Imaging & Deconvolution
I. Single Field

Jérôme PETY
(IRAM/Obs. de Paris)

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Scientific Analysis of a mm Interferometer Output

mm interferometer output:

Calibrated visibilities in the uv plane ($\simeq$ the Fourier plane).

2 possibilities:

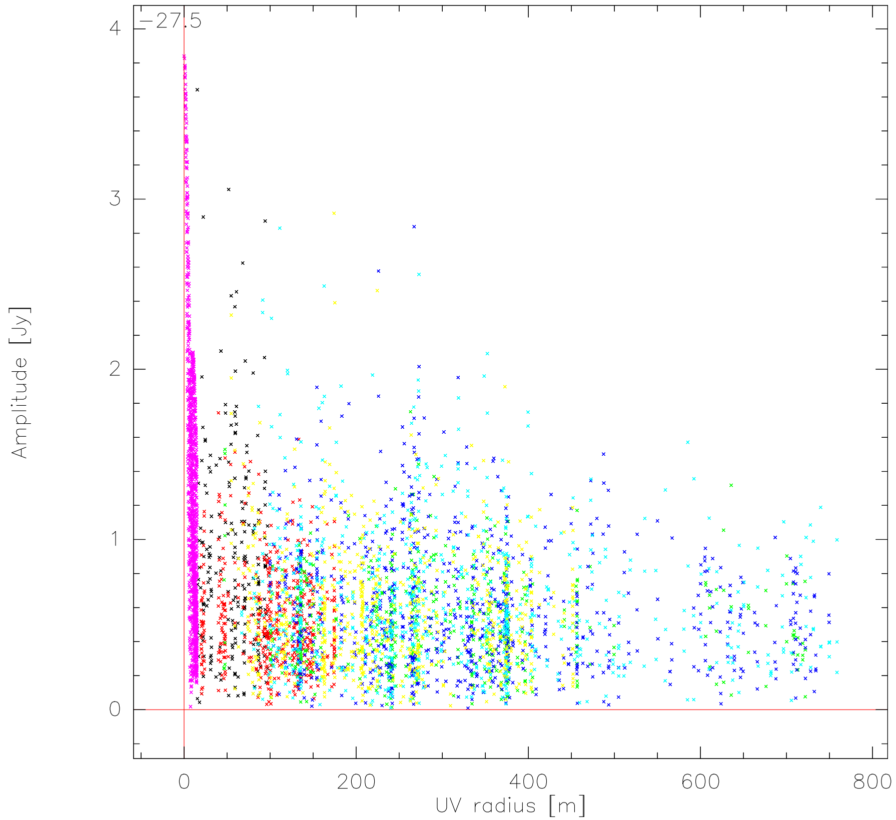
- uv plane analysis (cf. Lecture by M. Montargés):
  Always better . . . when possible!
  (in practice for "simple" sources as point sources or disks)
- Image plane analysis:
  $\Rightarrow$ Mathematical transforms to go from uv to image plane!

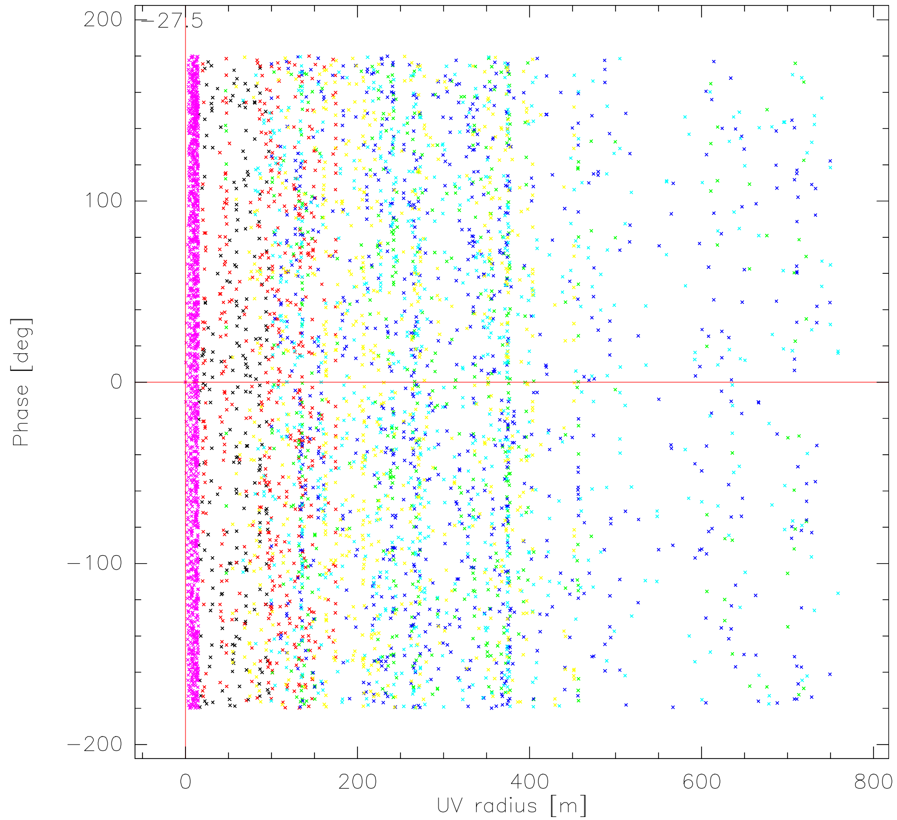
Goal: Understand effects of the imaging process on

- The resolution;
- The field of view (single pointing or mosaicing, cf. Lecture by F. Gueth);
- The reliability of the image;
- The noise level and repartition (cf. lecture by F. Gueth or V. Piétu).

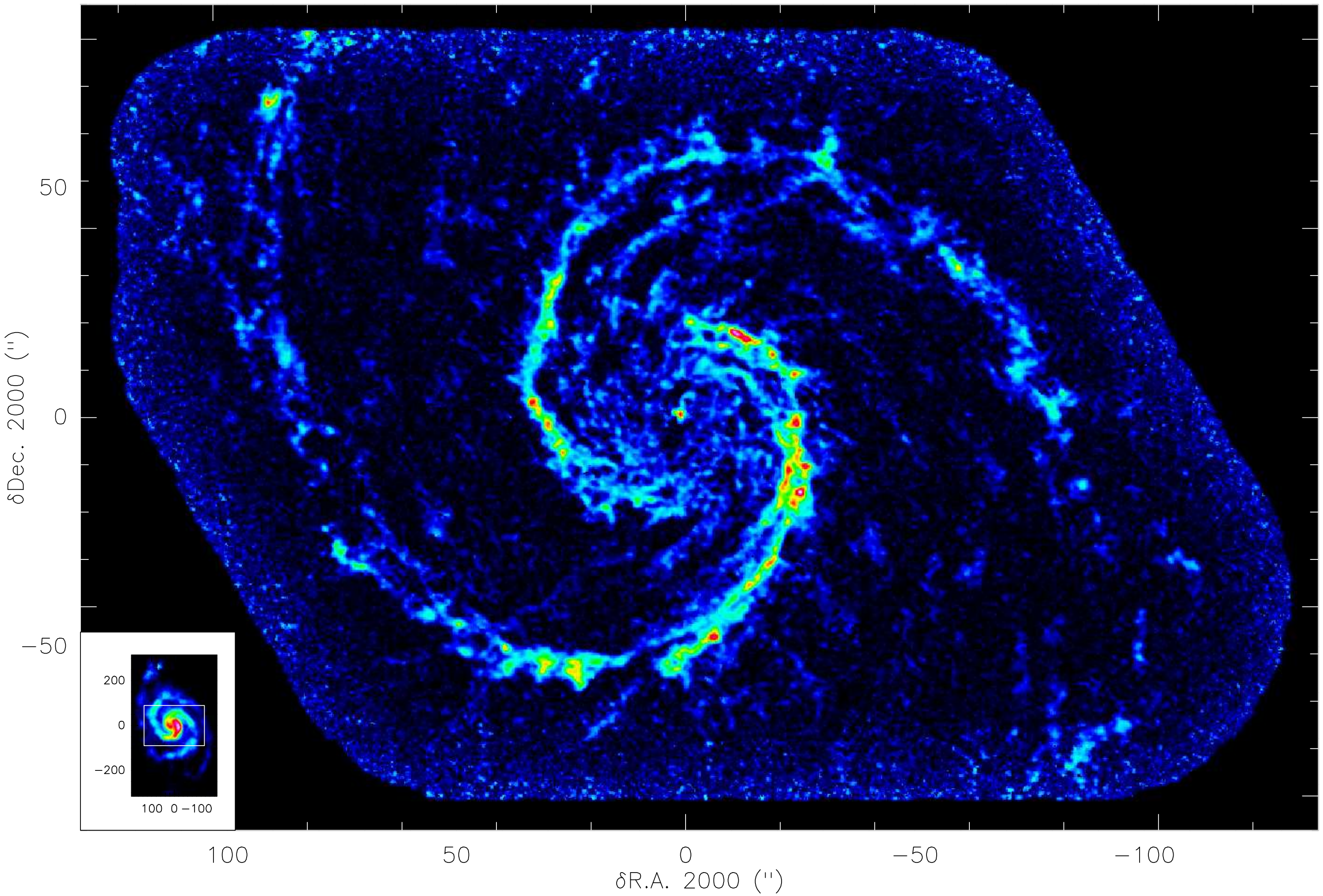
From Calibrated Visibilities

227 000 visibilities (amplitude & phase) per channels



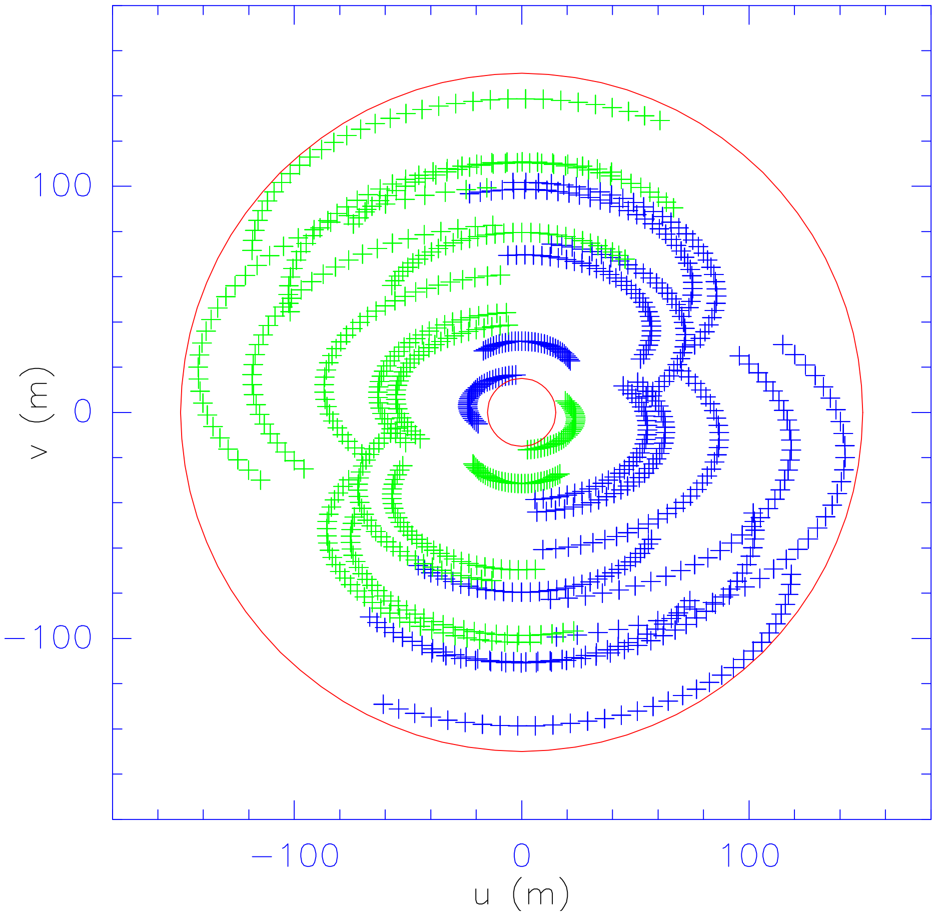


^{12}CO (J=1–0) emission of M51 at 1″ (IRAM Key program: PAWS)



From Calibrated Visibilities to Images:
I. Comparison Visibilities/Source Fourier Transform

$$V_{ij}(b_{ij}) = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}(b_{ij}) + N$$



- Primary Beam
  ⇒ Distorted source information.
- Noise ⇒ Sensitivity problems.
- Irregular, limited sampling
  ⇒ incomplete source information:
  - Support limited at:
    * High spatial frequency
      ⇒ limited resolution;
    * Low spatial frequency ⇒ problem of wide field imaging;
  - Inside the support, incomplete (*i.e.* Nyquist's criterion not respected) sampling ⇒ lost of information.

From Calibrated Visibilities to Images:
II. Effect of Irregular, Limited Sampling

Definitions:

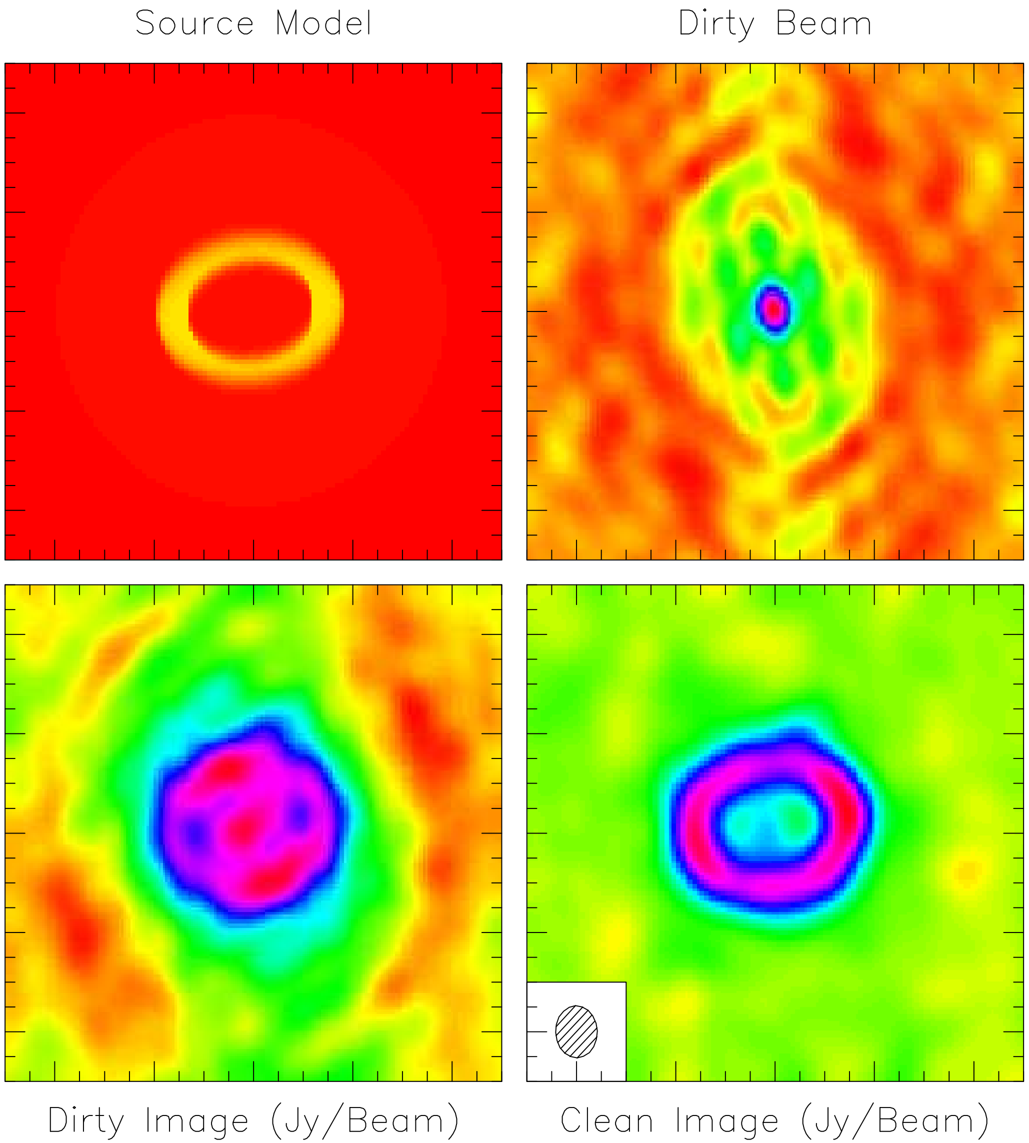
- $V = 2\text{D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$;
- Irregular, limited sampling function:
  - $S(u,v) = 1$ at (u,v) points where visibilities are measured;
  - $S(u,v) = 0$ elsewhere;
- $B_{\text{dirty}} = 2\text{D FT}^{-1}\{S\}$;
- $I_{\text{meas}} = 2\text{D FT}^{-1}\{S.V\}$.

Fourier Transform Property #1:

$I_{\text{meas}} = B_{\text{dirty}} * \left\{B_{\text{primary}}.I_{\text{source}}\right\}$.

B_{dirty}: Point Spread Function (PSF) of the interferometer
(*i.e.* if the source is a point, then $I_{\text{meas}} = I_{\text{tot}}.B_{\text{dirty}}$).

From Calibrated Visibilities to Images:
III. Why Deconvolving?



- Difficult to do science on dirty image.
- Deconvolution ⇒ a clean image compatible with the sky intensity distribution.

From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
$\Downarrow$ Fourier Transform	GO UVSTAT, GO UVMAP
Dirty beam & image	
$\Downarrow$ Deconvolution	GO CLEAN
Clean beam & image	
$\Downarrow$ Visualization	GO BIT, GO VIEW
$\Downarrow$ Image analysis	GO NOISE, GO FLUX, GO MOMENTS
Physical information on your source	

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Direct vs. Fast Fourier Transform

Direct FT:

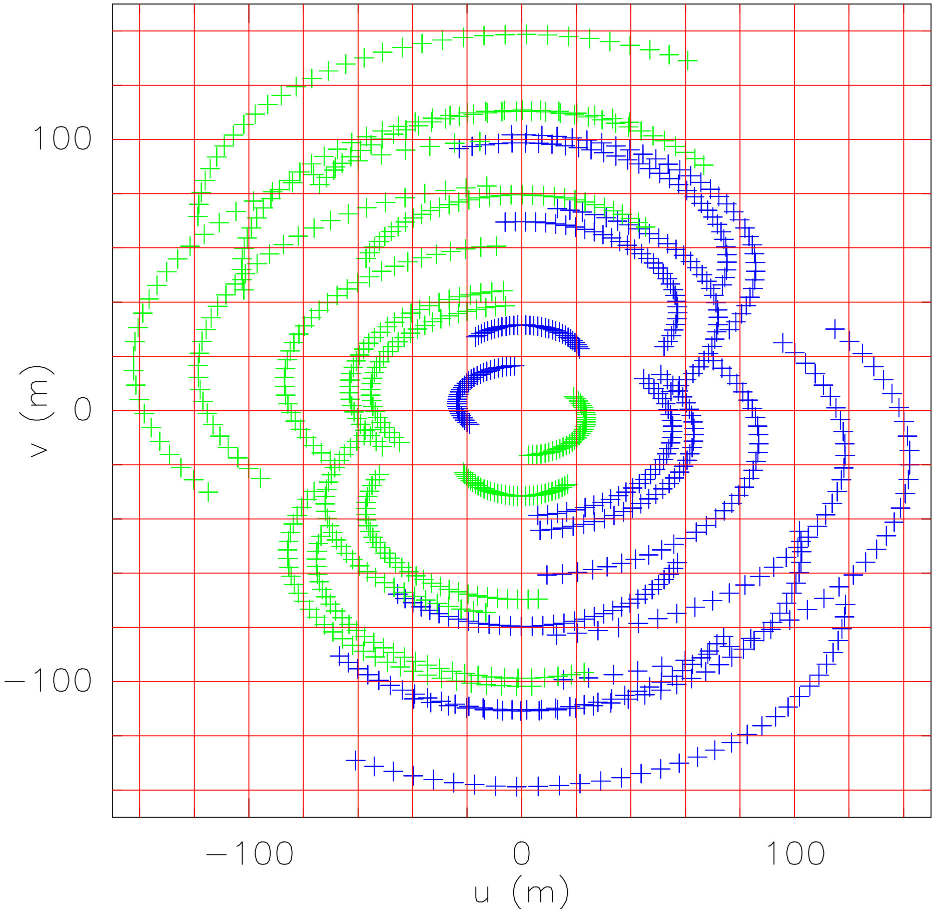
- Advantage: Direct use of the irregular sampling;
- Inconvenient: Slow.

Fast FT:

- Inconvenient: Needs a regular sampling $\Rightarrow$ Gridding;
- Advantage: Quick for images of size $2^M \times 2^N$.

$\Rightarrow$ In practice, everybody use FFT.

Gridding: I. Interpolation Scheme



Convolution because:

- Visibilities = noisy samples of a smooth function.
  ⇒ Some smoothing is desirable.
- Nearby visibilities are not independent.
  - $V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$
    $= \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}};$
  - FWHM(convolution kernel)
    $<$ FWHM($\tilde{B}_{\text{primary}}$)
    ⇒ No real information lost.

Gridding: II. Measurement Equation is Kept Through Gridding

Before Gridding

$$I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\}$$

After Gridding

- $I^{\text{grid}}_{\text{meas}} \overset{\text{2D FT}}{\rightleftharpoons} G * (S.V) \quad \Leftrightarrow \quad I^{\text{grid}}_{\text{meas}} = \tilde{G}.(\widetilde{S.V}) = \tilde{G}.(\tilde{S} * \tilde{V});$
- $B^{\text{grid}}_{\text{dirty}} \overset{\text{2D FT}}{\rightleftharpoons} G * S \quad \Leftrightarrow \quad B^{\text{grid}}_{\text{dirty}} = \tilde{G}.\tilde{S};$

$\Rightarrow I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\}$

with $I_{\text{meas}} = I^{\text{grid}}_{\text{meas}} / \tilde{G}$

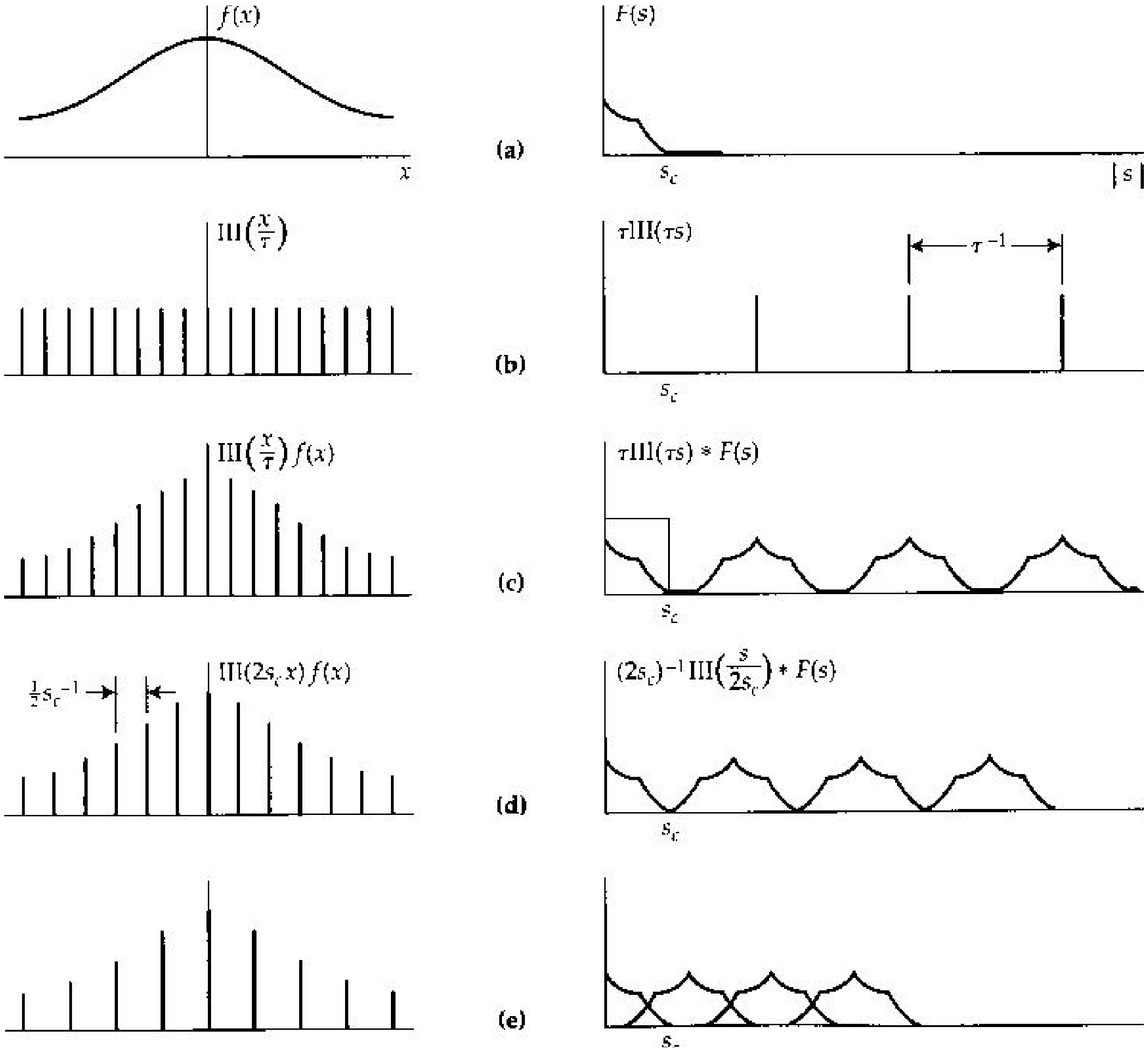
and $B_{\text{dirty}} = B^{\text{grid}}_{\text{dirty}} / \tilde{G}$.

Remark Gridding may be hidden in equations but it is still there.

$\Rightarrow$ Artifacts due to gridding! (cf. next transparencies)

Gridding:
III. Effect of a Regular Sampling (Periodic Replication)

uv Plane | Image Plane



$B_{\text{primary}}.I_{\text{source}}$

Regular Sampling function

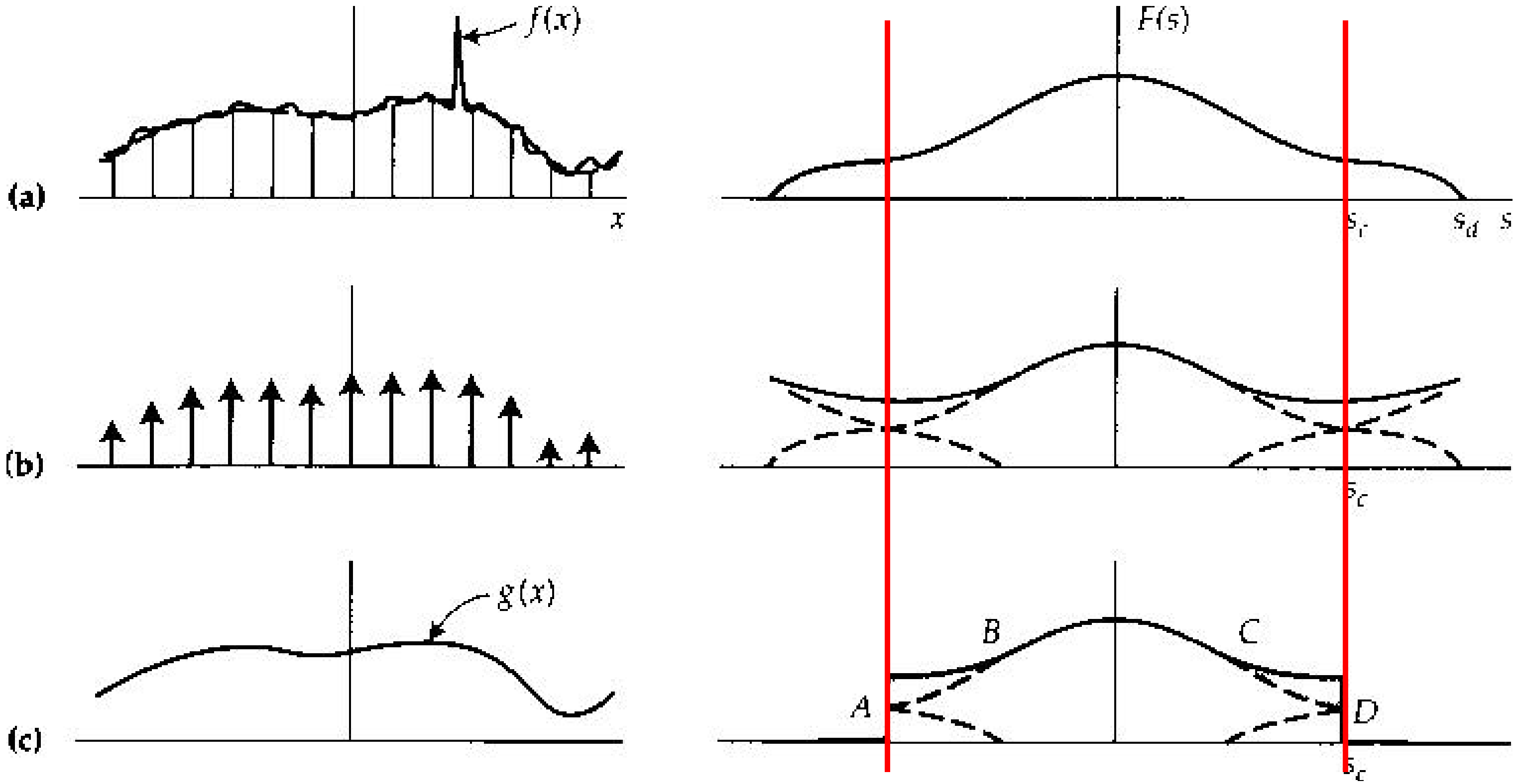
Result for a fine sampling

Result for critical sampling (Nyquist's criterion)

Result for a coarse sampling

Gridding: III. Effect of a Regular Sampling (Aliasing)

uv Plane | Image Plane



Aliasing = Folding of intensity outside the image size into the image.

⇒ Image size must be large enough.

Gridding: IV. Pixel and Image Sizes

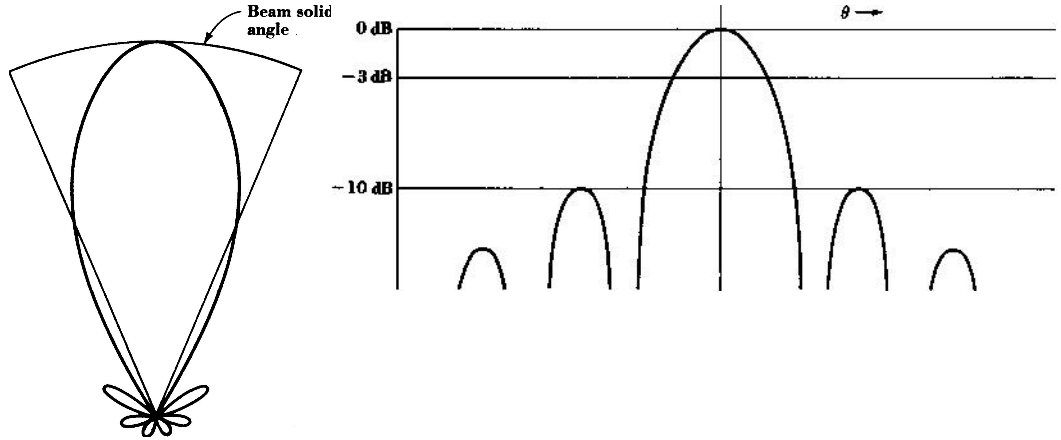
Pixel size: Between 1/4 and 1/5 of the synthesized beam size (*i.e.* more than the Nyquist's criterion in image plane to ease deconvolution).

Image size:

- $=$ uv plane sampling rate (FT property # 2);
- Natural resolution in the uv plane: $\tilde{B}_{\text{primary}}$ size;

$\Rightarrow$ At least twice the B_{primary} size (*i.e.* Nyquist's criterion in uv plane).

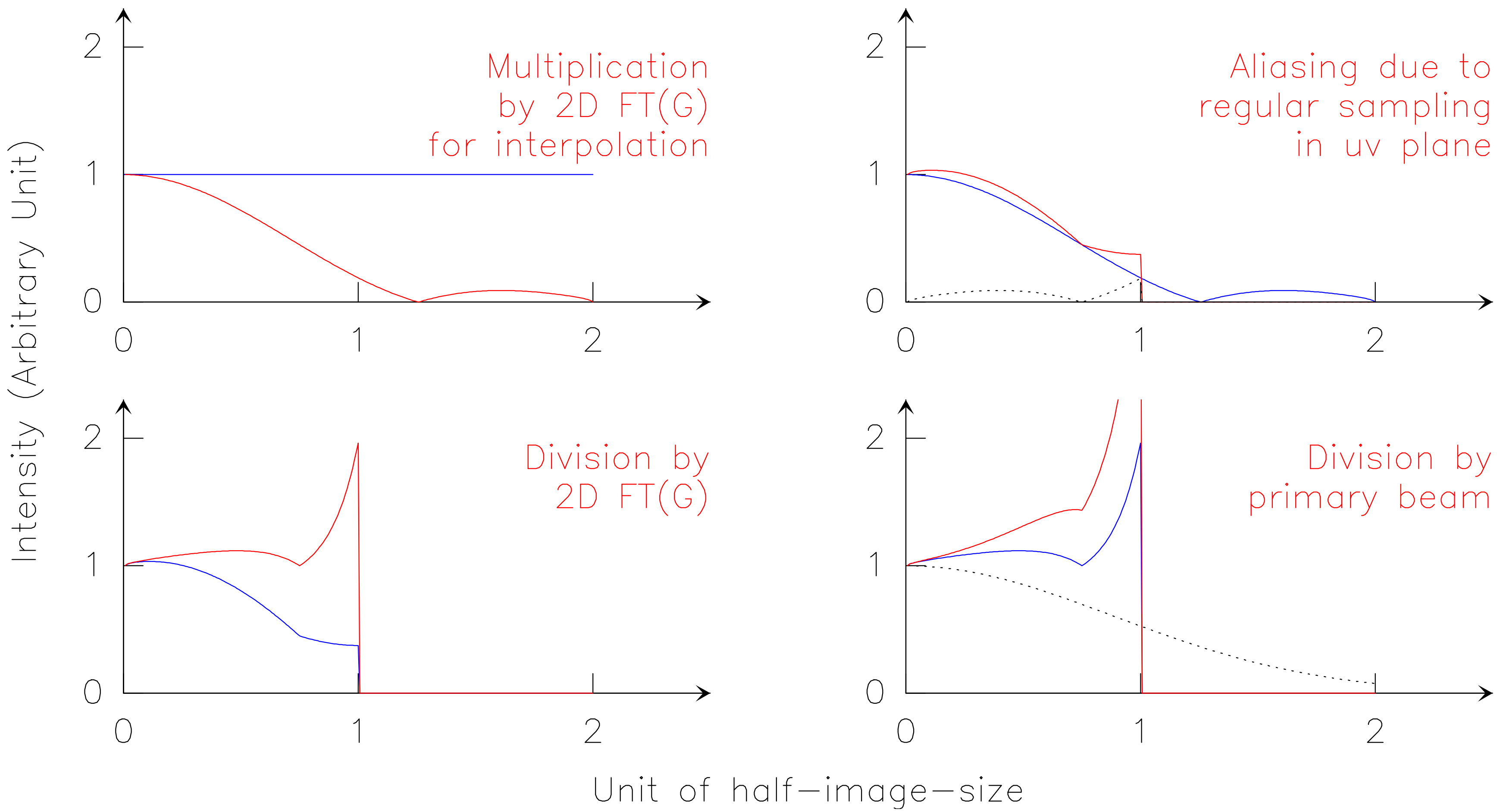
Gridding: V. Bright Sources in B_{primary} Sidelobes



Bright Sources in B_{primary} sidelobes
outside image size will be aliased into image.
⇒ Spurious source in your image!

Solution: Increase the image size.
(Be careful: only when needed for efficiency reasons!)

Gridding: VI. Noise Distribution



Gridding: VII. Choice of Gridding function

Gridding function must:

- Fall off quickly in image plane (to avoid noise aliasing);
- Fall off quickly in uv plane (to avoid too much smoothing).

⇒ Define a mathematical class of functions: Spheroidal functions.

