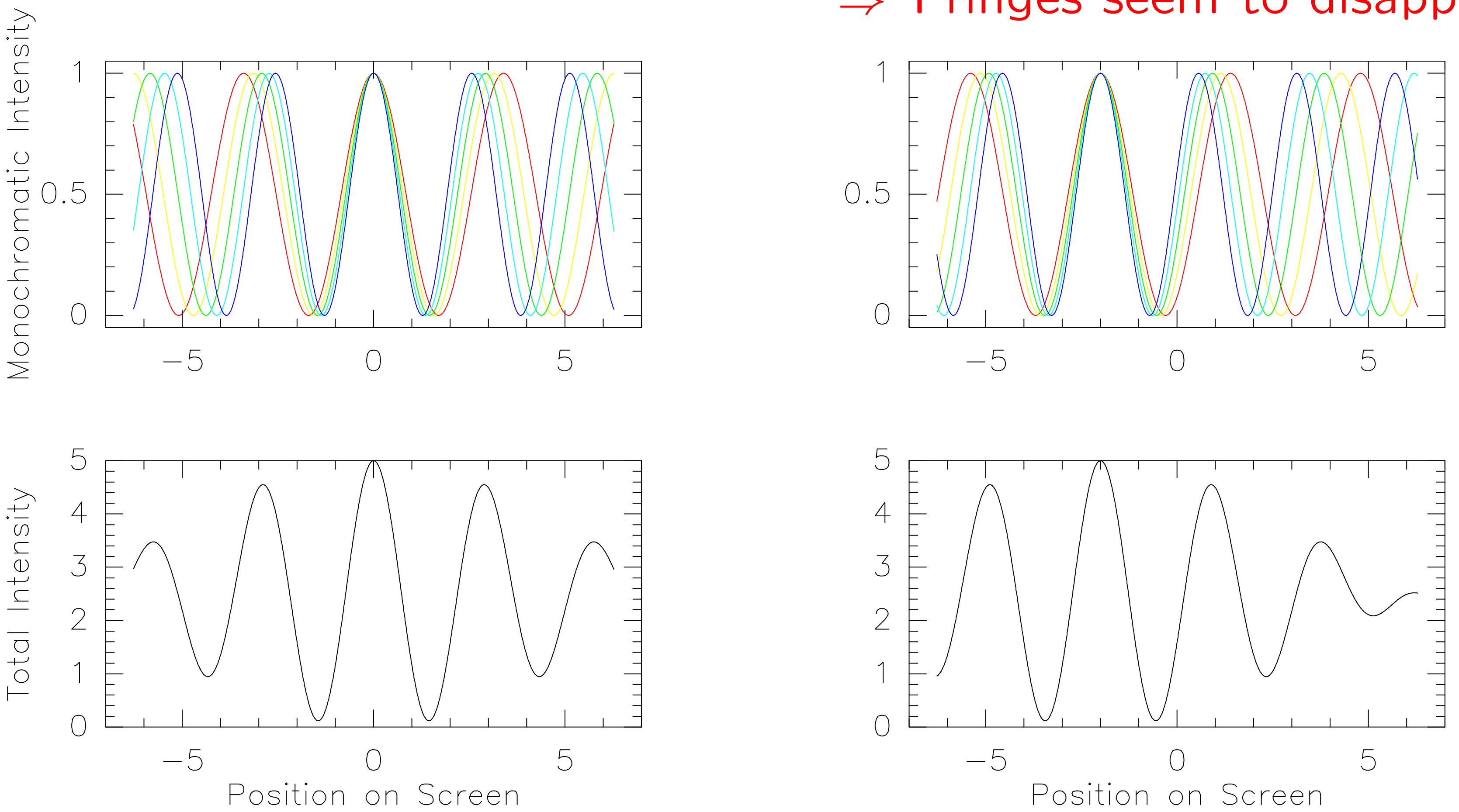
Worse and worse delay correction.

$\Rightarrow$ Translation of the fringe pattern.

$\Rightarrow$ Fringes seem to disappear.



