Imaging Principles

Jérôme PETY
(Obs. de Paris/IRAM)

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Scientific Analysis of a mm Interferometer Output

mm interferometer output:
Calibrated visibilities in the uv plane ($\simeq$ the Fourier plane).

2 possibilities:

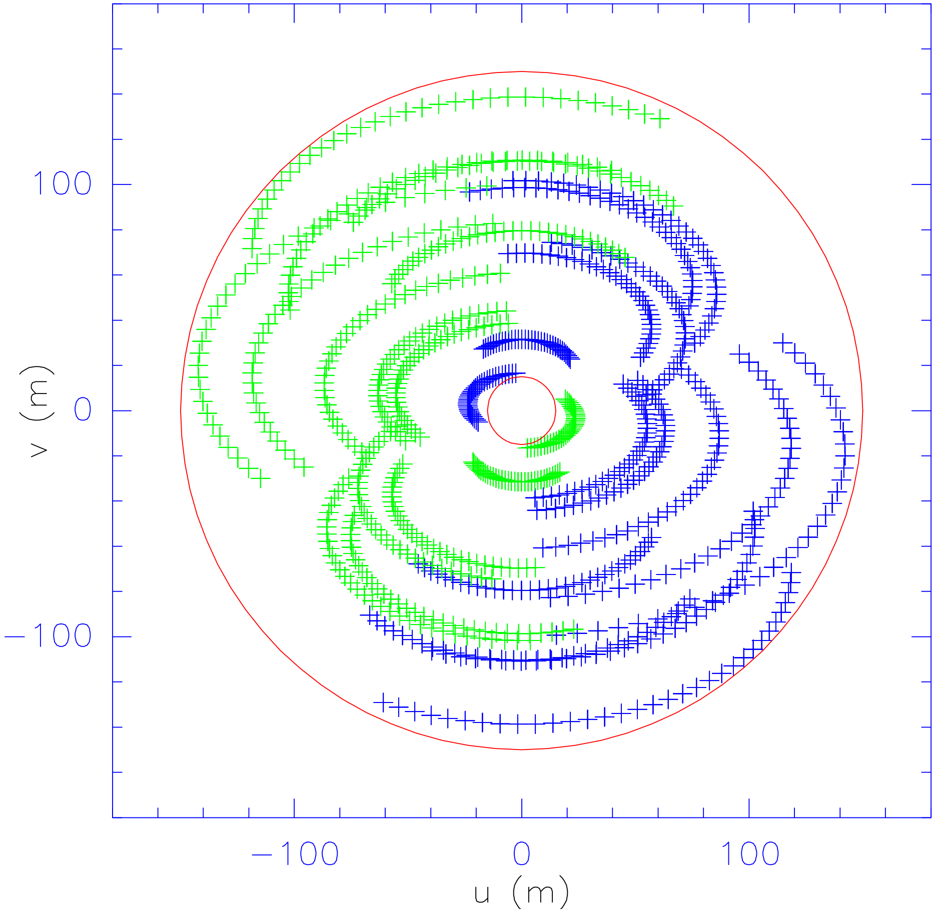
- uv plane analysis (cf. Lecture by S. Muller):
  Always better . . . when possible!
  (in practice for "simple" sources as point sources or disks)
- Image plane analysis:
  $\Rightarrow$ Mathematical transforms to go from uv to image plane!

Goal: Understand effects of the imaging process on

- The resolution;
- The field of view (single pointing or mosaicing, cf. Lecture by F. Gueth, tomorrow);
- The reliability of the image (cf. this lecture and next one);
- The noise level and repartition (cf. lecture by S.Guilloteau);

From Calibrated Visibilities to Images:
I. Comparison Visibilities/Source Fourier Transform

$$V_{ij}(b_{ij}) = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}(b_{ij}) + N$$



- Primary Beam
  ⇒ Distorted source information.
- Noise ⇒ Sensitivity problems.
- Irregular, limited sampling
  ⇒ incomplete source information:
  - Support limited at:
    * High spatial frequency
      ⇒ limited resolution;
    * Low spatial frequency ⇒ problem of wide field imaging;
  - Inside the support, incomplete (*i.e.* Nyquist's criterion not respected) sampling ⇒ lost of information.