GILDAS implementation: In `GO UVMAP`

- Spheroidal functions = Default gridding function;
- Tabulated values are used for speed reasons.

Dirty Beam Shape and Image Quality

$$B_{\text{dirty}} = \text{2D FT}^{-1}\{S\}.$$

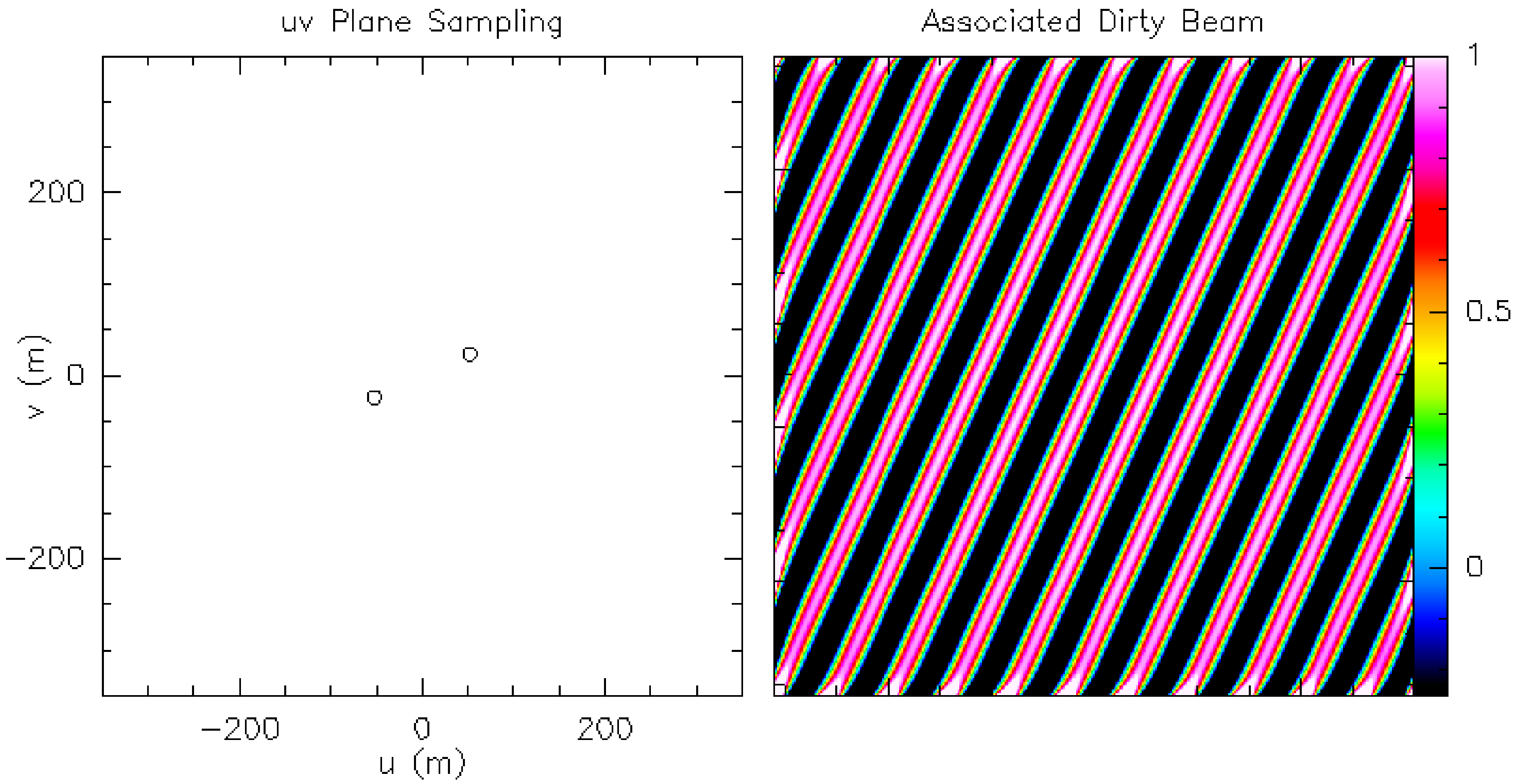
Importance of the Dirty Beam Shape:

- Deconvolving a dirty image is a delicate stage;
- The closest to a Gaussian B_{dirty} is, the easier the deconvolution;
- Extreme case:
  B_{dirty} = Gaussian $\Rightarrow$ No deconvolution needed at all!

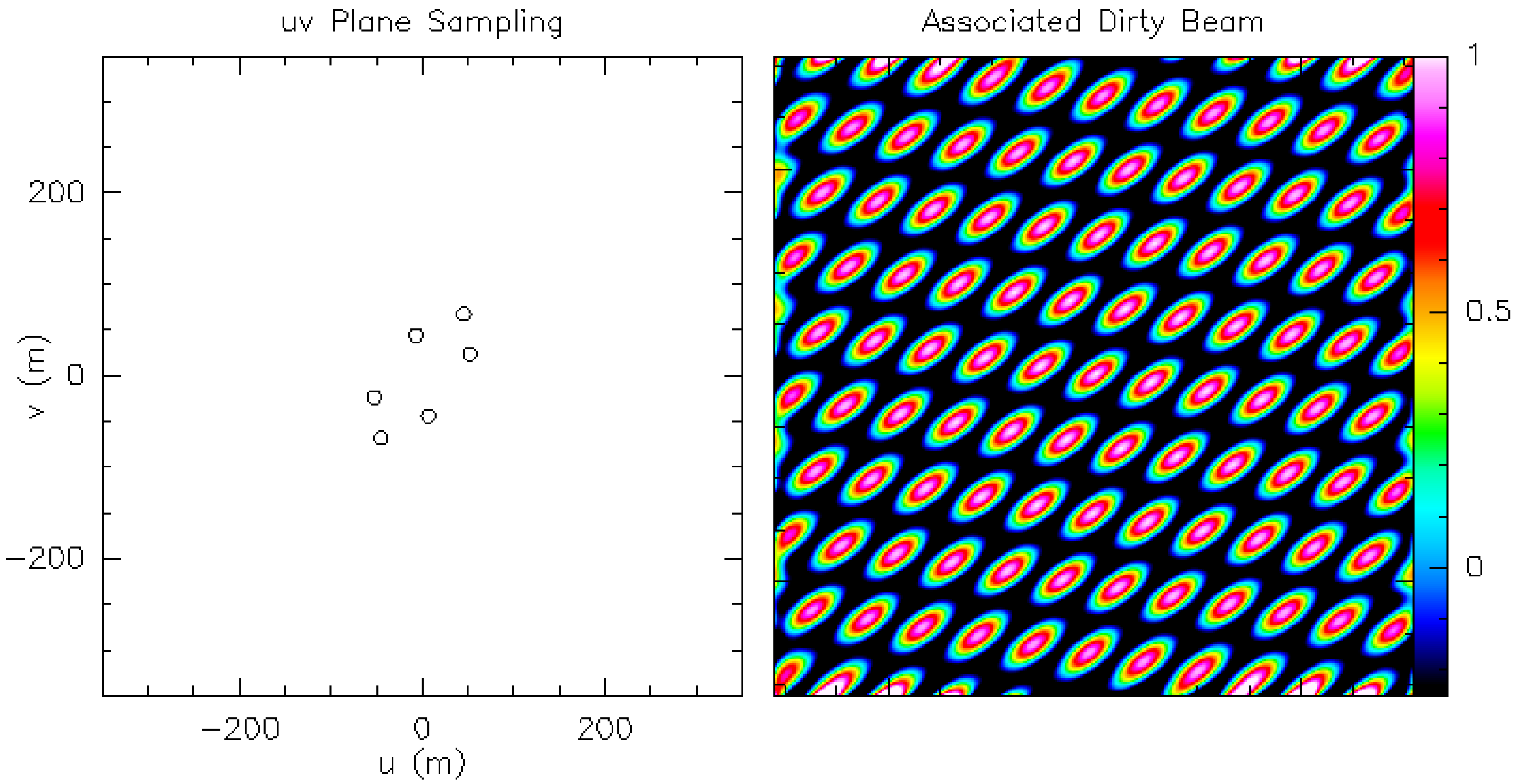
Ways to improve (at least change) B_{dirty} shape:

- Increase the number of antenna (costly).
- Change the antenna layout (technically difficult).
- Weight the irregular, limited sampling function S (the only thing you can do in practice).

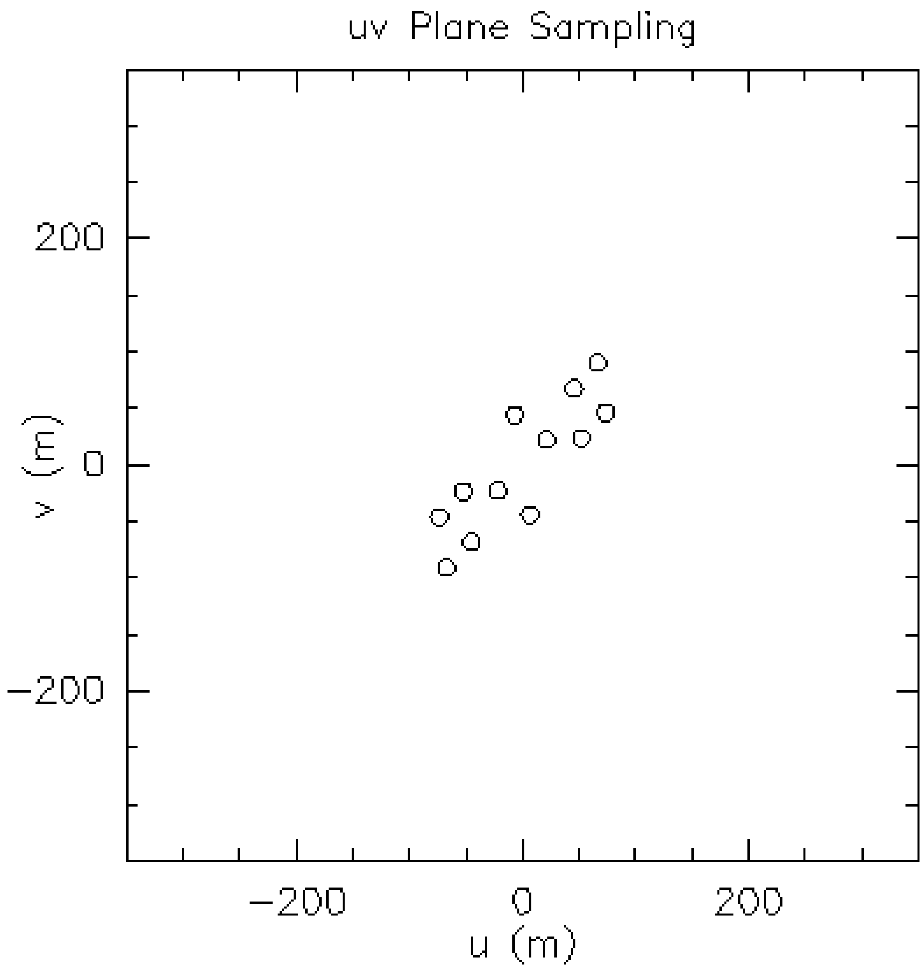
Dirty Beam Shape and Number of Antenna:
2 Antenna

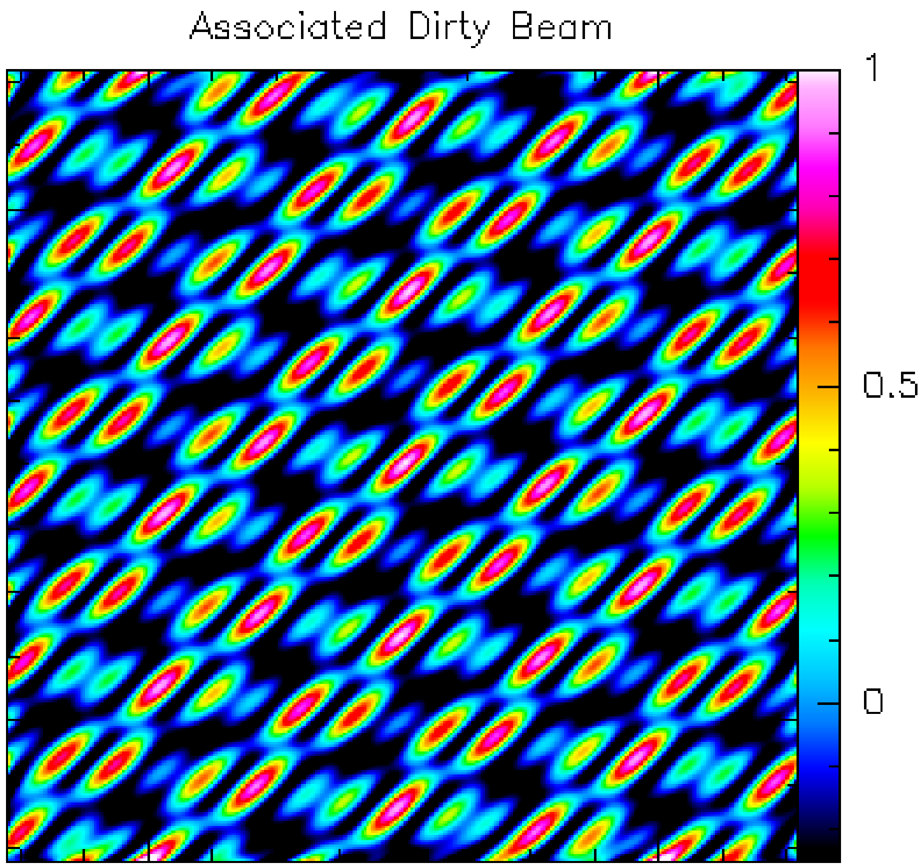


Dirty Beam Shape and Number of Antenna:
3 Antenna

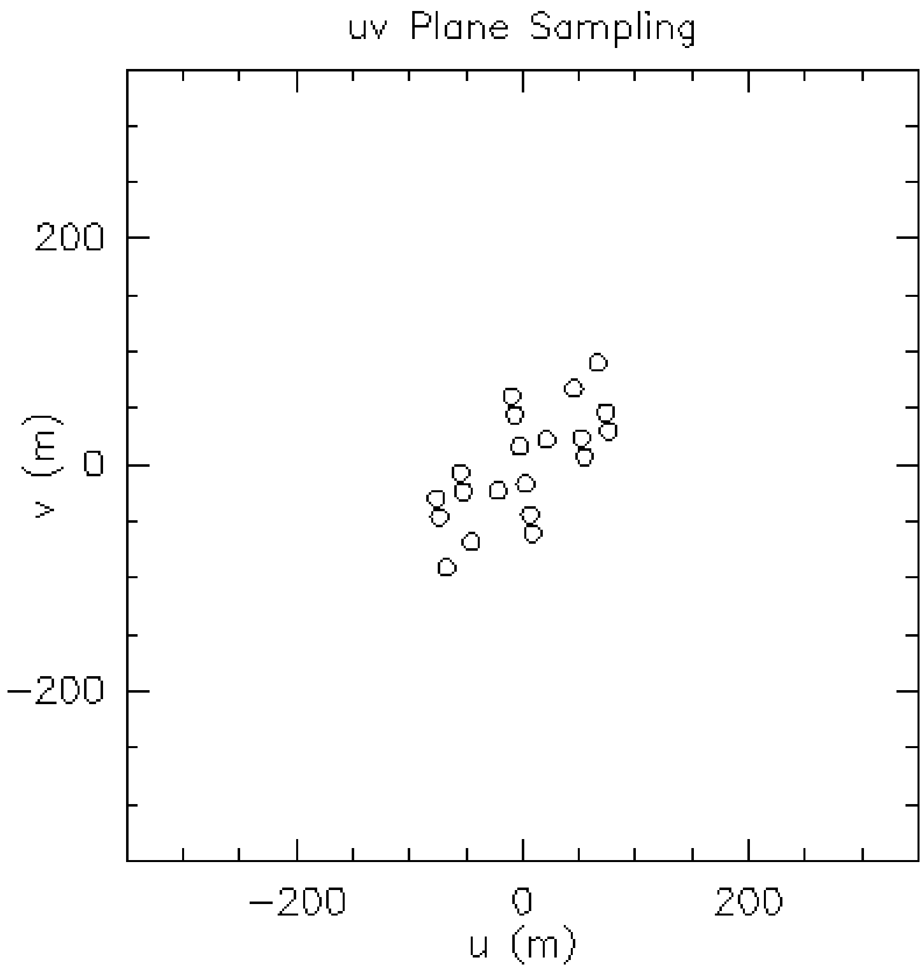


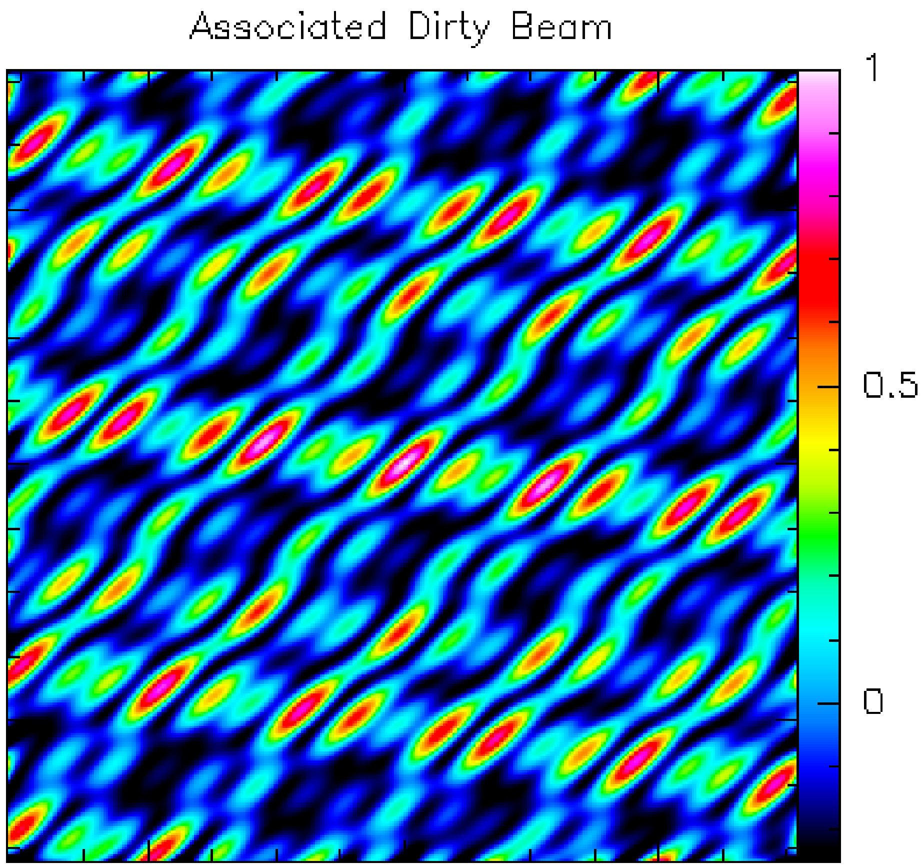
Dirty Beam Shape and Number of Antenna:
4 Antenna



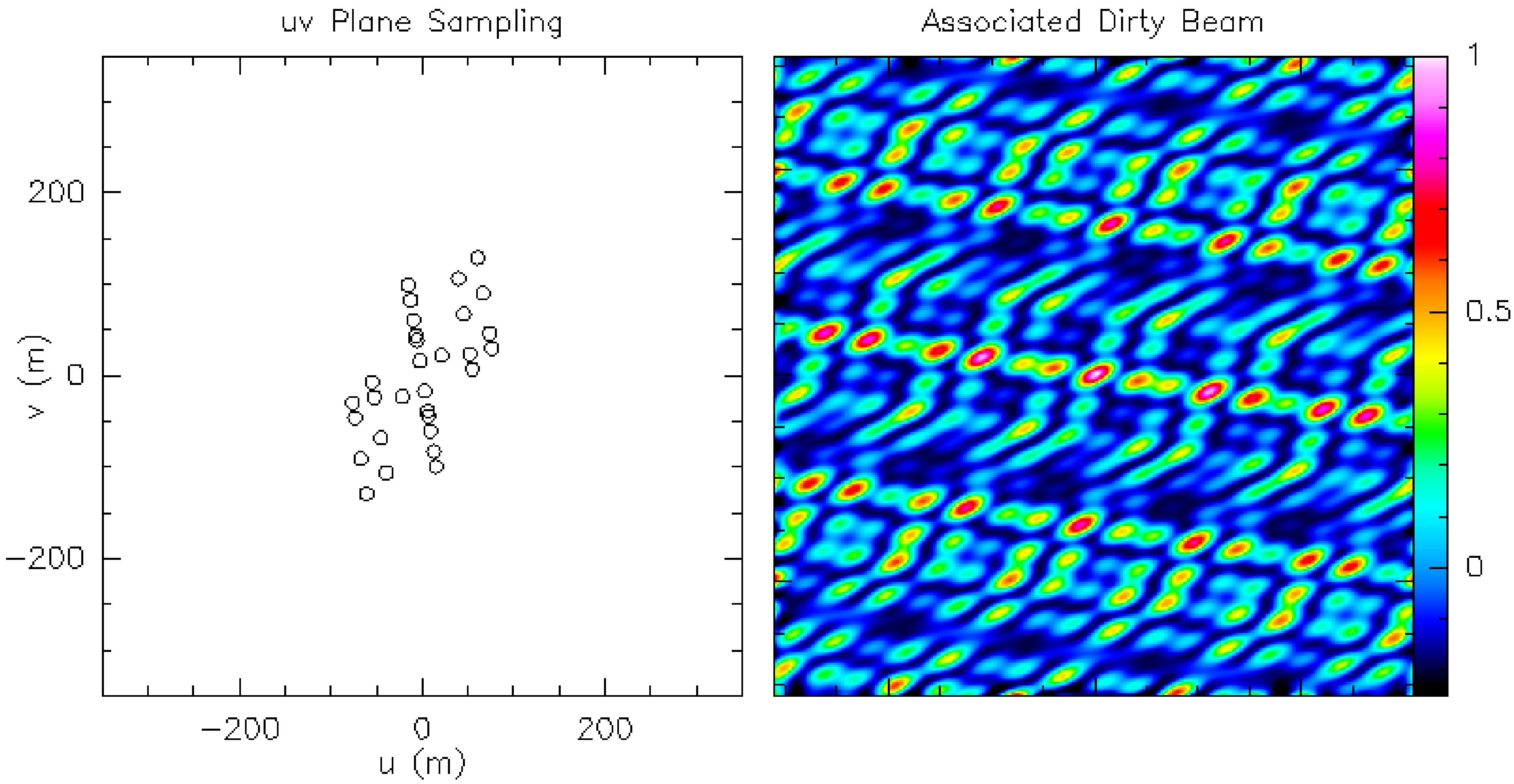


Dirty Beam Shape and Number of Antenna:
5 Antenna

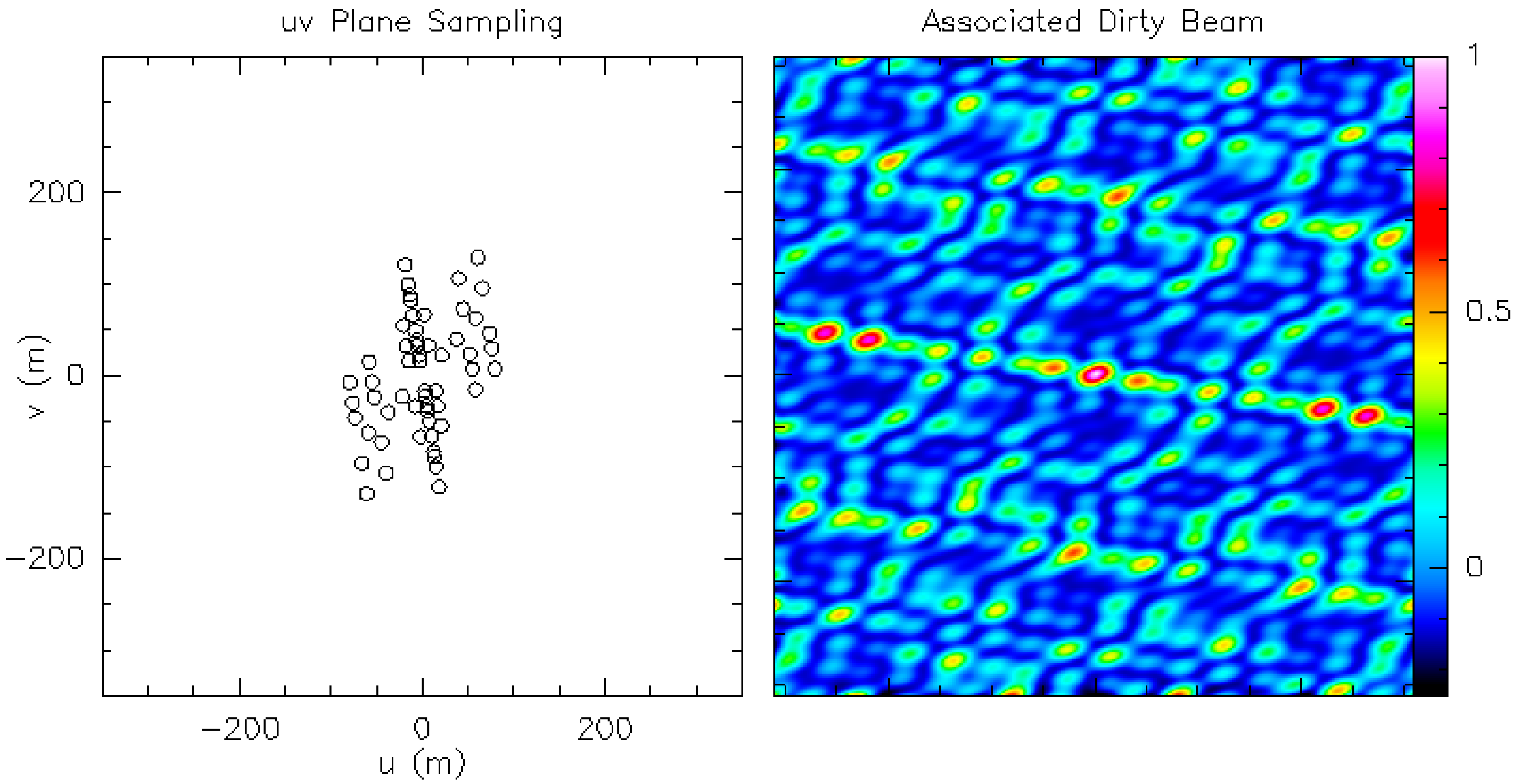




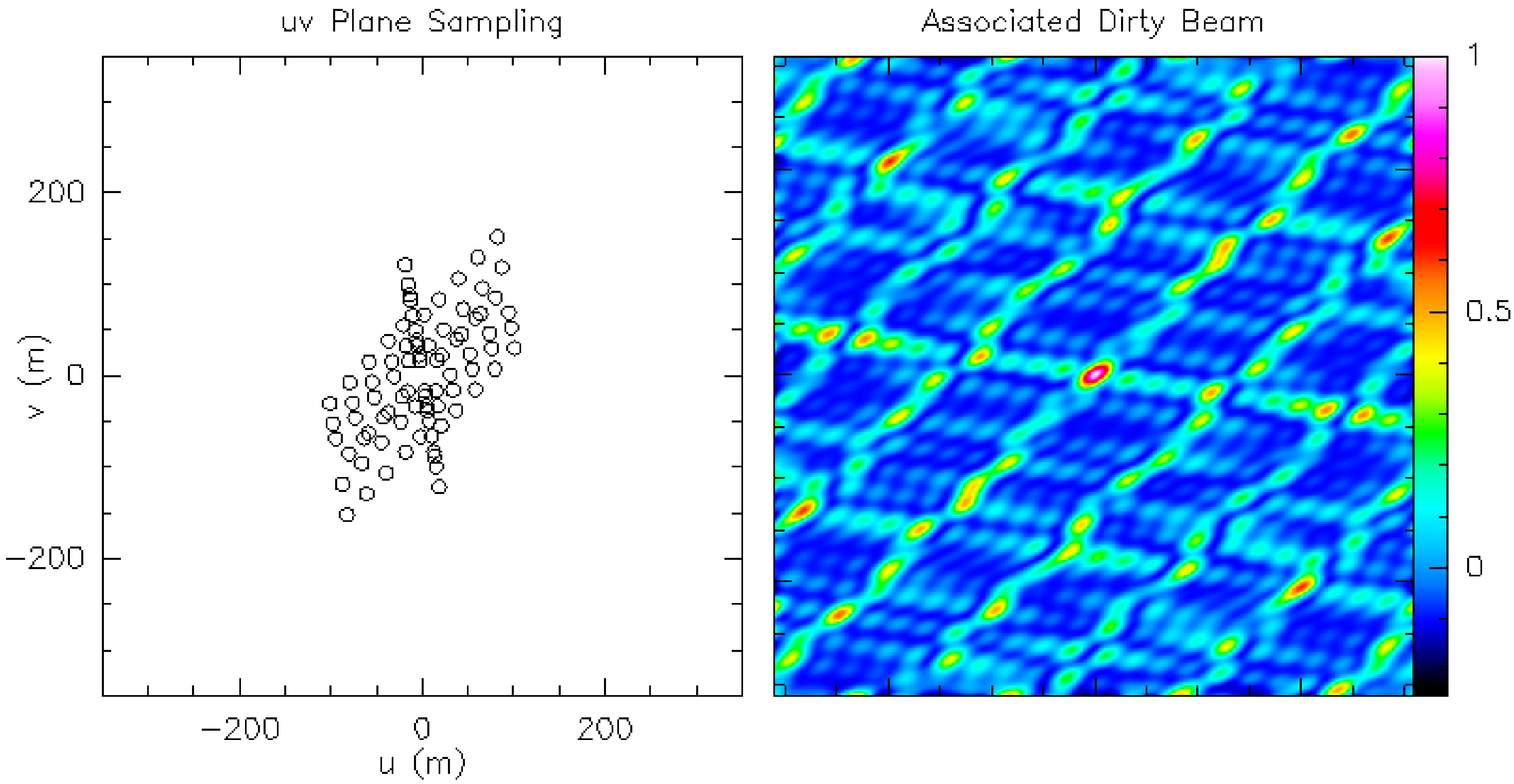
Dirty Beam Shape and Number of Antenna: 6 Antenna



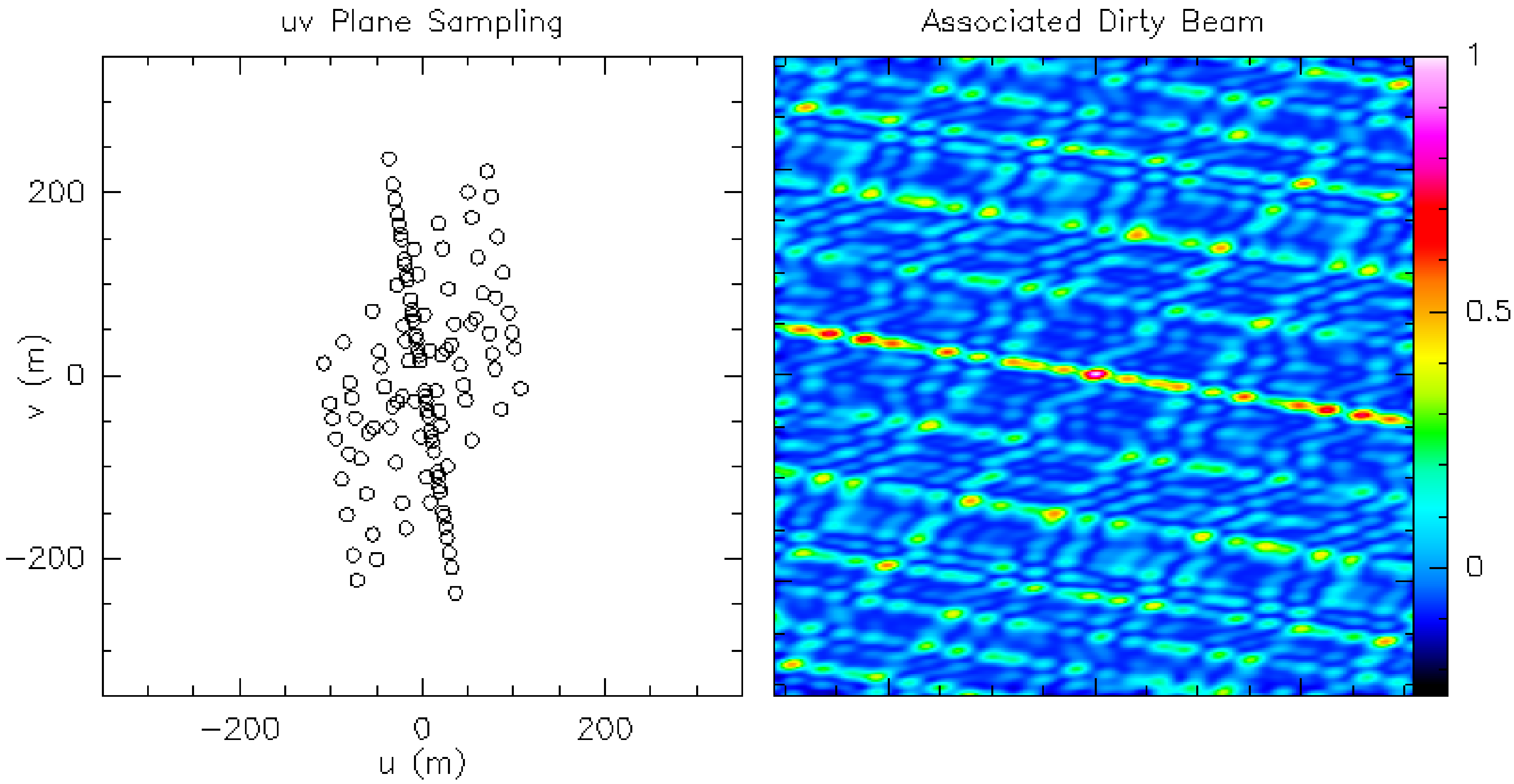
Dirty Beam Shape and Number of Antenna:
8 Antenna



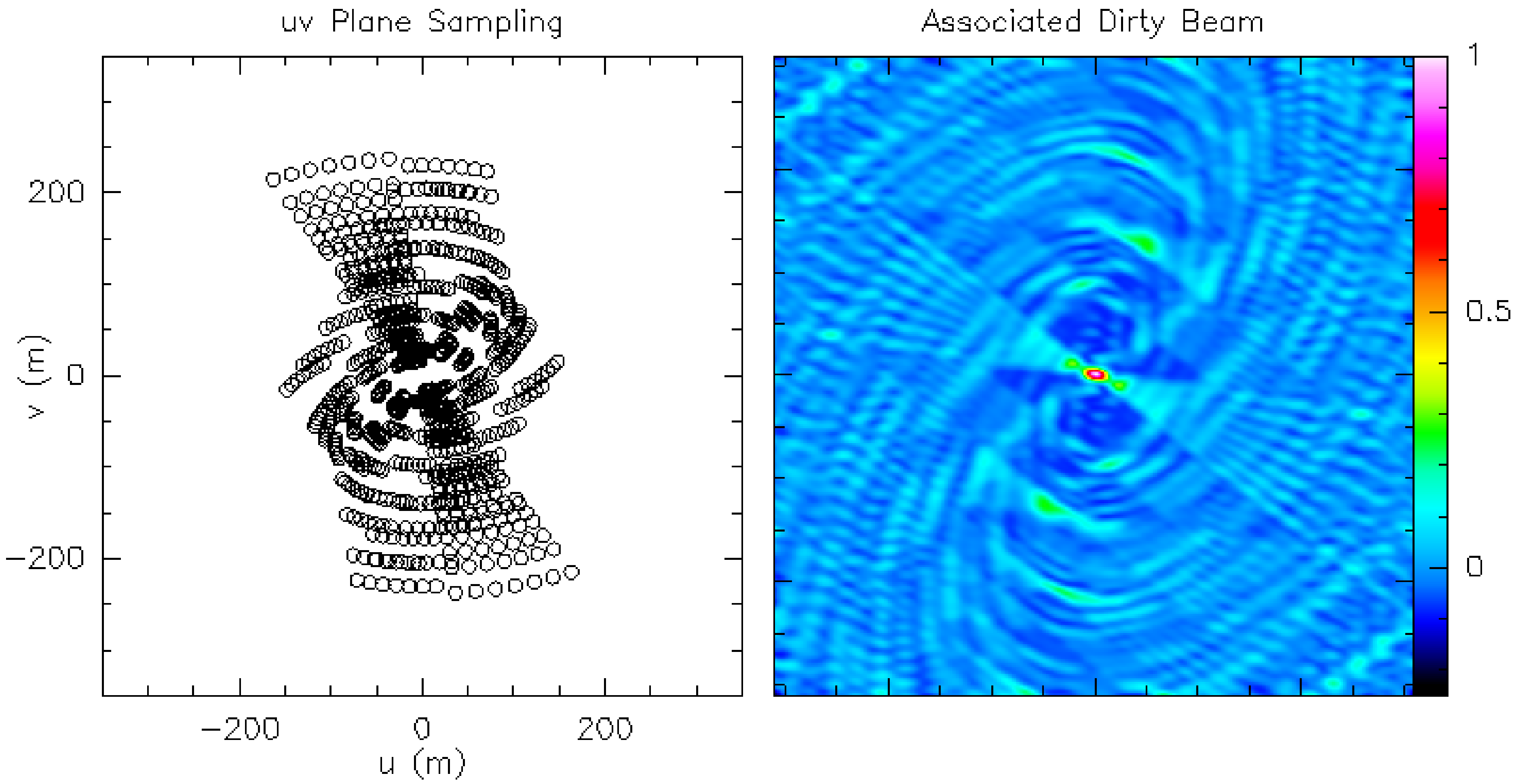
Dirty Beam Shape and Number of Antenna:
10 Antenna