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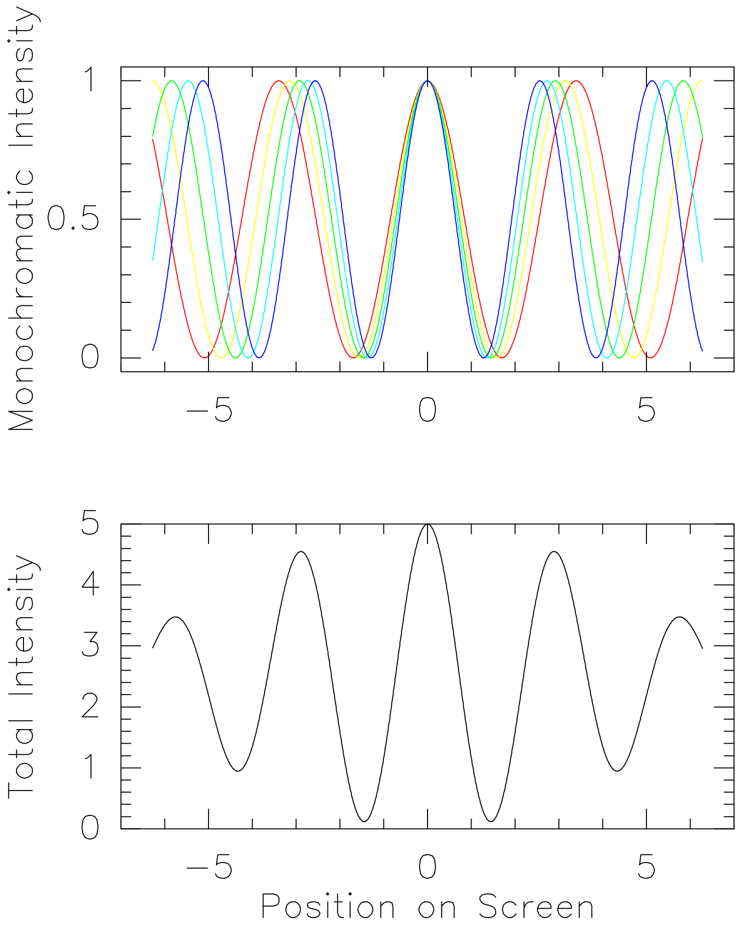

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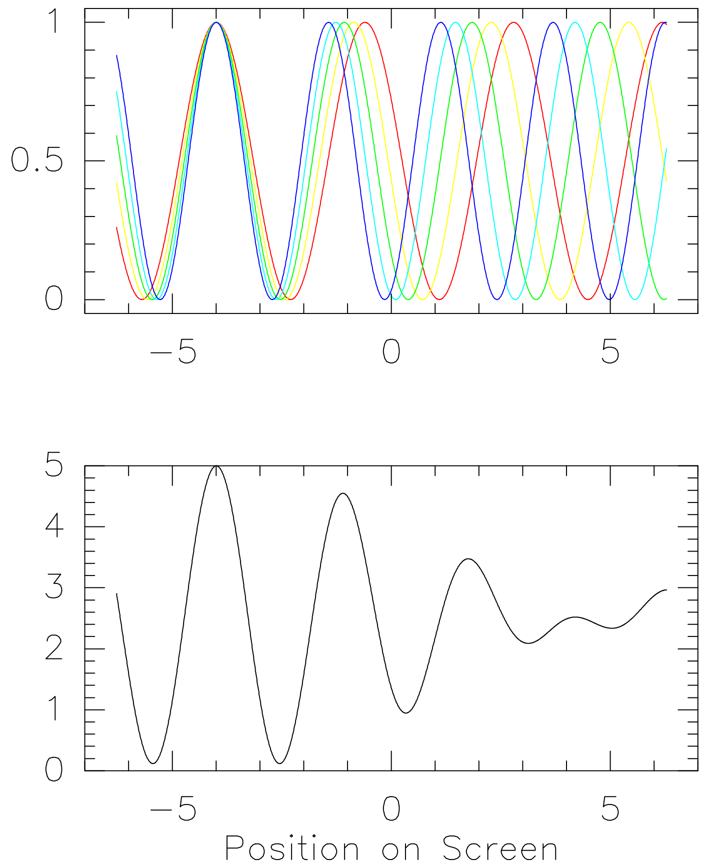

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Real life: Observation of finite bandwidth.

