

A Sightseeing Tour of mm Interferometry

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7th IRAM Millimeter Interferometry School
Oct. 4 - Oct. 8 2010, Grenoble

Towards Higher Resolution:

I. Problem

Telescope resolution:

- $\sim \lambda/D$;
- IRAM-30m: $\sim 11''$ @ 1 mm.

Needs to:

- increase D ;
 - increase precision of telescope positioning;
 - keep high surface accuracy.
- ⇒ Technically difficult (perhaps impossible?).

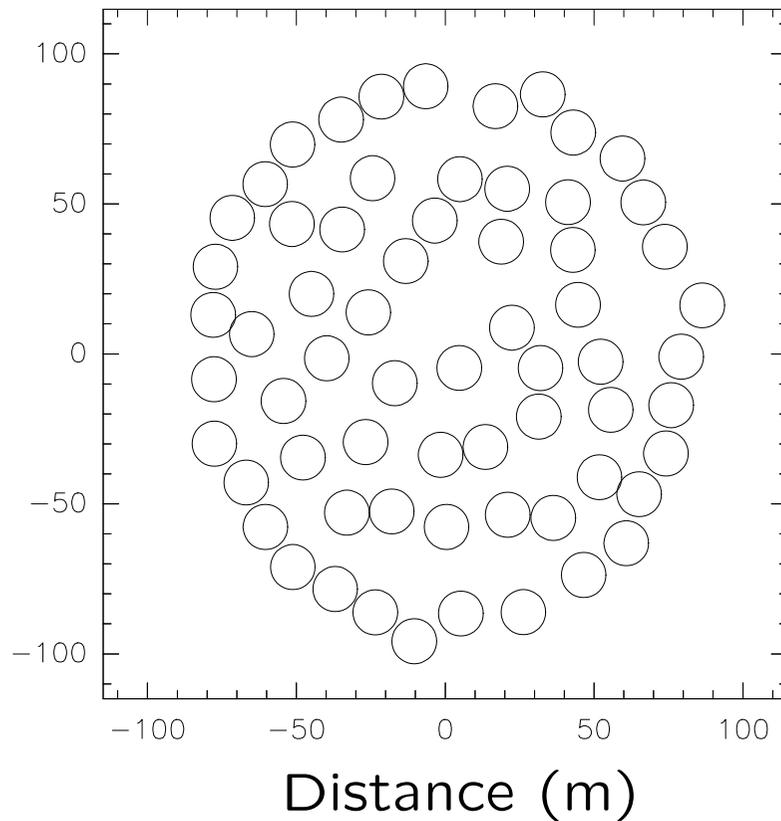
Towards Higher Resolution:

II. Solution

Aperture Synthesis: Replacing a single large telescope by a collection of small telescope “filling” the large one.

⇒ Technically difficult but **feasible**.

ALMA



Vocabulary and notations:

Baseline Line segment between two antenna.

b_{ij} Baseline length between antenna i and j .

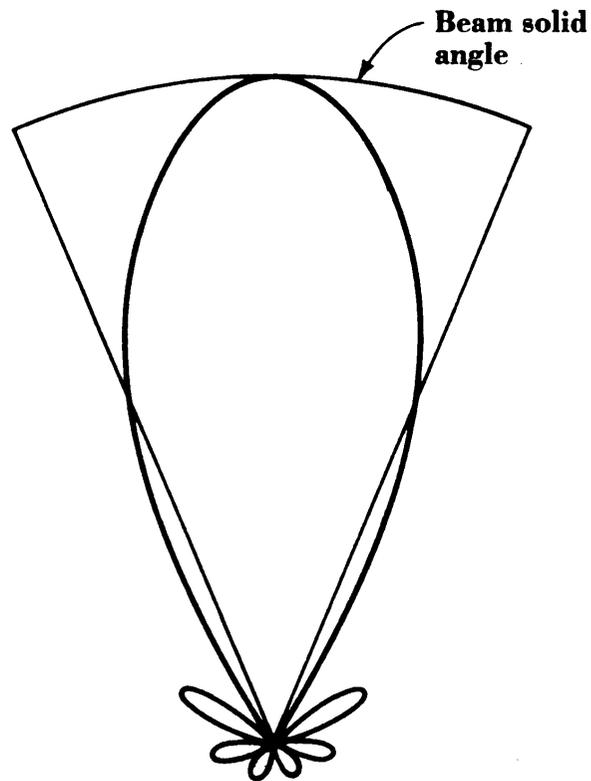
Configuration Antenna layout (e.g. compact configuration).

D configuration size (e.g. 150 m).

Primary beam resolution of one antenna (e.g. $27''$ @ 1 mm).

Synthesized beam resolution of the array (e.g. $2''$ @ 1 mm).

Parenthesis: PSF = Diffraction Pattern = Beam Pattern



Single-Dish sensitivity
in polar coordinates.

Combination of:

- Antenna properties;
- Optical system (*i.e.* how the waves are feeding the receiver).

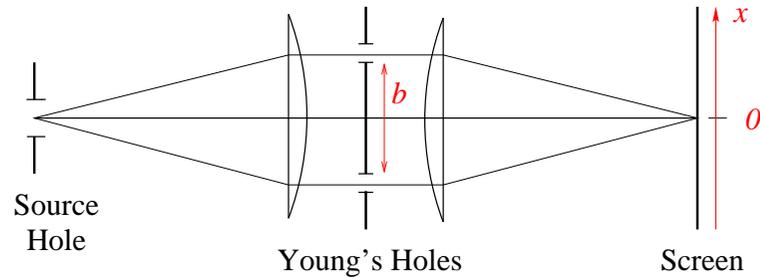
Typical kind:

Optic/IR Airy function;
Radio Gaussian function.

(Lecture by P. Hily-Blant)

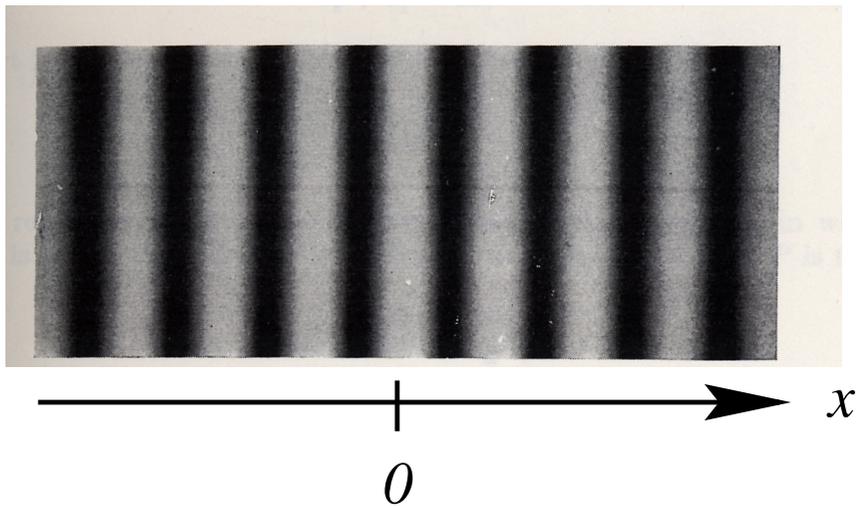
Young's Experiment

Setup



Lens \Rightarrow Fraunhofer conditions
(i.e. Plane waves as if the source were placed at infinity).

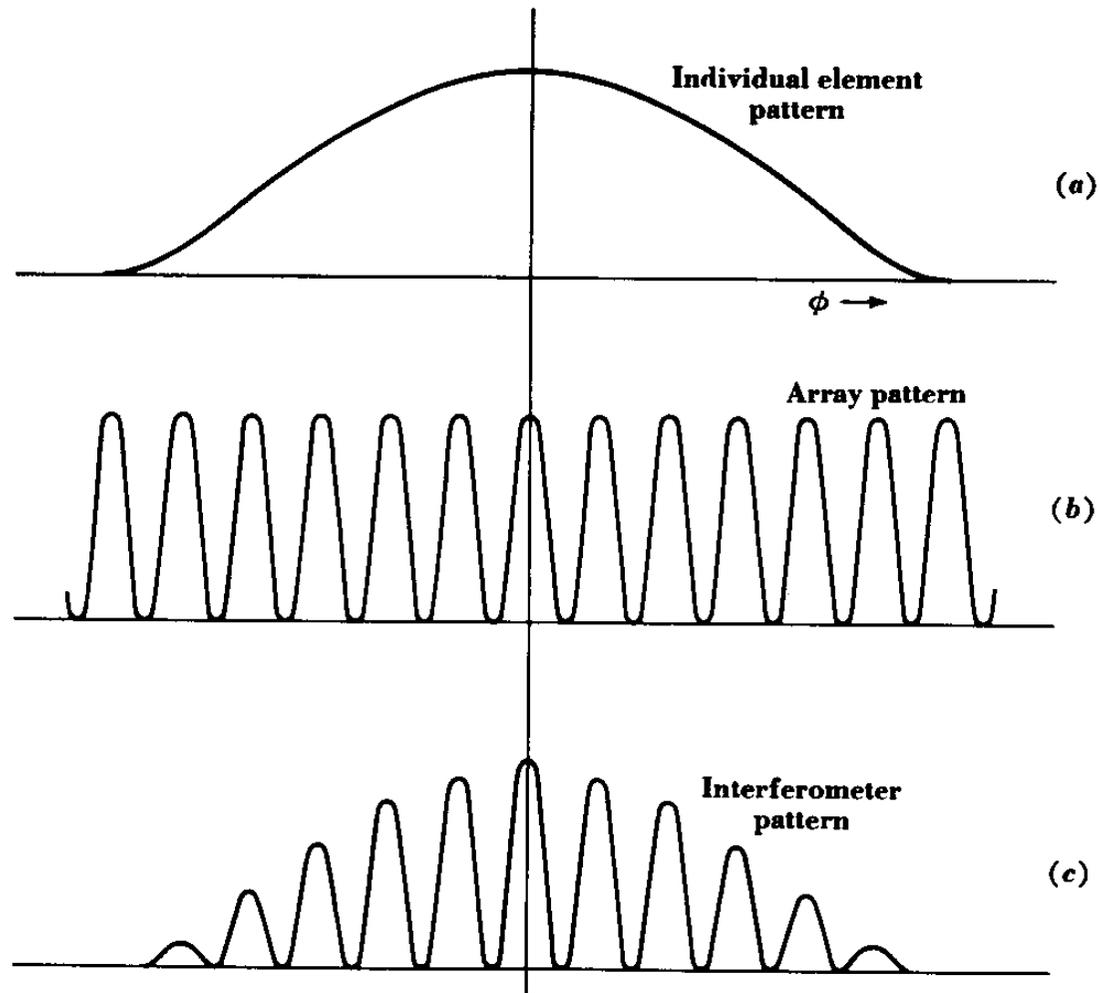
Obtained image of interference: fringes



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\frac{bx}{\lambda}\right)$$

with $\left\{ \begin{array}{l} \lambda \text{ Source wavelength;} \\ b \text{ Distance between the} \\ \text{two Young's holes;} \\ x \text{ Distance from the opti-} \\ \text{cal center on the screen.} \end{array} \right.$

Effect of the Antenna Diffraction Pattern



$$I(x) = B(x) \cdot \left\{ I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\left(\frac{bx}{\lambda}\right) \right\}$$

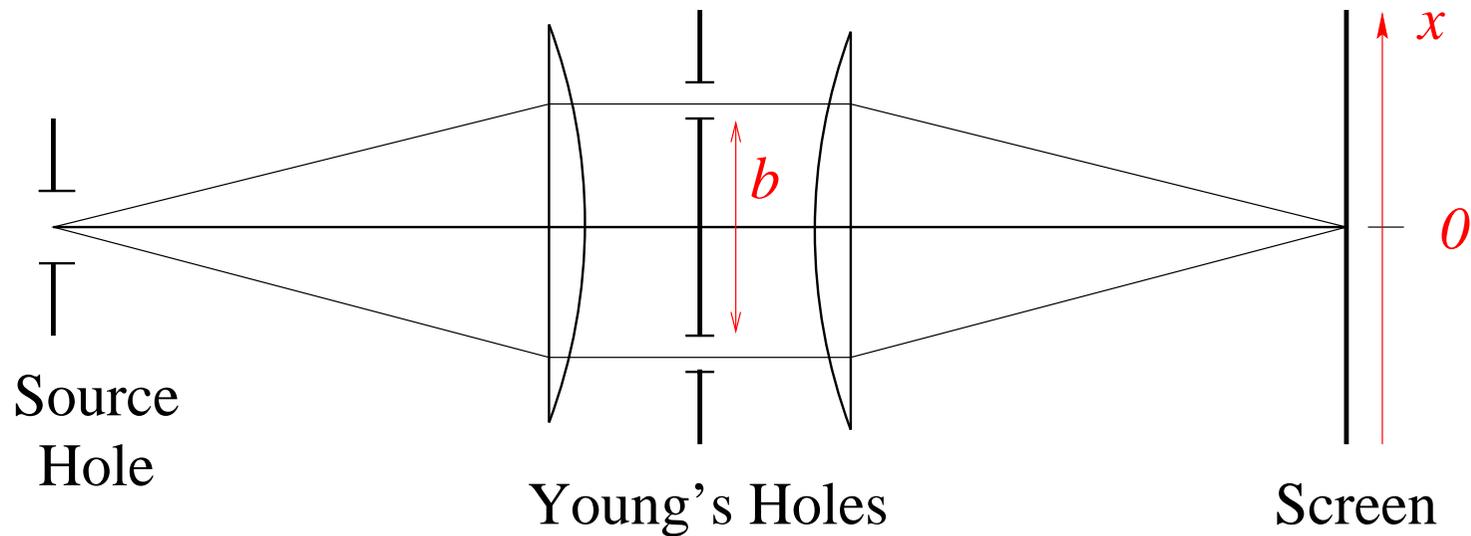
Effect of the Source Hole Size:

I. Description

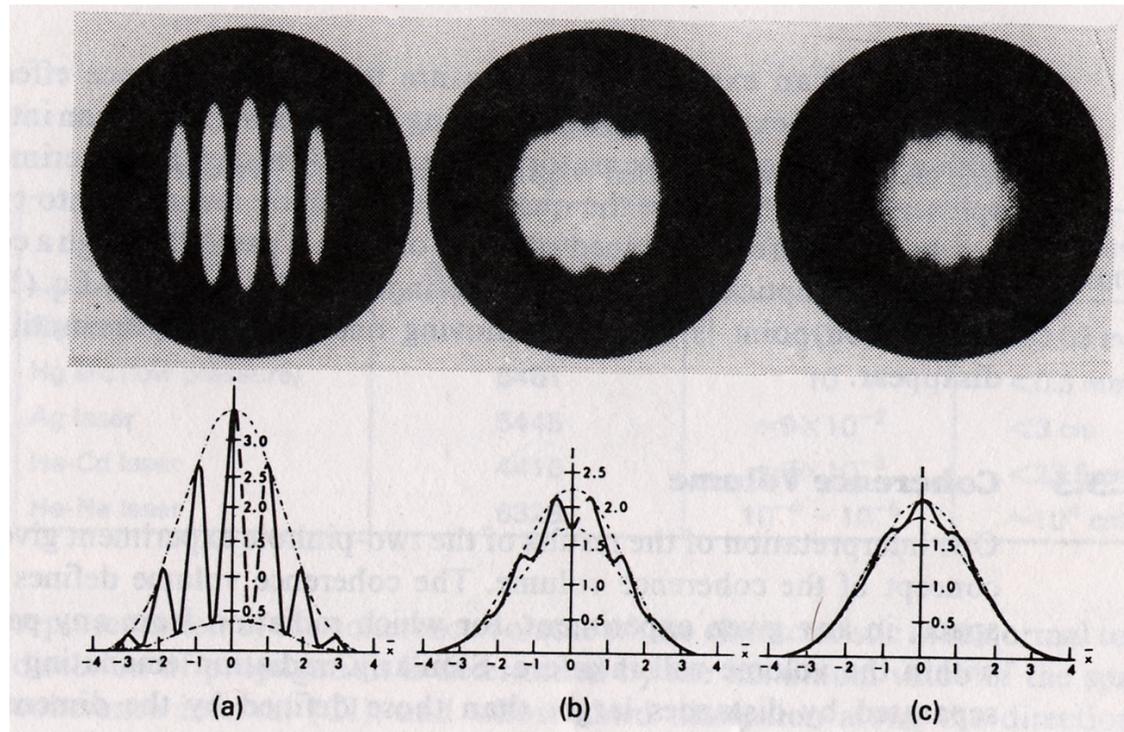
Hypothesis: Monochromatic source (but not a laser).