Dirty Beam Shape and Number of Antenna:
12 Antenna



Dirty Beam Shape and Super Synthesis



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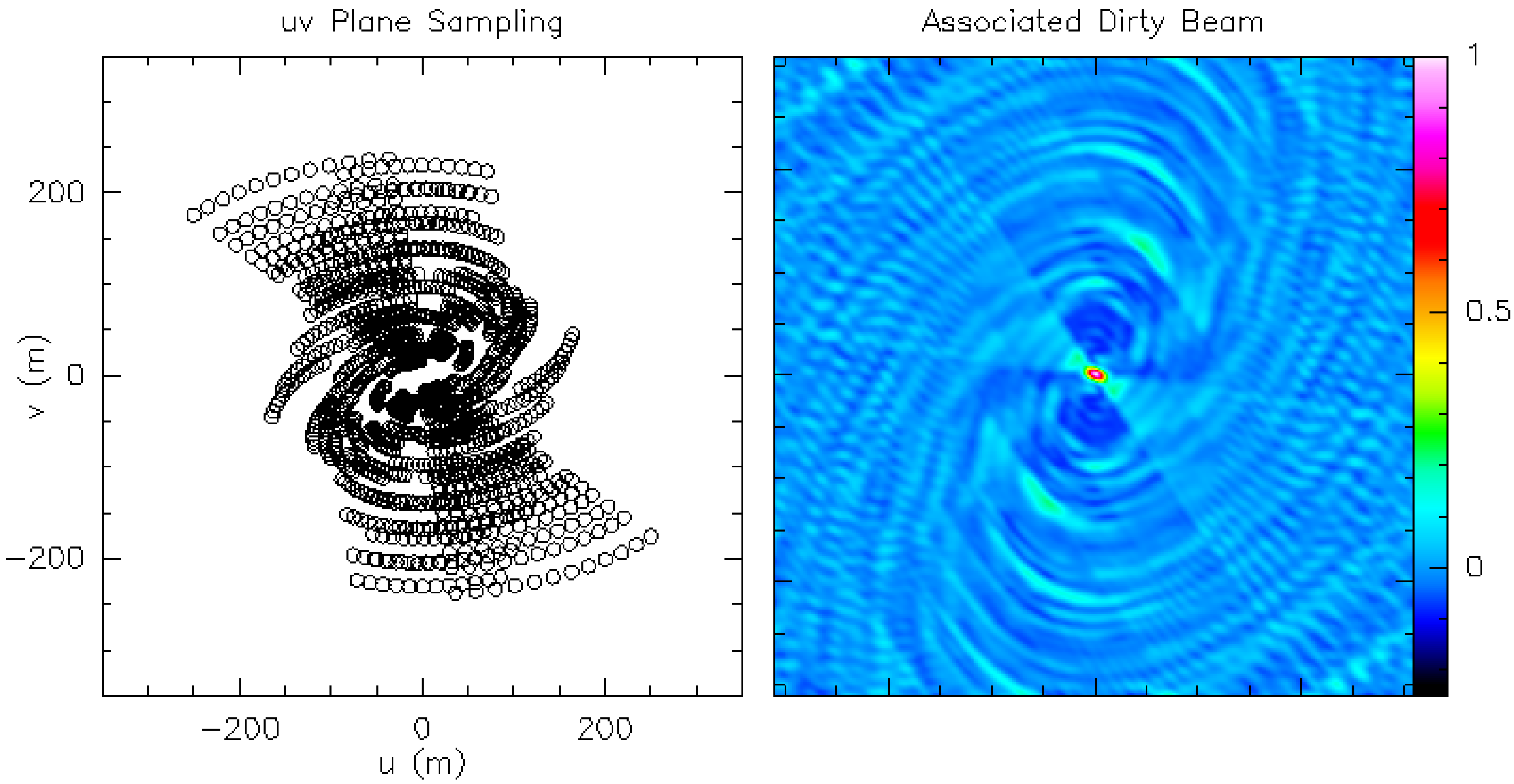

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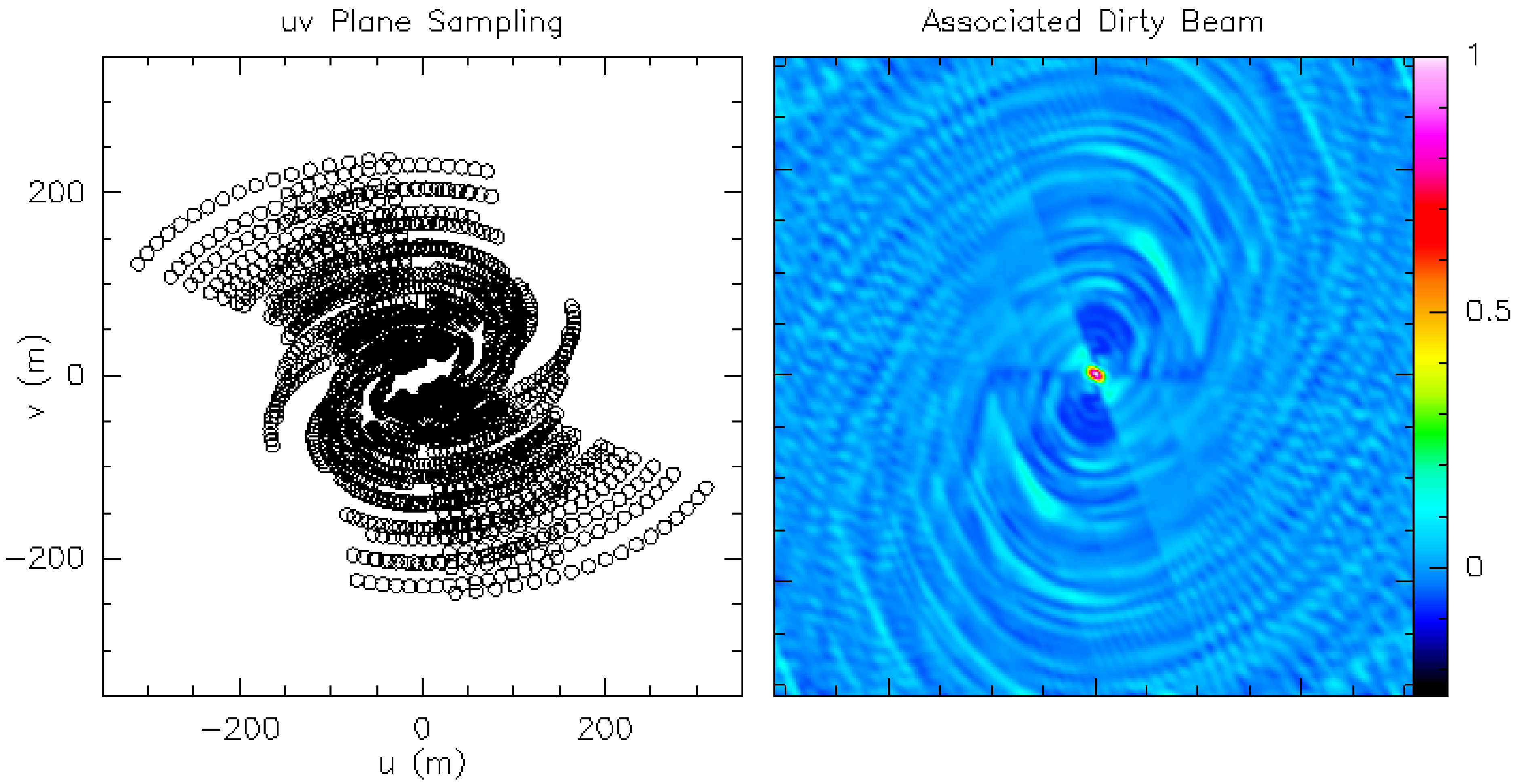

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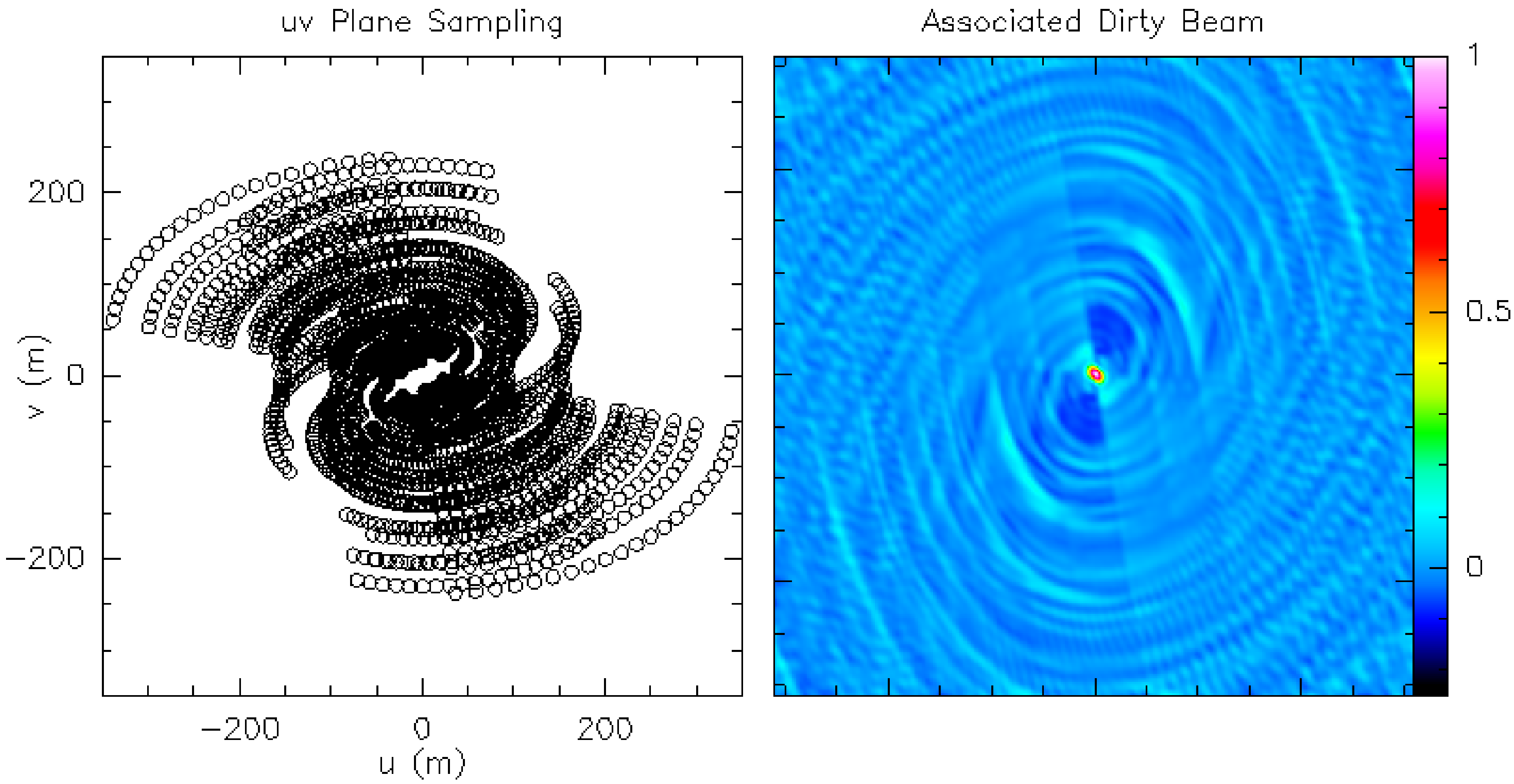

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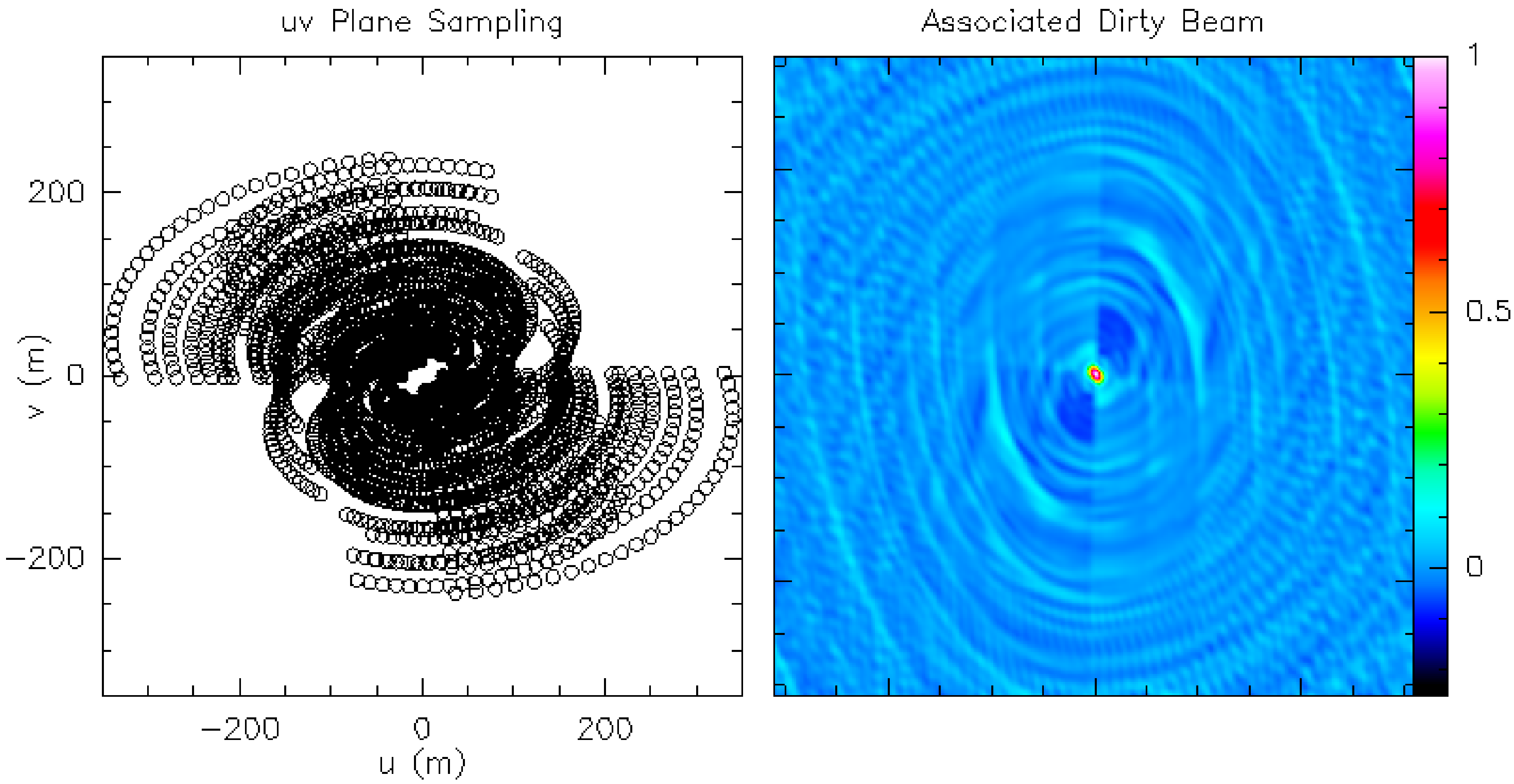

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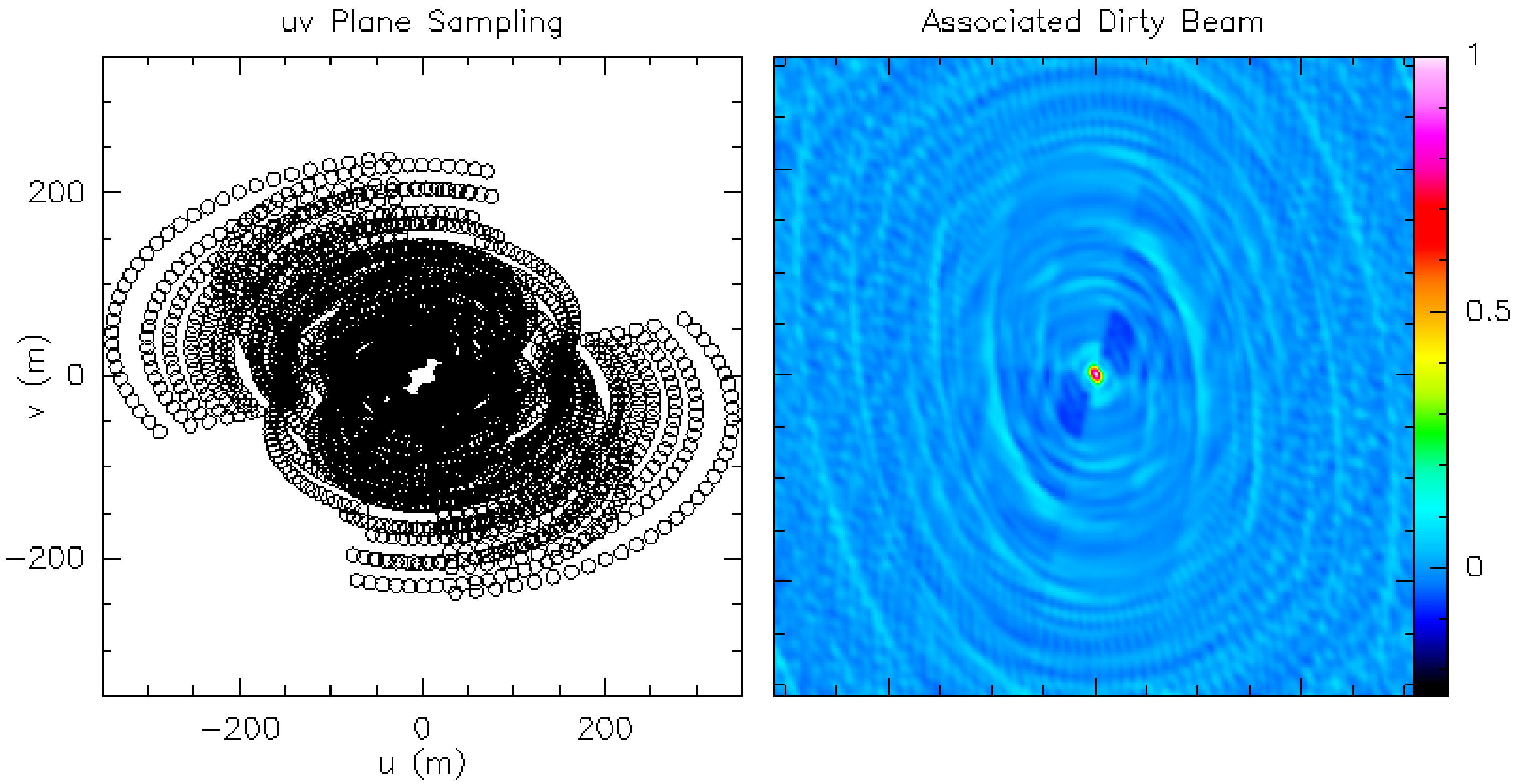

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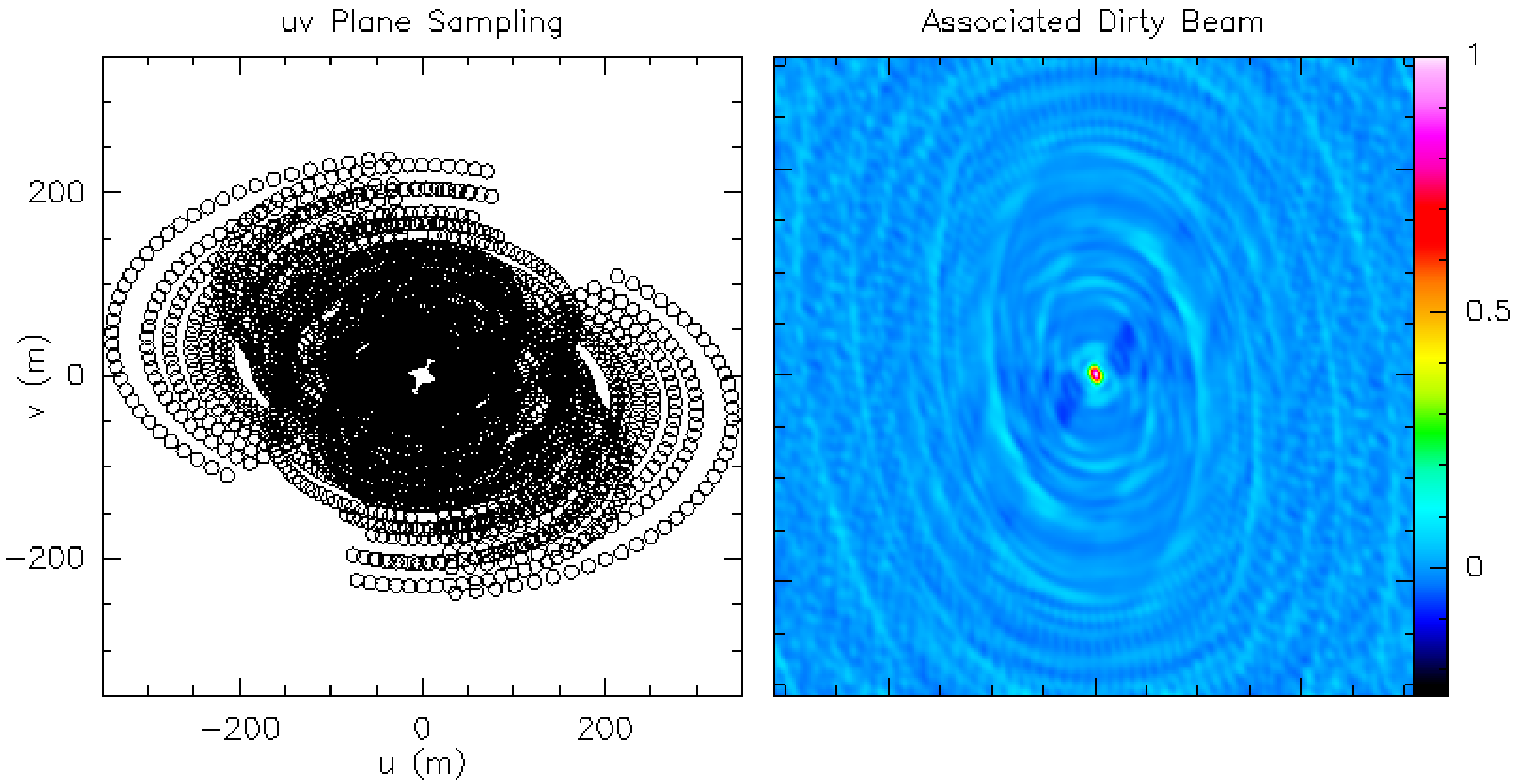

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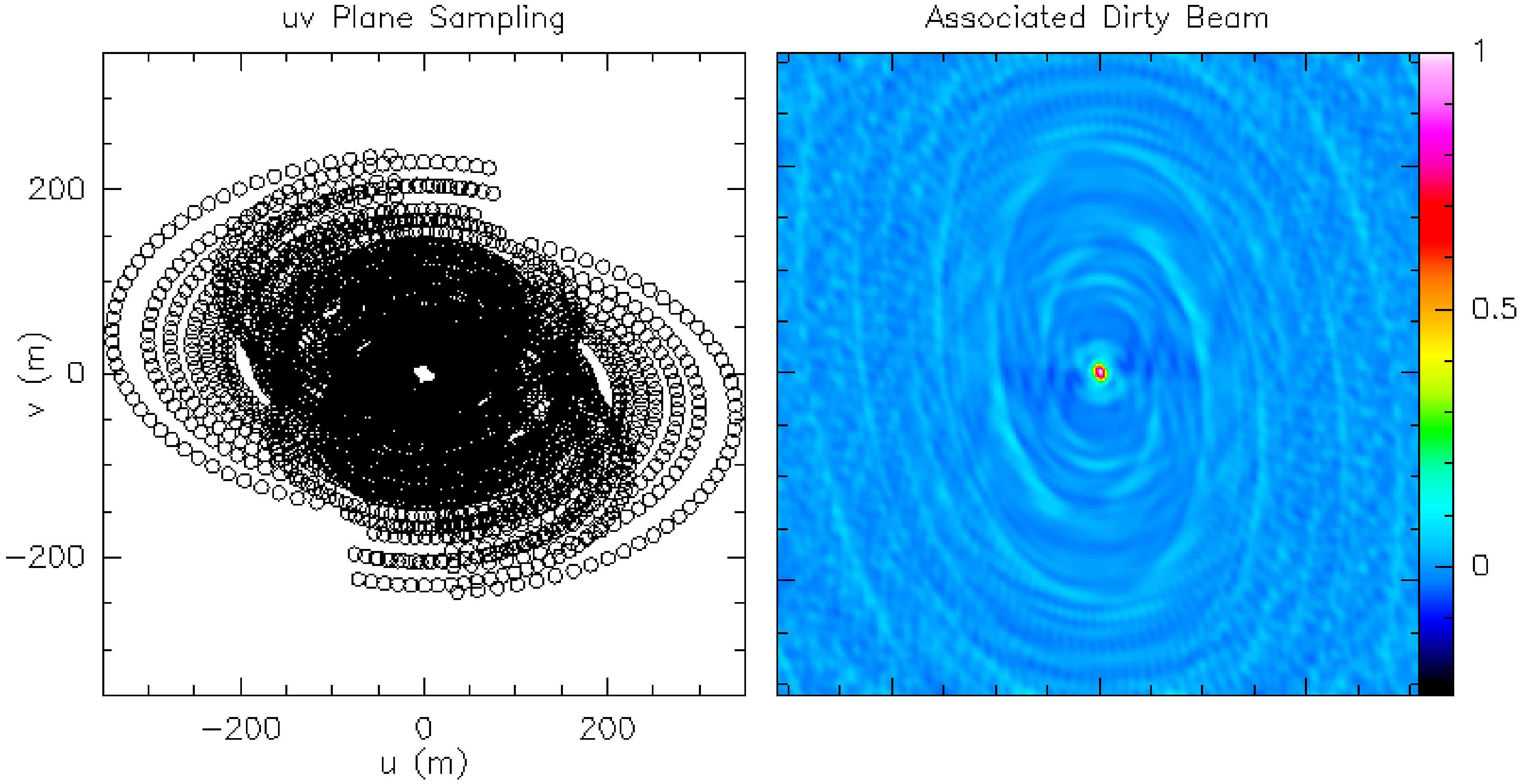


Dirty Beam Shape and Weighting

Natural Weighting: Default definition of the irregular sampling function at uv table creation.

- $S(u,v) = 1/\sigma^2$ at (u,v) points where visibilities are measured;
- $S(u,v) = 0$ elsewhere;

with $\sigma^2(u,v)$ the noise variance of the visibility.

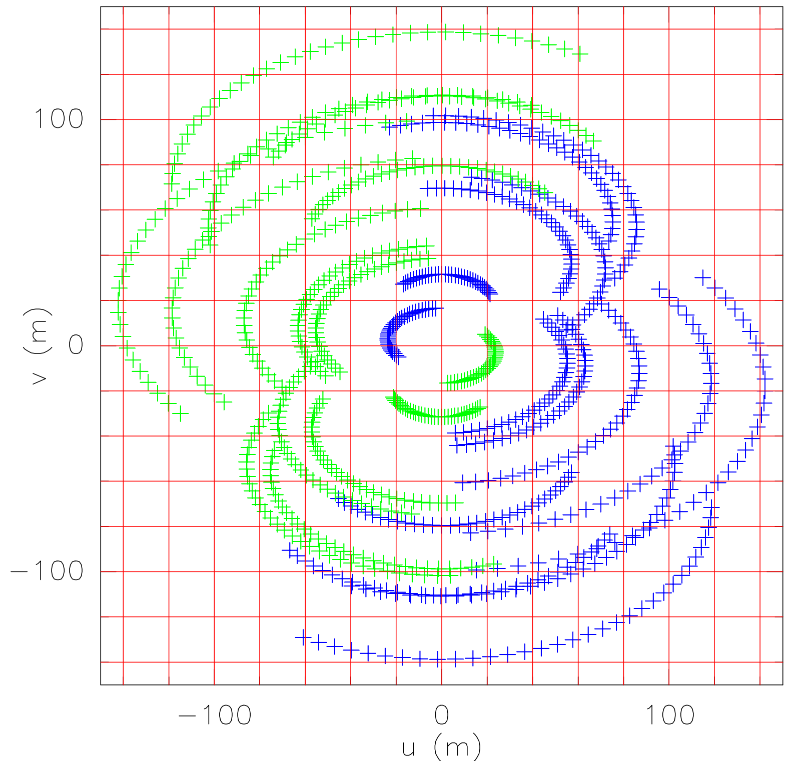
Introduction of a weighting function $W(u,v)$:

- $B_{\mathrm{dirty}} = \mathrm{2D\ FT}^{-1}\{W.S\}$;
- Robust weighting: W enhance the large baseline contribution;
- Tapering: W enhance the small baseline contribution.

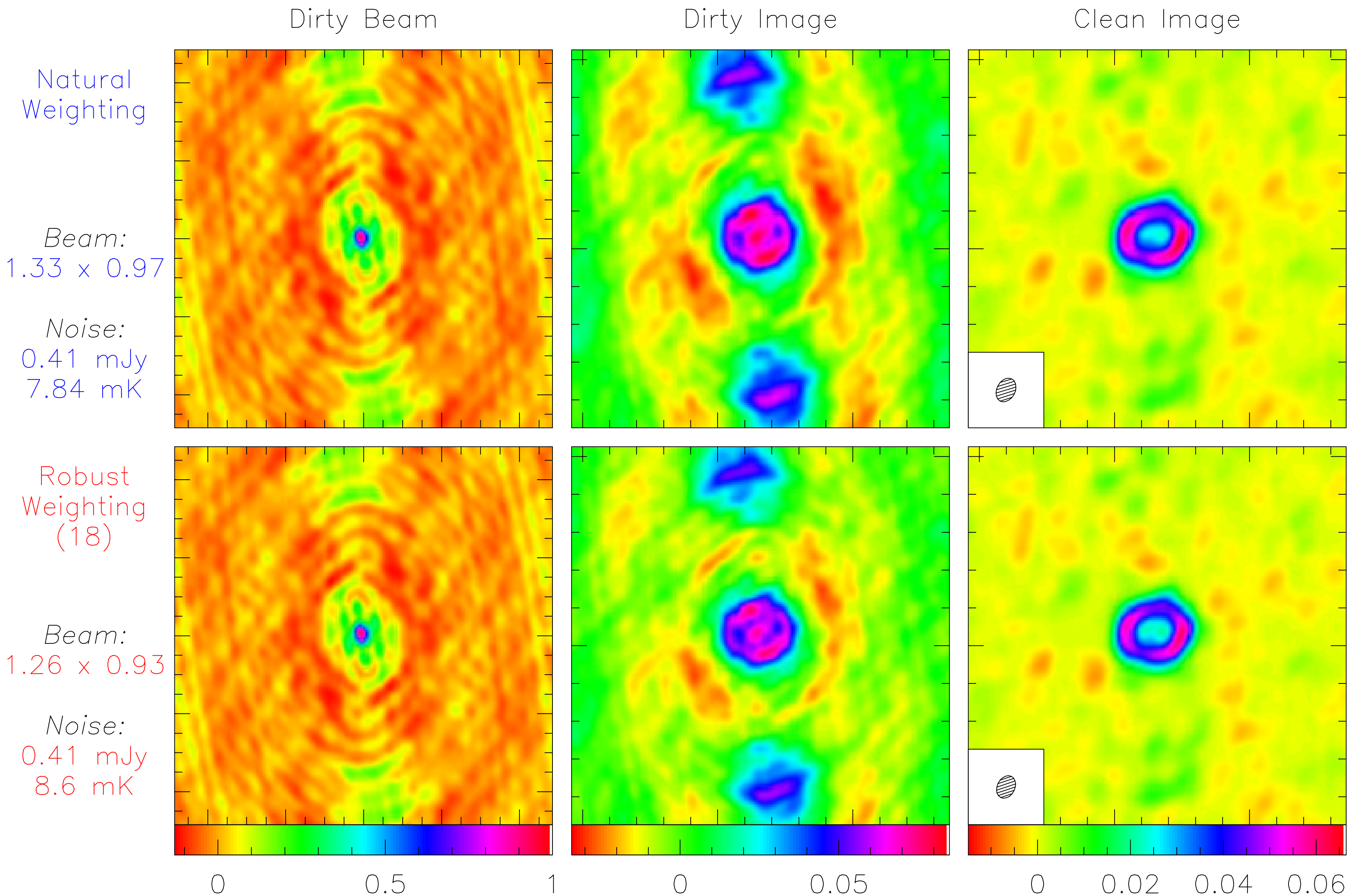
Robust Weighting: I. Definition

Definitions:

- Natural $= \sum_{(u,v)\in\text{Cell}} S$;
- $\sum_{(u,v)\in\text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \geq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is $0.5D$ where D is the single-dish antenna diameter (*i.e.* 15m for PdBI).