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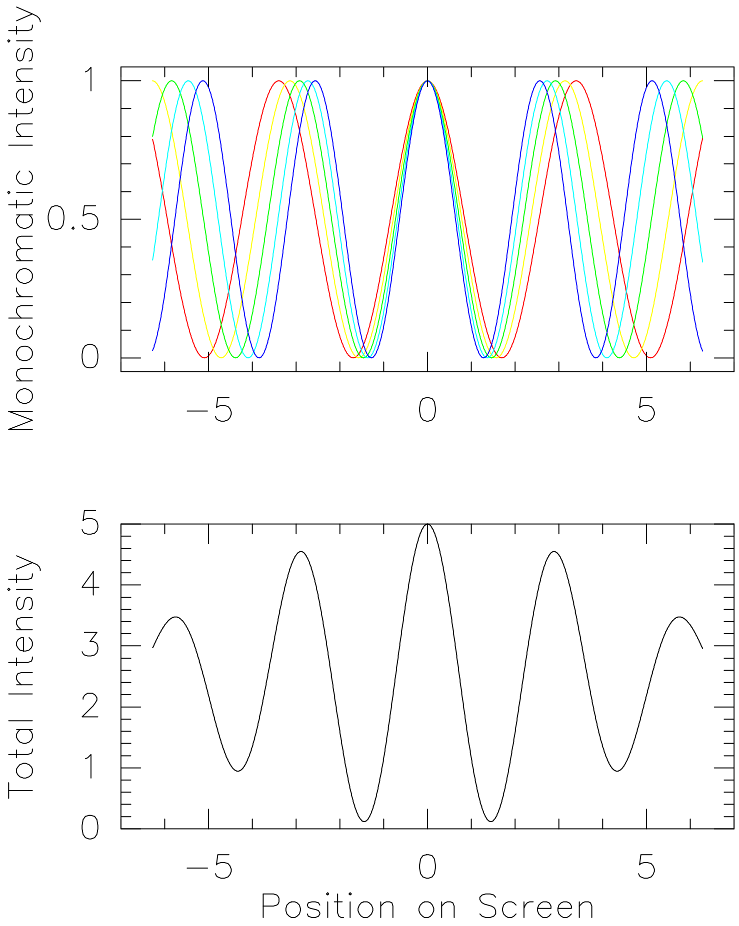
Perfect delay correction

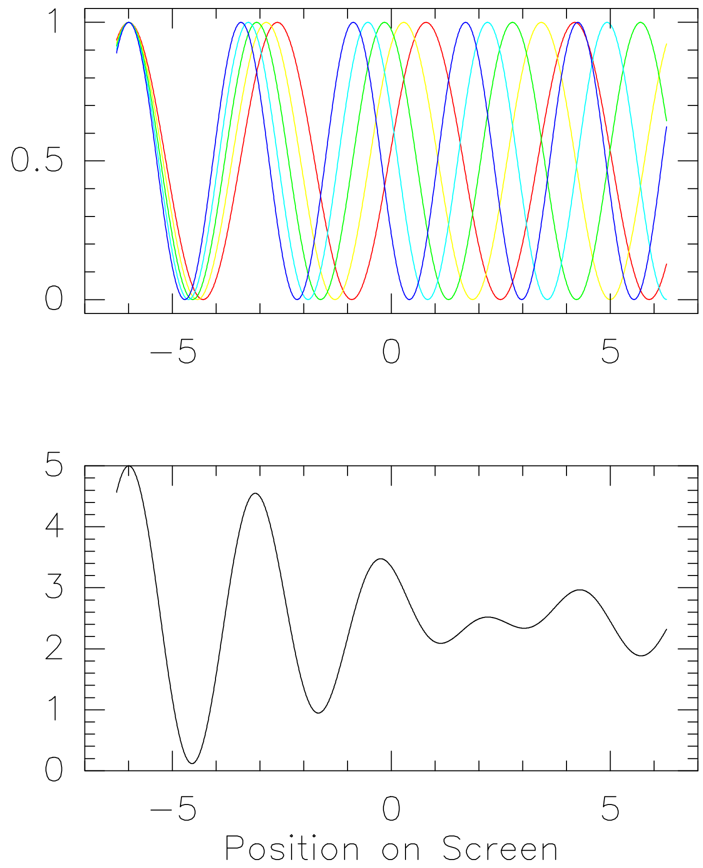
$\Rightarrow$ White fringes in 0.

Worse and worse delay correction.

$\Rightarrow$ Translation of the fringe pattern.

$\Rightarrow$ Fringes seem to disappear.