Description:

- The Source Hole Size is increased.
- Everything else is kept equal.



Effect of the Source Hole Size: II. Results



Fringes disappear! \Rightarrow { Fringe contrast is linked to the
spatial properties of the source.

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2} |C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

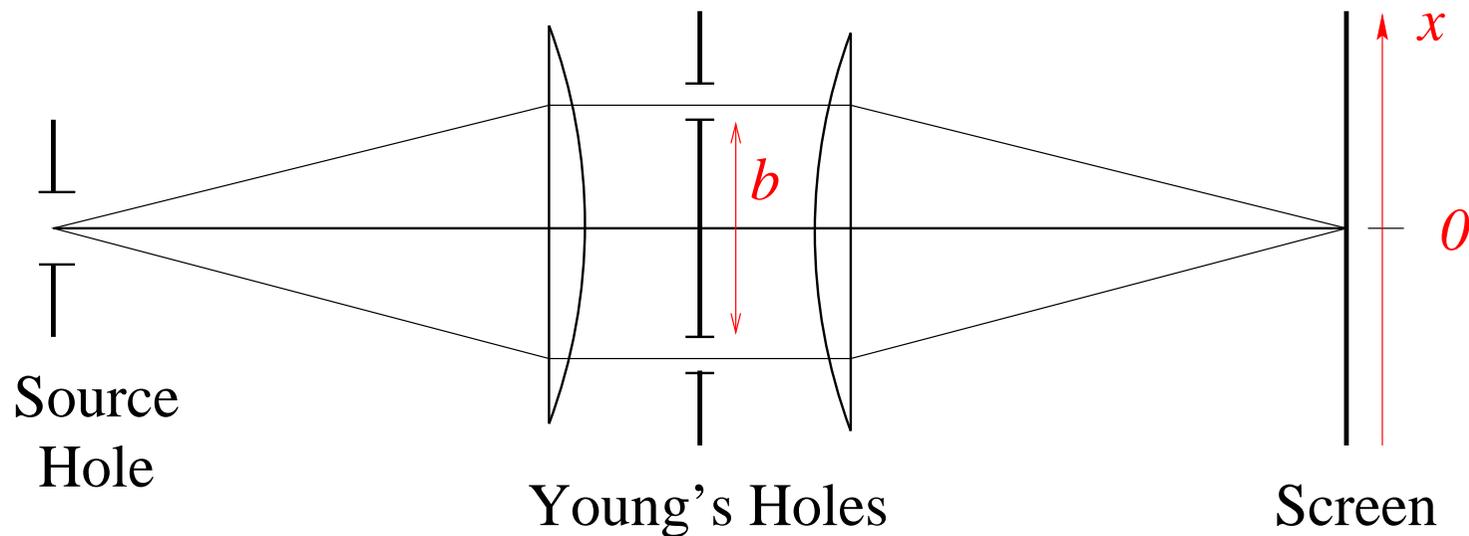
Effect of the Distance Between Young's Holes: I. Description

Hypothesis:

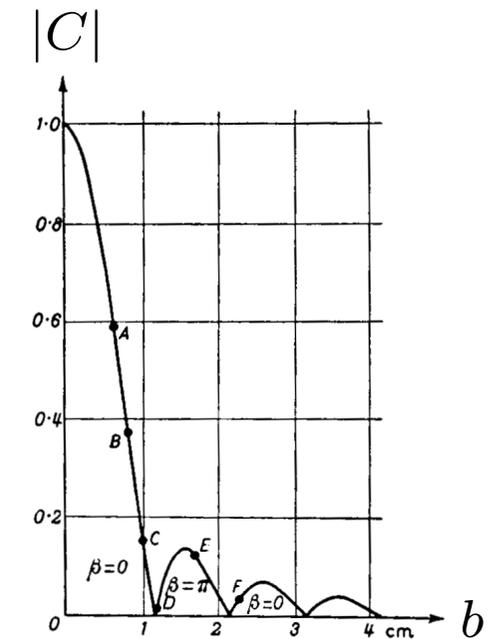
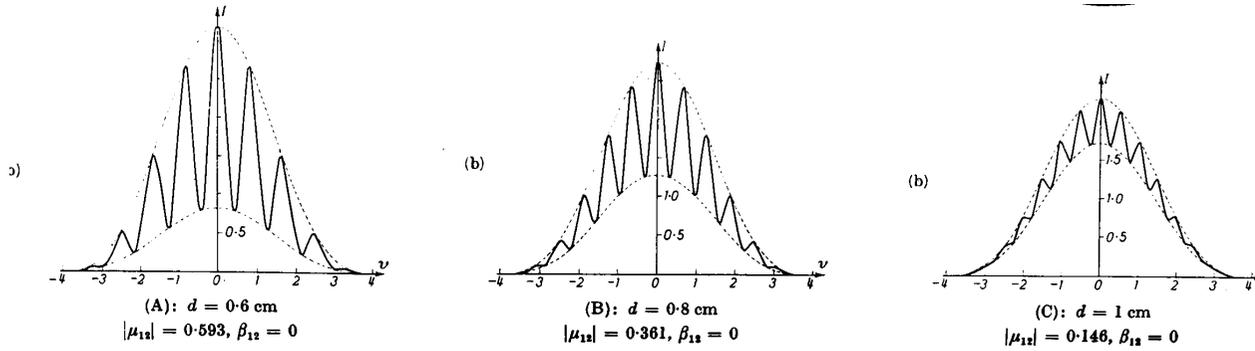
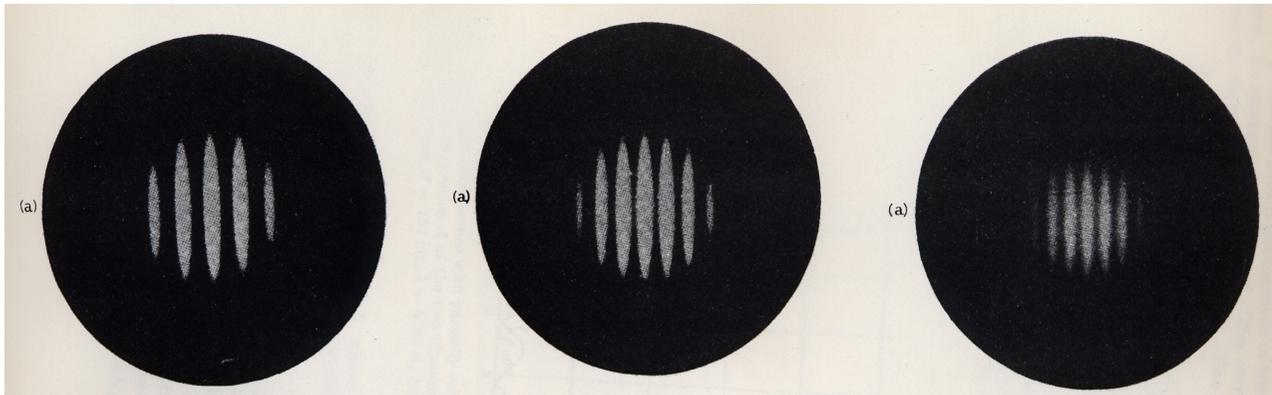
- Monochromatic source (but not a laser).
- The source hole is a circular disk.

Description:

- The distance between the two Young's holes is increased.
- Everything else is kept equal (in particular the hole size).

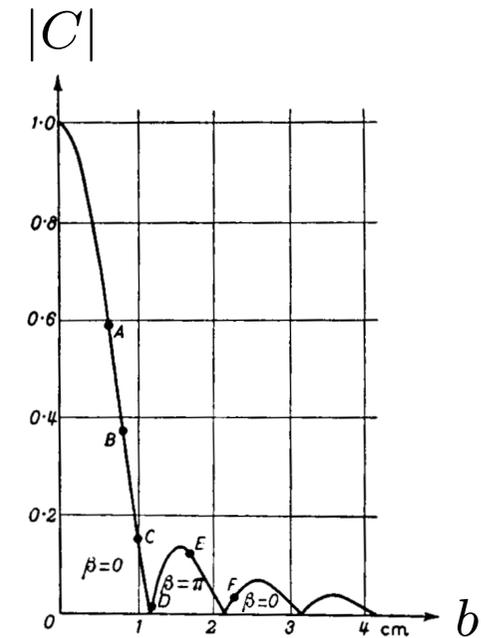
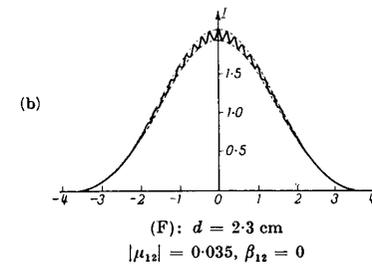
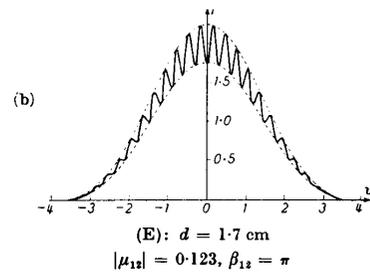
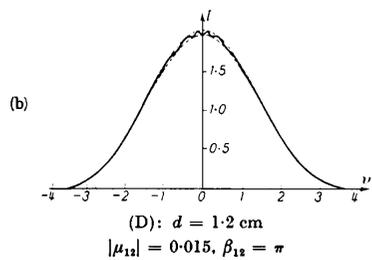
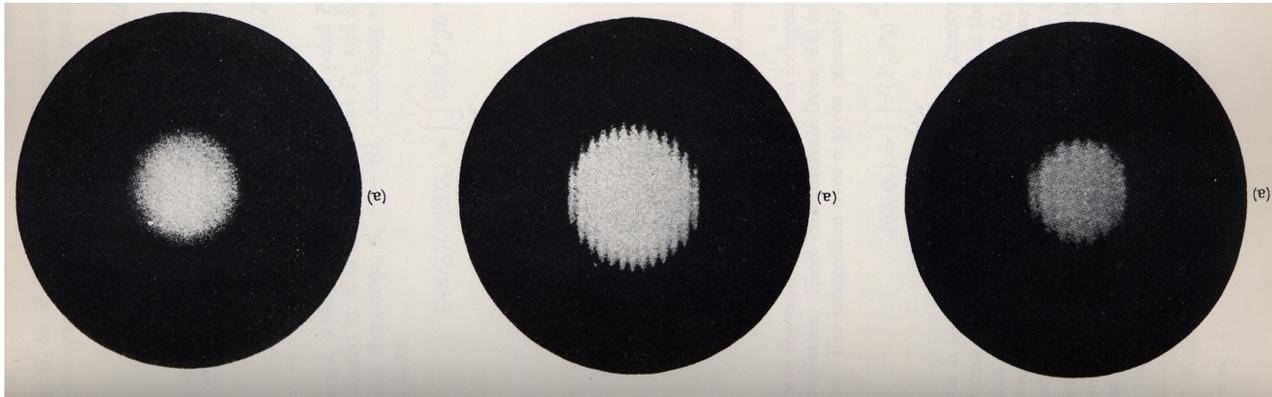


Effect of the Distance Between Young's Holes: II. Results



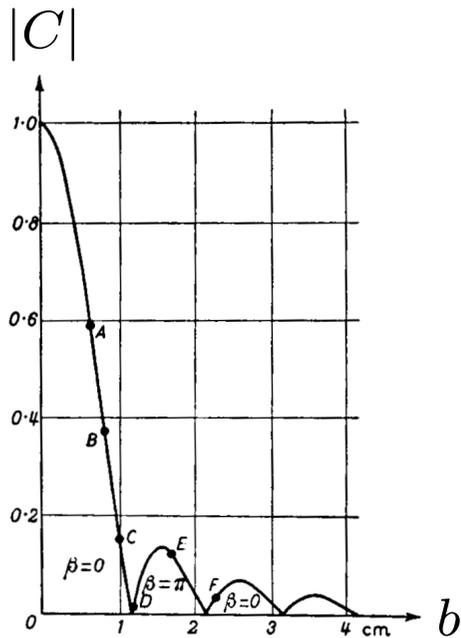
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Effect of the Distance Between Young's Holes: II. Results (Continued)

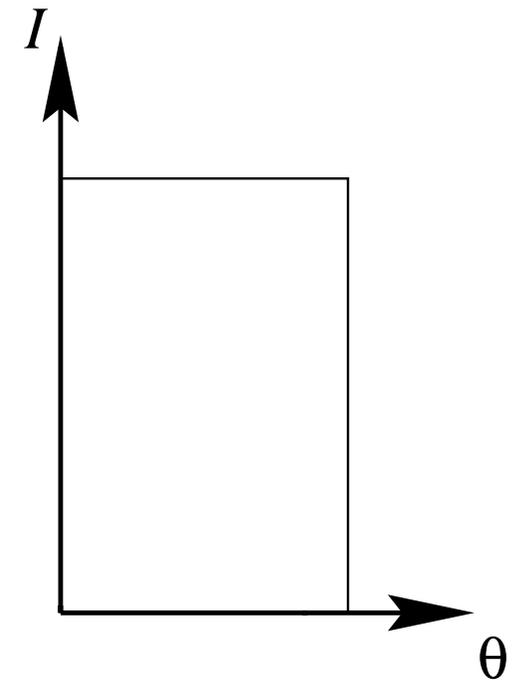


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Measured Curve = 2D Fourier Transform of the Source



$$\frac{J_1(b)}{b} \stackrel{\text{2D FT}}{\Leftrightarrow} \text{Heaviside}(\theta)$$



Source = Uniformly illuminated disk.