Robust Weighting: II. Examples



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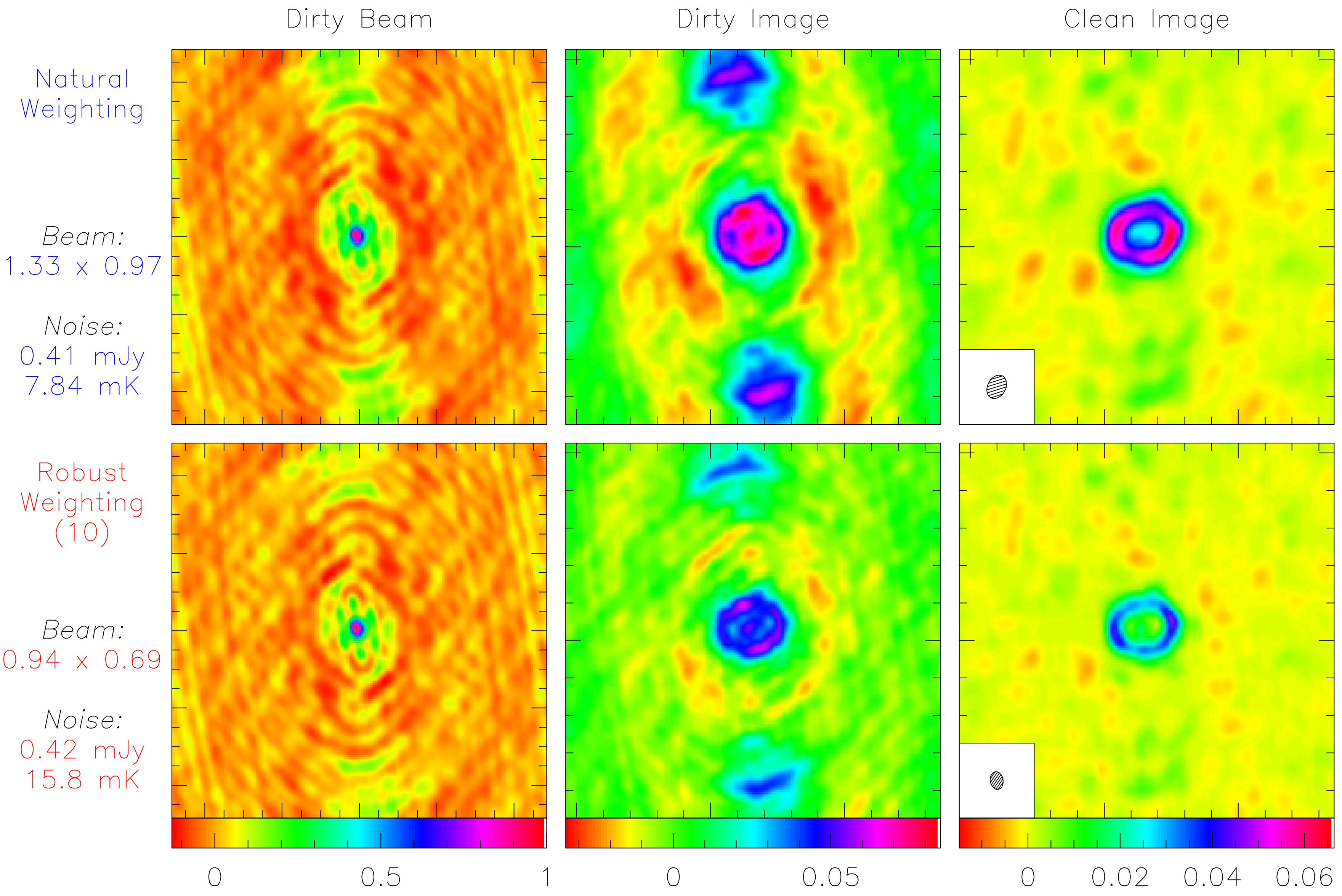

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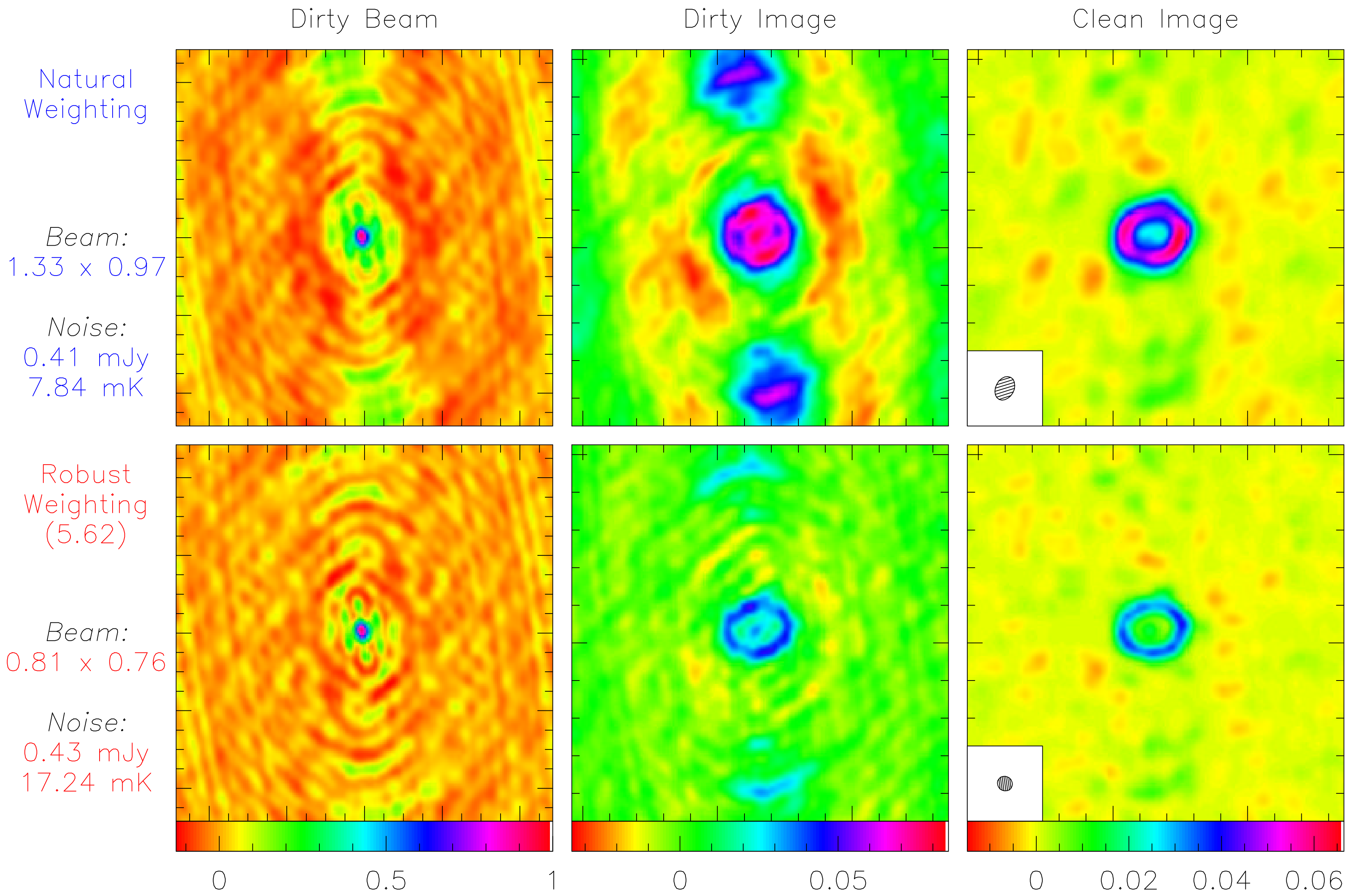

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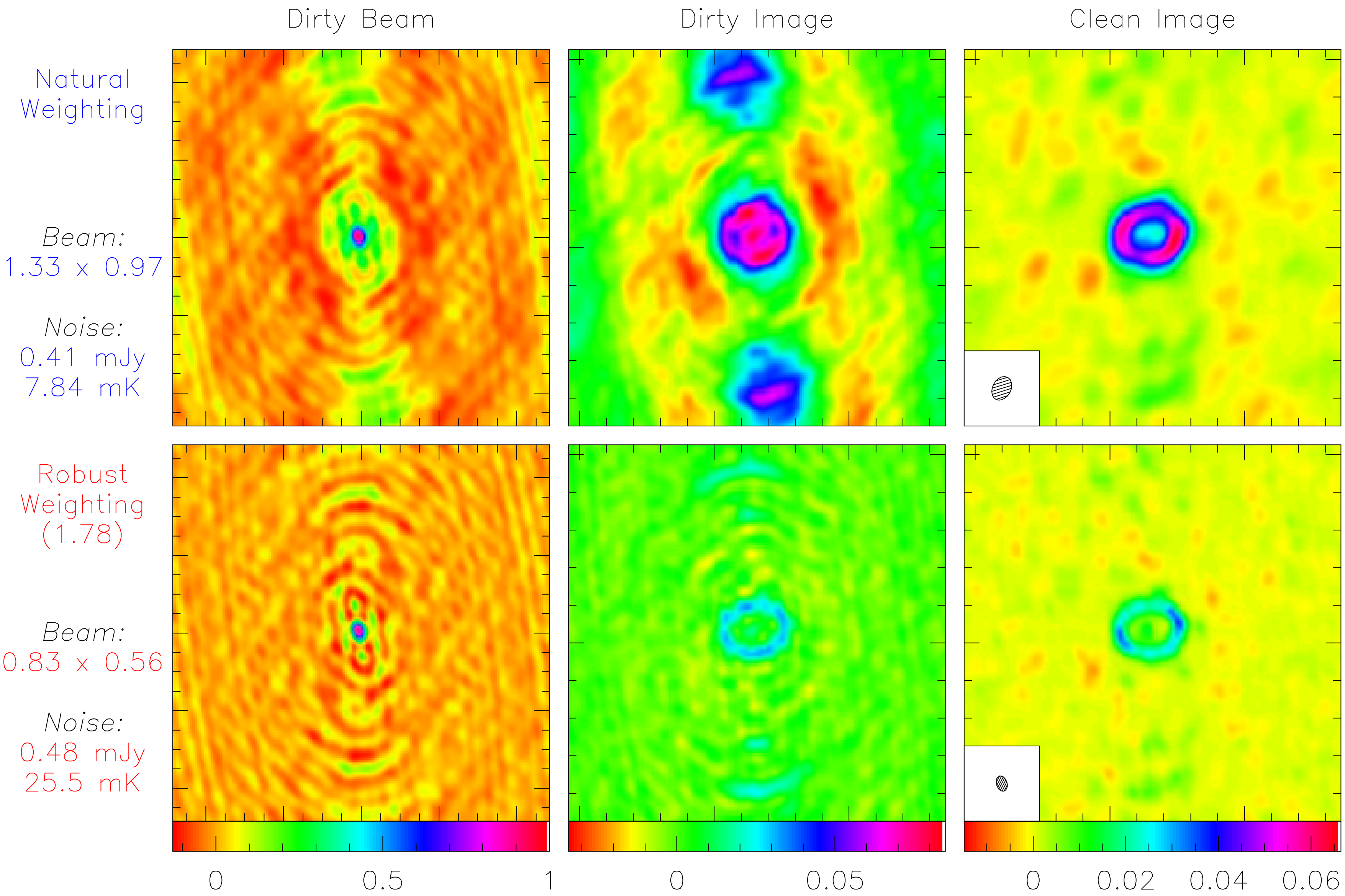


Robust Weighting: III. Definition and Properties

Definitions:

- Natural $= \sum_{(u,v)\in\text{Cell}} S$;
- $\sum_{(u,v)\in\text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \leq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is $0.5D$.

Properties:

- Increase the resolution;
- Lower the sidelobes;
- Degrade point source and brightness sensitivity.

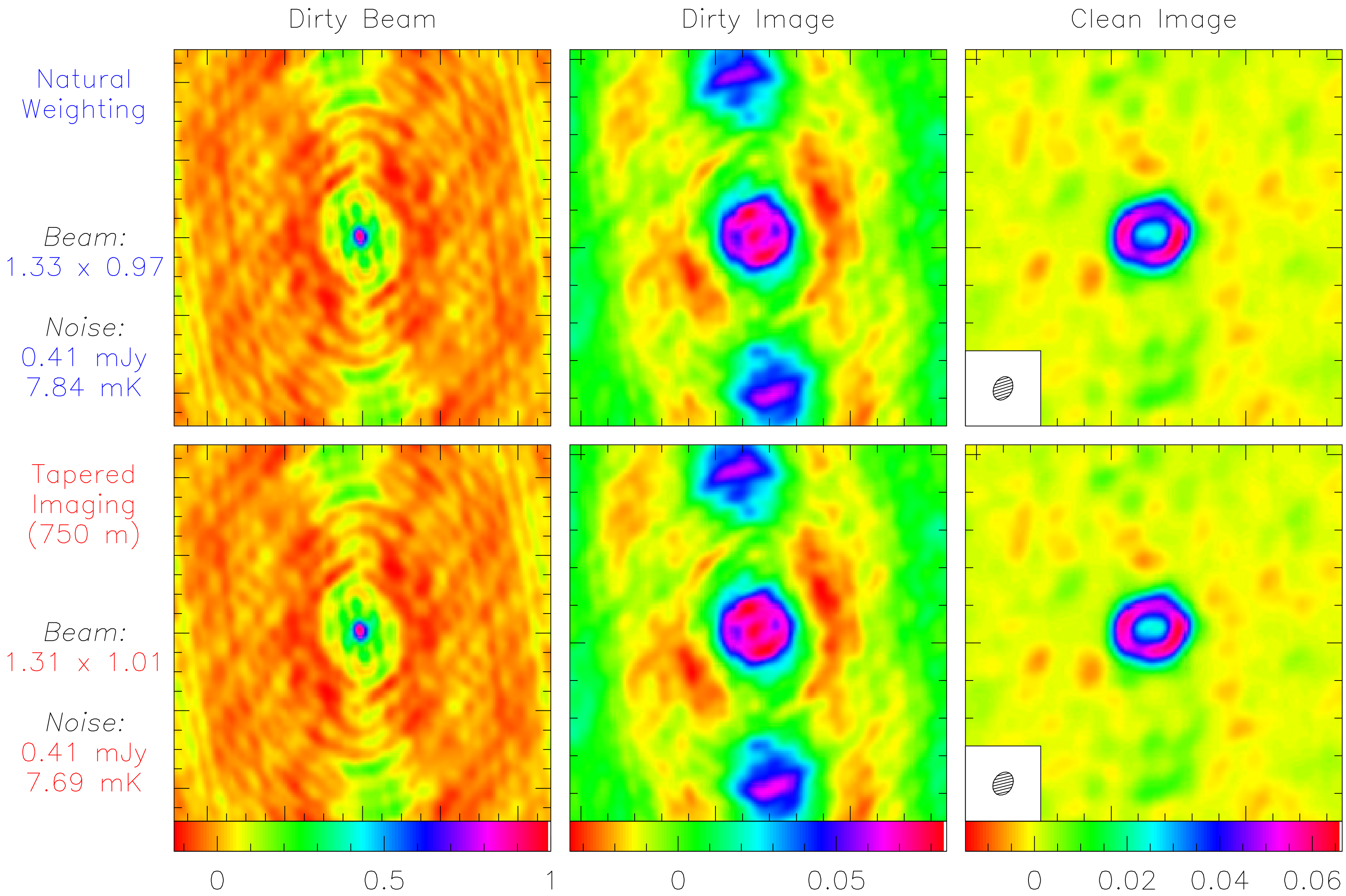
Tapering: I Definition

Definition:

- Apodization of the uv coverage in general by a Gaussian;
- $W = \exp\left\{-\frac{\left(u^2+v^2\right)}{t^2}\right\}$ where t = tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Tapering: II. Examples



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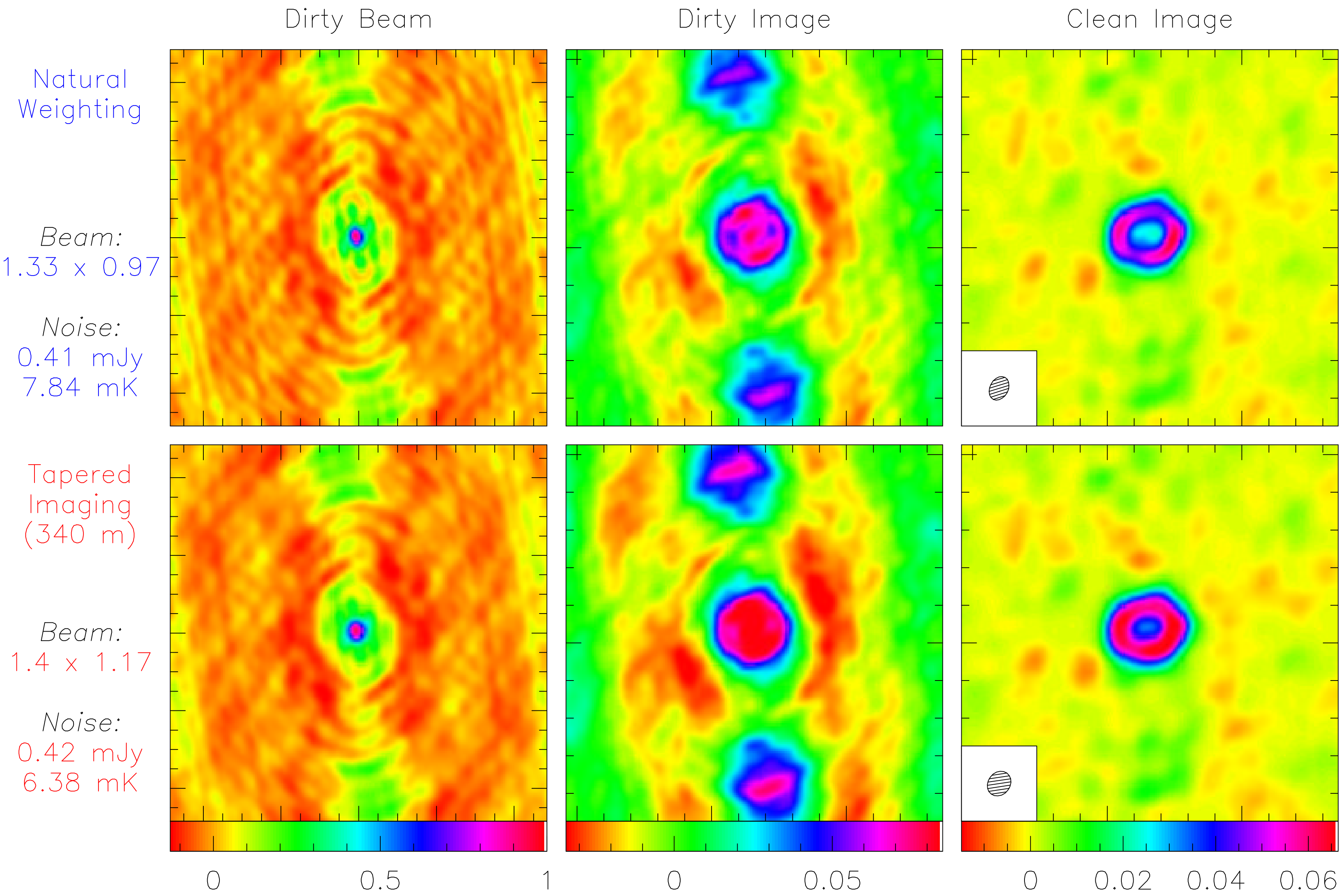
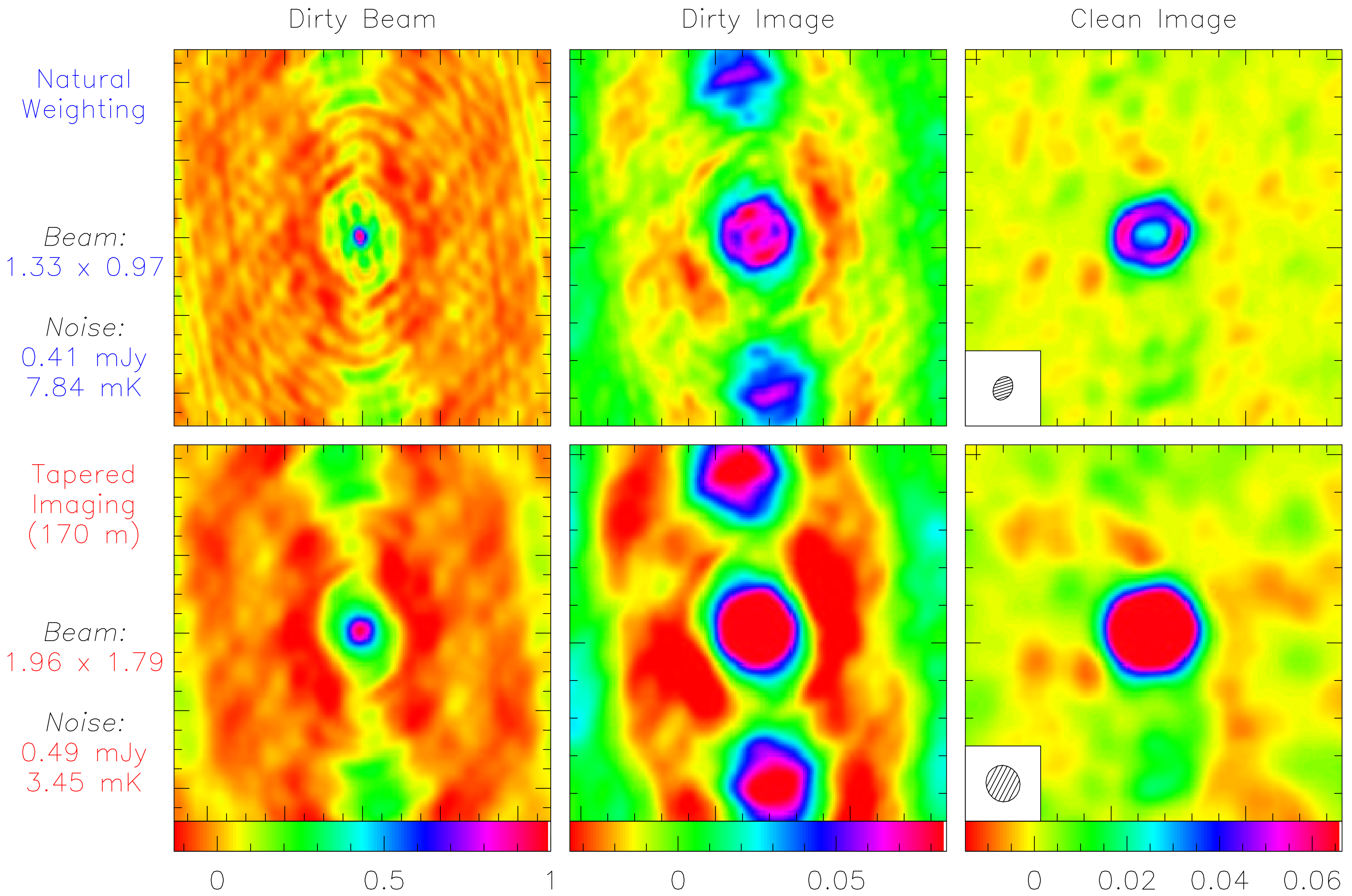


Tapering: II. Examples



Tapering: III. Definition and Properties

Definition:

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- $W = \exp\left\{-\frac{\left(u^2+v^2\right)}{t^2}\right\}$ where t = tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Properties:

- Decrease the resolution;
- Degrade point source sensitivity;
- Increase brightness sensitivity to “medium size” structures.

Inconvenient: Throw out some information.

$\Rightarrow$ To increase sensitivity to extended sources, use compact arrays not tapering.

Weighting and Tapering: Summary

	Robust	Natural	Tapering
Resolution	High	Medium	Low
Side Lobes	$\searrow$	Medium	?
Point Source Sensitivity	$\searrow$	Maximum	$\searrow$
Extended Source Sensitivity	$\searrow$	Medium	$\nearrow$

Non-circular tapering:

Sometimes $\Rightarrow$ Better (*i.e.* more circular) beams.

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Physical information on your source	

Deconvolution: I. Philosophy

$$I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\} + N.$$

Information lost:

- Irregular, incomplete sampling $\Rightarrow$ convolution by B_{dirty};
- Noise $\Rightarrow$ Low signal structures undetected.

$\Rightarrow$ 1. Impossible to recover the intrinsic source structure!

$\Rightarrow$ 2. Infinite number of solutions!

$$\left\{ \begin{array}{c} S \text{ solution } (\textit{i.e. } I_{\text{meas}} = B_{\text{dirty}} * S + N) \\ B_{\text{dirty}} * R = 0 \end{array} \right\} \Rightarrow (S + R) \text{ solution.}$$

Deconvolution: I. Philosophy (continued)

$$I_{\rm meas} = B_{\rm dirty} * \left\{ B_{\rm primary}.I_{\rm source} \right\} + N.$$

Information lost:

⇒ 1. Impossible to recover the intrinsic source structure!

⇒ 2. Infinite number of solutions!

Deconvolution goal: Finding a sensible intensity distribution compatible with the intrinsic source one.

Deconvolution needs:

- Some *a priori* assumptions about the source intensity distribution;
- As much as possible knowledge of
  - $B_{\rm dirty}$ (OK in radioastronomy);
  - Noise properties.

The best solution: A Gaussian $B_{\rm dirty}$ ⇒ No deconvolution needed!

Deconvolution: II. The Basic CLEAN Algorithm

a priori assumption: Source = Collection of point sources.

Idea: "Matching pursuit".

Algorithm:

1 Initialize

- the residual map to the dirty map;
- the Clean component list to an empty (NULL) value;

2 Identify pixel of $|I_{\text{max}}|$ in residual map as a point source;

3 Add $\gamma.I_{\text{max}}$ to clean component list;

4 Subtract $\gamma.I_{\text{max}}$ from residual map;

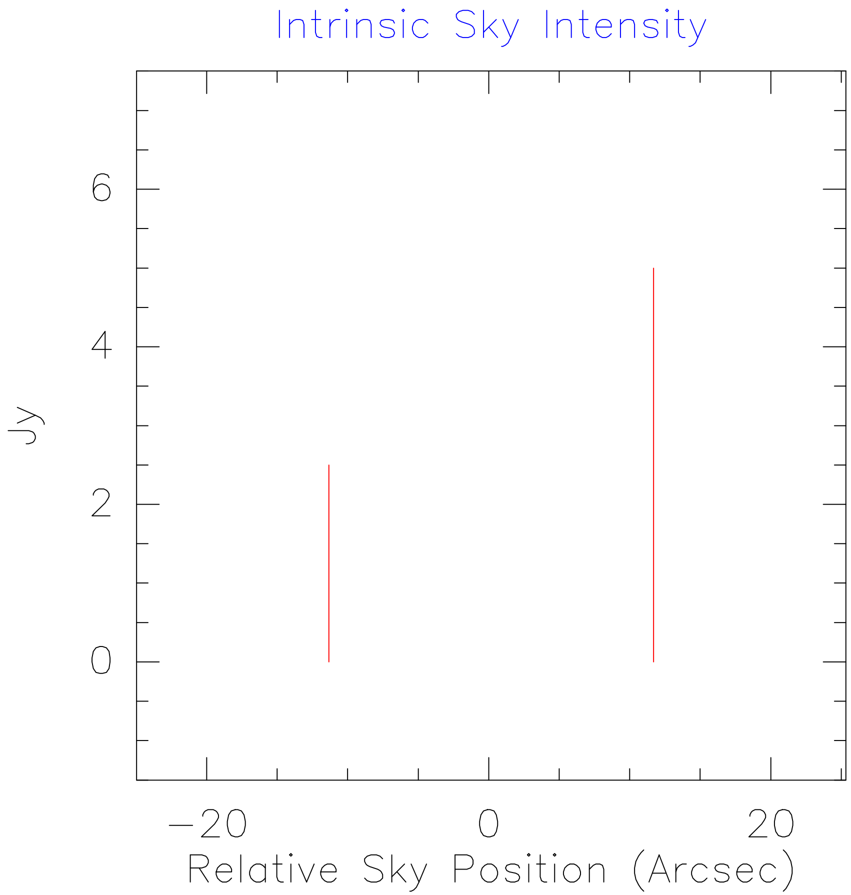
5 Go back to point 2 while stopping criterion is not matched;

6 Convolution by Clean beam (*a posteriori* regularization);

5 Addition of residual map to enable:

- Correction when cleaning is too superficial;
- Noise estimation.

Deconvolution: II. The Basic Clean Algorithm
1. First Illustration



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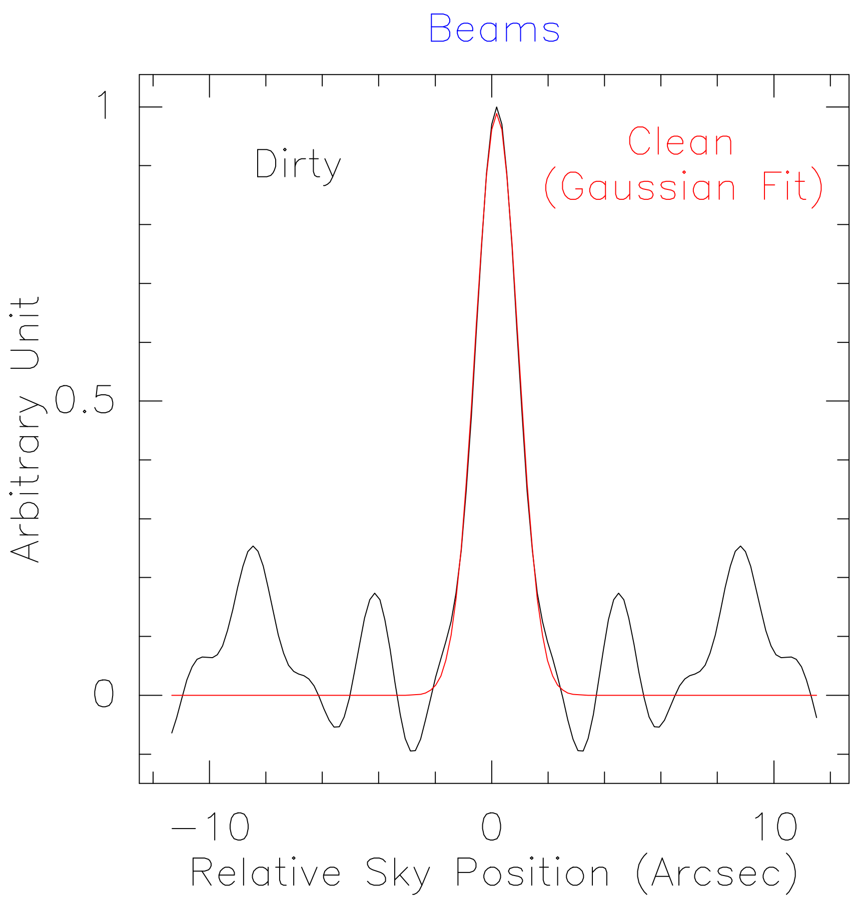

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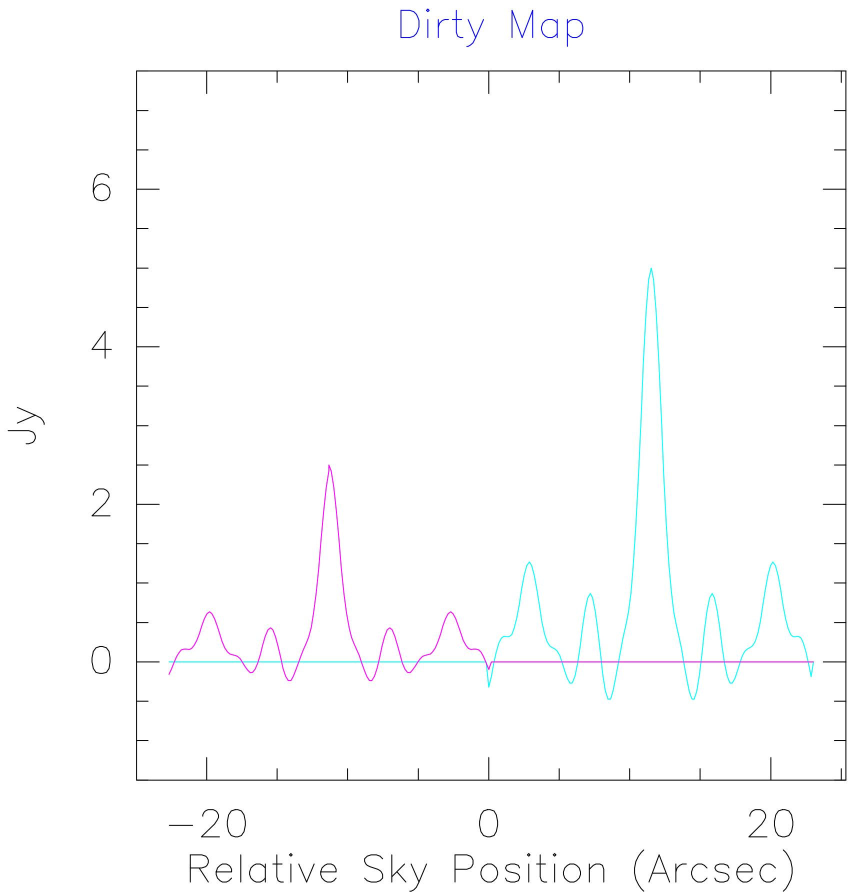
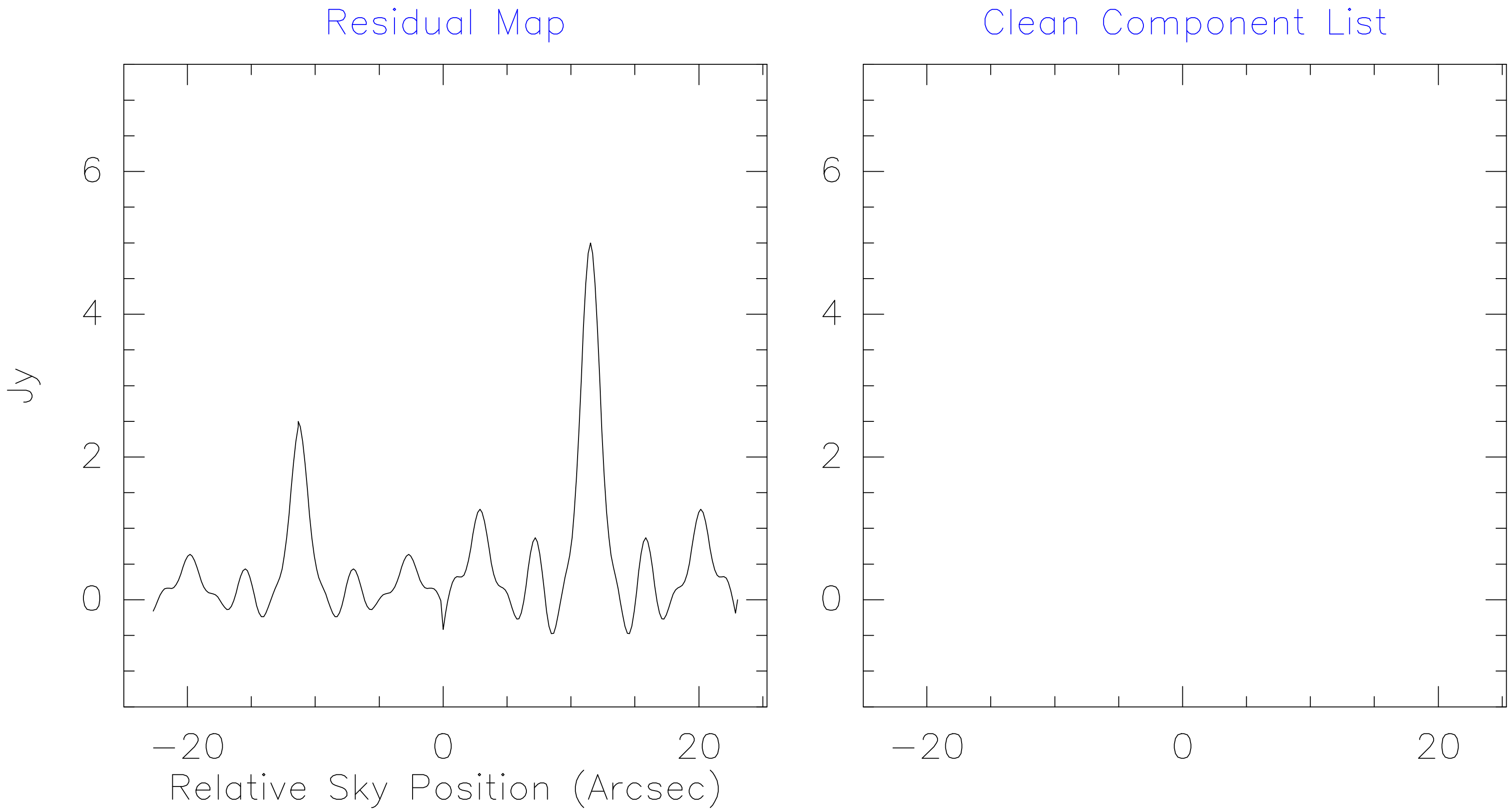