Delay Correction: III. Finite Bandwidth

Real life: Observation of finite bandwidth.
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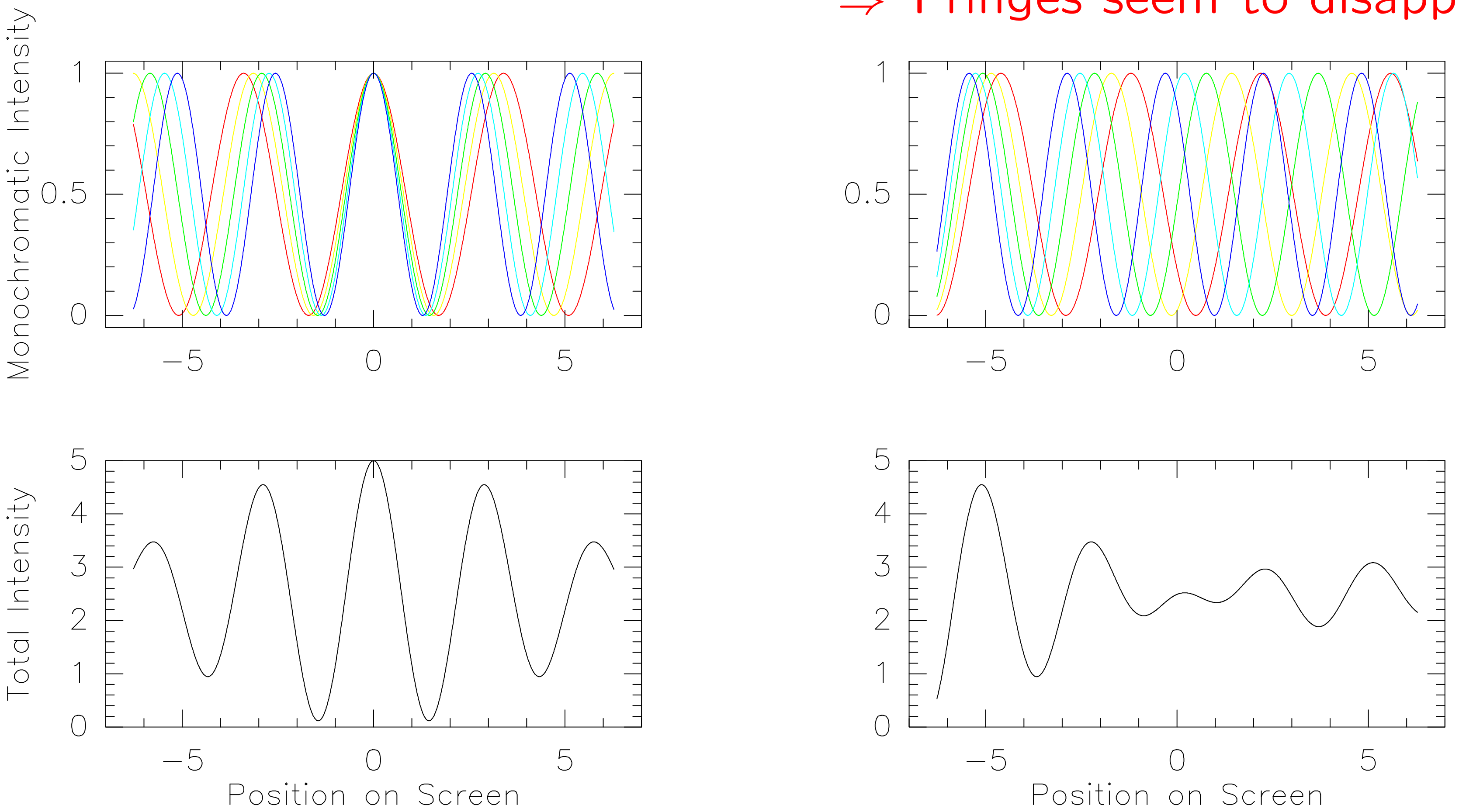