Theoretical Basis of the Aperture Synthesis

The van Citter-Zernike theorem

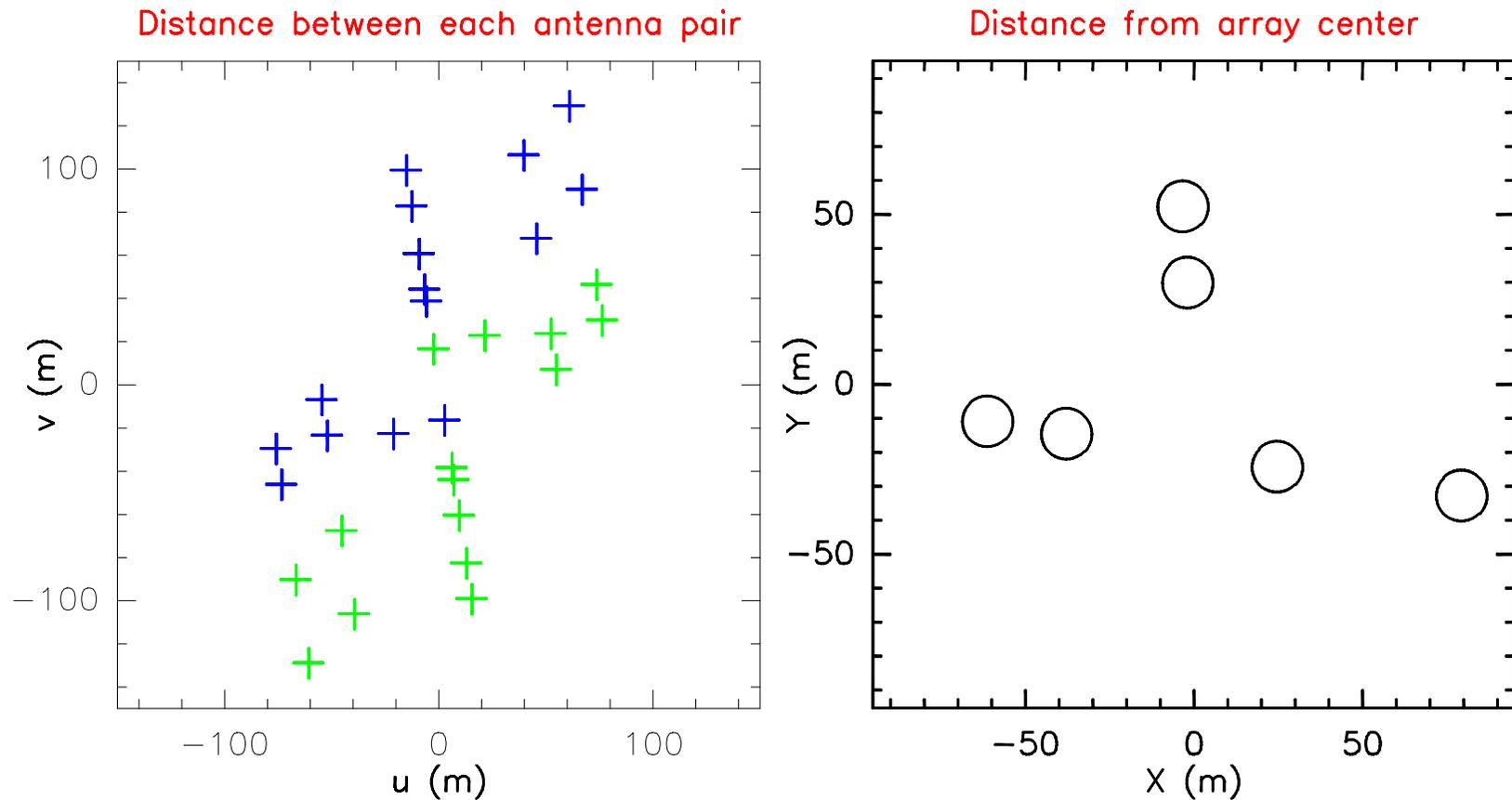
$$V_{ij}(b_{ij}) = C_{ij}(b_{ij}) \cdot I_{\text{tot}} \stackrel{\text{2D FT}}{\Leftrightarrow} B_{\text{primary}} \cdot I_{\text{source}}$$

- Young's holes = Telescopes;
 - Signal received by telescopes are combined by pairs;
 - Fringe visibilities are measured.
- ⇒ One Fourier component of the source (*i.e.* one visibility) is measured by baseline (or antenna pair).
- ⇒ Each baseline length b_{ij} = a spatial frequency.
 - ⇒ Convention: Spatial frequencies are measured in meter.

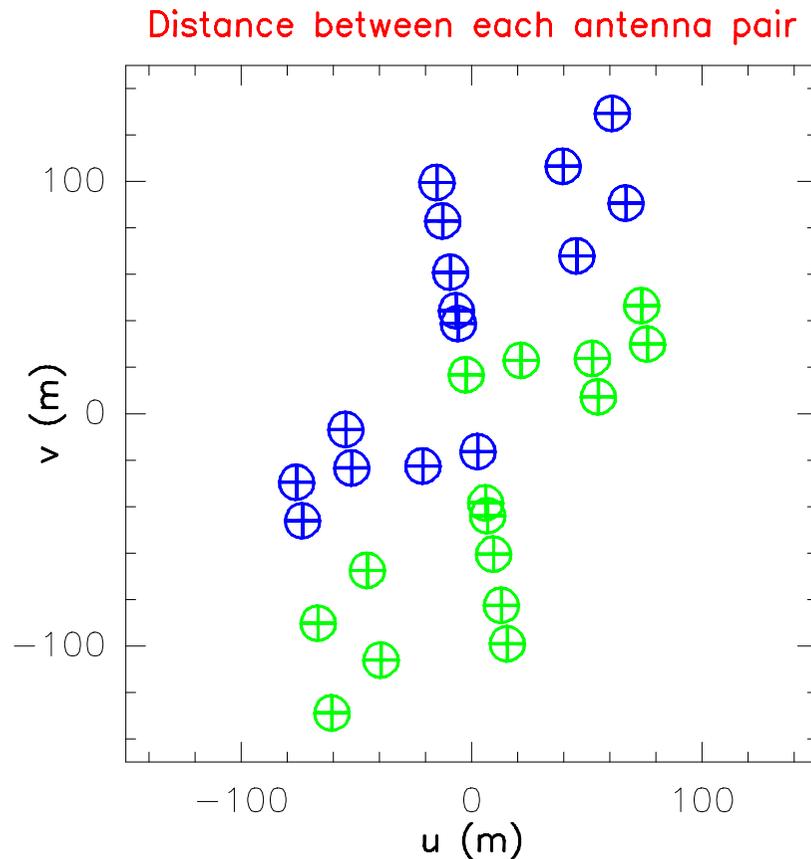
An Example: The PdBI

Number of baselines: $N(N - 1) = 30$ for $N = 6$ antennas.

Convention: Fourier plane = uv plane.



Each Visibility is a Weighted Sum of the Fourier Components of the Source



$$V_{ij}(b_{ij}) \stackrel{2D \text{ FT}}{\Leftrightarrow} B_{\text{primary}} \cdot I_{\text{source}}$$

i.e. $V_{ij}(b_{ij}) = \{ \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}} \} (b_{ij})$

with $\tilde{B}_{\text{primary}}$ a Gaussian of FWHM=15 m.

\Rightarrow { Indirect information on the source
(important for mosaicing).

Mathematical Properties of Fourier Transform

- 1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \xLeftrightarrow{\text{FT}} \tilde{F}_1 \text{ and } F_2 \xLeftrightarrow{\text{FT}} \tilde{F}_2), \text{ then } F_1 \cdot F_2 \xLeftrightarrow{\text{FT}} \tilde{F}_1 * \tilde{F}_2.$$

- 2 Sampling size $\xLeftrightarrow{\text{FT}}$ Image size.

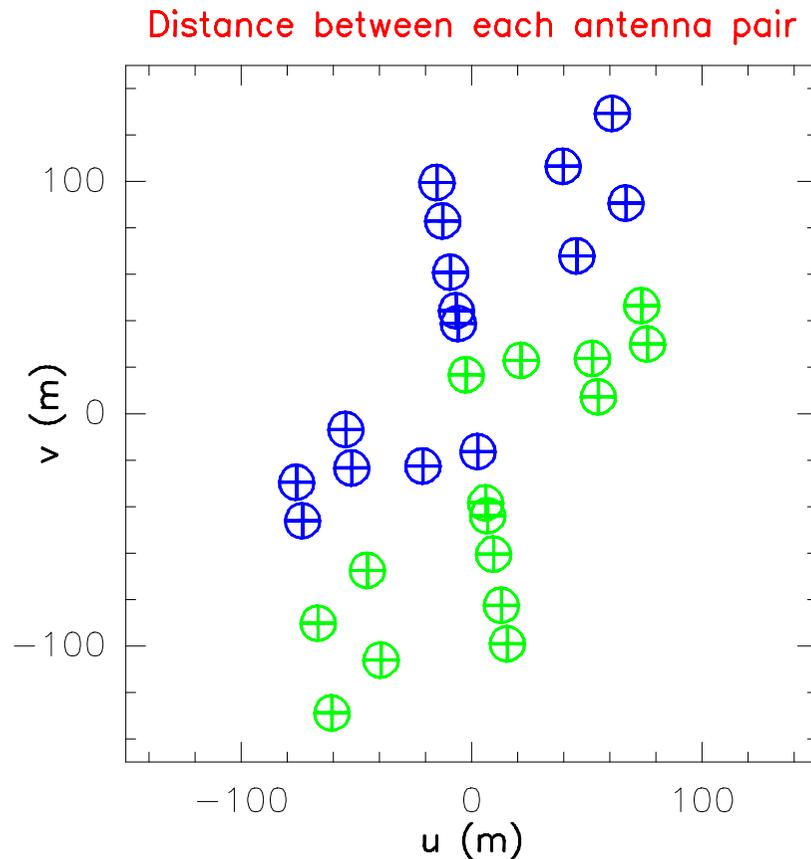
- 3 Bandwidth size $\xLeftrightarrow{\text{FT}}$ Pixel size.

- 4 Finite support $\xLeftrightarrow{\text{FT}}$ Infinite support.

- 5 Fourier transform evaluated at zero spacial frequency
= Integral of your function.

$$V(u = 0, v = 0) \xLeftrightarrow{\text{FT}} \sum_{ij \in \text{image}} I_{ij}.$$

Each Visibility is a Weighted Sum of the Fourier Components of the Source



$$V_{ij}(b_{ij}) \stackrel{2D\ FT}{\rightleftharpoons} B_{\text{primary}} \cdot I_{\text{source}}$$

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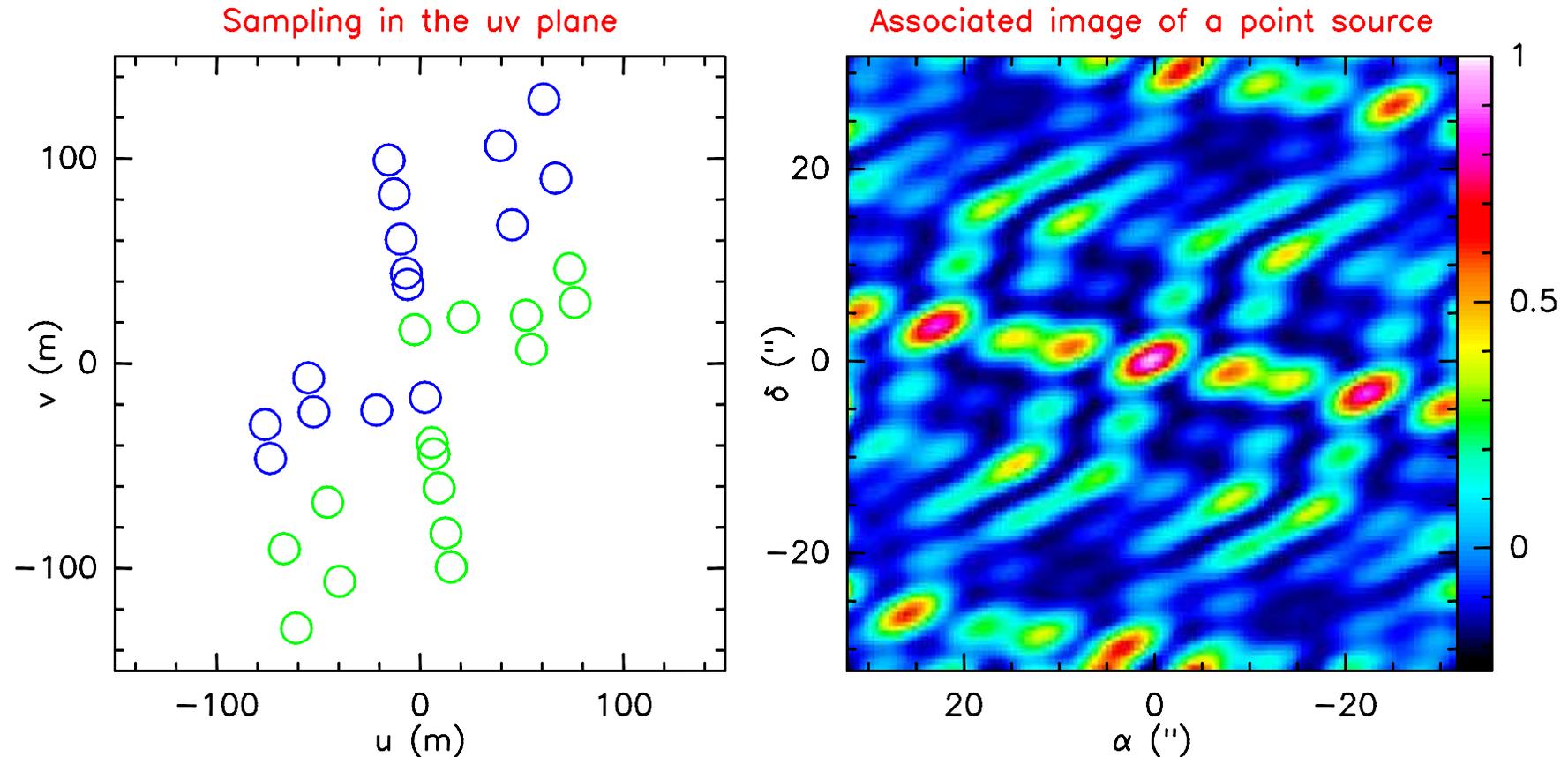
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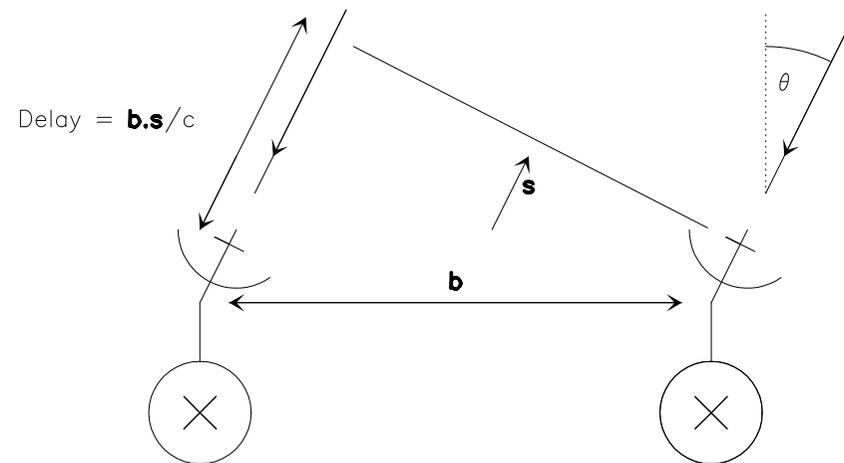
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Incomplete uv plane coverage \Rightarrow difficult to make a reliable image
(Lectures by A. Castro-Carrizo, J. Pety and F. Gueth).