Deconvolution: II. The Basic Clean Algorithm
1. First Illustration



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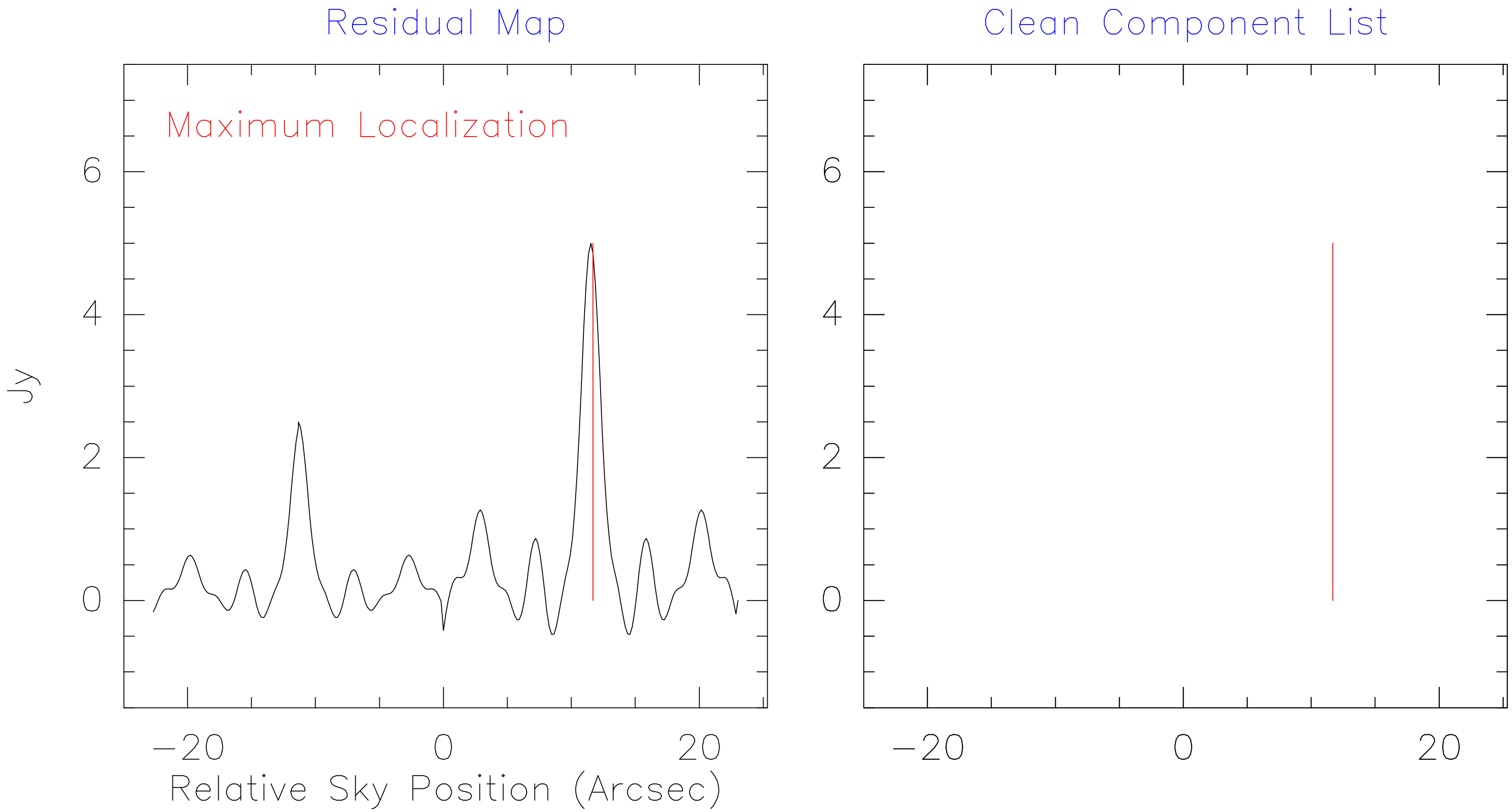

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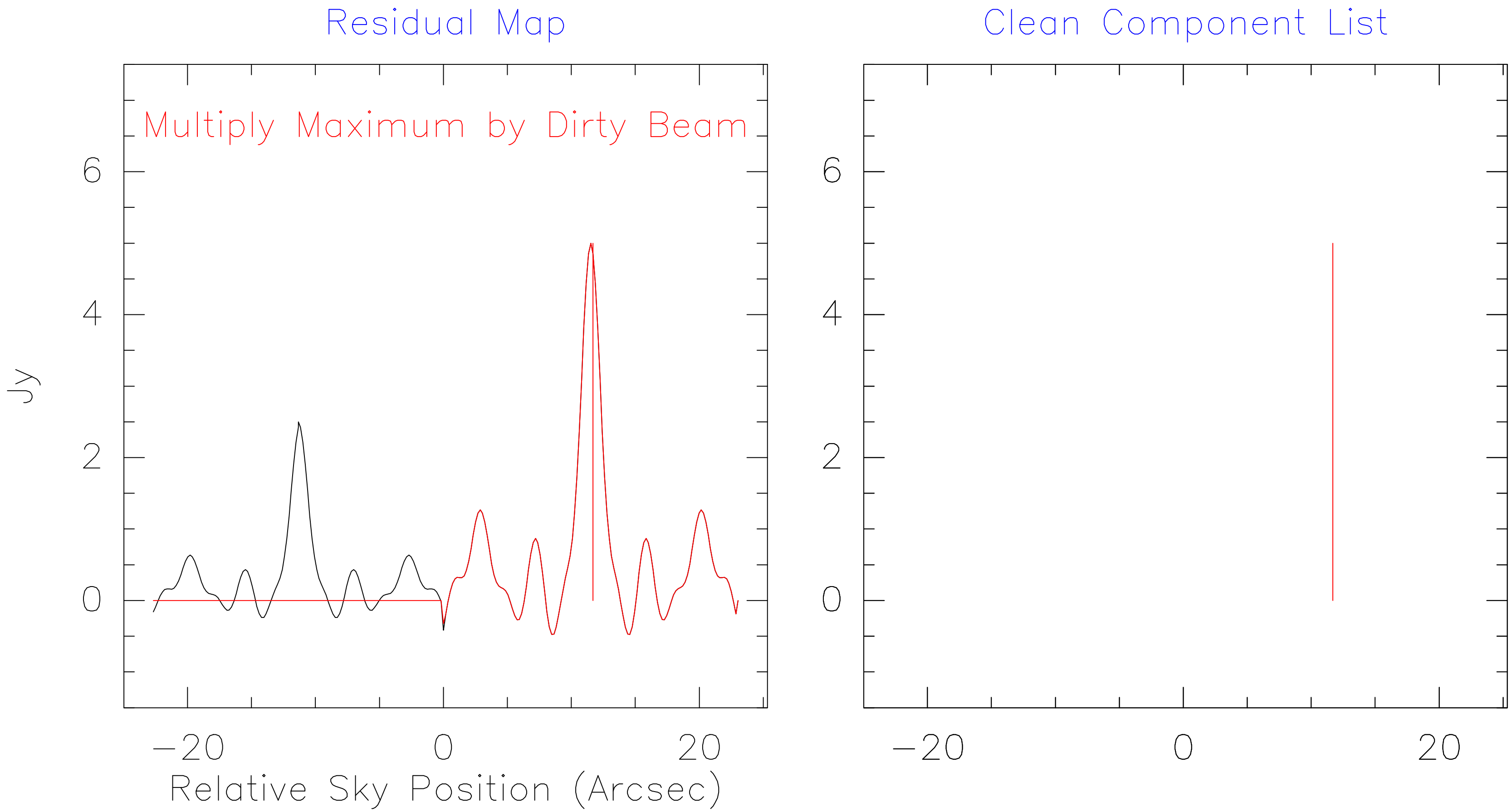

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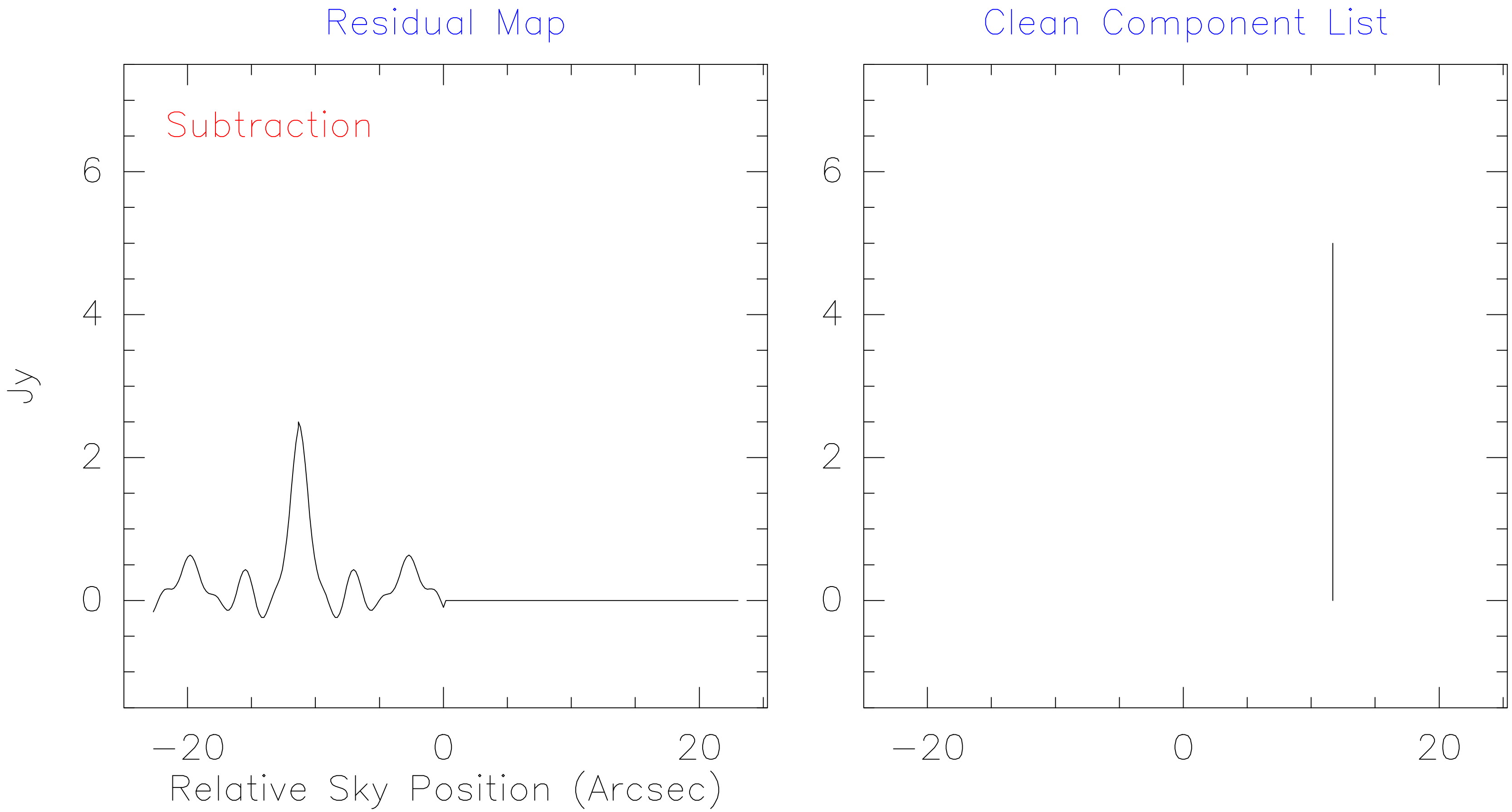
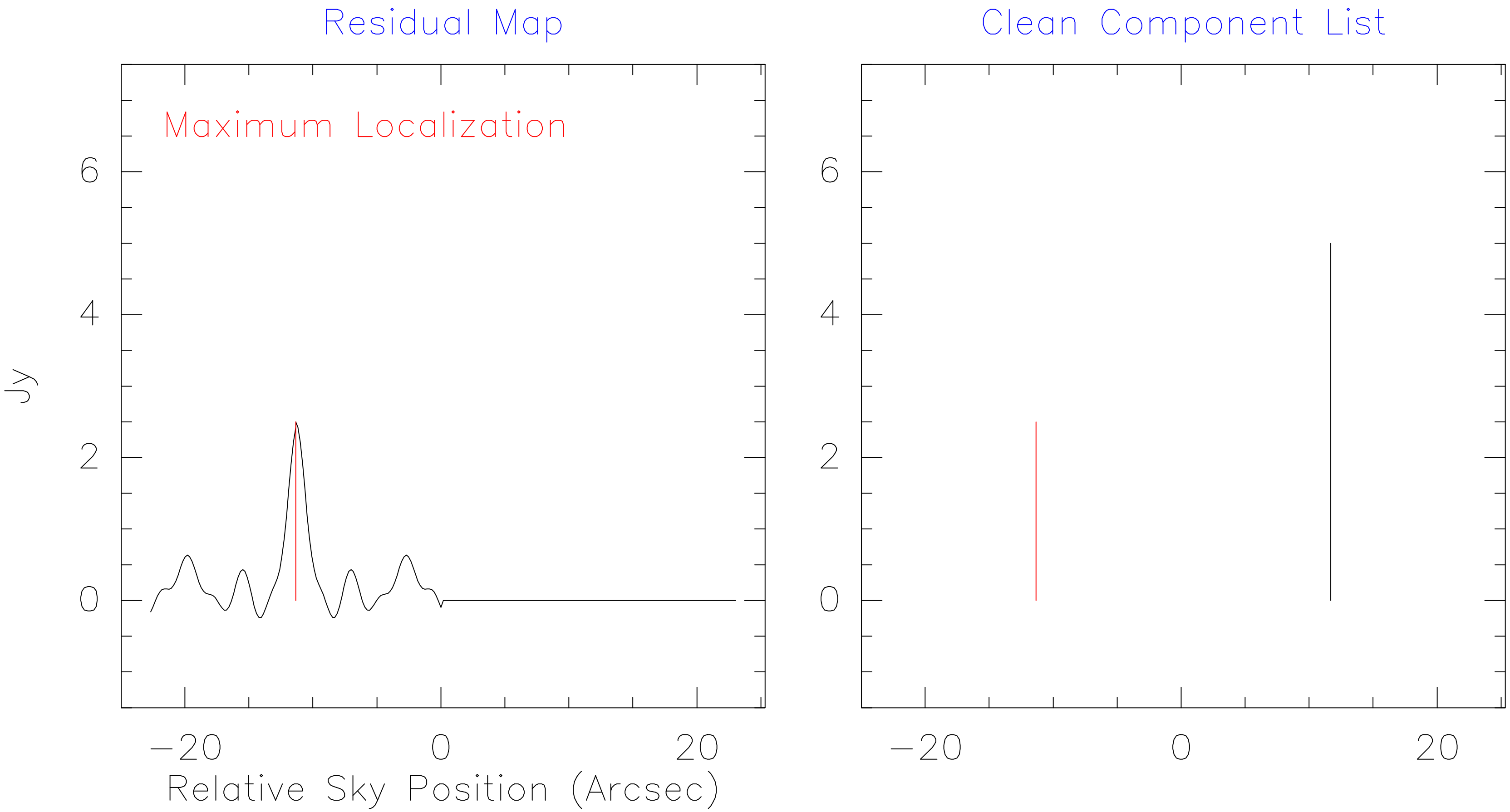


Deconvolution: II. The Basic Clean Algorithm
1. First Illustration



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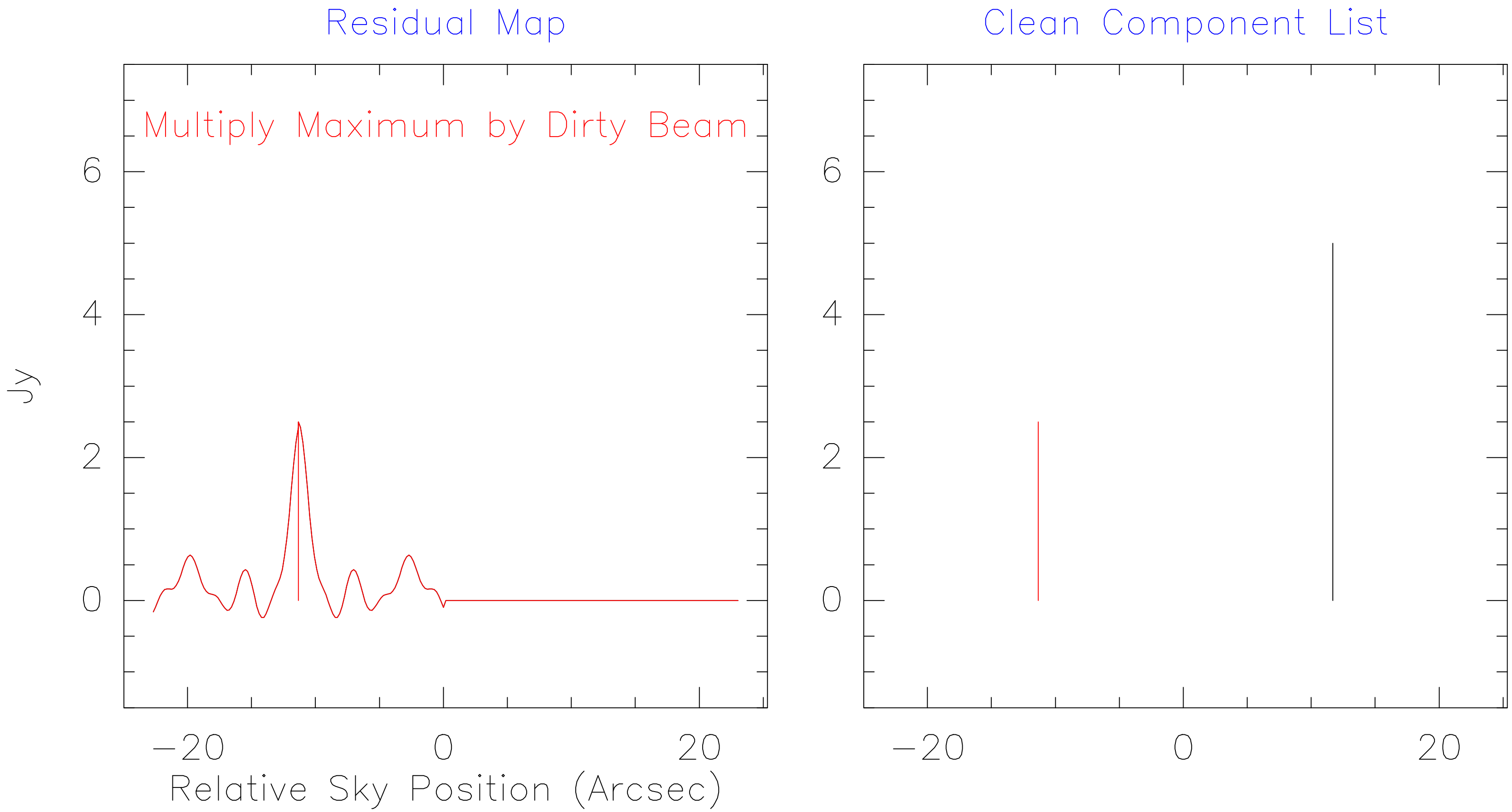

Deconvolution: II. The Basic Clean Algorithm
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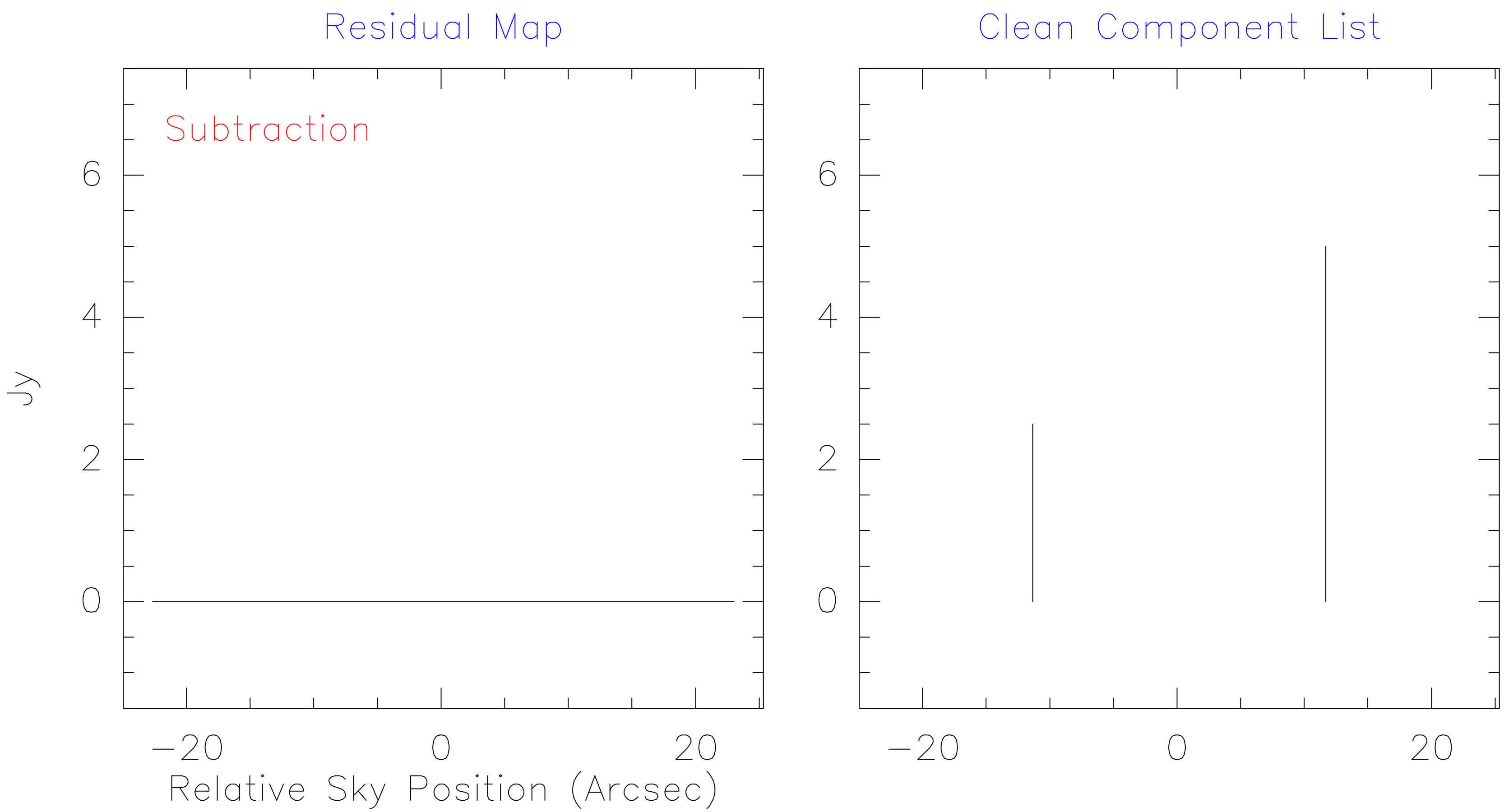

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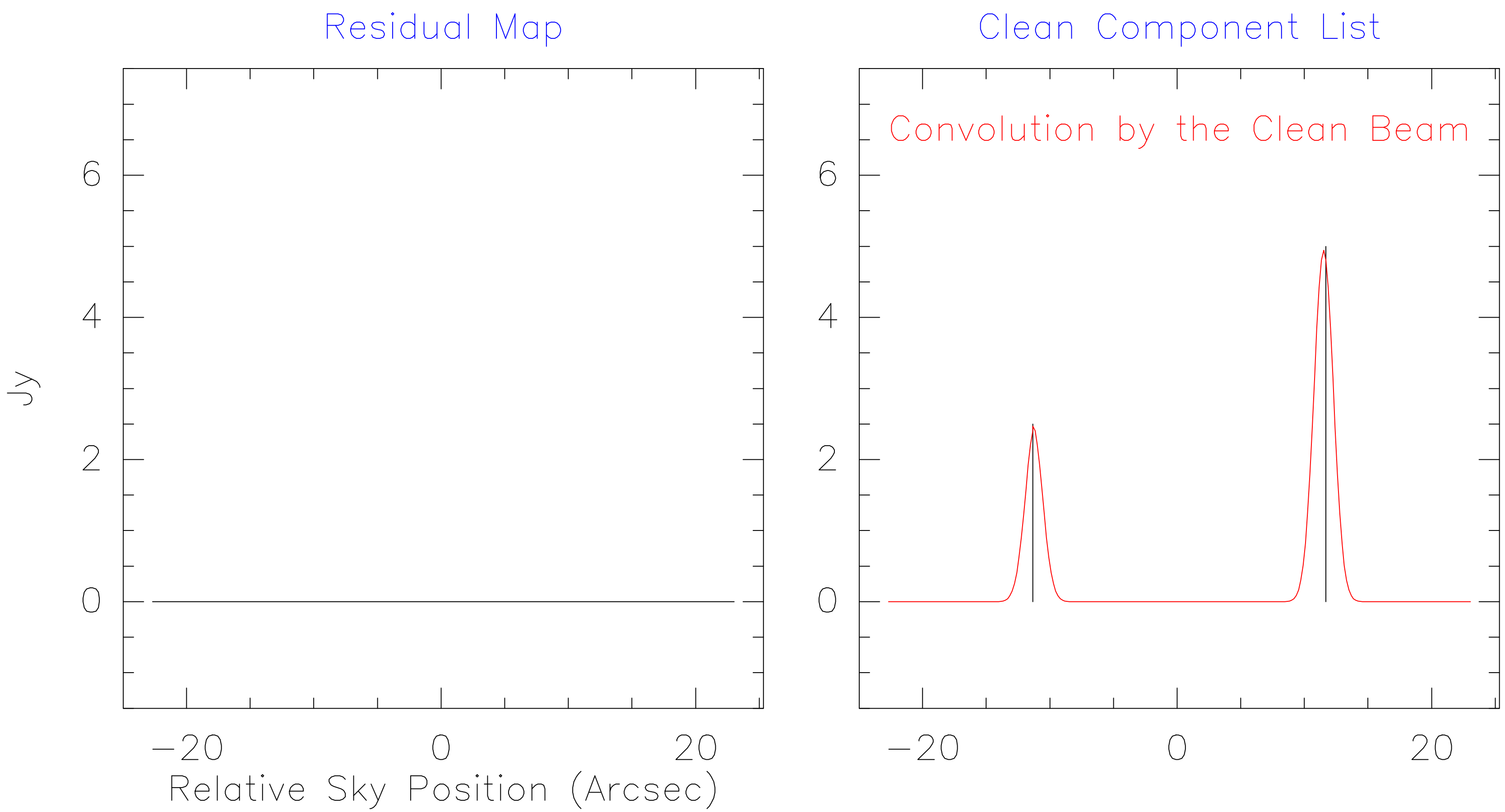
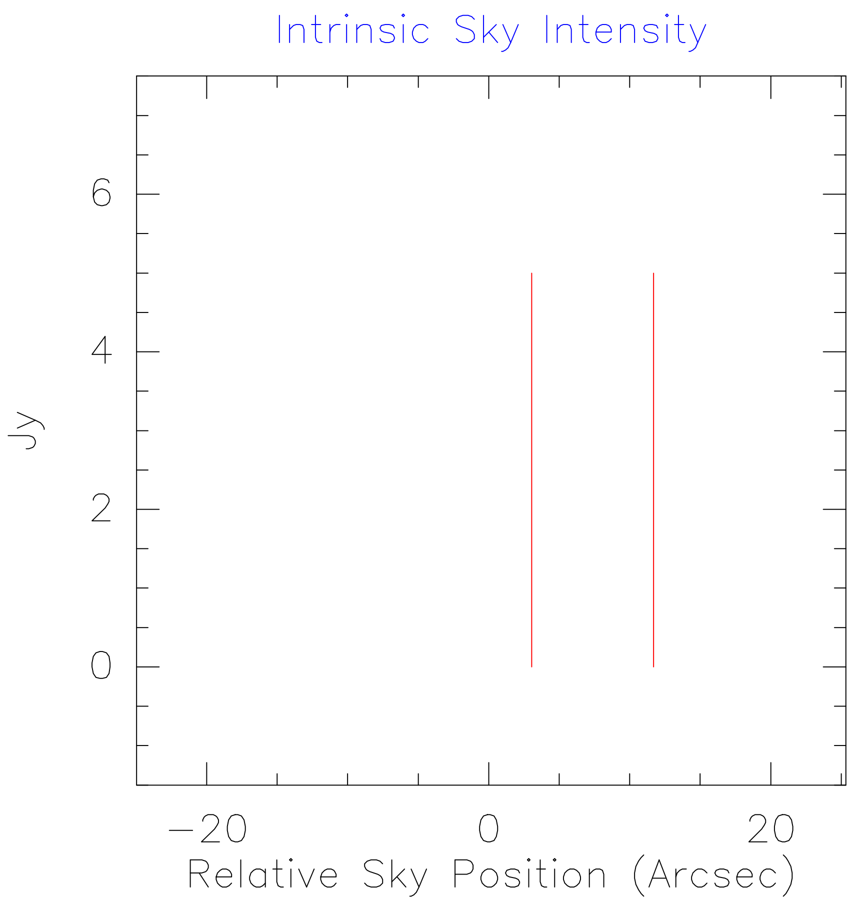


Deconvolution: II. The Basic Clean Algorithm
2. Second Illustration



Deconvolution: II. The Basic Clean Algorithm
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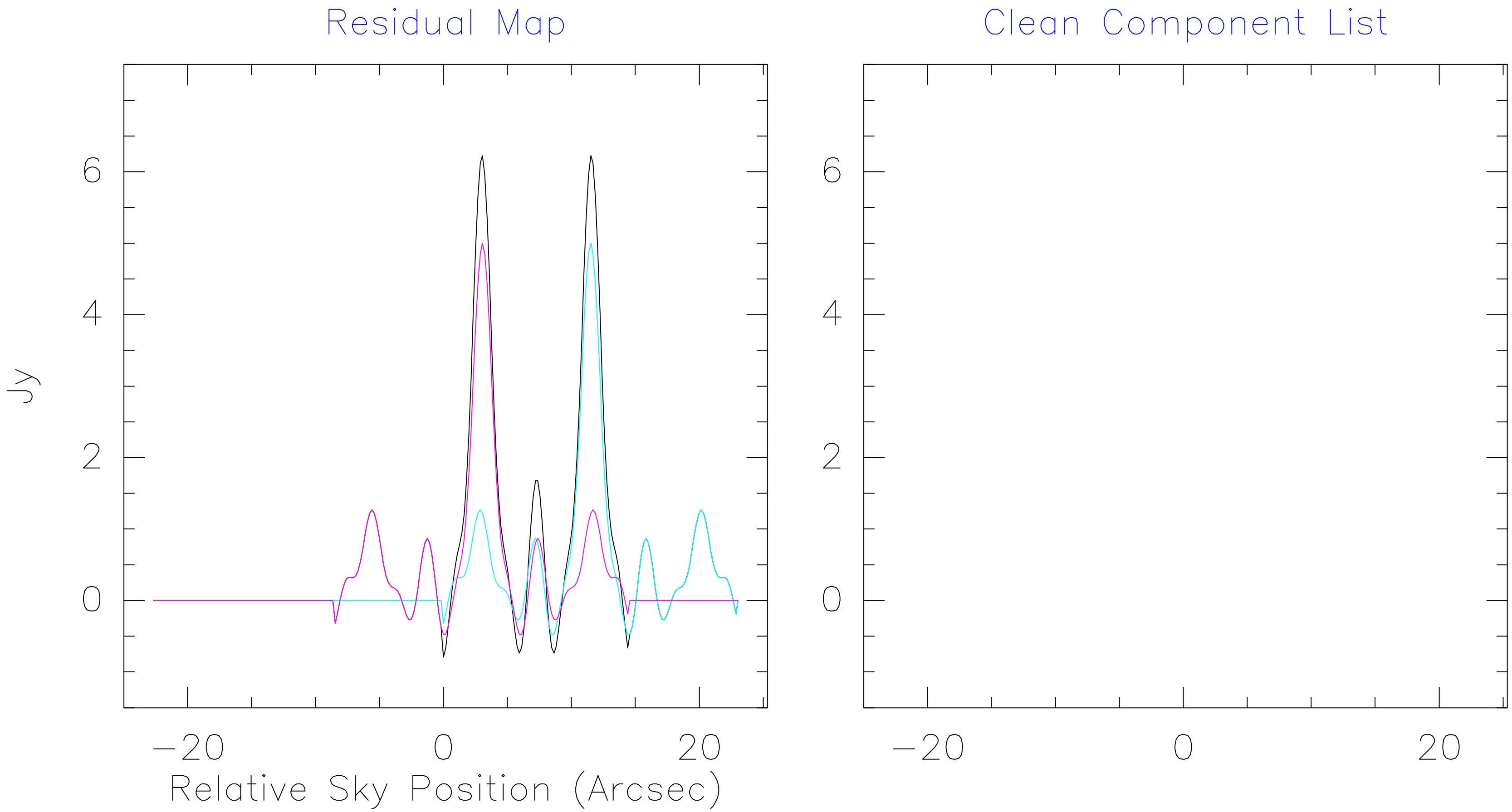

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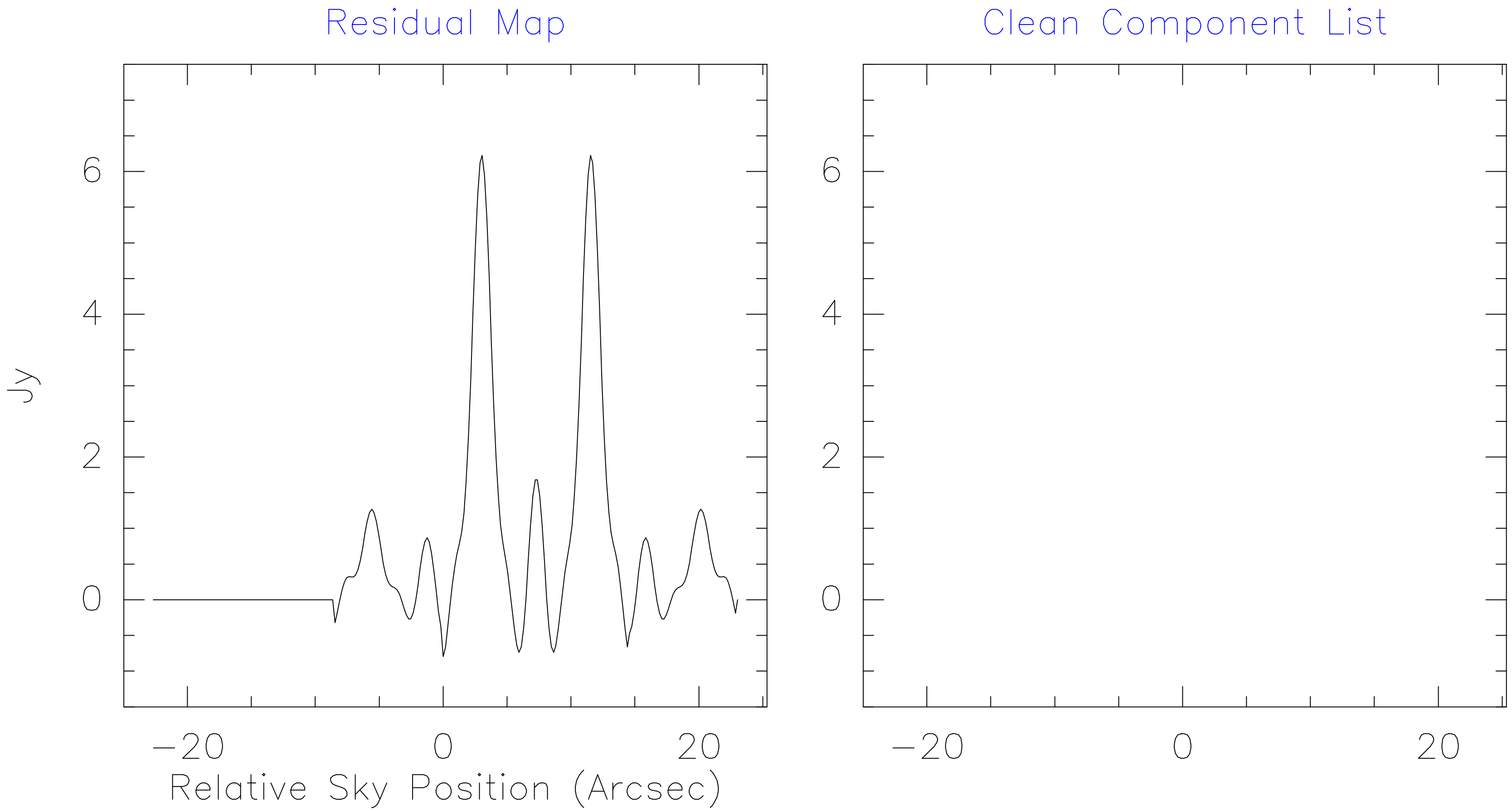