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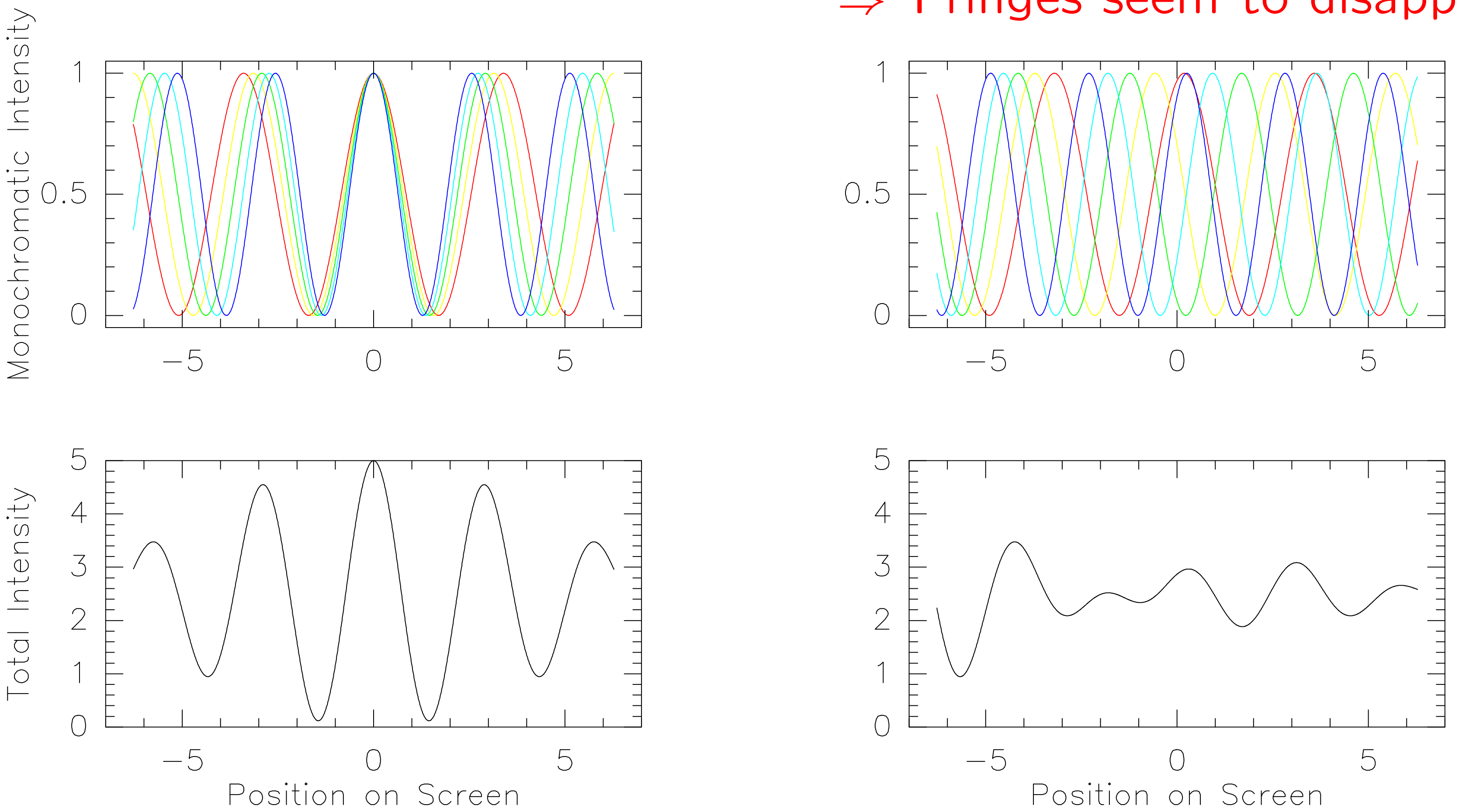
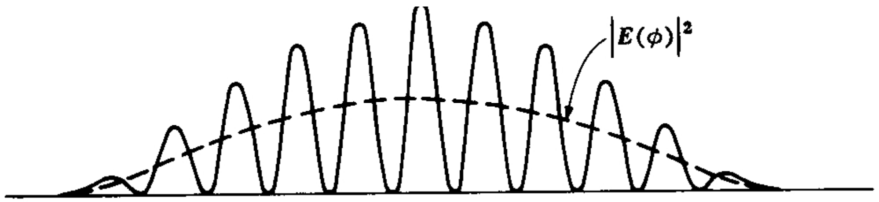
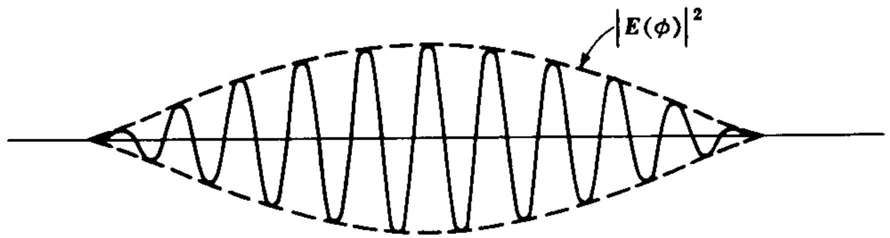