Earth Rotation and Super Synthesis

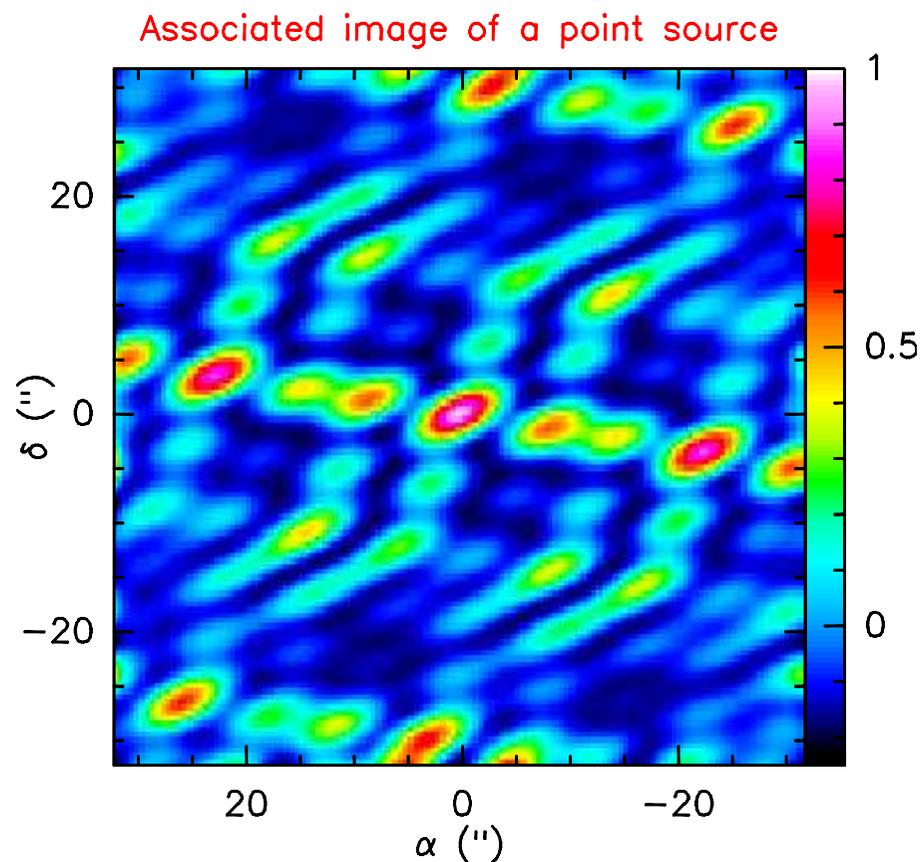
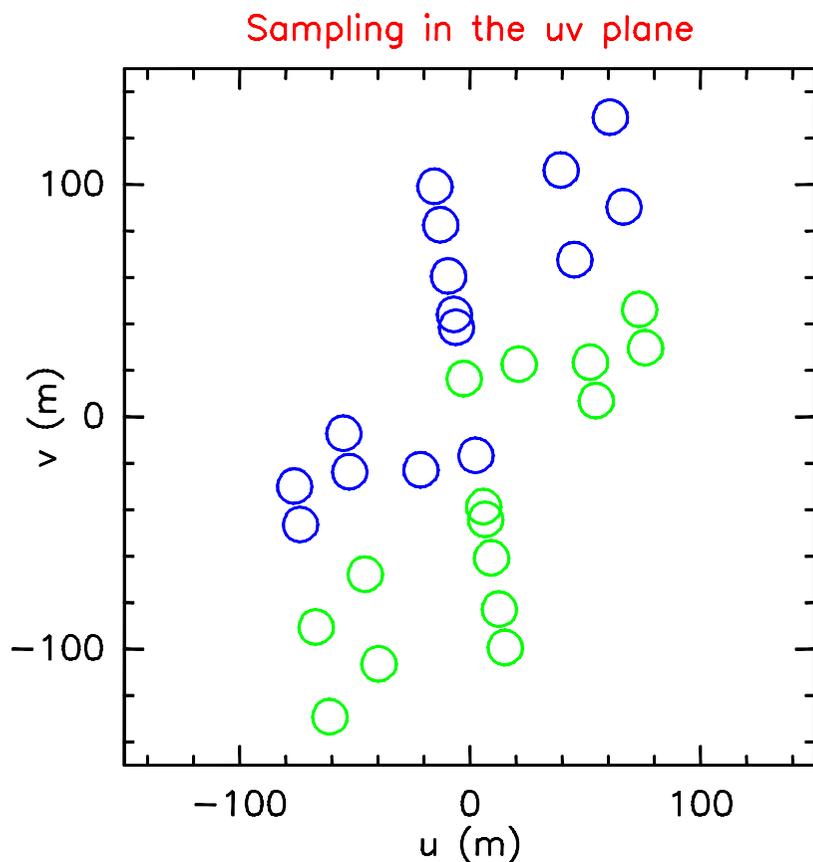
Precision: Spatial frequencies = baseline lengths **projected** in a plane perpendicular to the source mean direction.



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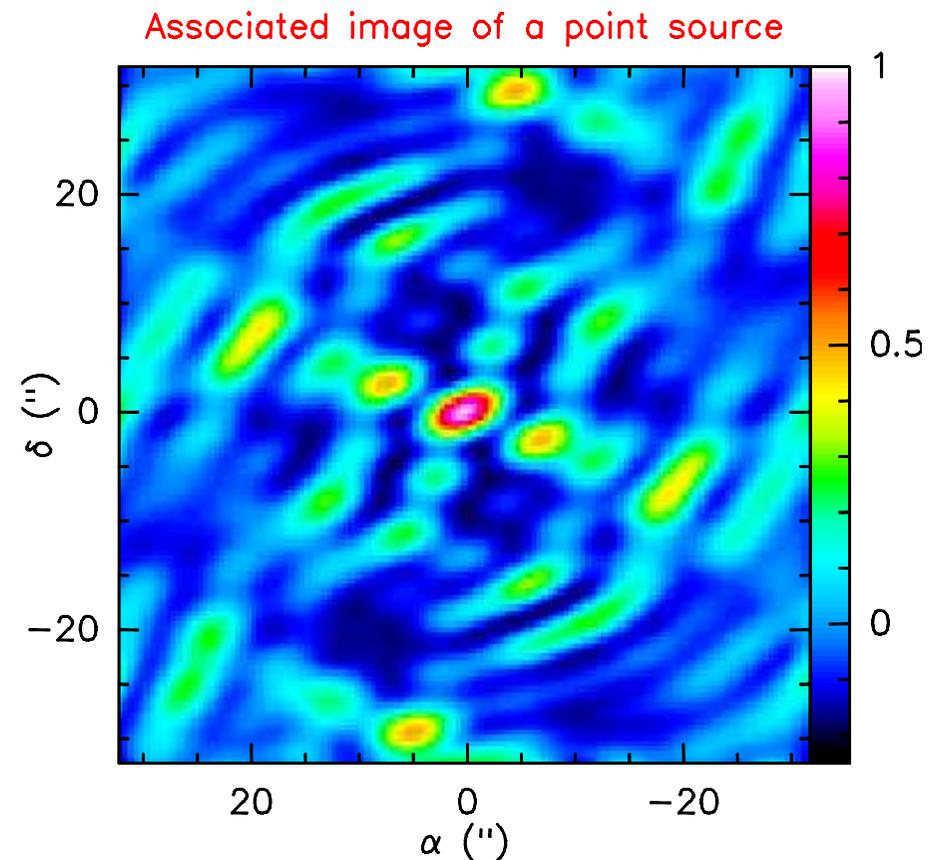
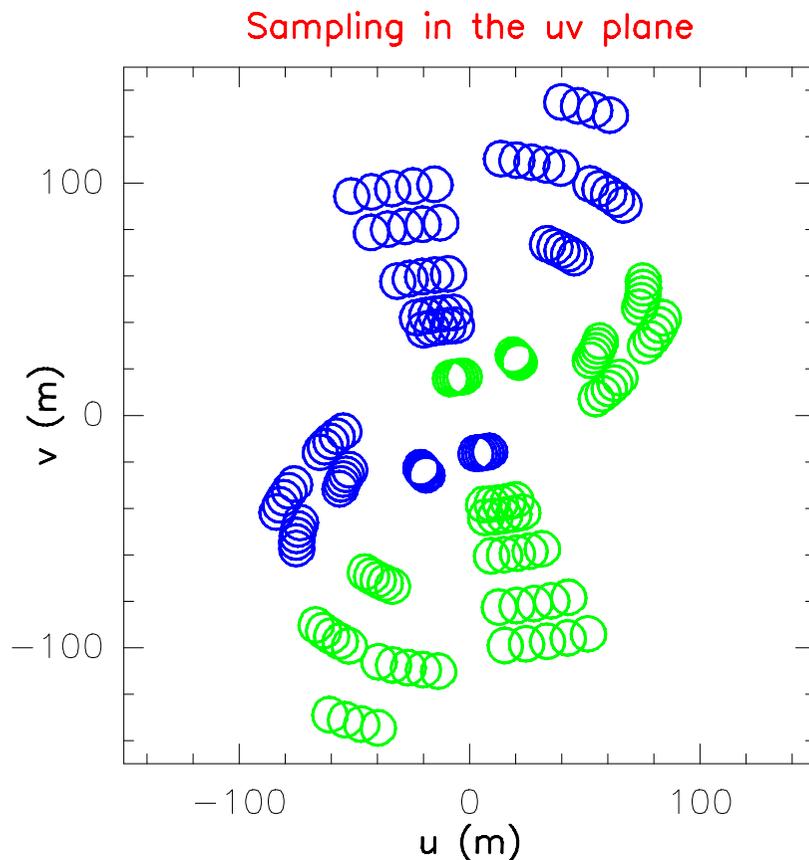
Advantage: Possibility to measure different Fourier components without moving antennas!