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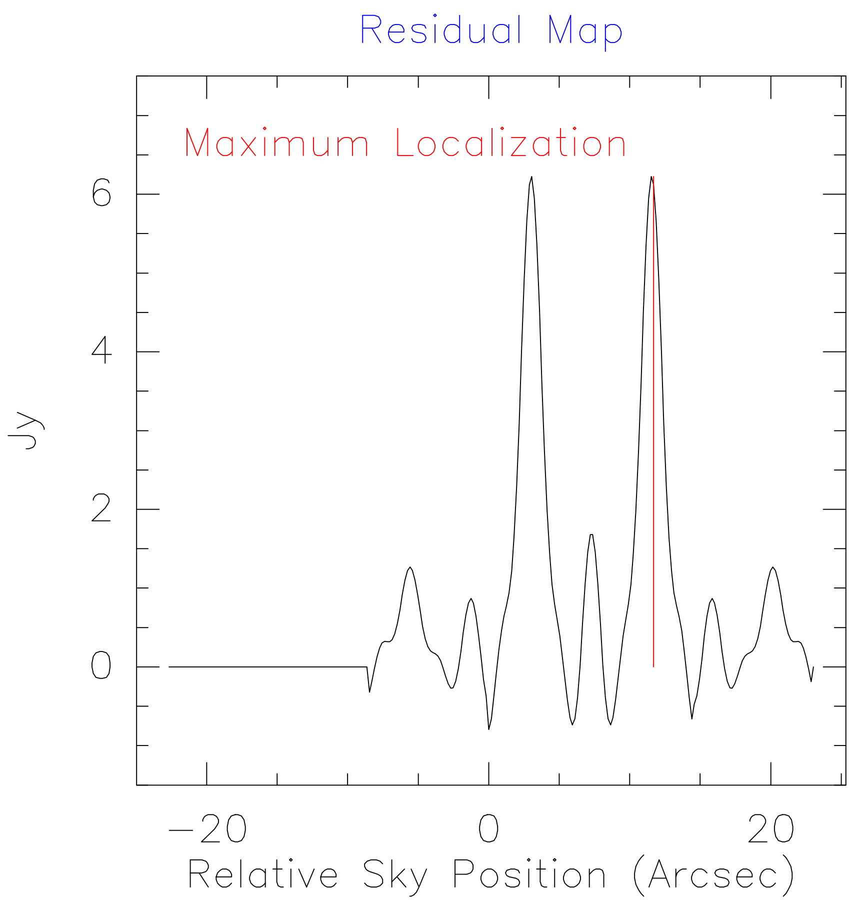

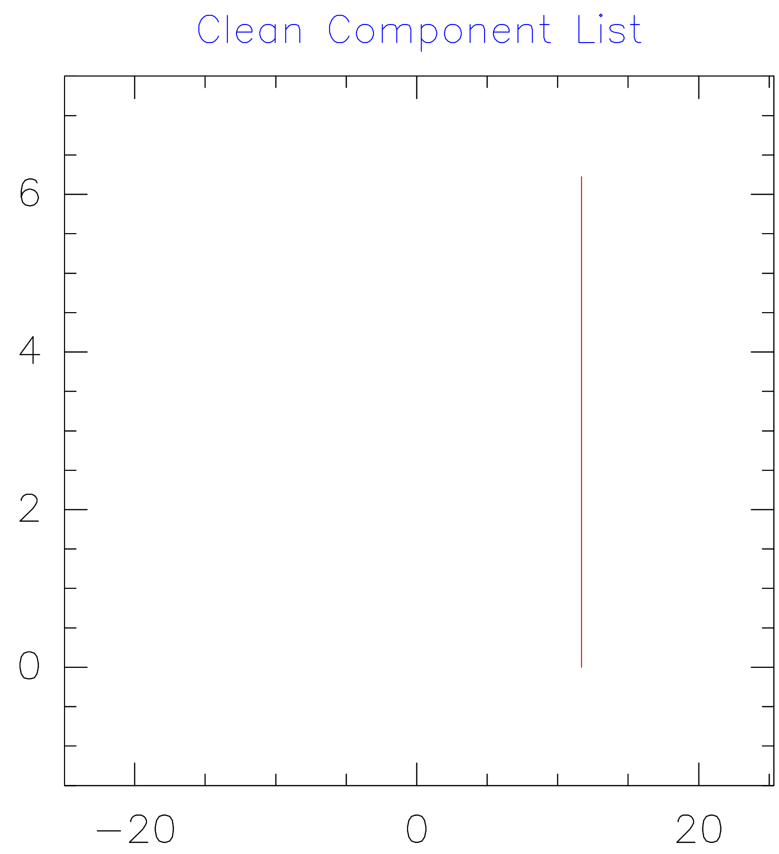

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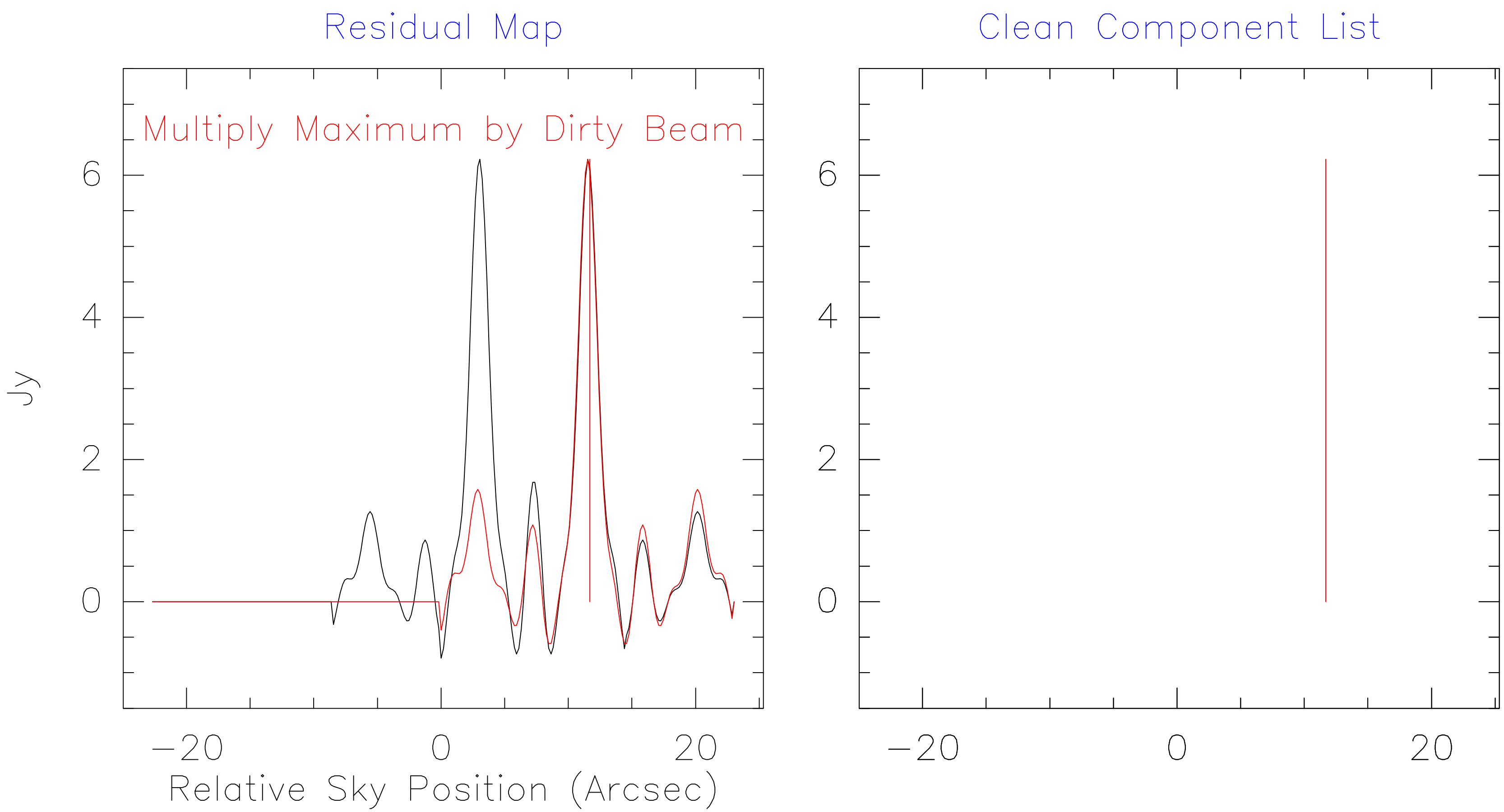

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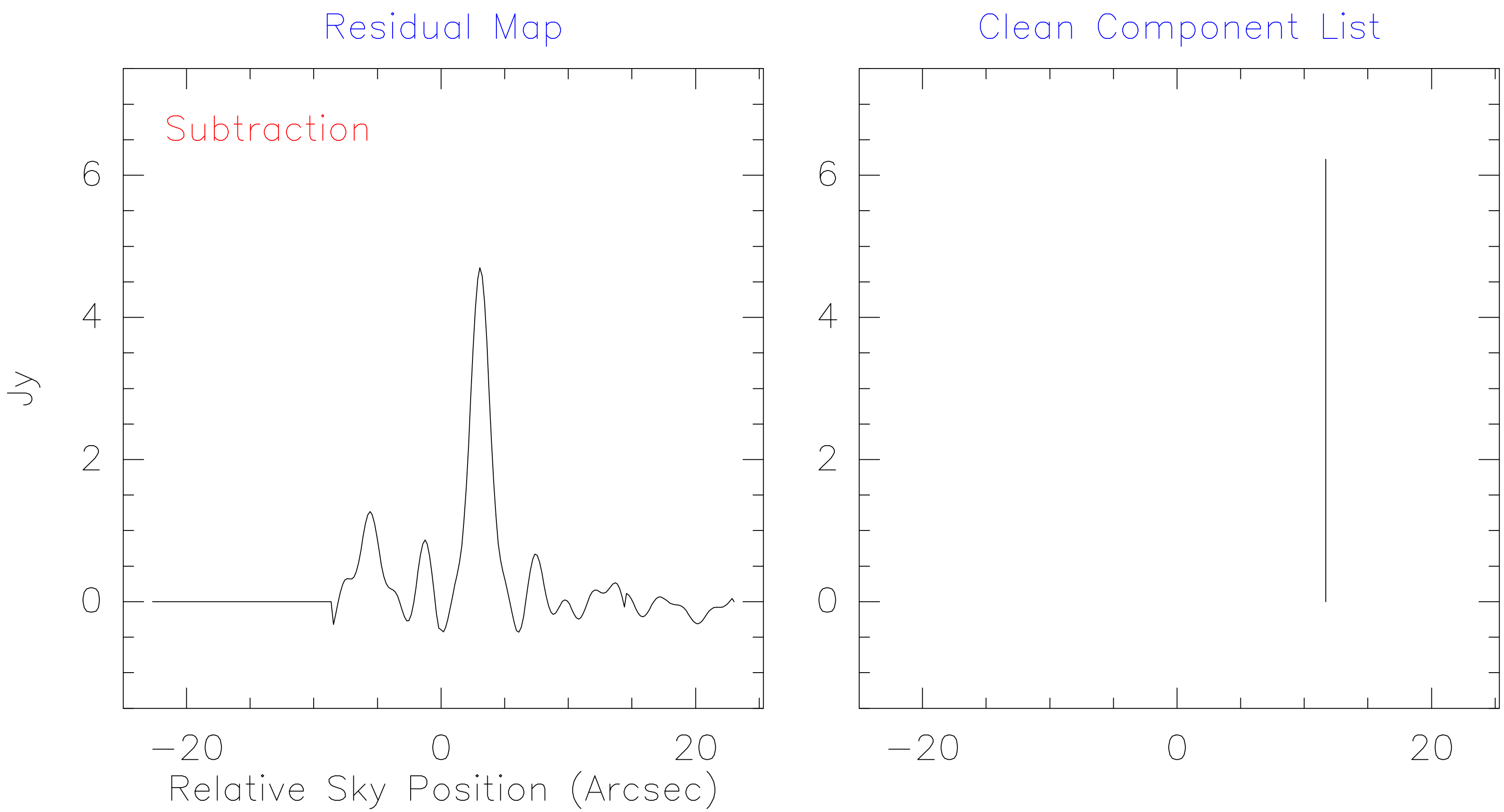

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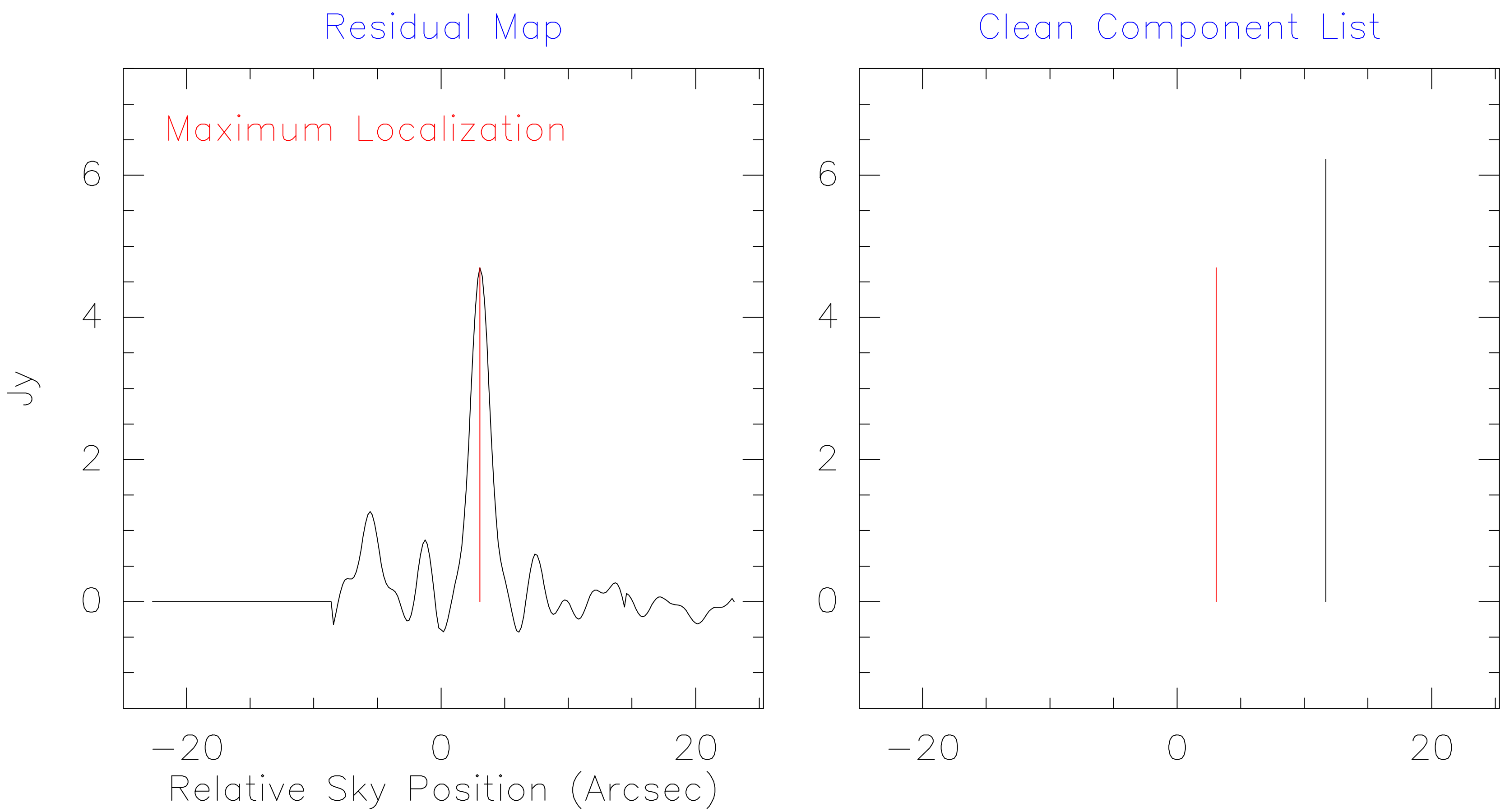

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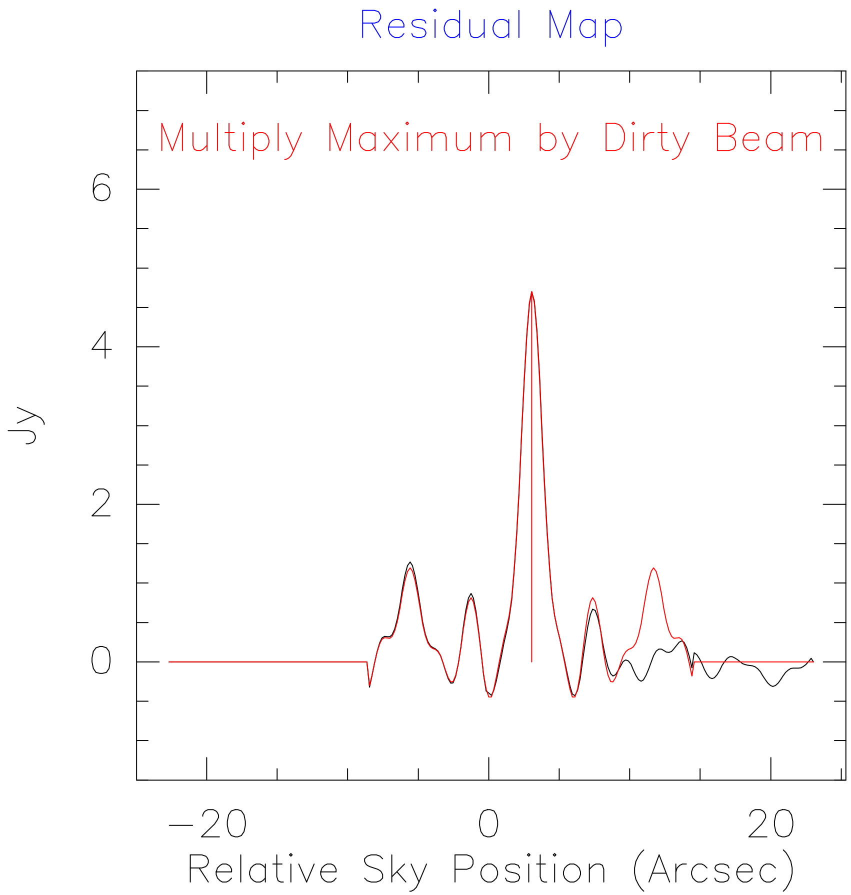

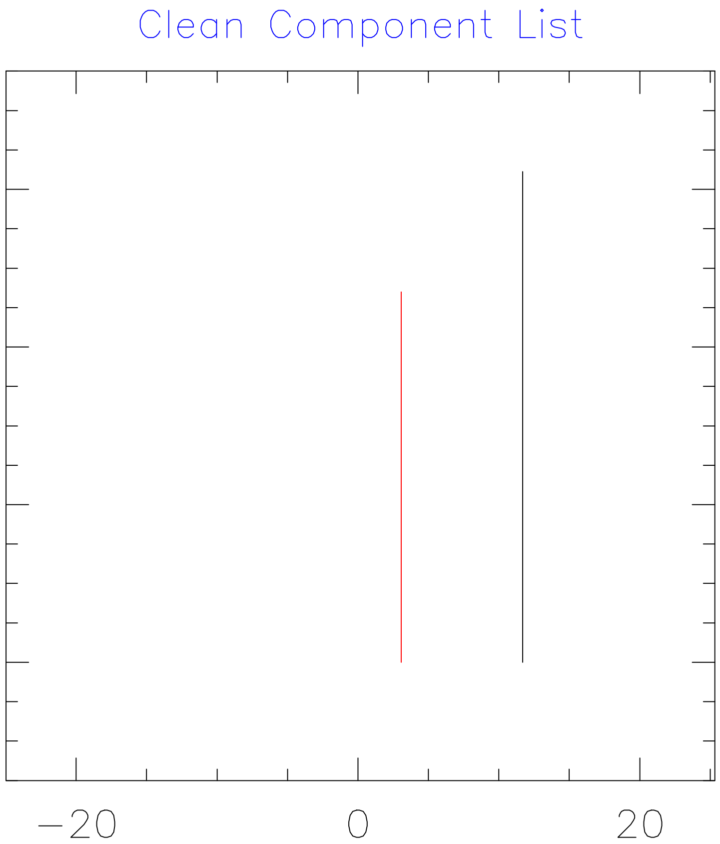

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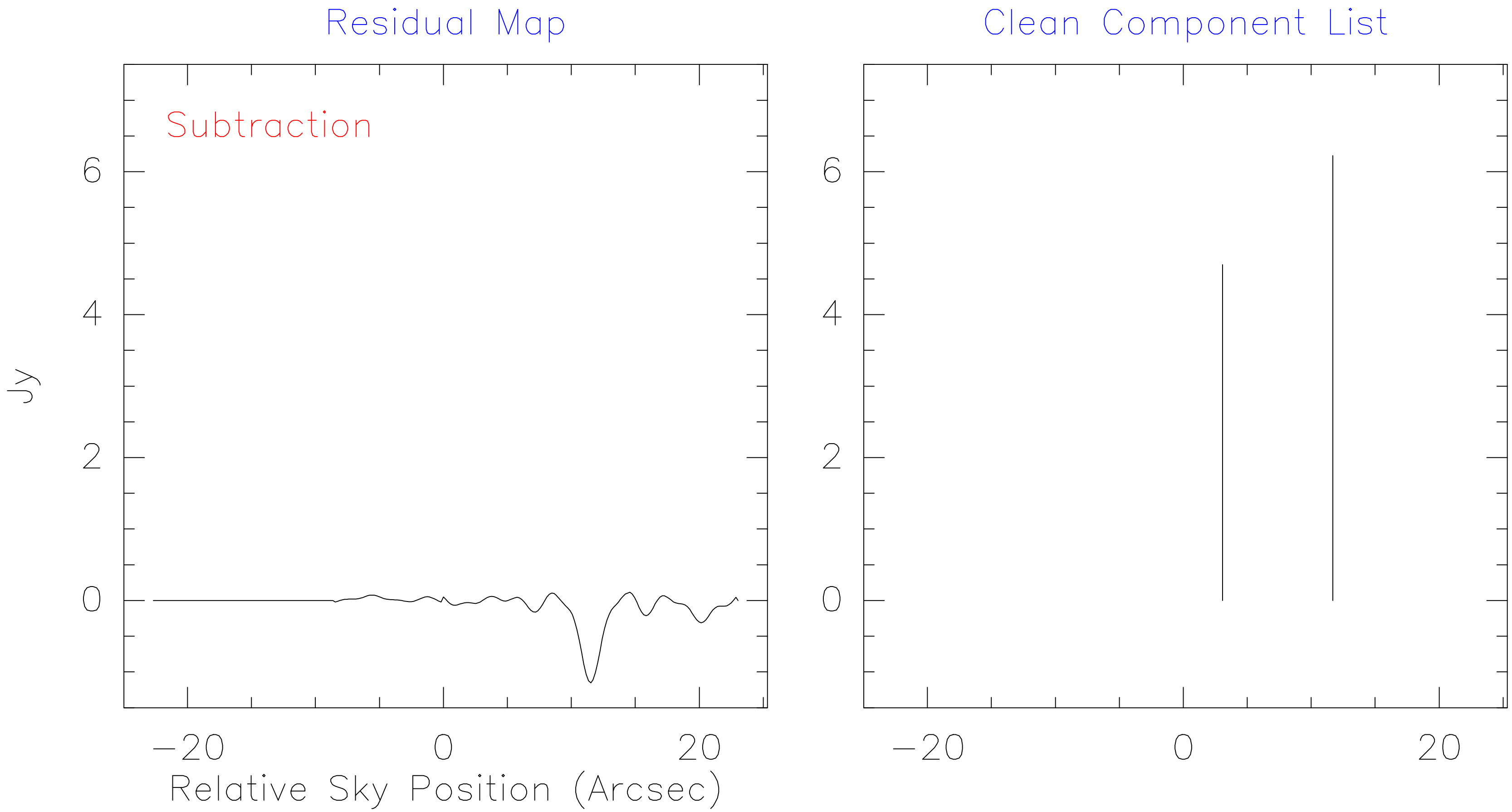
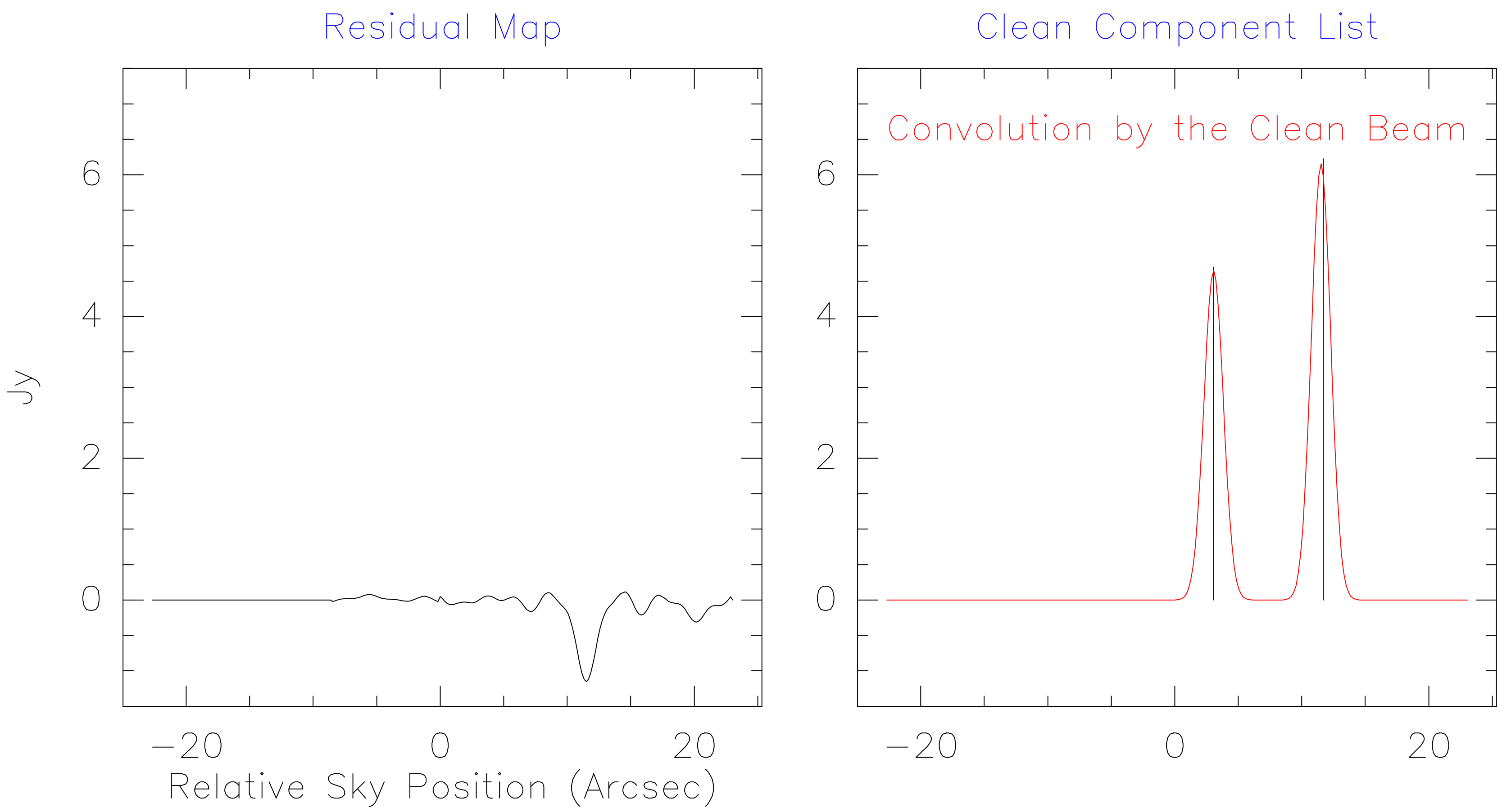
2. Second Illustration



Deconvolution: II. The Basic Clean Algorithm
2. Second Illustration

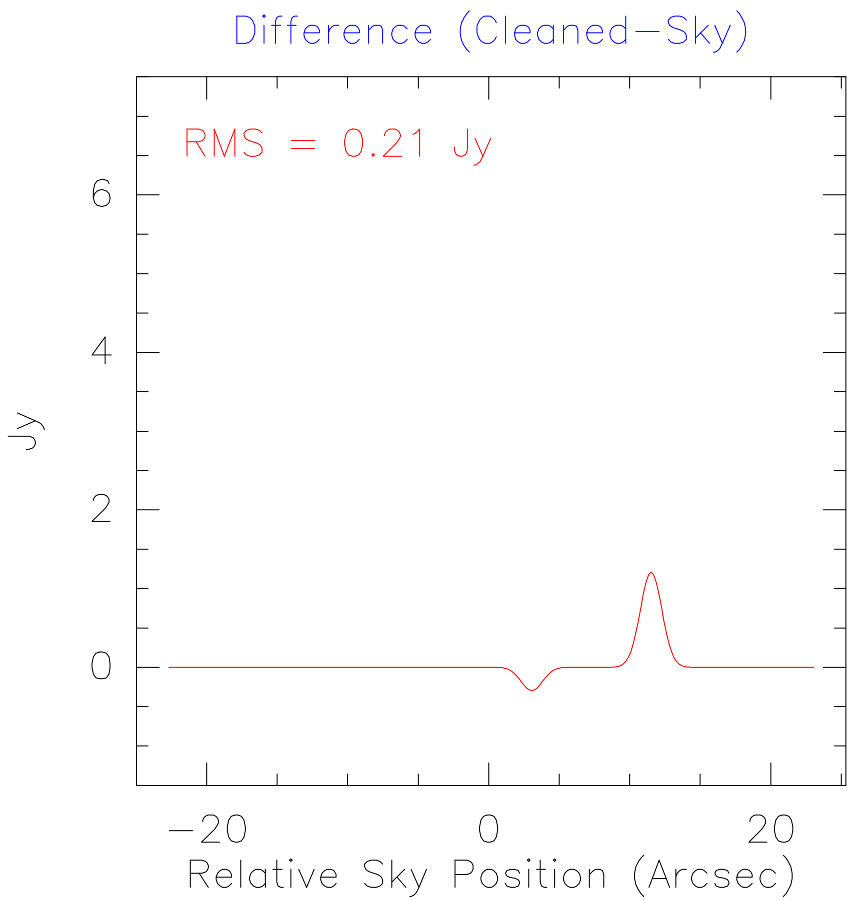


Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

Convergence:

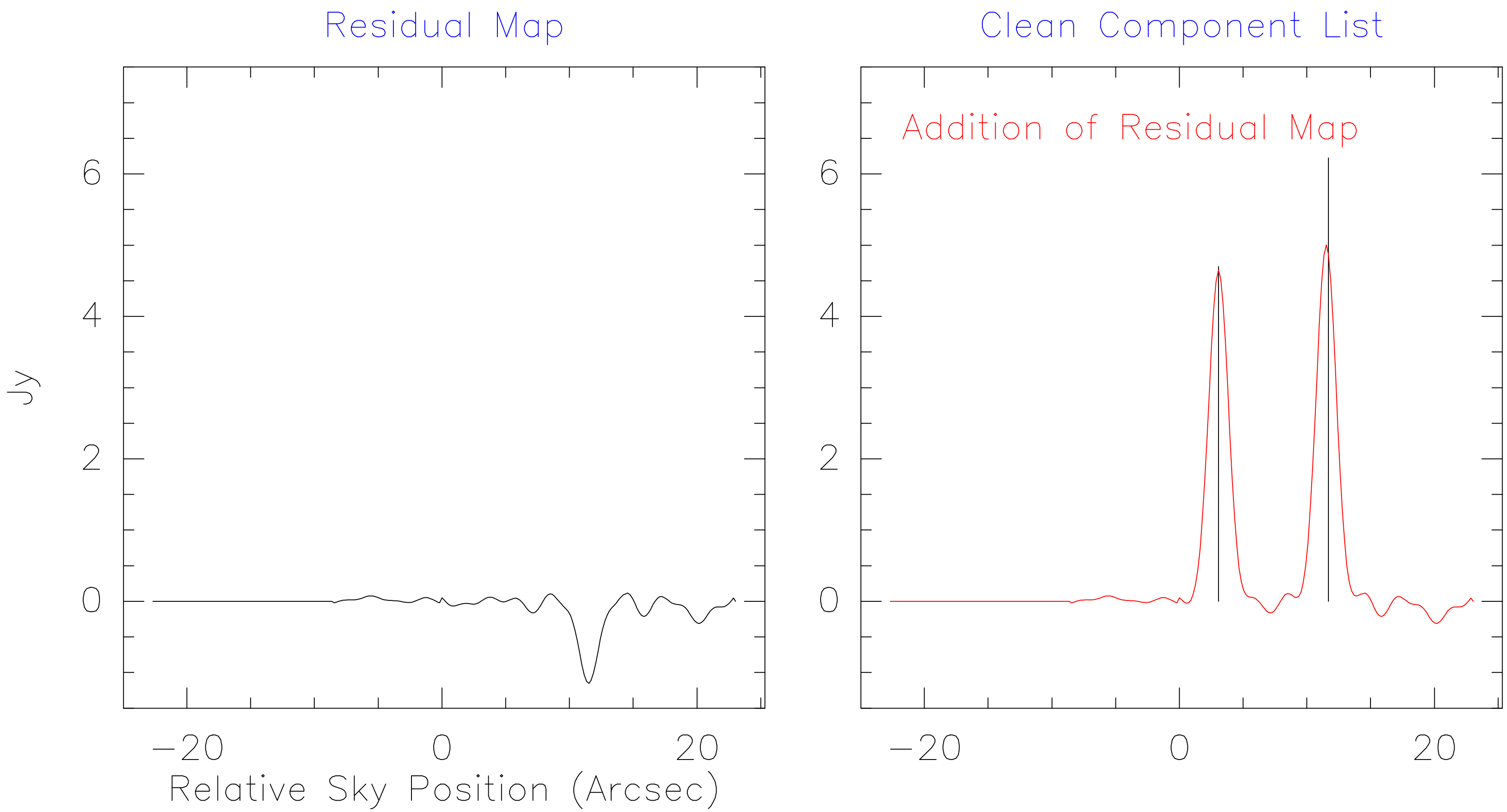
Too superficial cleaning $\Rightarrow$ Approximate results.

Too deep cleaning $\Rightarrow$ Divergence.



Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:
Improvement when convergence not reached;
Noise estimation.



Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:

Improvement when convergence not reached;

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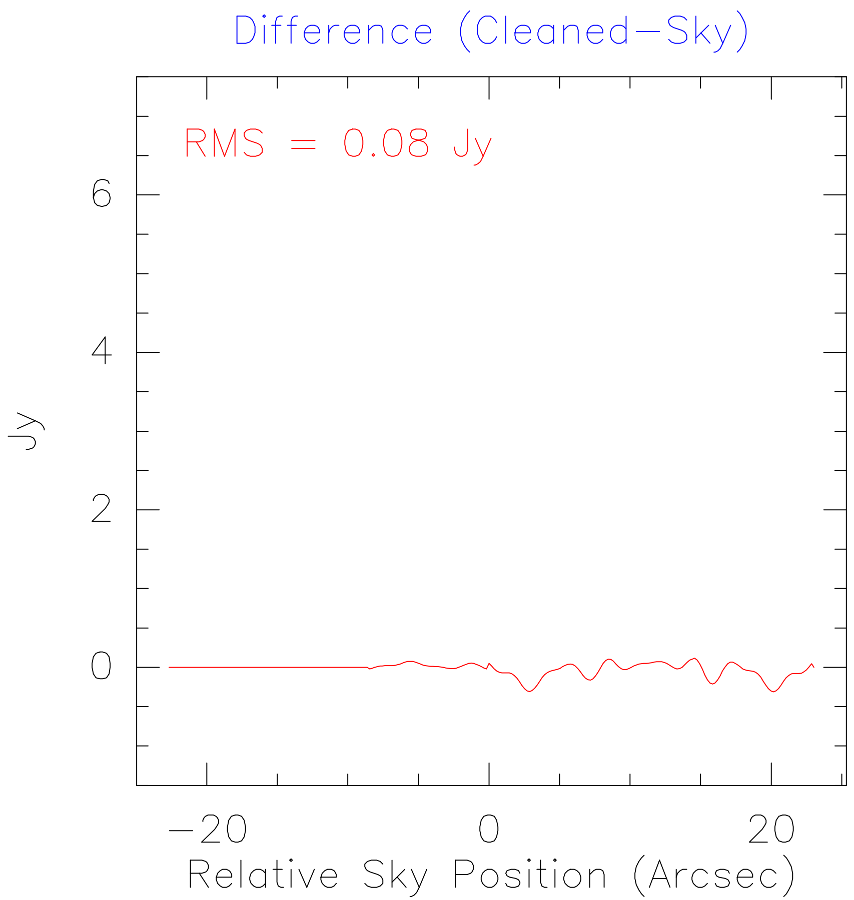
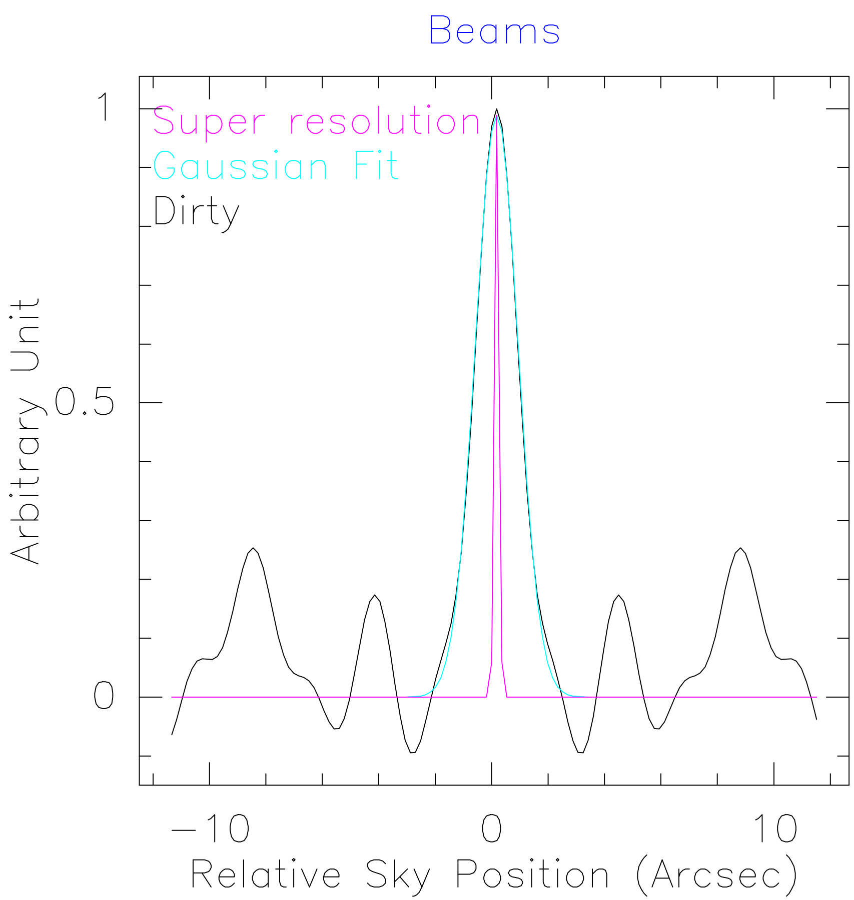


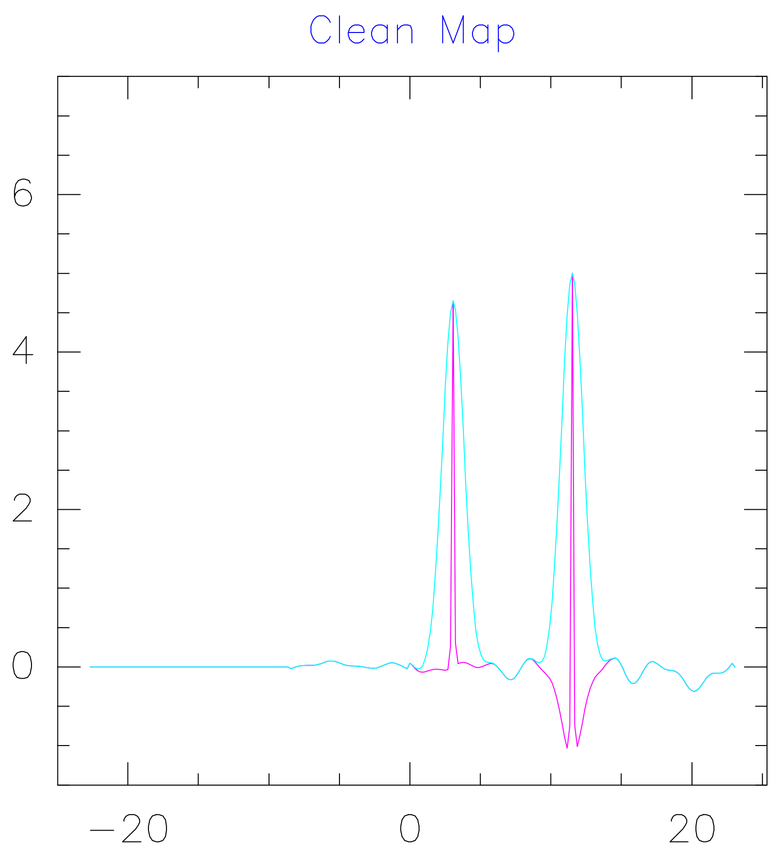
Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

Choice of clean beam:

Gaussian of FWHM matching the synthesized beam size.

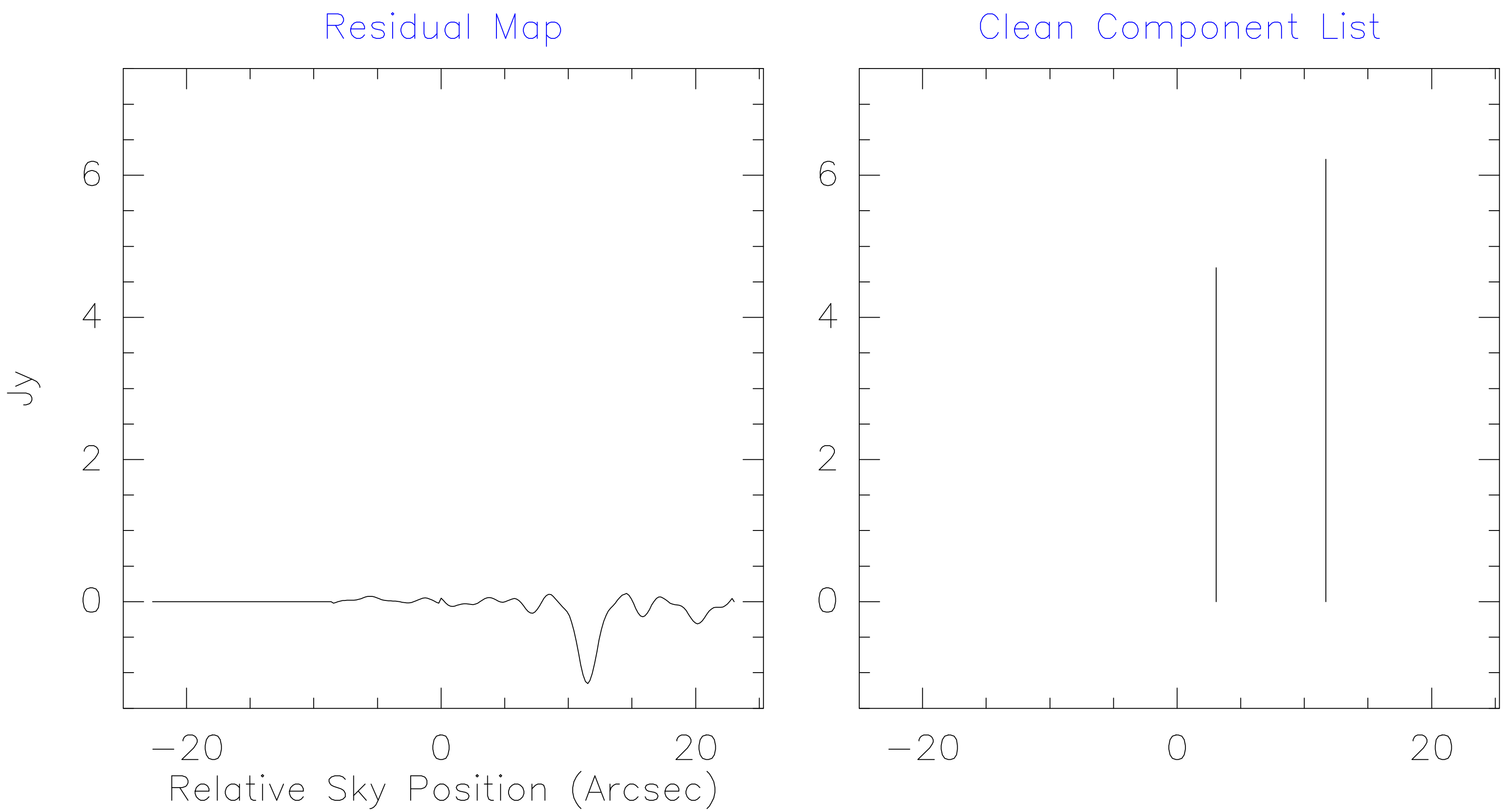
⇒ Super resolution strongly discouraged.





Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.



Deconvolution: II. The Basic Clean Algorithm
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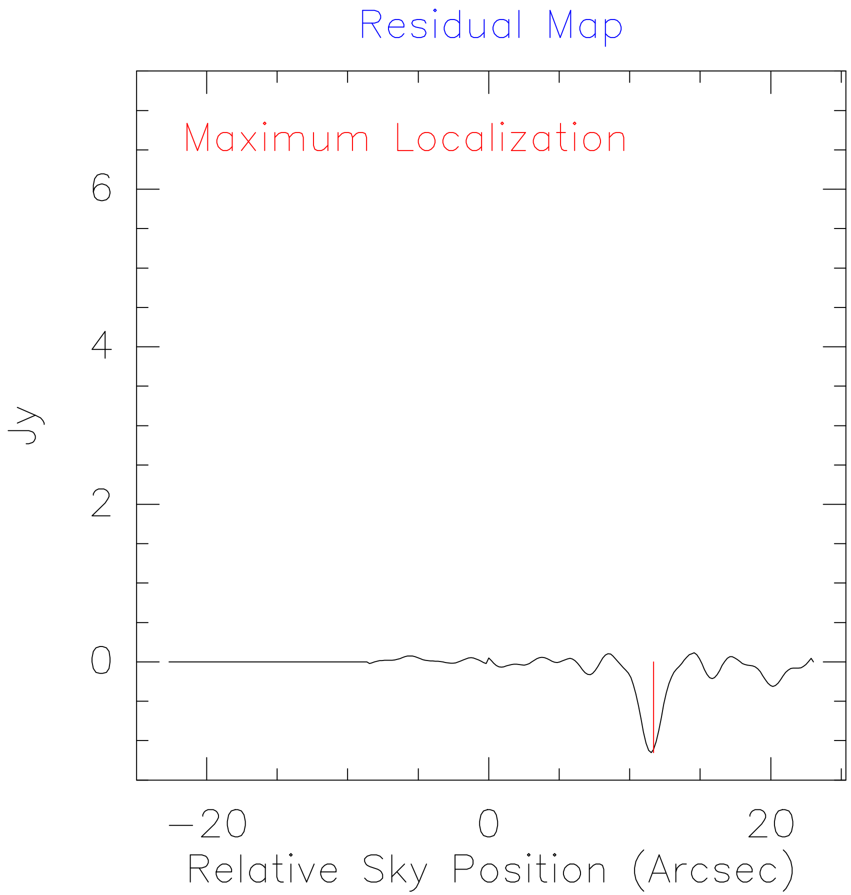

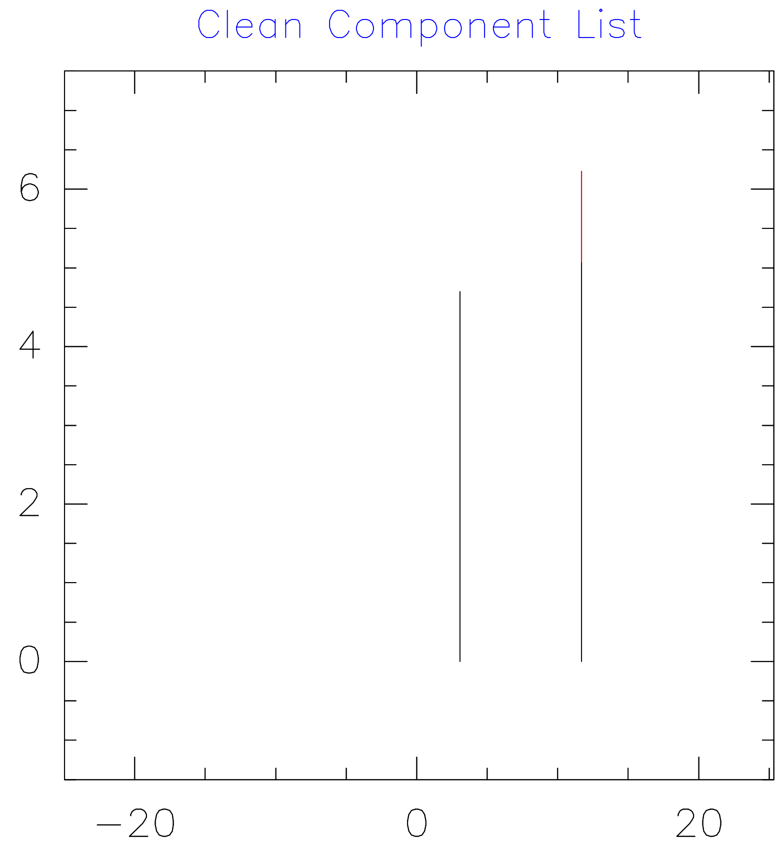

Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

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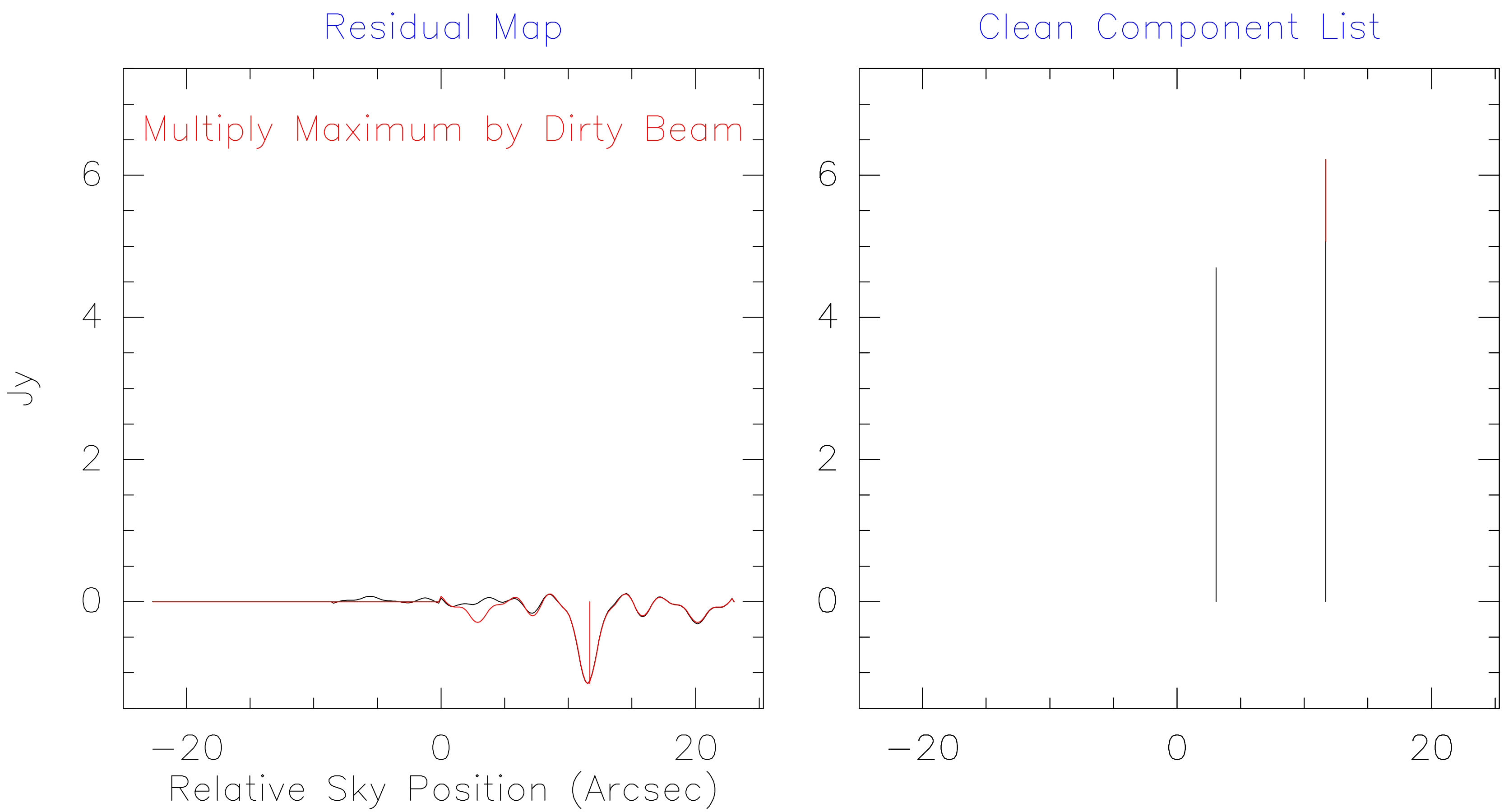

Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

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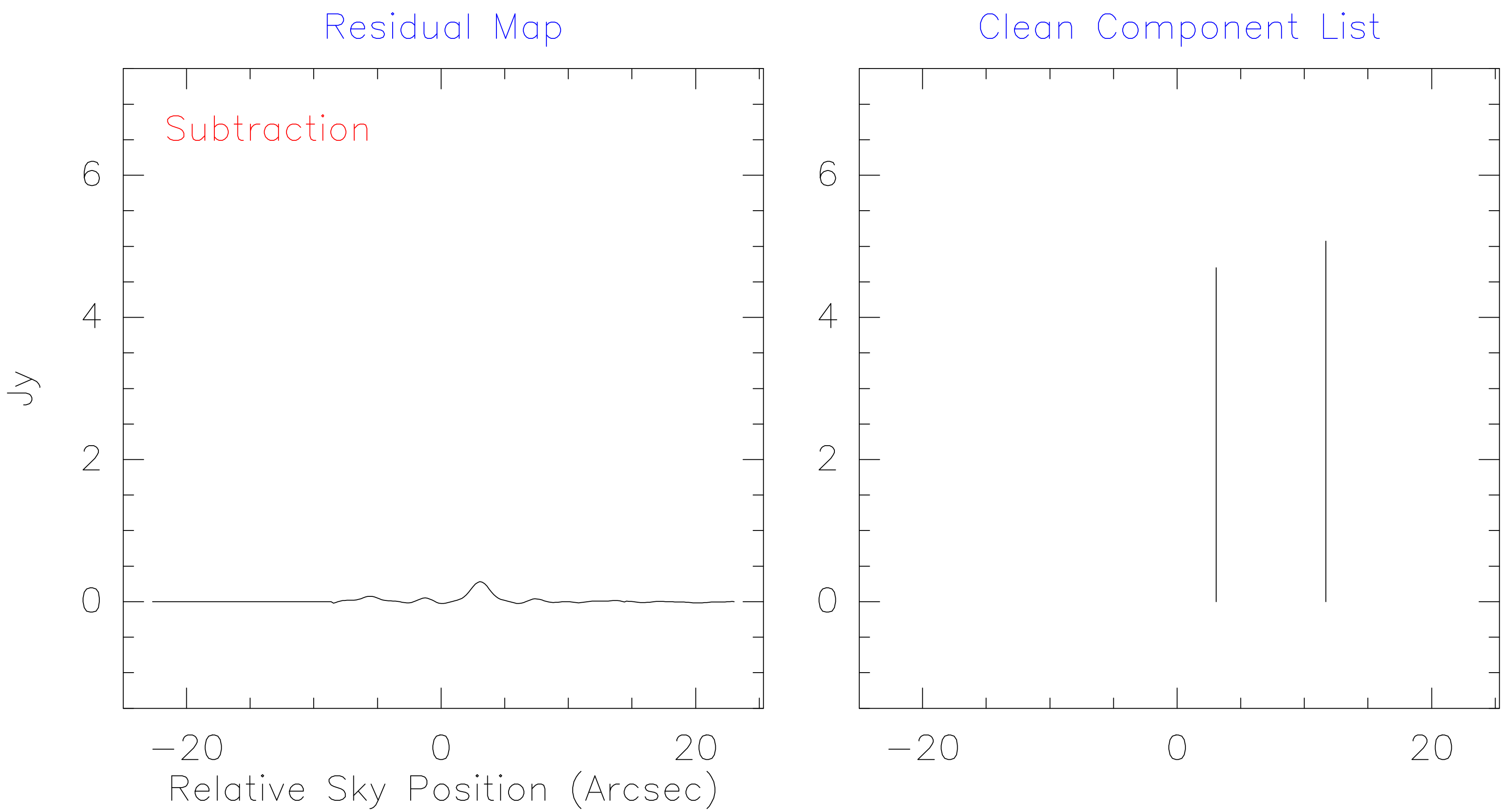

Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

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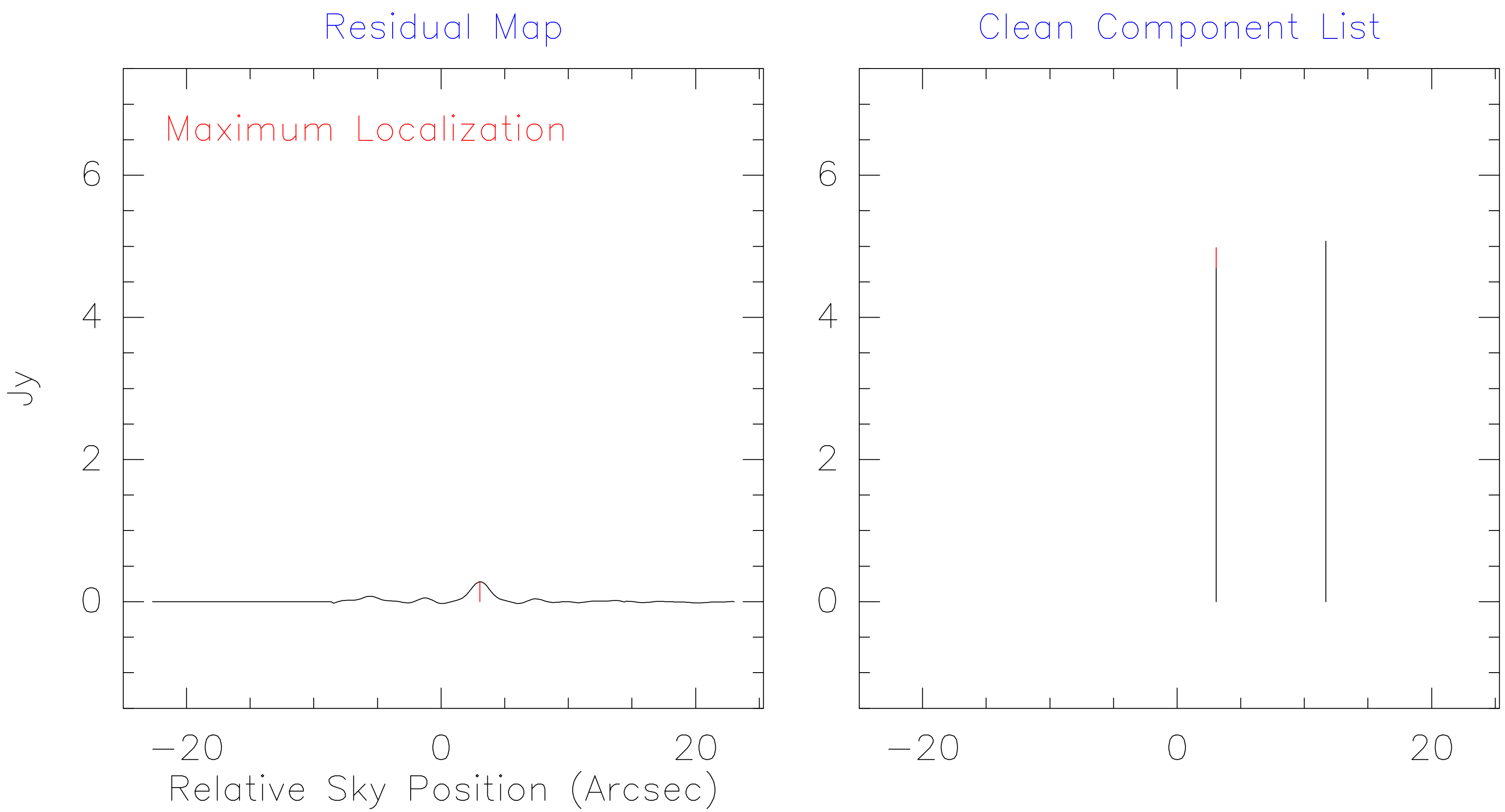

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3. Little Secrets

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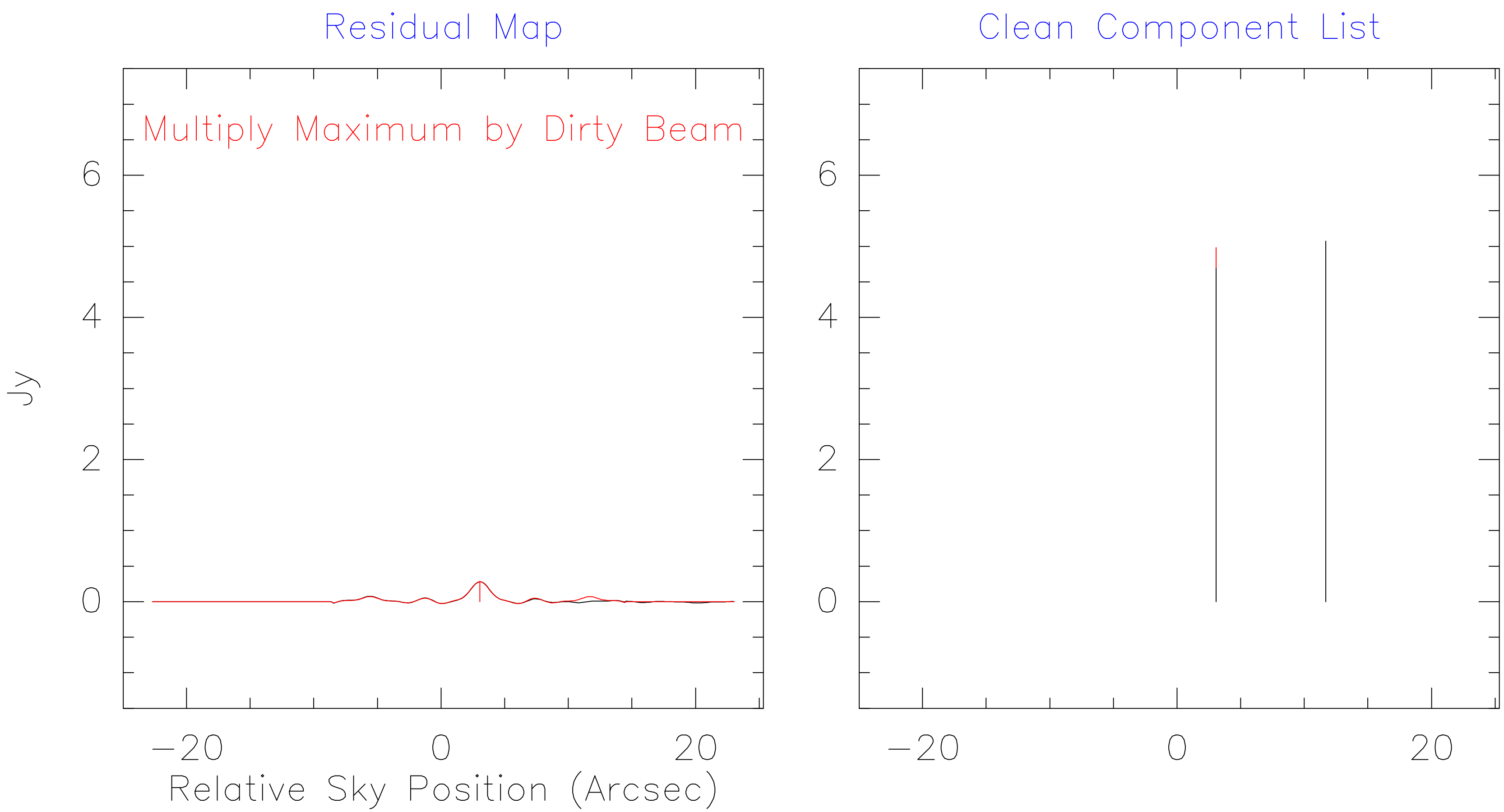

Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

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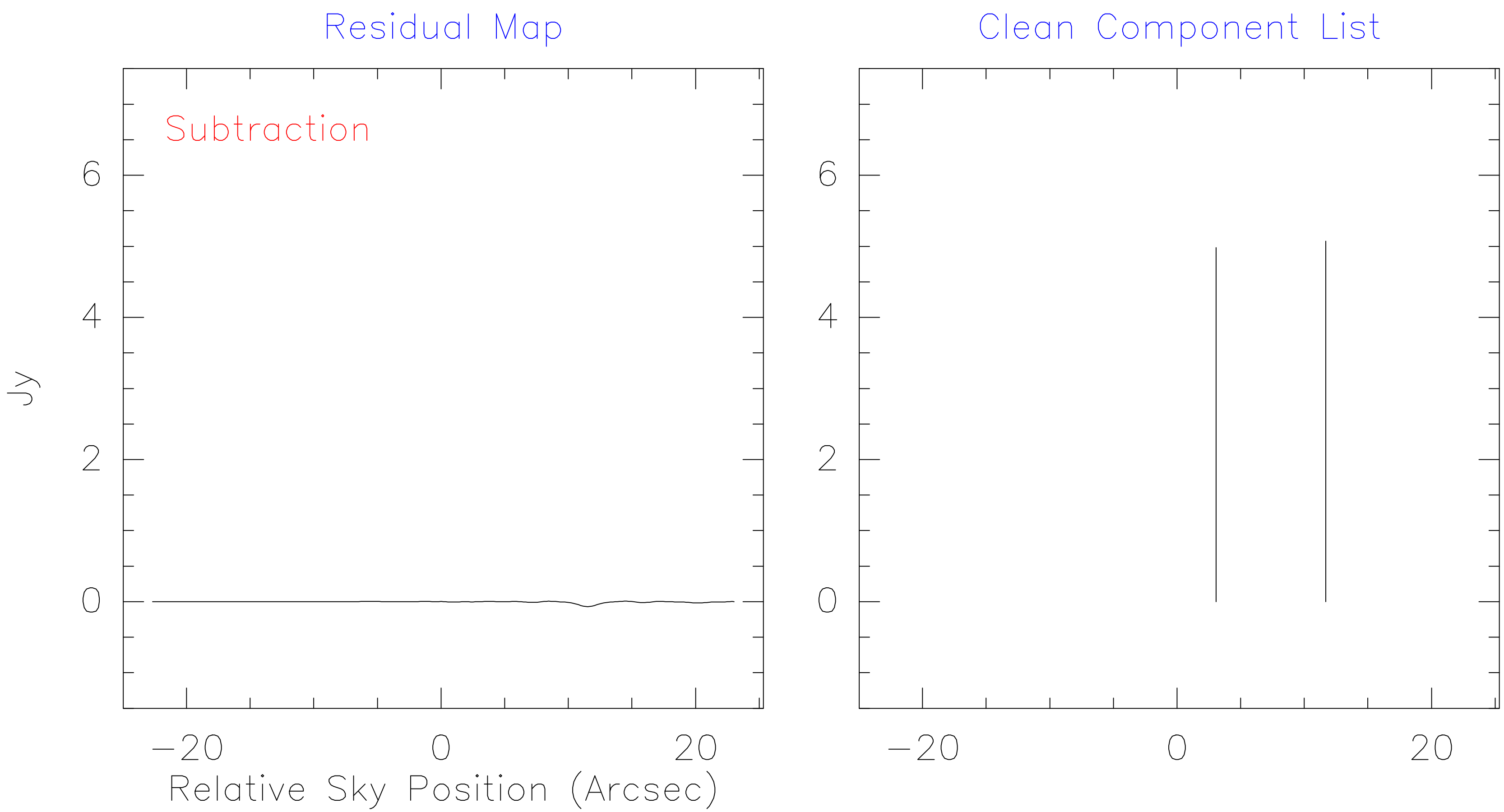

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3. Little Secrets

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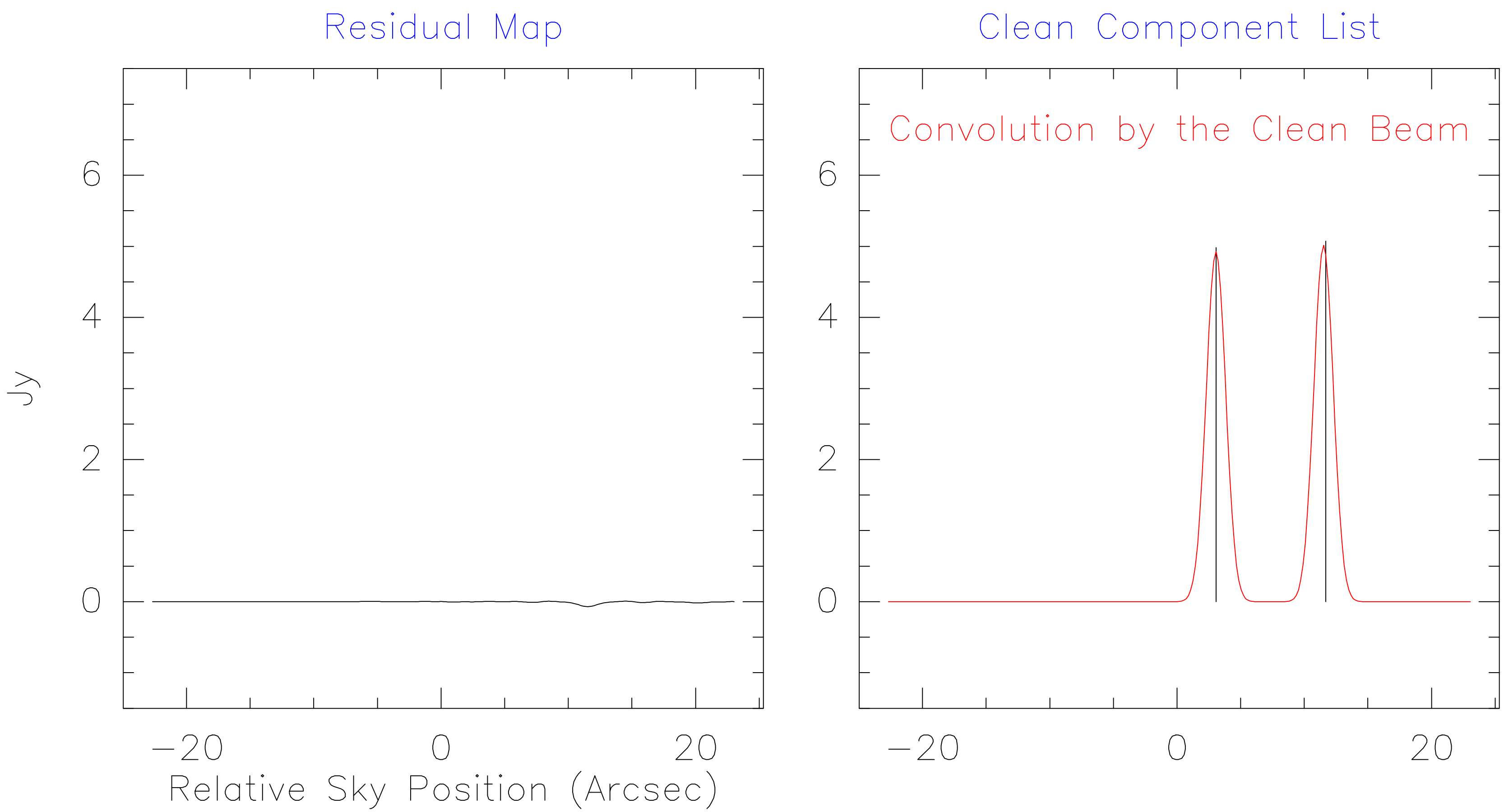