Optic vs Radio Interferometer: I. Measurement Method

		Optic	Radio
Detector	Kind	Quadratic	Linear (Heterodyne)
	Observable	$I = \|EE^\star\|$	$\|E\|\,\exp(i\psi)$
Measure	Method	Optical fringes	Electronic correlation
	Quantity	$\|C\| = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$	$\|V\|\exp(i\phi_V) = < E_1.E_2 >$
Interferometer kind		Additive	Multiplicative





$$I_1 + I_2 + 2\sqrt{I_1 I_2}\,|C|\cos\left(\frac{bx}{\lambda} + \phi_C\right)$$

$$\overbrace{|E_1|\,|E_2|\,|C|}^{|V|}\cos\left(\frac{bx}{\lambda} + \overbrace{\phi_C}^{\phi_V}\right)$$

(Heterodyne: lecture by F. Gueth)

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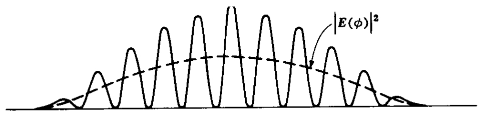
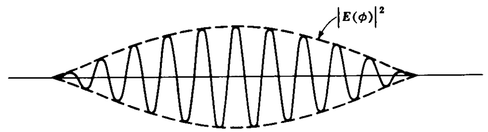


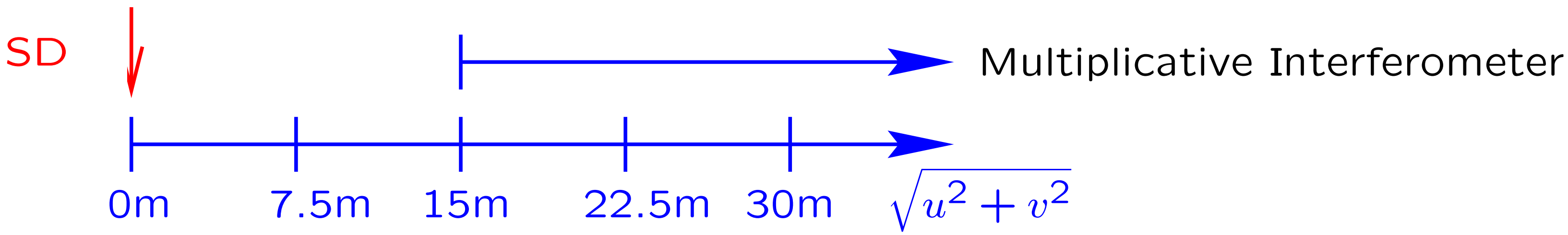


Multiplicative Interferometer

Avantage: all offsets are irrelevant $\Rightarrow$ Much easier;

Inconvenient: Radio interferometer = bandpass instrument;

$\Rightarrow$ Low spatial frequencies are filtered out.



Optic vs Radio Interferometer: II. Atmospheric Influence

Atmosphere emits and absorbs: (Lecture by M. Bremer).

Signal = Transmission * Source + Atmosphere.

- Optic: $\left\{\begin{array}{l}\text{Source} \gg \text{Atmosphere} \\ \text{Transmission} \sim 1\end{array}\right\} \Rightarrow$ transparent;
- Radio: $\left\{\begin{array}{l}\text{Source} \ll \text{Atmosphere} \\ \text{Transmission can be small}\end{array}\right\} \Rightarrow$ fog.

Good news: Atmospheric noise uncorrelated

$\Rightarrow$ Correlation suppresses it!

Bad news: Transmission depends on weather and frequency.

$\Rightarrow$ Astronomic sources needed to calibrate the flux scale!

(Lectures by F. Gueth and V. Piétu)

Atmosphere is turbulent: $\Rightarrow$ Phase noise (Lecture by M. Bremer).

Timescale of atmospheric phase random changes:

- Optic: 10-100 milli secondes;
- Radio: 10 minutes.