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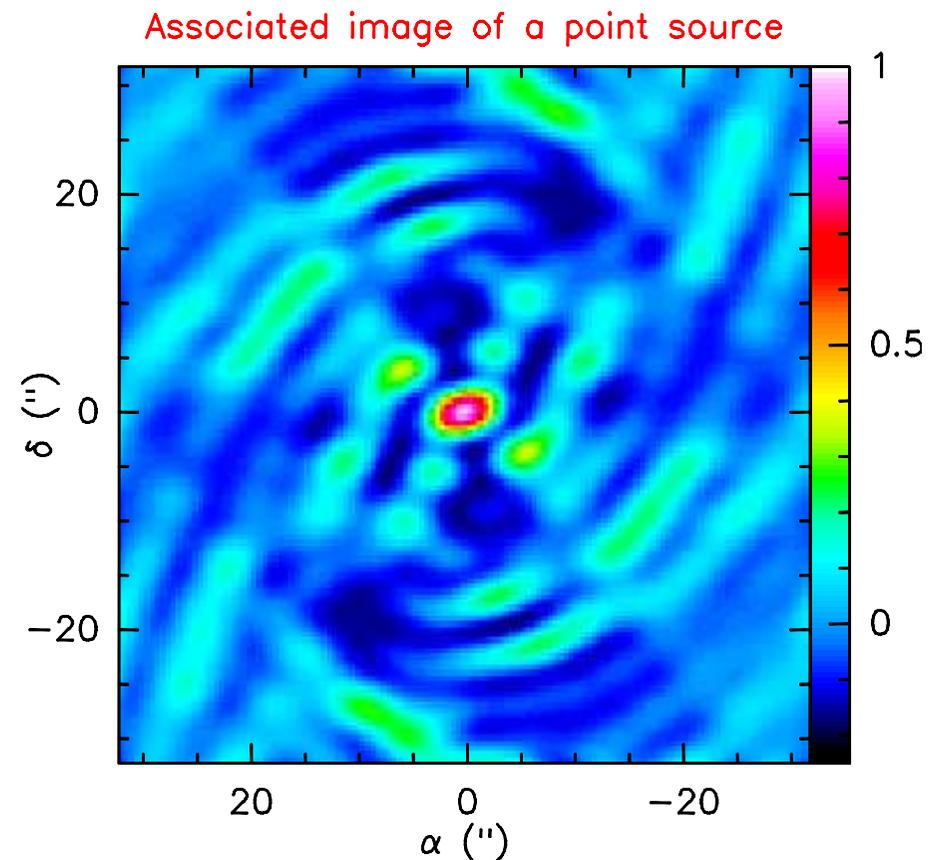
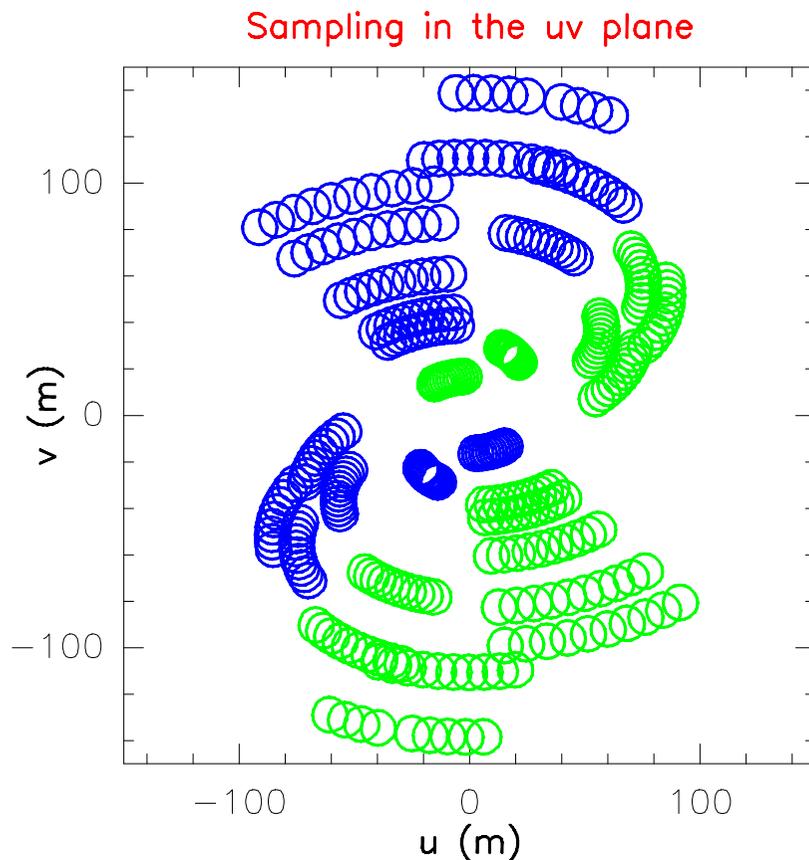
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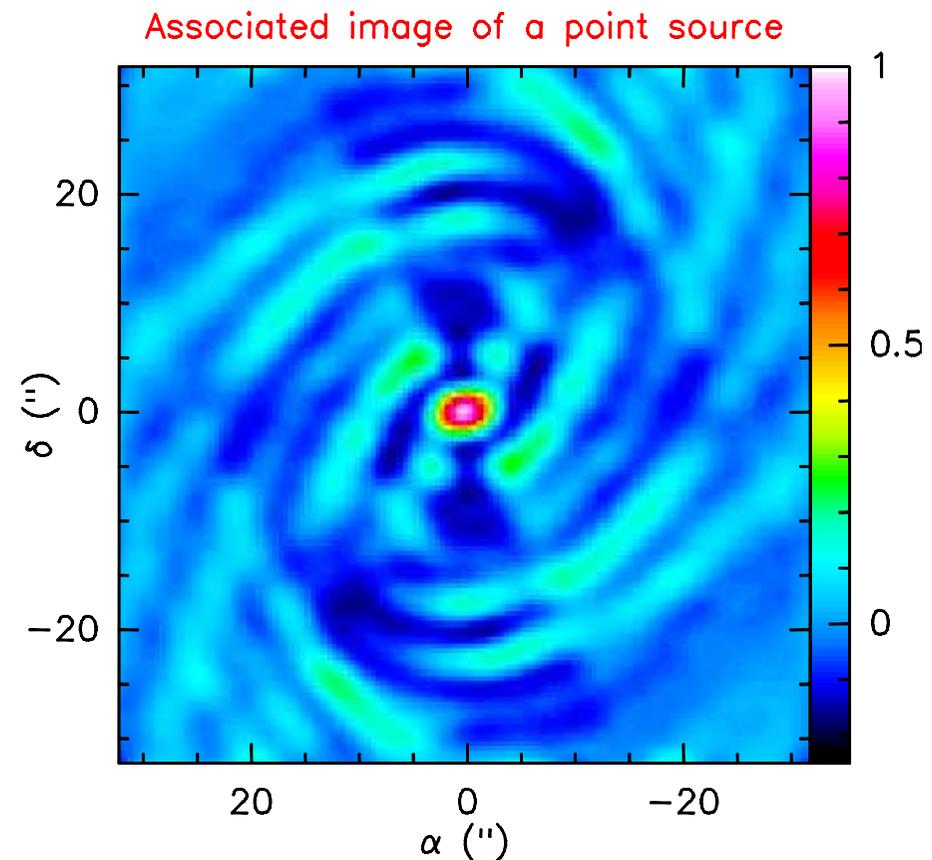
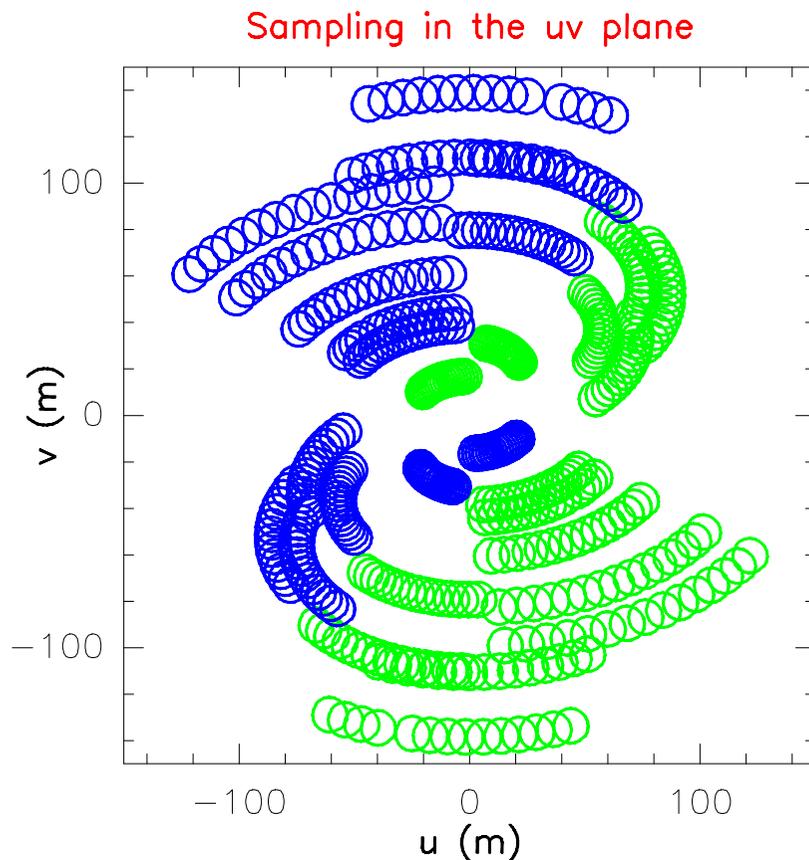
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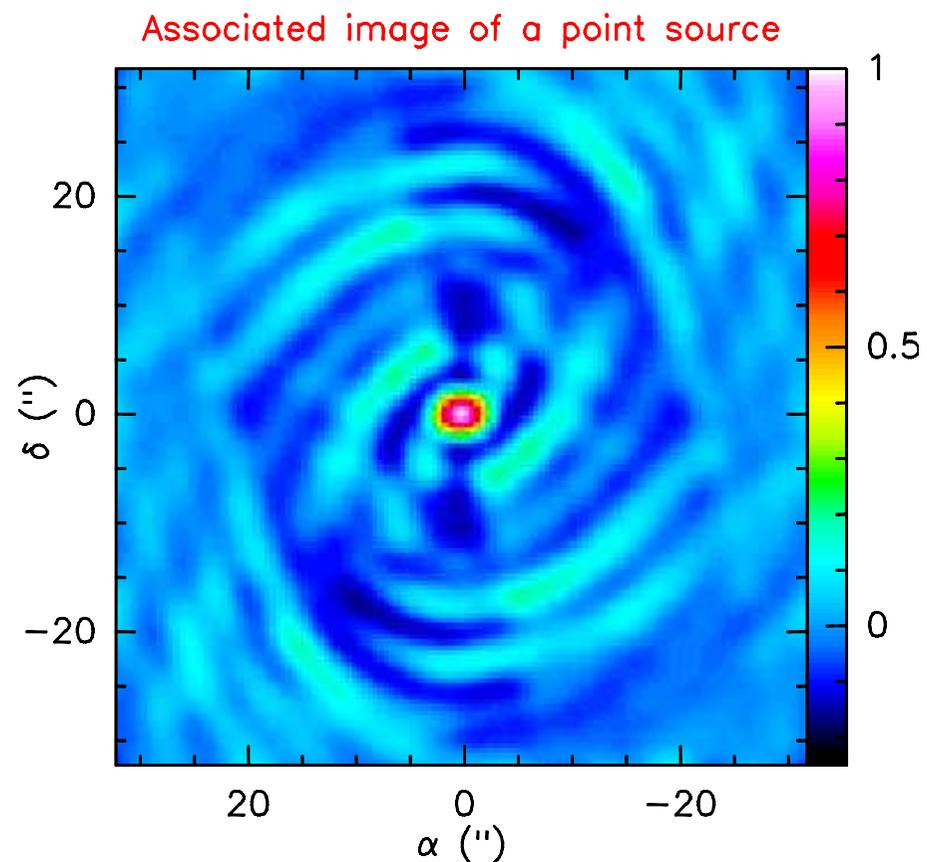
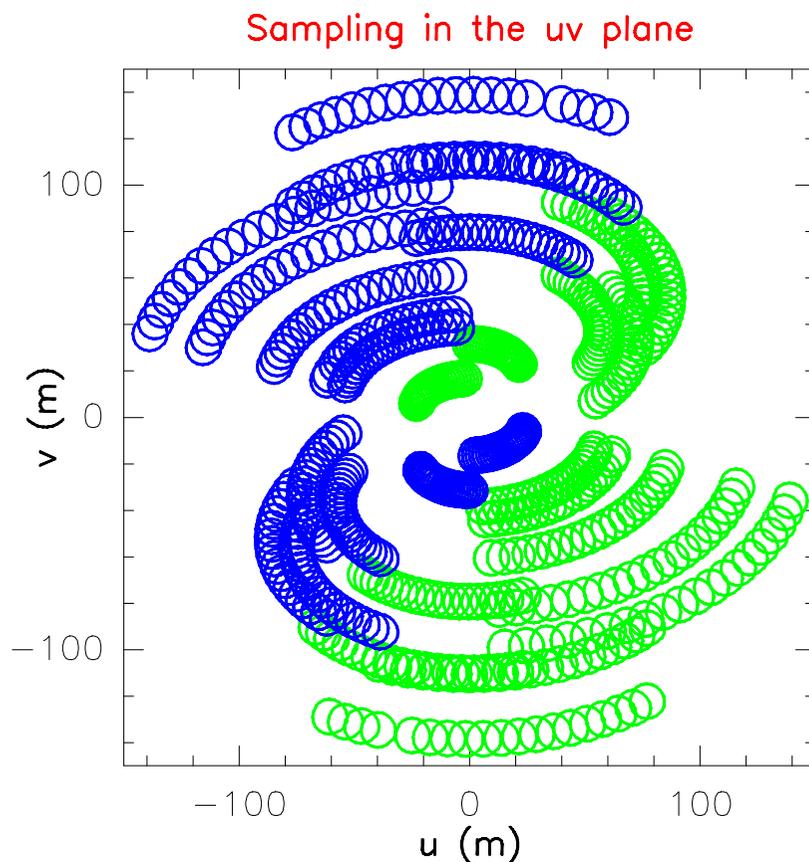
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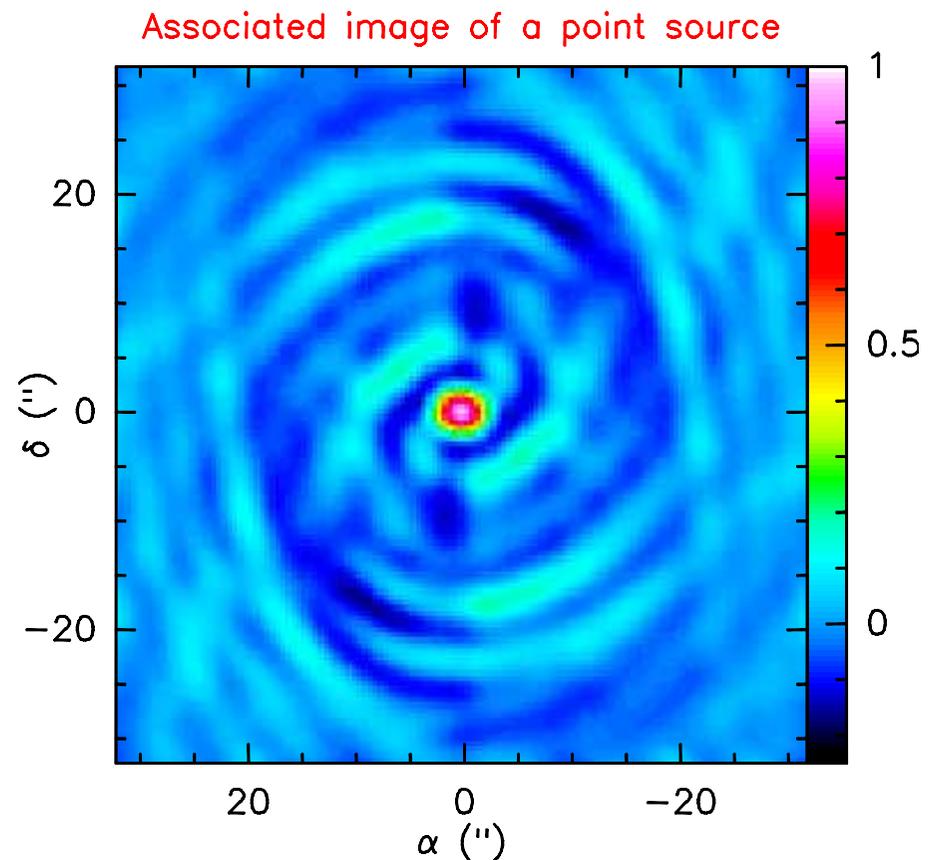
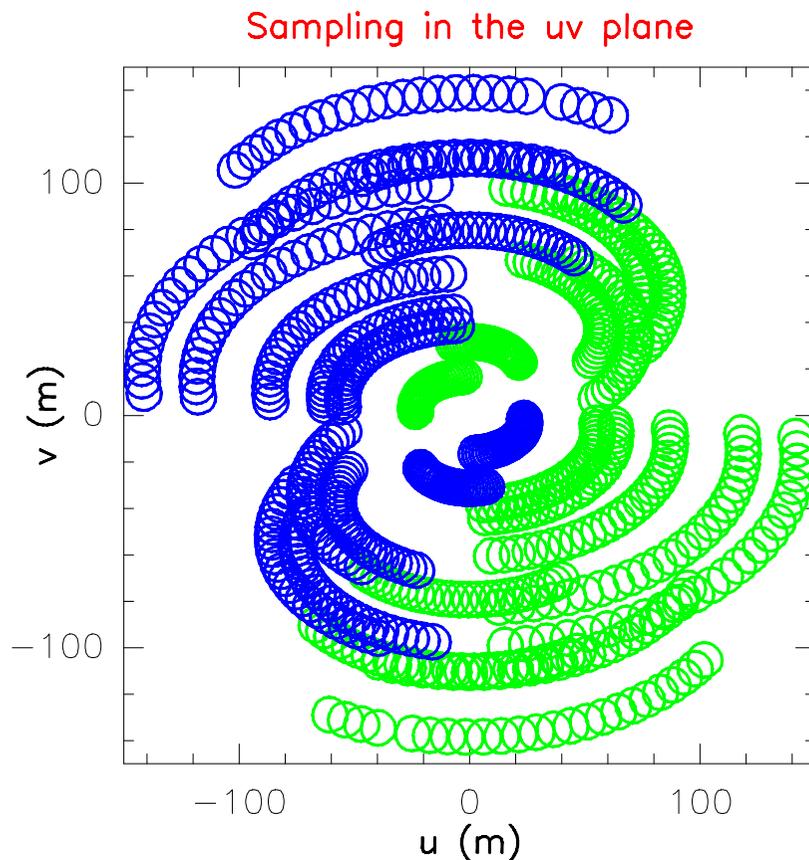
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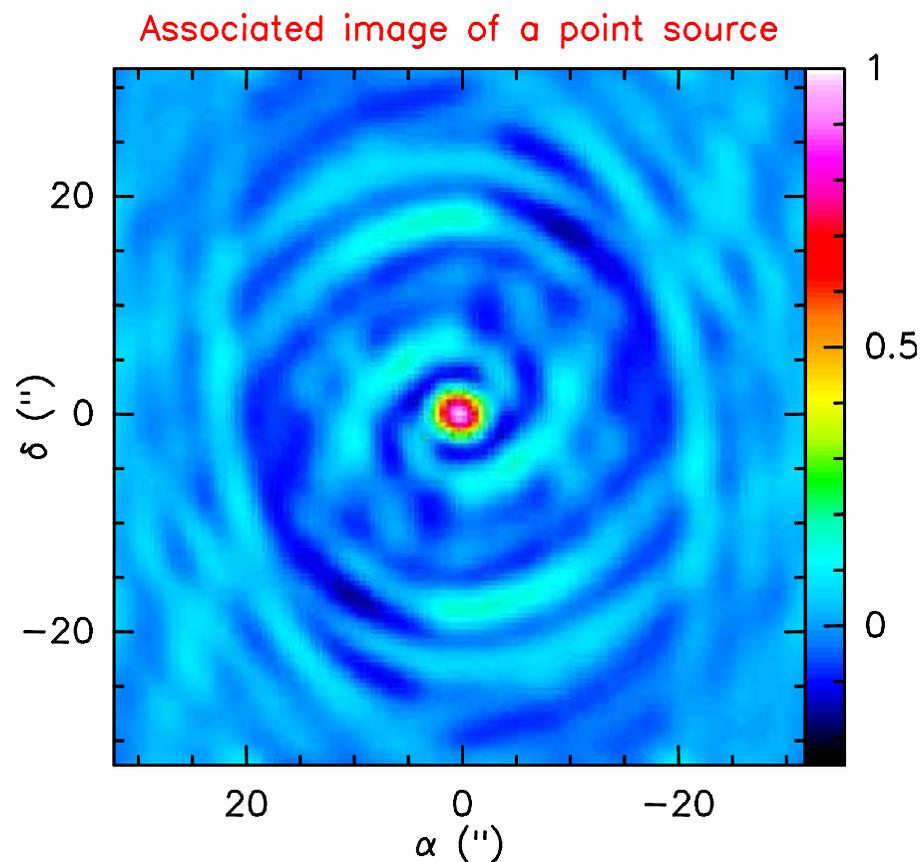
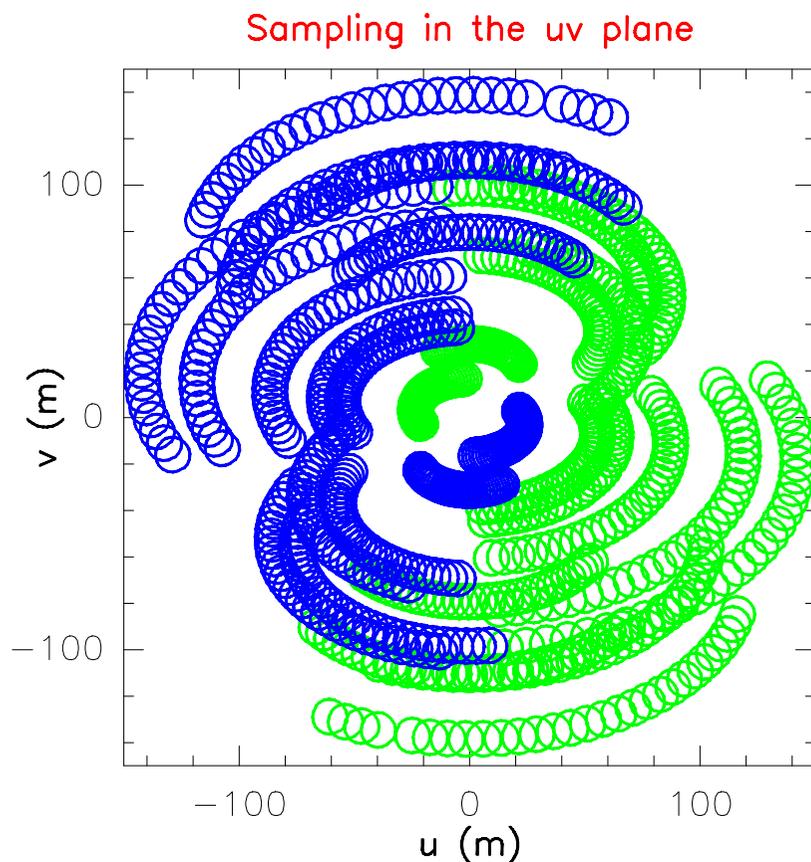
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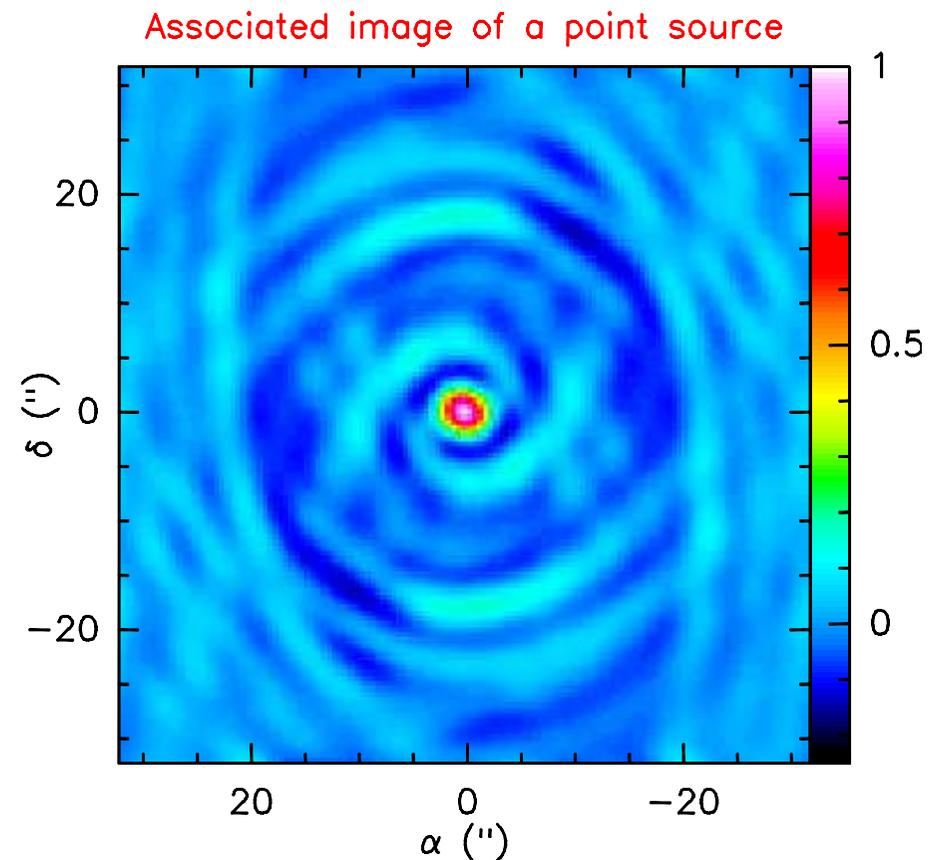
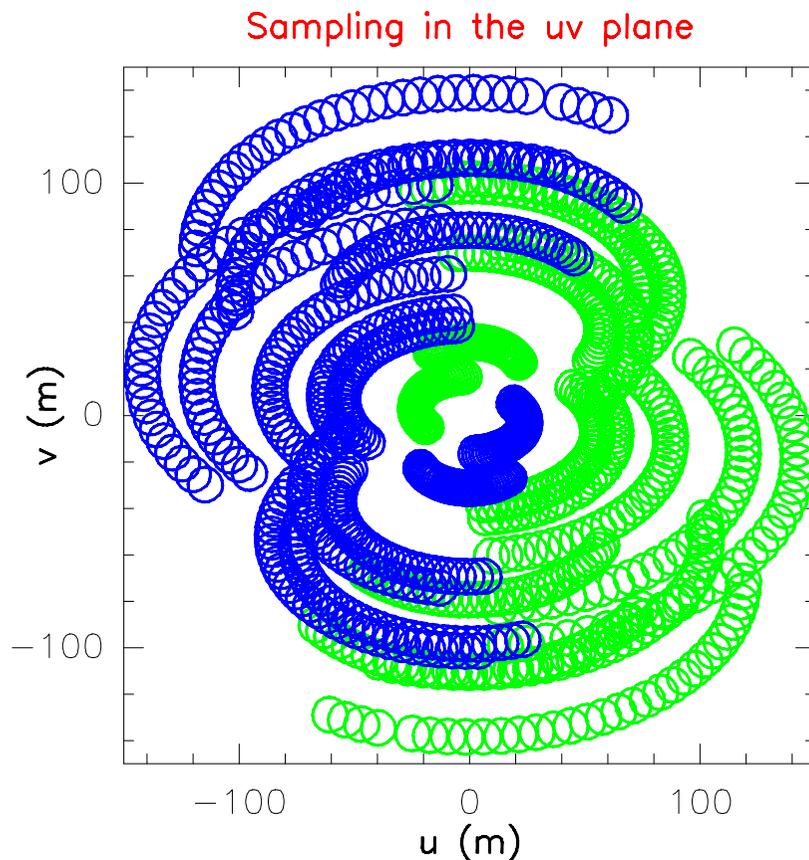
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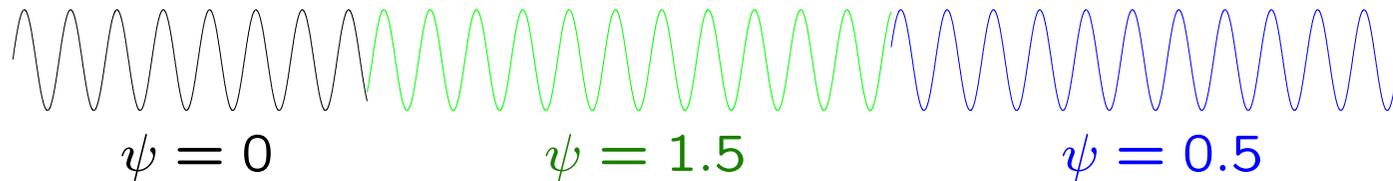
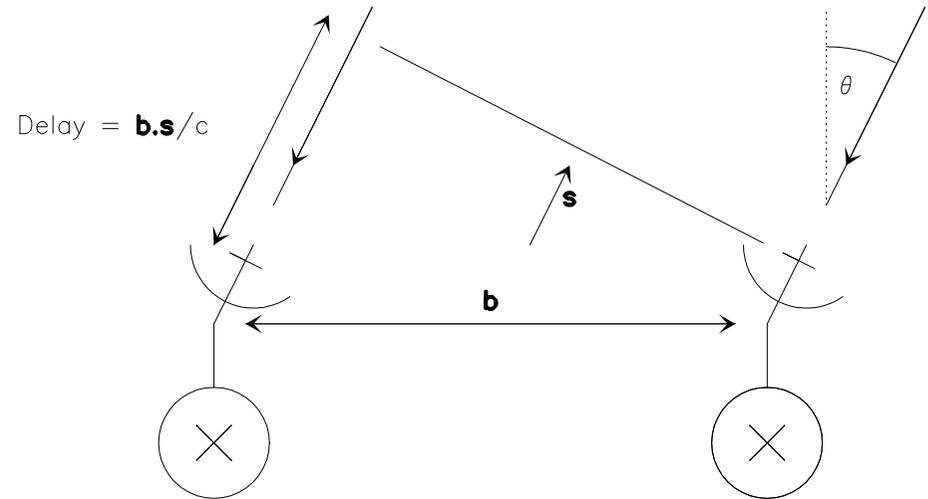
Delay Correction: I. Why?