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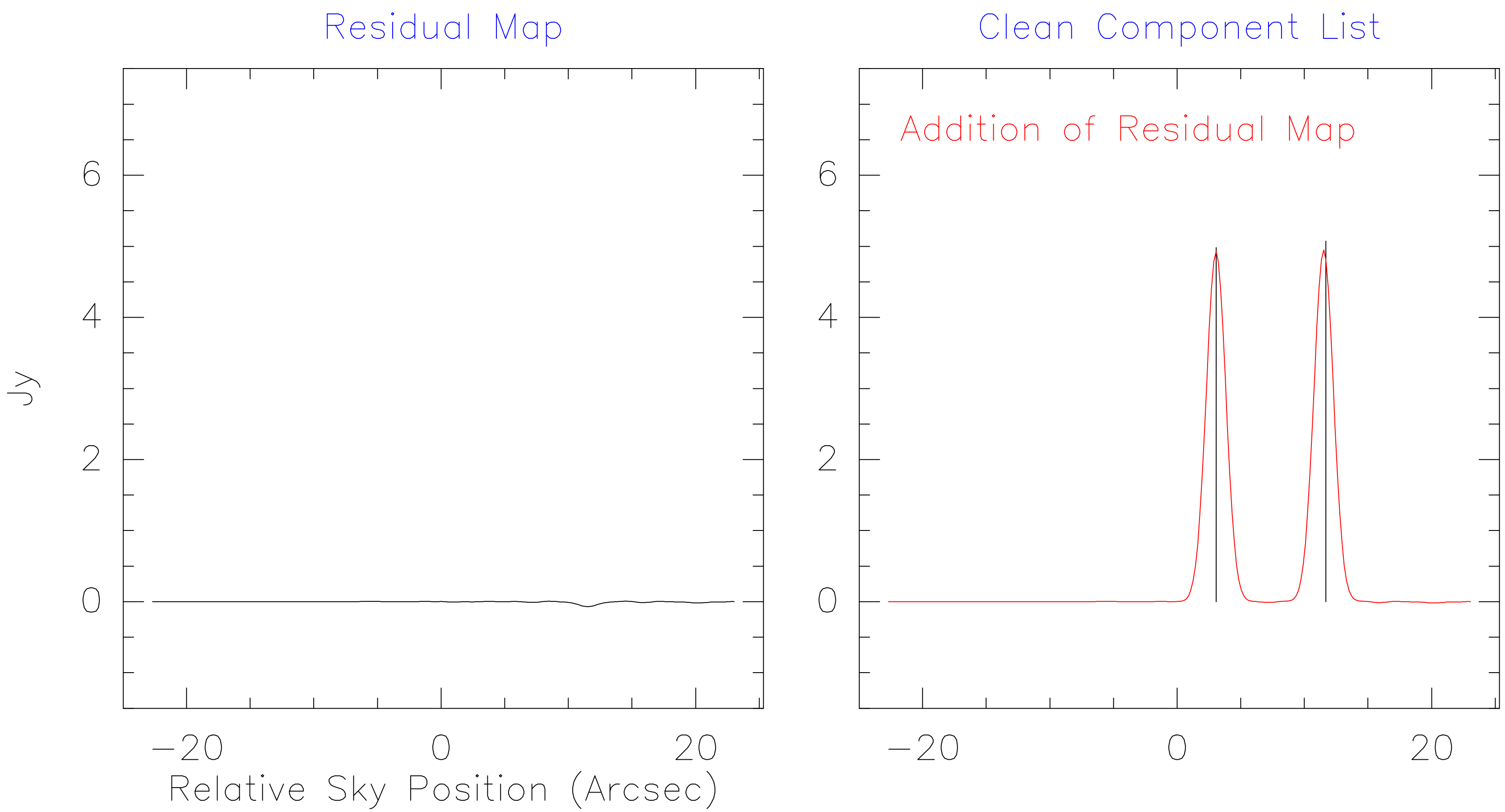

Deconvolution: II. The Basic Clean Algorithm
3. Little Secrets

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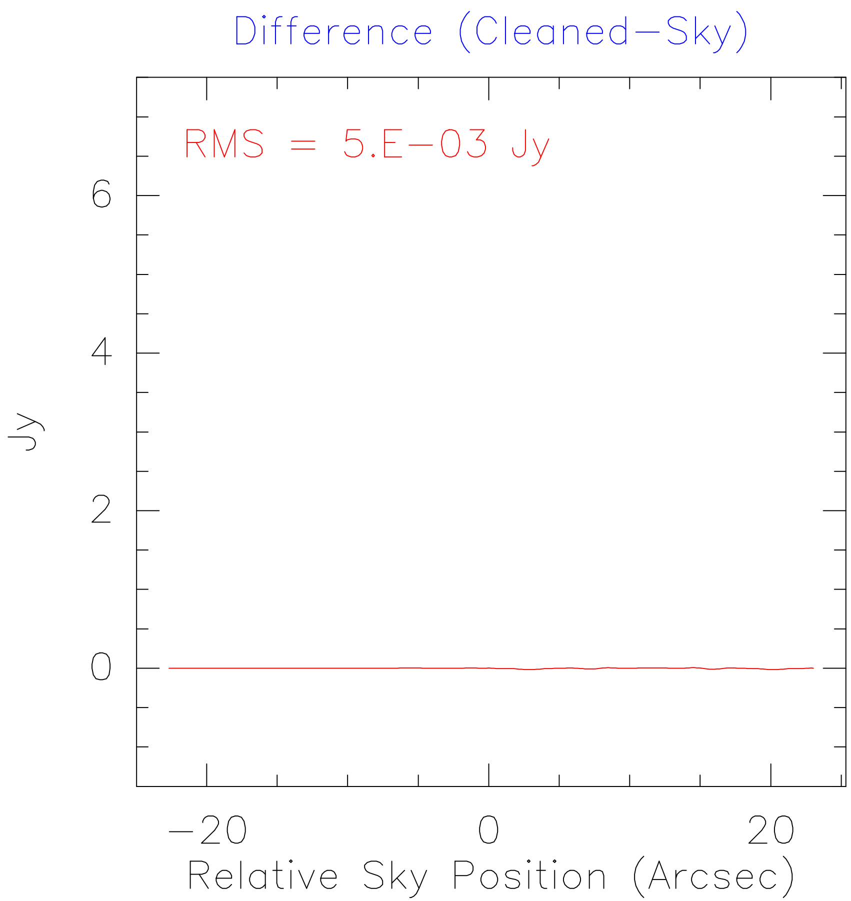
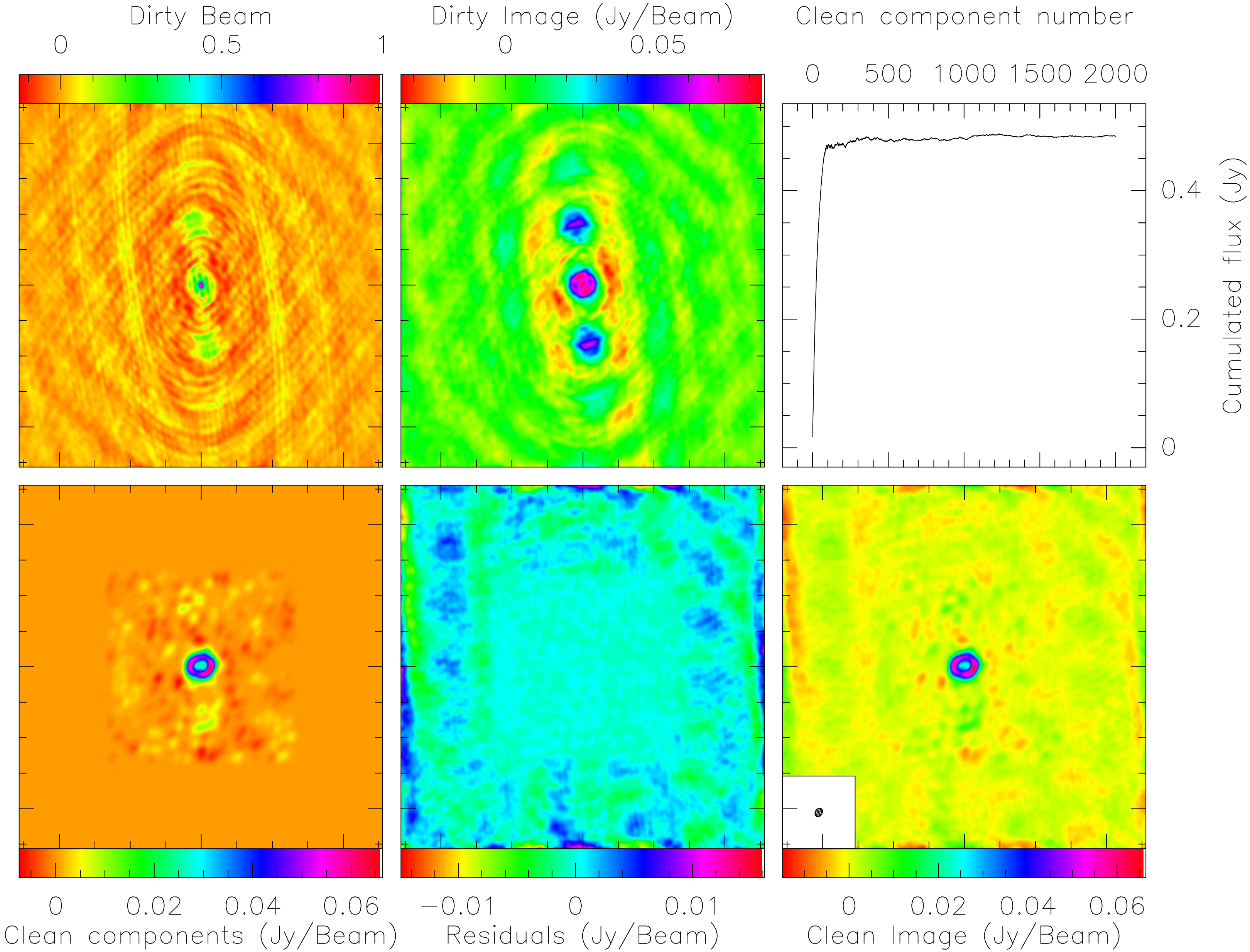


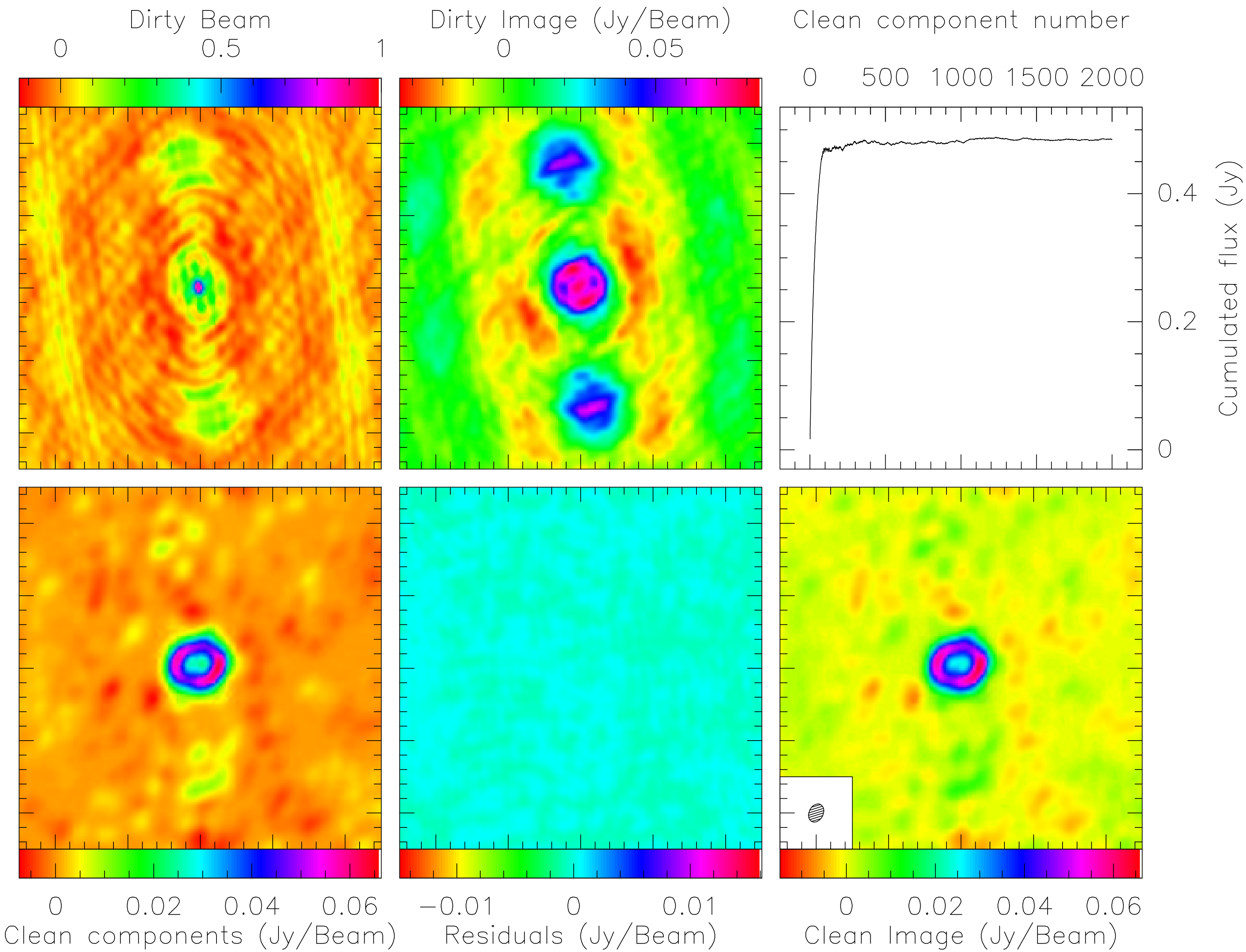
Deconvolution: II. The Basic Clean Algorithm
4. Other Little Secrets

- Stopping criterions:
  - Total number of Clean components;
  - $|I_{\text{max}}| <$ fraction of noise (when noise limited);
  - $|I_{\text{max}}| <$ fraction of dirty map max (when dynamic limited).
- Loop gain: Good results when $\gamma \sim 0.1 - 0.3$.
- Cleaned region: Only the inner quarter of the dirty image.
- Support: Definition of a region where CLEAN components are searched.
  - *A priori* information $\Rightarrow$ Help CLEAN convergence.
  - But bias if support excludes signal regions
    $\Rightarrow$ Be wise!

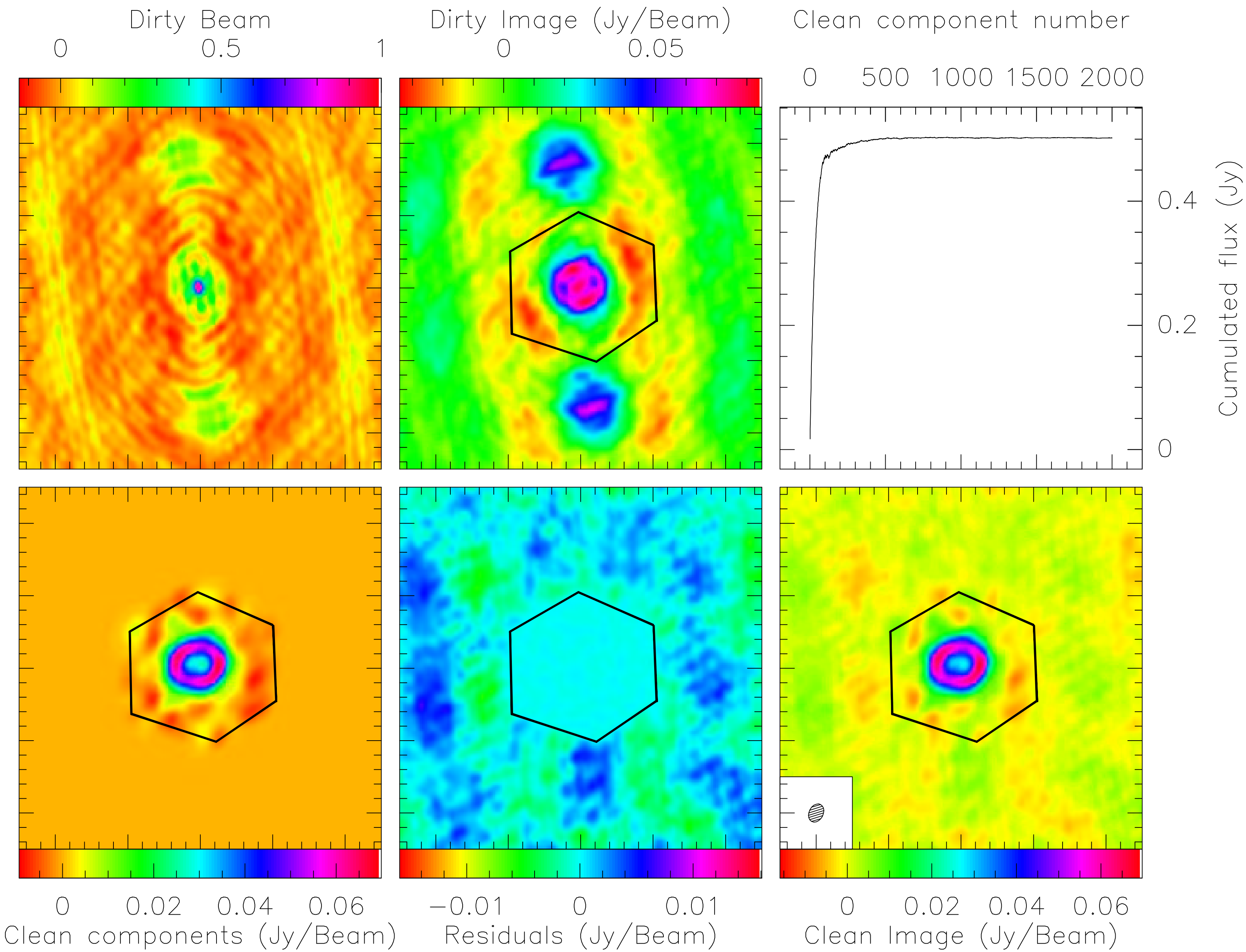
Deconvolution: II. The Basic Clean Algorithm
5. A True Example without support



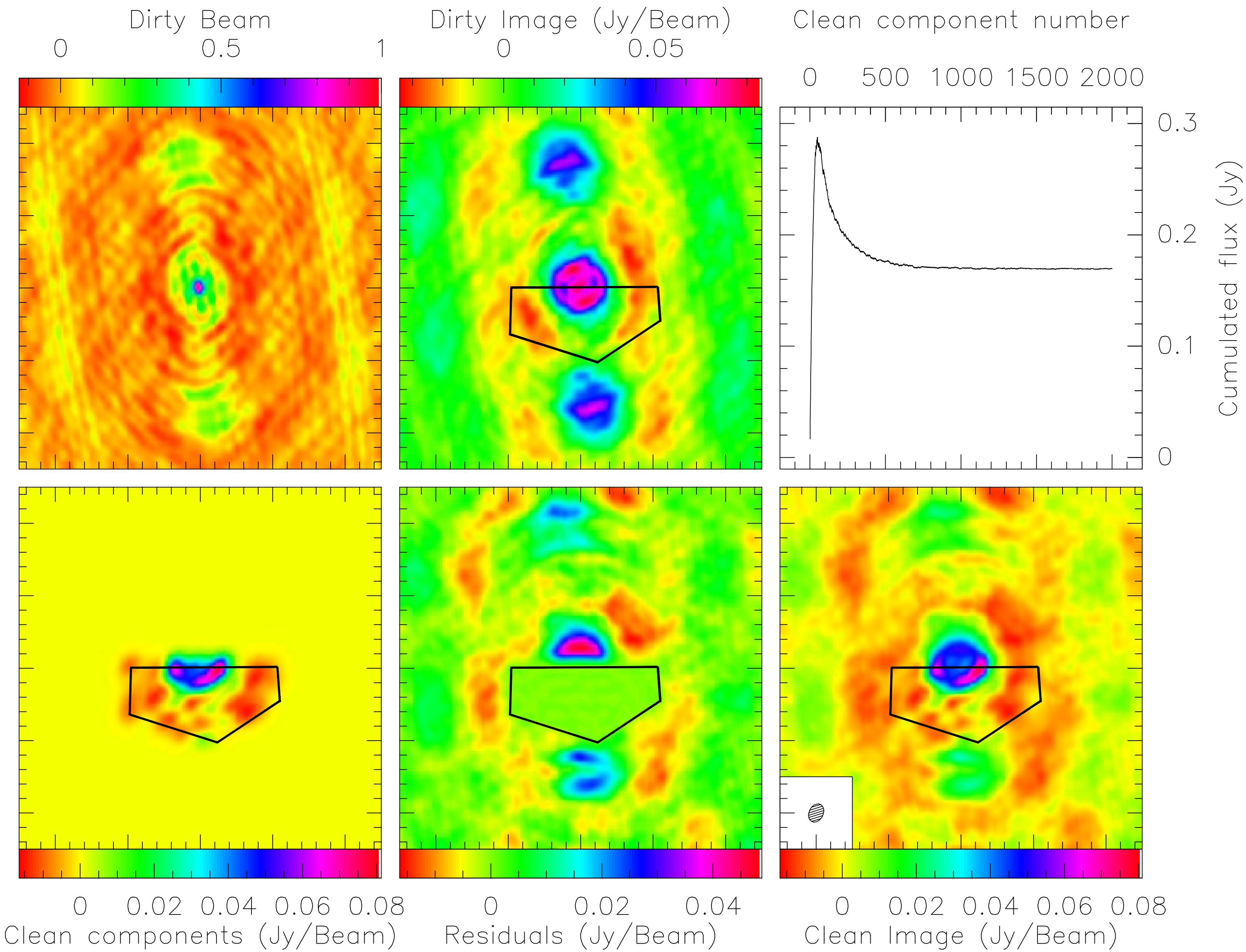
Deconvolution: II. The Basic Clean Algorithm
5. A True Example without support (zoom)



Deconvolution: II. The Basic Clean Algorithm
5. A True Example with right support



Deconvolution: II. The Basic Clean Algorithm
5. A True Example with wrong support



Deconvolution: III. CLEAN Variants

Basic:

- `HOGBOM` (Hogböm 1974)
  Robust but slow.

Faster Search Algorithms:

- `CLARK` (Clark 1980)
  Fast but instable (when sidelobes are high).
- `MX` (Cotton& Schwab 1984)
  Better accuracy (Source removal in the uv plane), but slower (gridding steps repeated).

Better Handling of Extended Sources:

- `MULTI` (Multi-Scale Clean by Cornwell 1998)
  Multi-resolution approach.

Exotic use at PdBI:

- `SDI` (Steer, Dewdney, Ito 1984)
  Created to minimize stripes.
- `MRC` (Multi-Resolution Clean by Wakker & Schwarz 1988)
  Too simple multi-resolution approach.

Deconvolution: IV. Recommended Practices

- Method: Start with `CLARK` and turn to `HOGBOM` in case of high side-lobes.
- Support:
  - Start without one.
  - Define one on your first clean image if really needed (*i.e.* difficulties of convergence).
- Stopping criterion:
  - Use a large enough number of iterations to ensure convergence.
  - Clean down to the noise level unless a very strong source is present.
- Misc: Consult an expert until you become one.

Deconvolution: V. Current research

Sparcity

- A point source is sparce in the image.
- A constant flux is sparce in the uv plane, i.e., it appears compact.
  ⇒ There exist transforms (Φ) that makes your source sparce, i.e., easy to describe.

Game rules

Minimize distance between data and source model
Find I that minimizes $\sum_k |V(u_k,v_k) - \tilde{I}(u_k,v_k)|^2$.

Constraint ΦI is sparce.

Mathematical formulation Lagrangian minimization

$$\min \left\{ \sum_k |V(u_k,v_k) - \tilde{I}(u_k,v_k)|^2 + \lambda \left(\sum_i |\Phi I|^p \right)^{\frac{1}{p}} \right\} \quad (1)$$

Caveats Devil hides in applied mathematical details (which function Φ, which value of p, which minimization routine, which noise model, how to fix the regularization parameter...)

Visualization and Image Analysis

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
⇓ Fourier Transform	`GO UVSTAT, GO UVMAP`
Dirty beam & image	
⇓ Deconvolution	`GO CLEAN`
Clean beam & image	
⇓ Visualization	`GO BIT, GO VIEW`
⇓ Image analysis	`GO NOISE, GO FLUX, GO MOMENTS`
Physical information on your source	

Photometry: I Generalities

- Brightness = Intensity (e.g. Power $= I_\nu(\alpha,\beta) dA d\Omega d\nu$)
- Flux unit: 1 Jy = 10^{-26} W m^{-2} Hz^{-1}.
- Source flux measured by a single–dish antenna:
  $F_\nu = B * I_\nu$ with B the antenna beam.
- Relationship between measured flux and temperature scales:
  $T_A = \frac{\lambda^2}{2k\Omega_A} F_\nu$, $T_A^\star = \frac{\lambda^2}{2k\Omega_{2\pi}} F_\nu$ and $T_{mb} = \frac{\lambda^2}{2k\Omega_{mb}} F_\nu$ because
  - $P_\nu = \frac{1}{2} A_e F_\nu$ Power detected by the single–dish antenna.
  - $P'_\nu = kT$ Power emitted by a resistor at temperature T.
  - $P_\nu = P'_\nu \Rightarrow T_A = \frac{A_e}{2k} F_\nu$.
  - $\lambda^2 = A_e \Omega_A$ (diffraction).
  - $\Omega_{2\pi} = F_{\text{eff}} \Omega_A$ or $F_{\text{eff}} = \frac{\text{Forward beam}}{\text{Total beam}}$.
  - $\Omega_{mb} = B_{\text{eff}} \Omega_A$ or $B_{\text{eff}} = \frac{\text{Main beam}}{\text{Total beam}}$.

Visibility unit: Jy because:

$$V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$$
$$= \iint B_{\text{primary}}(\sigma).I_{\text{source}}(\sigma)\,\exp(-i2\pi\mathbf{b}.\sigma/c)d\Omega.$$

Effect of flux calibration errors on your image:

- Multiplicative factor if uniform in uv plane.
- Convolution (*i.e.* distorsion) else.

Photometry: III Dirty map

Ill–defined because:

- $S(u = 0, v = 0) = 0 \Rightarrow$ Area of the dirty beam is 0!
- $V(u = 0, v = 0) = 0 \Rightarrow$ Total flux of the dirty image is 0!
  $\Rightarrow$ A source of constant intensity will be fully filtered out.
- A single point source of 1 Jy appears with peak intensity of 1.
- Several close-by point sources of 1 Jy appears with peak intensities different of 1.

Photometry: IV Clean map
(my dream: Don't take it seriously)

$I_{\text{clean}} = \frac{1}{\Omega_{\text{clean}}}\left(B_{\text{clean}} * I_{\text{point}}\right)$: *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* 1 steradian).

Unit: Jy/sr

Consequences:

- Source flux computation by integration inside a support:

$$\text{Flux} = \sum_{ij\,\in\,\mathcal{S}} I_{\text{clean}}\; d\Omega$$

[Jy] [Jy/sr] [sr]

  with $d\Omega$ the image pixel surface.

- From Brightness to temperature: $T_{\text{clean}} = \frac{\lambda^2}{2k} I_{\text{clean}}$

Photometry: IV Clean map (reality)

$I_{\text{clean}} = B_{\text{clean}} * I_{\text{point}}$: *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* clean beam).

Unit: Jy/beam with 1 beam = Ω_{clean} sr.

Consequences:

- Source flux computation by integration inside a support:

$$\text{Flux} = \sum_{ij \in \mathcal{S}} I_{\text{clean}} \cdot \frac{d\Omega}{\Omega_{\text{clean}}}$$

[Jy] [Jy/beam] [beam]

  with $\frac{d\Omega}{\Omega_{\text{clean}}}$ the nb of beams in the surface of an image pixel.

- From Brightness to temperature: $T_{\text{clean}} = \frac{\lambda^2}{2k\Omega_{\text{clean}}} I_{\text{clean}}$

Photometry: IV Clean map

Consequences of a Gaussian clean beam shape:

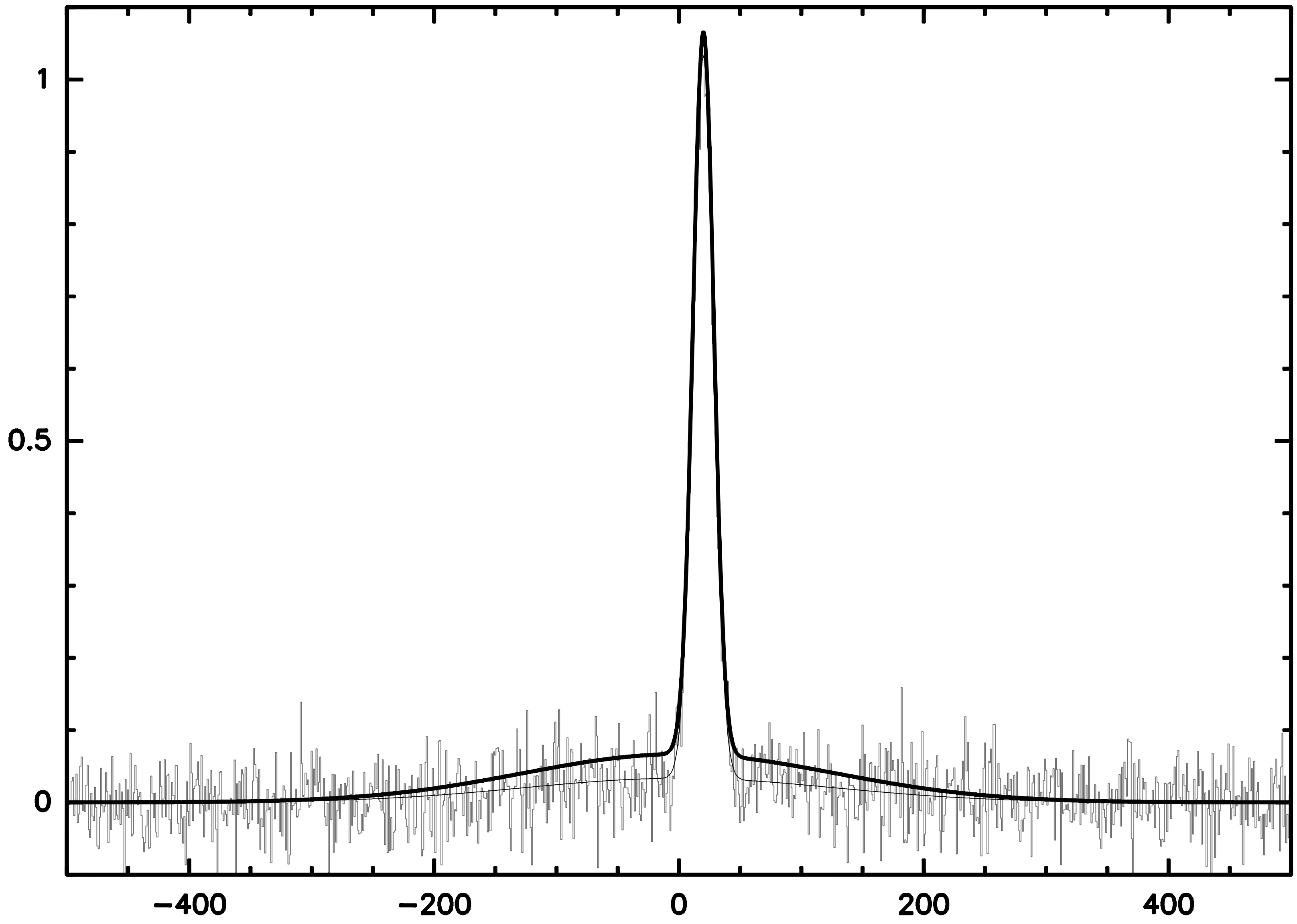
- No error beams, no secondary beams.
- T_{clean} is a main beam temperature.

Natural choice of clean beam size: Synthesized beam size (*i.e.* fit of the central peak of the dirty beam).
$\Rightarrow$ Minimize unit problems when adding the dirty map residuals.

Caveats of flux measurements:

- CLEAN does not conserve flux
  (*i.e.* CLEAN extrapolates unmeasured short spacings).
- Large scales are filtered out (source size $>$ 1/3 primary beam size $\Rightarrow$ need of short spacings, cf. lecture by F. Gueth).
- $I_{\text{clean}} = B_{\text{primary}}.I_{\text{source}} + N$
  $\Rightarrow$ Primary beam correction may be needed:
  $$I_{\text{clean}}/B_{\text{primary}} = I_{\text{source}} + N/B_{\text{primary}} \Rightarrow \text{Varying noise!}$$
- Seeing scatters flux.

Photometry: V Importance of Extended, Low Level Intensity



Noise: I. Formula

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)A}}$$

δT Brightness noise [K].
λ Wavelenght.
k Boltzmann constant.
Ω Synthesized beam solid angle.
A Antenna area.

σ Flux noise [Jy].
T_{sys} System temperature.
Δt On-source integration time.
$\Delta \nu$ Channel bandwidth.
N_{ant} Number of antennas.

and η Global efficiency (= Quantum x Antenna x Atm. Decorrelation).

Noise: III. σ to compare instruments

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\mathrm{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\mathrm{ant}}(N_{\mathrm{ant}}-1)A}}$$

Wavelenght: 1 mm. $T_{\mathrm{sys}} = 150$ K. Decorrelation = 0.8.

Instrument	Bandwidth	σ	On-source time
PdBI 2009	8 GHz	1.0 mJy/Beam	3 min
ALMA 2012	16 GHz	1.0 mJy/Beam	3 sec
ALMA 2012	16 GHz	0.12 mJy/Beam	3 min

One order of magnitude ($\sim 8\times$) sensitivity increase in continuum.

Noise: III. δT to prepare observations: 1. Continuum

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)A}}$$

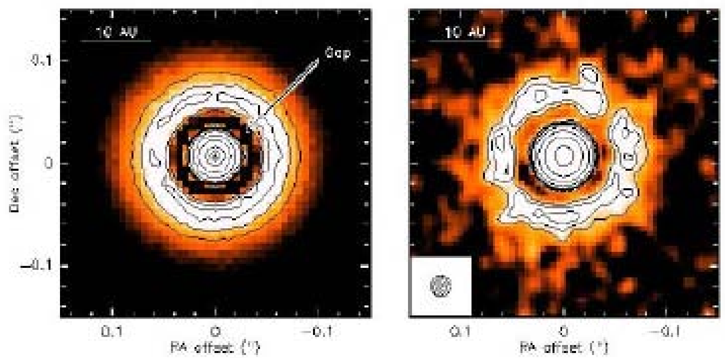
Wavelenght: 1 mm. $T_{\text{sys}} = 150$ K. Decorrelation = 0.8.

Instrument	Bandwidth	Resol.	δT	On time	Comment
PdBI 2009	8 GHz	0.30″	30 mK	3 hrs	
ALMA 2012	16 GHz	0.30″	30 mK	3 min	Low contrast, many objects
ALMA 2012	16 GHz	0.30″	4 mK	3 hrs	High contrast, same object
ALMA 2012	16 GHz	0.03″	30 mK	500 hrs	5.7% of a civil year
ALMA 2012	16 GHz	0.03″	400 mK	3 hrs	Intermediate sensitivity
ALMA 2012	16 GHz	0.10″	30 mK	3 hrs	Intermediate resolution

Almost one order of magnitude ($\sim 8\times$) sensitivity increase

$\Rightarrow$ A factor ~ 3 resolution increase (same integration time, same noise level).

Wolf et al. 2002, 0.02″ in 3 hrs.



Noise: III. δT to prepare observations: 2. Line

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)}A}$$

Channel width: $0.8\,\mathrm{km\,s^{-1}}$. Wavelenght: 1 mm. Decorrelation = 0.8.

Instrument	Resolution	δT	On-source time	Comment
PdBI now	$1''$	0.3 K	2 hrs	
ALMA 2012	$1''$	0.3 K	3.5 min	Same line, many objects
ALMA 2012	$1''$	0.05 K	2 hrs	Fainter lines, same objec
ALMA 2012	$0.1''$	0.3 K	575 hrs	6.5% of a civil year!
ALMA 2012	$0.1''$	5 K	2 hrs	Intermediate sensitivity
ALMA 2012	$0.4''$	0.3 K	2 hrs	Intermediate resolution

A factor $\sim$ 6 sensitivity increase
$\Rightarrow$ A factor $\sim$ 2.4 resolution increase
(same integration time, same noise level).

Noise: IV. Advices

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)}A}$$

- For your estimation:
  - Use the official sensitivity estimator!
  - Use δT not σ.

Writing the Paper: Your job!

Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} \text{and} F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_2}), \text{ then } F_1.F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} * \tilde{F_2}.$$

2 Sampling size $\overset{\text{FT}}{\rightleftharpoons}$ Image size.

3 Bandwidth size $\overset{\text{FT}}{\rightleftharpoons}$ Pixel size.

4 Finite support $\overset{\text{FT}}{\rightleftharpoons}$ Infinite support.

5 Fourier transform evaluated at zero spacial frequency
= Integral of your function.

$$V(u=0, v=0) \overset{\text{FT}}{\rightleftharpoons} \sum_{ij \,\in\, \text{image}} I_{ij}.$$

Photographic Credits and References

- R. N. Bracewell, "The Fourier Transform and its Applications".
- J. D. Kraus, "Radio Astronomy".