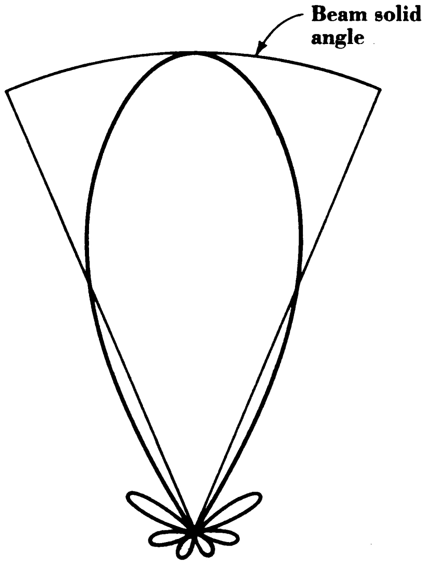
$\Rightarrow$ Radio permits phase calibration on a nearby point source (*e.g.* quasar).

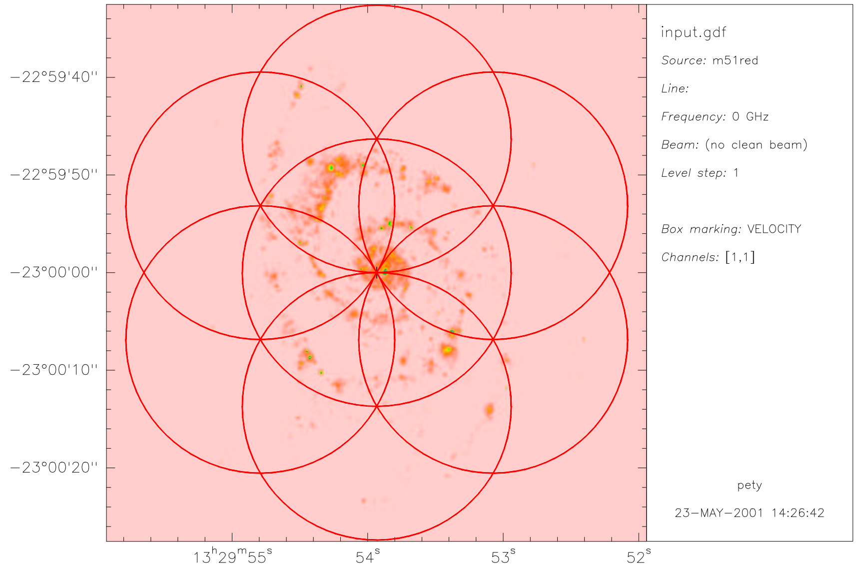
Instantaneous Field of View

One pixel detector:

- Single Dish: one image pixel/telescope pointing;
- Interferometer: numerous image pixels/telescope pointing
  - Field of view = Primary beam size;
  - Image resolution = Synthesized beam size.

Wide-field imaging: ⇒ mosaicing (Lecture by F. Gueth).





Conclusion

mm interferometry:

- A bit more of theory;
- Lot's of experimental details (*e.g.* lecture by F. Gueth and R. Neri).

Why caring about technical details: Some of them must be understood to know whether you can trust your data.

By the end of this week, you should be ready to use PdBI!
(Lecture by R. Neri, J. M. Winters and examples from users)

Bibliography

- "Synthesis Imaging". Proceedings of the NRAO School. R. Perley, F. Schwab and A. Bridle, Eds.
- "Proceedings from IMISS2", A. Dutrey Ed.
- "Interferometry and Synthesis in Radio Astronomy", R. Thompson, J. Moran and G. W. Swenson, Jr.

Photographic Credits

- M. Born & E. Wolf, "Principles of Optics".
- J. W. Goodman, "Statistical Optics".
- J. D. Kraus, "Radio Astronomy".

Lexicon

- Beam: Antenna diffraction pattern.
- Primary Beam: Instantaneous field of view (Single-Dish Beam).
- Synthesized Beam: Image resolution (Interferometer Beam).
- Configuration: Antenna layout of interferometer.
- Baseline: Distance between two antenna.
- uv-plane: Fourier plane.
- Visibilities: $\sim$ Fourier components of the source.
- Fringe stopping: Temporal variation of delay correction needed to avoid translation of the white fringe.
- Heterodyne: Principle of linear detection.
- Correlator: Where visibilities are measured by correlation of signal coming from pairs of antenna.