Real life: Source **not** at zenith.

⇒ { Wave plane arrives at different moment on each antenna.

Temporal coherence:

- $E(t) = E_0 \cos(\omega t + \psi)$
- Temporally Incoherent Source = random phase changes.
- Coherence time: mean time over which wave phase = constant.

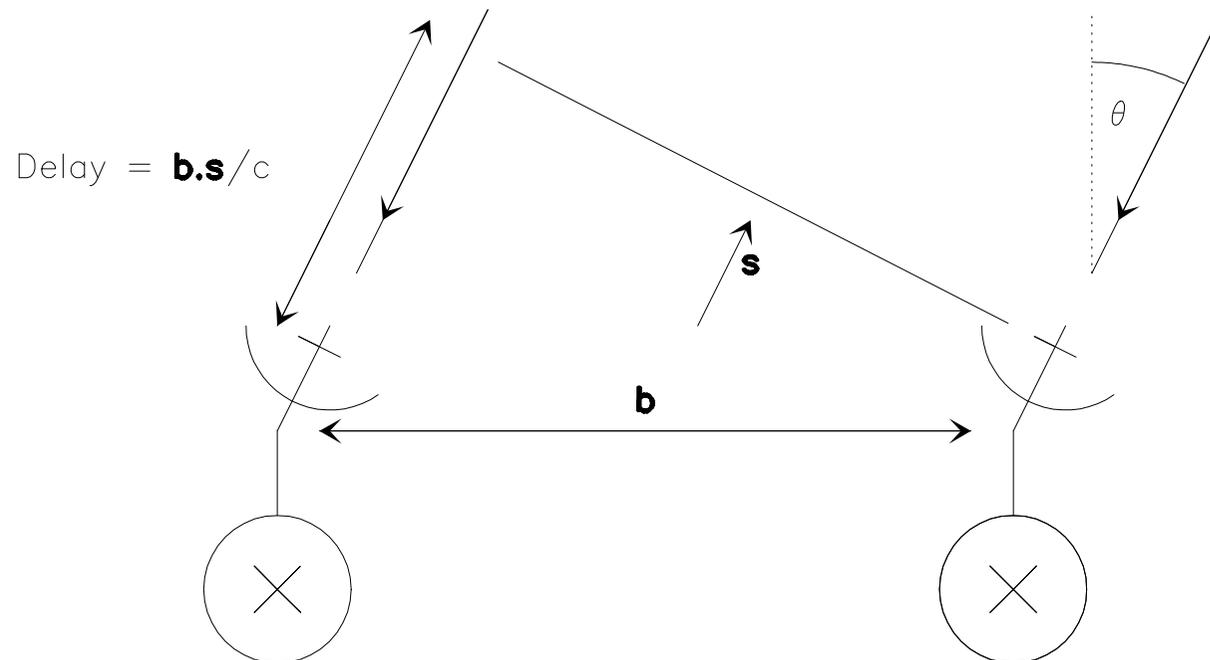


Problem: (Coherence time \lesssim delay) ⇒ fringes disappear!

Delay Correction: II. Earth rotation

Earth rotation:

- Advantage: Super synthesis;
- Inconvenient: Delay correction varies with time!



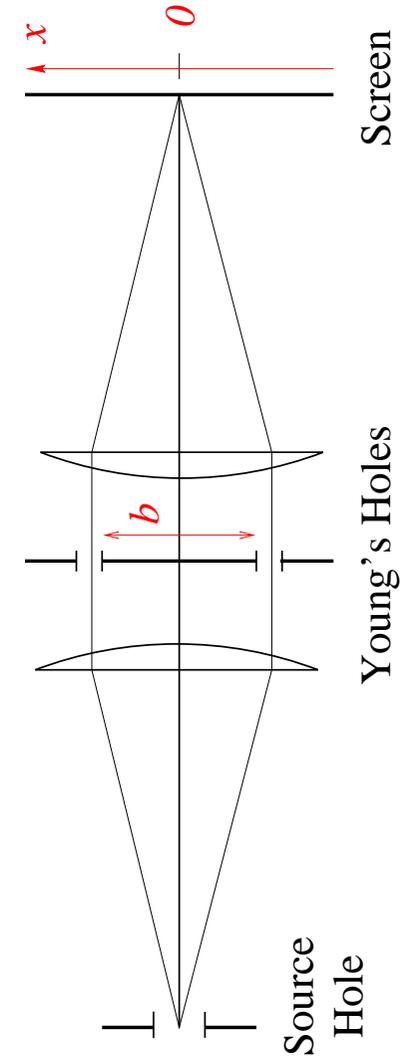
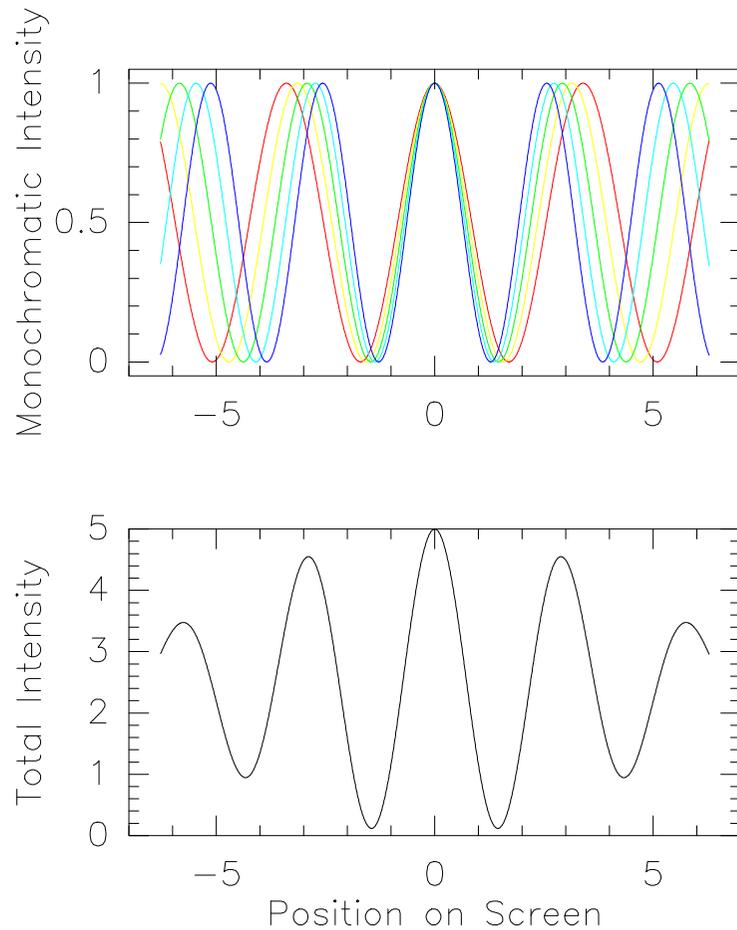
Delay Correction: III. Finite Bandwidth

Real life: Observation of finite bandwidth.

⇒ polychromatic light.

Perfect delay correction

⇒ White fringes in 0.



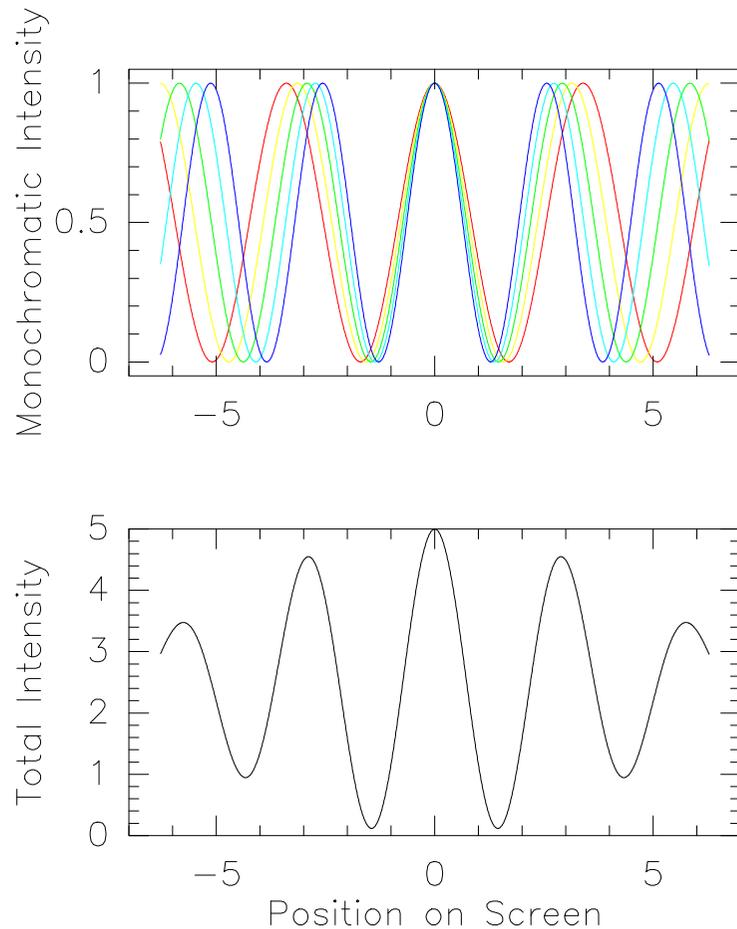
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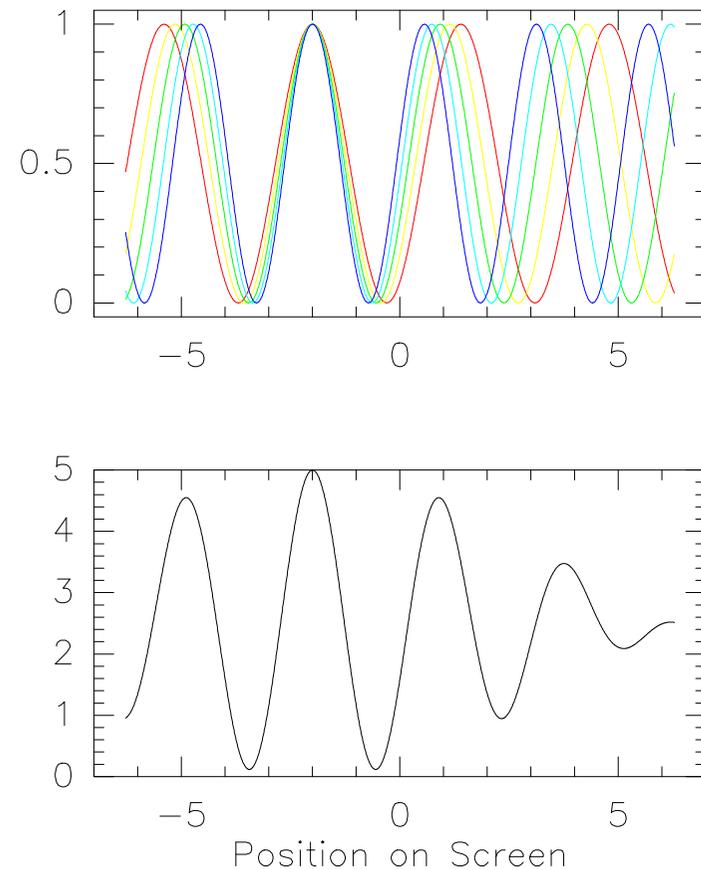
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Worse and worse delay correction.

⇒ Translation of the fringe pattern.

⇒ Fringes seem to disappear.



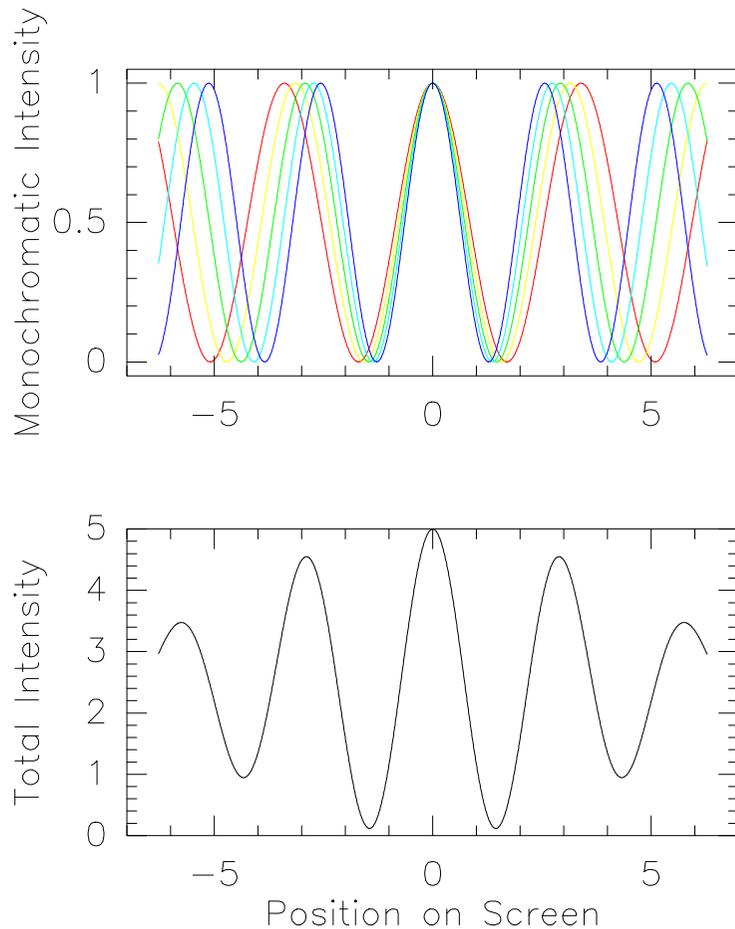
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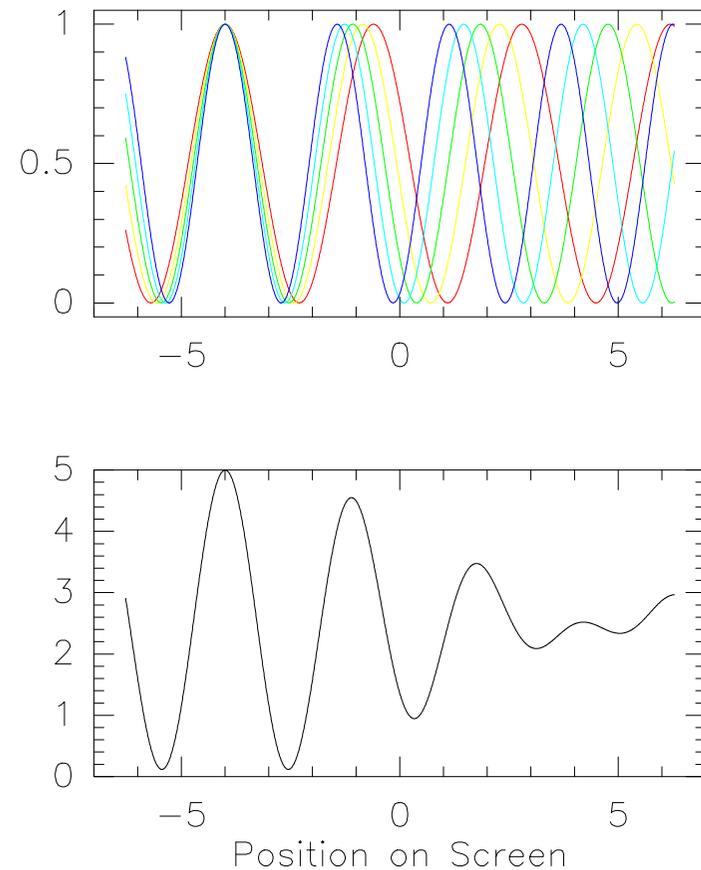
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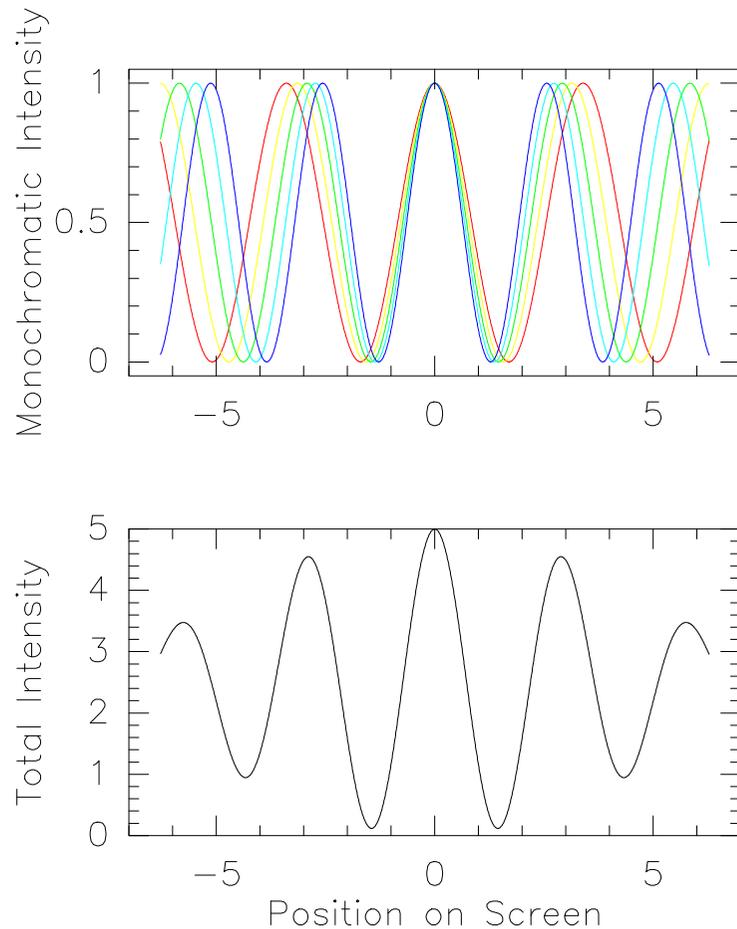
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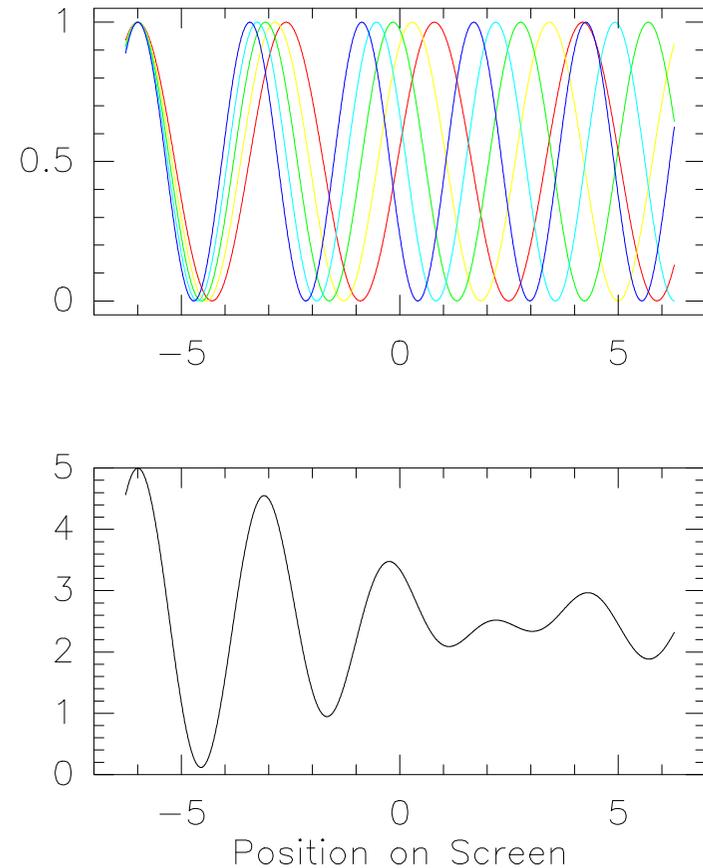
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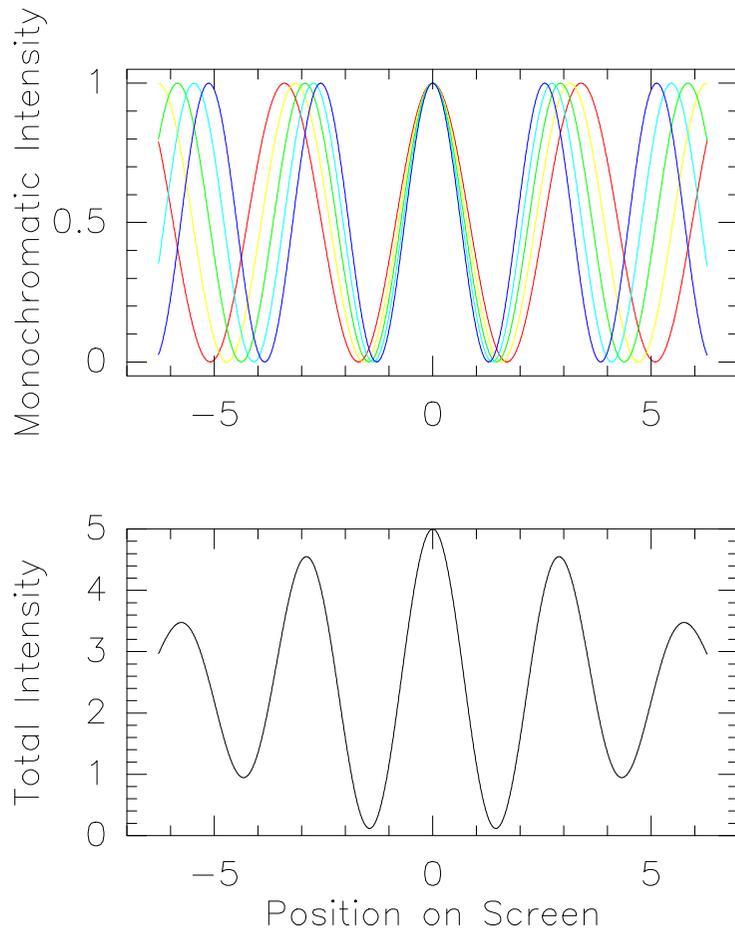
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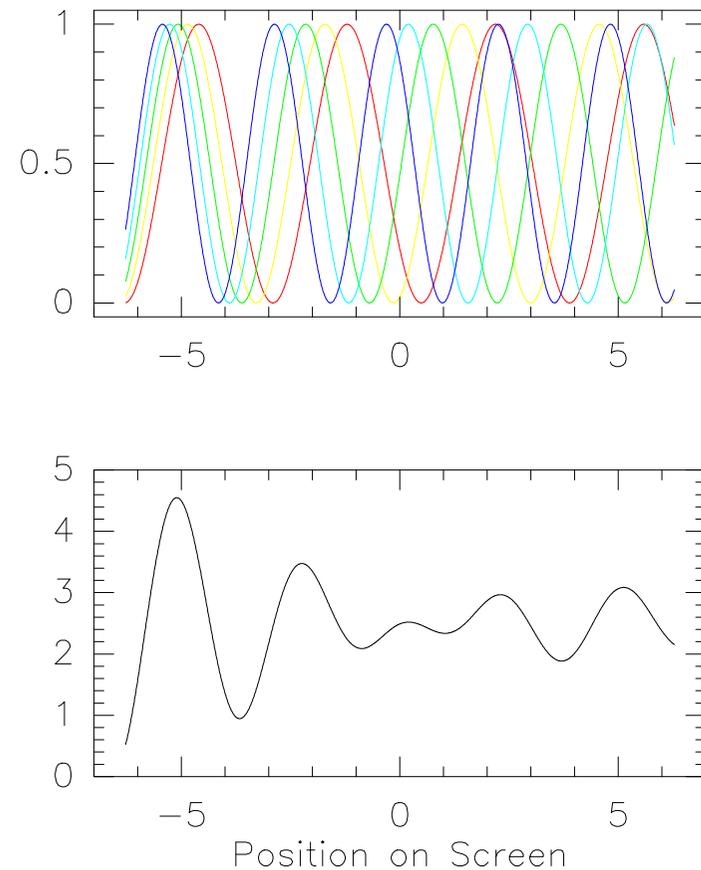
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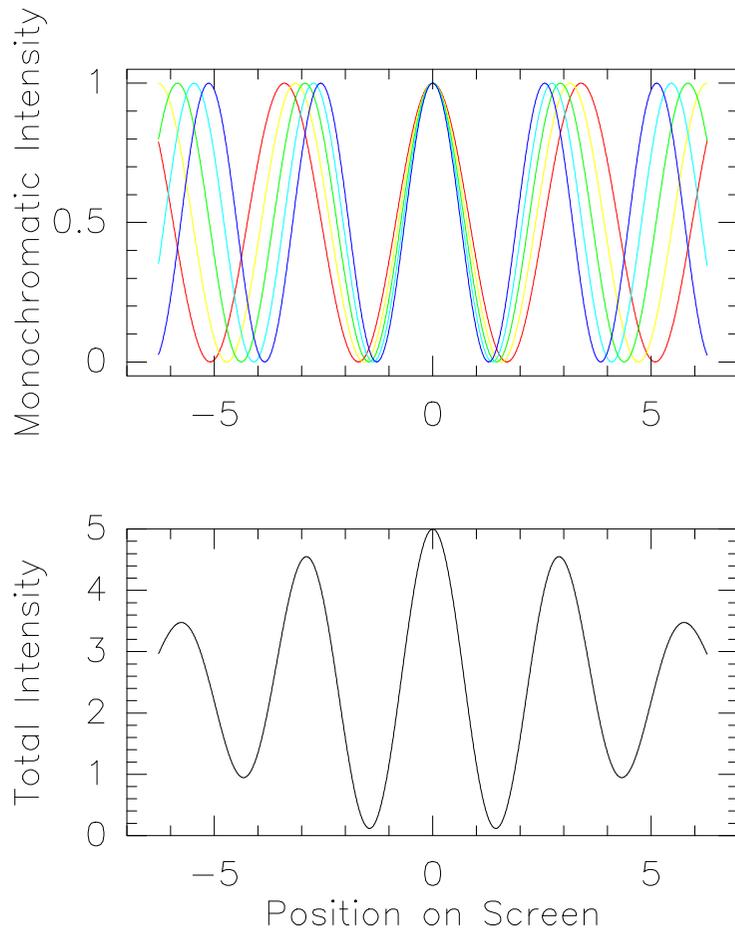
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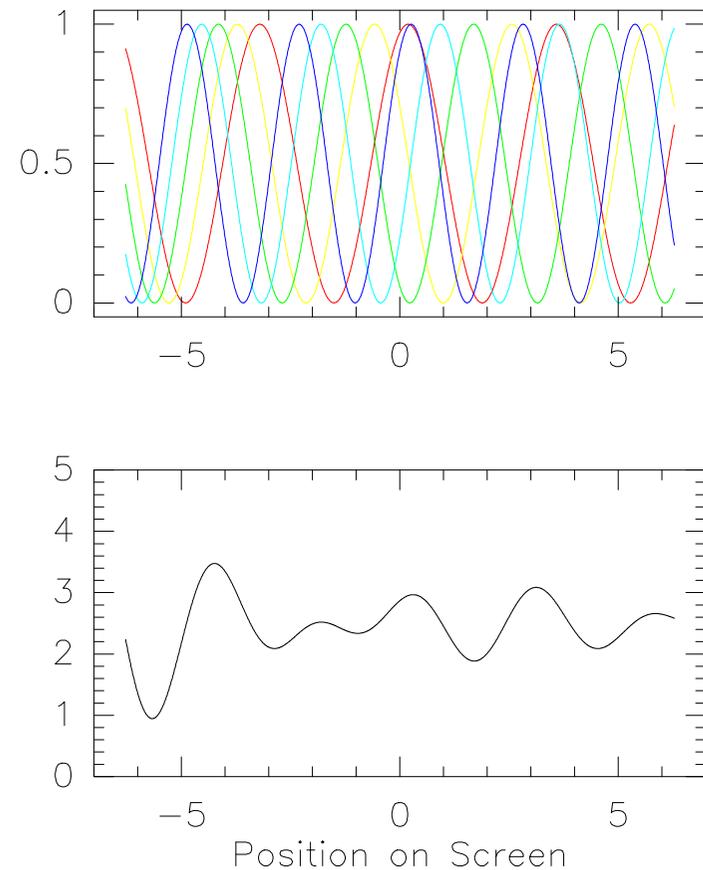
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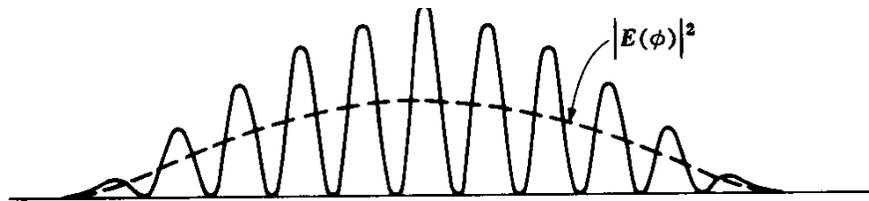
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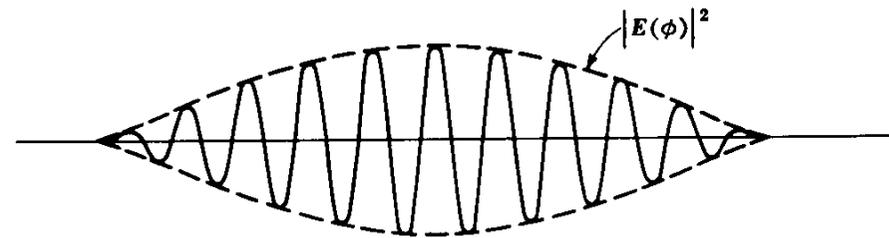


Optic vs Radio Interferometer: I. Measurement Method

	Optic	Radio
Detector { Kind Observable	Quadratic $I = EE^* $	Linear (Heterodyne) $ E \exp(i\psi)$
Measure { Method Quantity	Optical fringes $ C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$	Electronic correlation $ V \exp(i\phi_V) = \langle E_1 \cdot E_2 \rangle$
Interferometer kind	Additive	Multiplicative



$$I_1 + I_2 + 2\sqrt{I_1 I_2} |C| \cos\left(\frac{bx}{\lambda} + \phi_C\right)$$

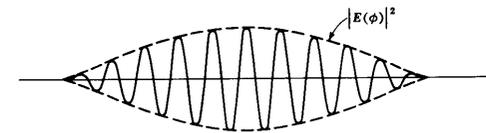
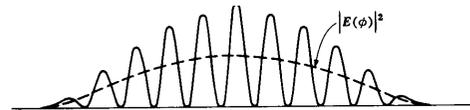


$$\overbrace{|E_1| |E_2| |C|}^{|V|} \cos\left(\frac{bx}{\lambda} + \phi_C\right)$$

(Heterodyne: lectures by F. Gueth and V. Piétu)

Optic vs Radio Interferometer: I. Measurement Method

	Optic	Radio
Detector { Kind Observable	Quadratic $I = EE^* $	Linear (Heterodyne) $ E \exp(i\psi)$
Measure { Method Quantity	Optical fringes $ C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$	Electronic correlation $ V \exp(i\phi_V) = \langle E_1 \cdot E_2 \rangle$
Interferometer kind	Additive	Multiplicative

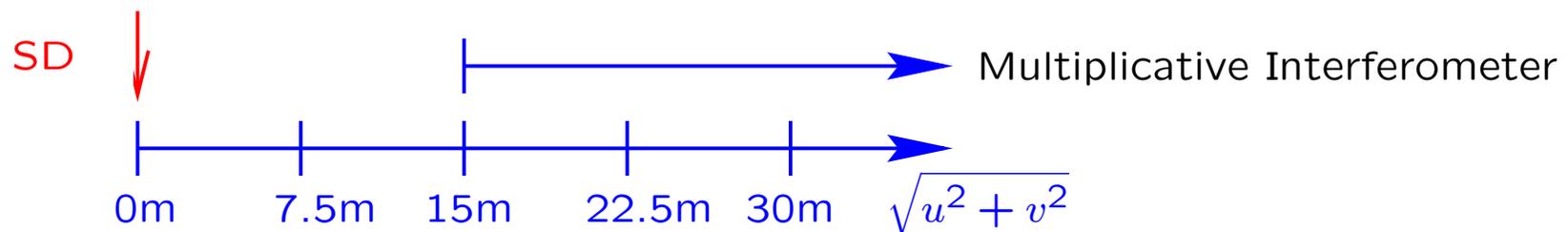


Multiplicative Interferometer

Avantage: all offsets are irrelevant \Rightarrow Much easier;

Inconvenient: Radio interferometer = bandpass instrument;

\Rightarrow Low spatial frequencies are filtered out.



Optic vs Radio Interferometer: II. Atmospheric Influence

Atmosphere emits and absorbs: (Lecture by M. Bremer).

Signal = Transmission * Source + Atmosphere.

- Optic: $\left\{ \begin{array}{l} \text{Source} \gg \text{Atmosphere} \\ \text{Transmission} \sim 1 \end{array} \right\} \Rightarrow \text{transparent};$
- Radio: $\left\{ \begin{array}{l} \text{Source} \ll \text{Atmosphere} \\ \text{Transmission can be small} \end{array} \right\} \Rightarrow \text{fog}.$

Good news: Atmospheric noise uncorrelated

\Rightarrow Correlation suppresses it!

Bad news: Transmission depends on weather and frequency.

\Rightarrow Astronomical sources needed to calibrate the flux scale!

(Lecture by M. Krips)

Atmosphere is turbulent: \Rightarrow Phase noise (Lecture by M. Bremer).

Timescale of atmospheric phase random changes:

- Optic: 10-100 milli secondes;
- Radio: 10 minutes.

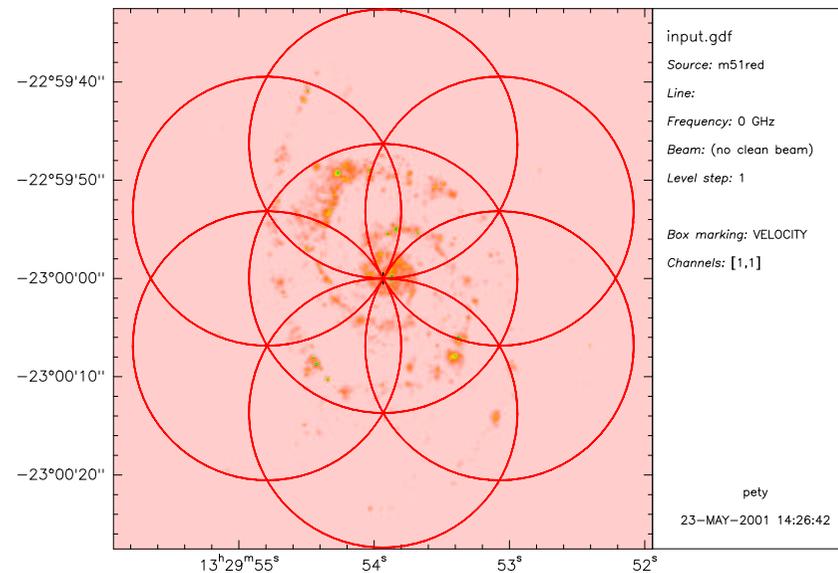
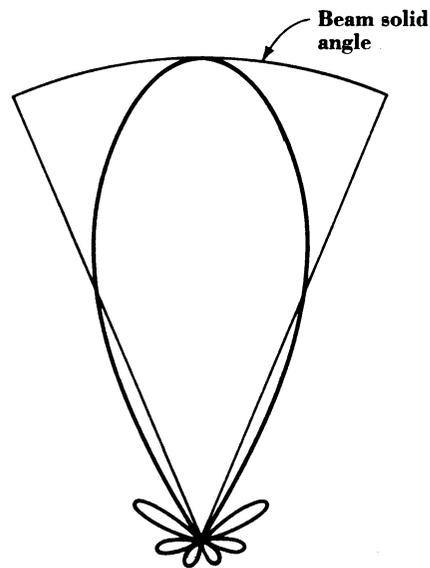
\Rightarrow Radio permits phase calibration on a nearby point source
(e.g. quasar).

Instantaneous Field of View

One pixel detector:

- Single Dish: one image pixel/telescope pointing;
- Interferometer: numerous image pixels/telescope pointing
 - Field of view = Primary beam size;
 - Image resolution = Synthesized beam size.

Wide-field imaging: \Rightarrow mosaicing (Lecture by F. Gueth).



Conclusion

mm interferometry:

- A bit more of theory;
- Lot's of experimental details (*e.g.* lecture by F. Gueth and R. Neri).

Why caring about technical details: Some of them must be understood to know whether you can trust your data.

By the end of this week, you should be ready to use PdBI!
(Lectures by R. Neri and J. M. Winters)

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Photographic Credits

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Lexicon

- Beam: Antenna diffraction pattern.
- Primary Beam: Instantaneous field of view (Single-Dish Beam).
- Synthesized Beam: Image resolution (Interferometer Beam).
- Configuration: Antenna layout of interferometer.
- Baseline: Distance between two antenna.
- uv -plane: Fourier plane.
- Visibilities: \sim Fourier components of the source.
- Fringe stopping: Temporal variation of delay correction needed to avoid translation of the white fringe.
- Heterodyne: Principle of linear detection.
- Correlator: Where visibilities are measured by correlation of signal coming from pairs of antenna.