From Calibrated Visibilities to Images:
II. Effect of Irregular, Limited Sampling

Definitions:

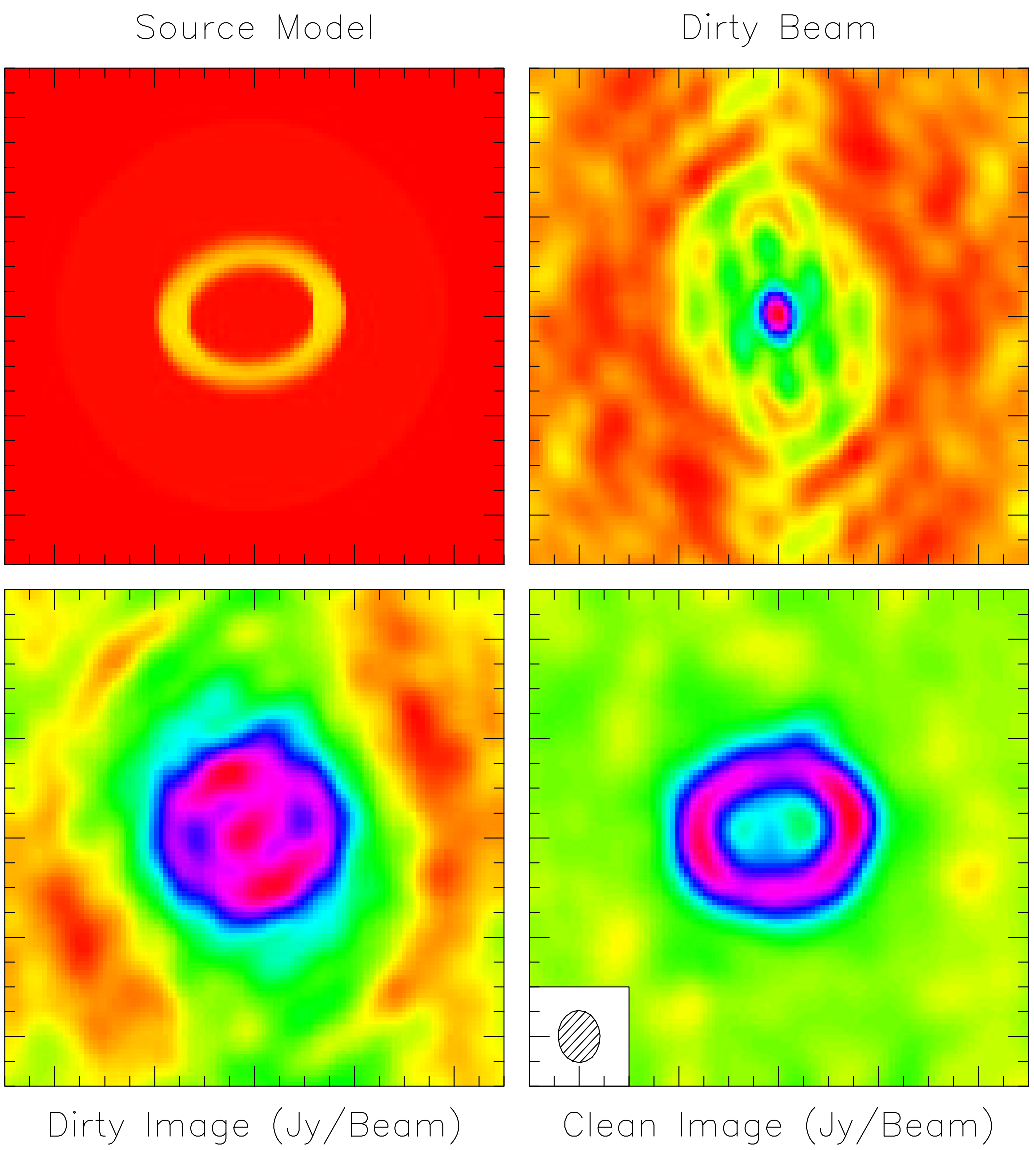
- $V = 2\text{D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$;
- Irregular, limited sampling function:
  - $S(u,v) = 1$ at (u,v) points where visibilities are measured;
  - $S(u,v) = 0$ elsewhere;
- $B_{\text{dirty}} = 2\text{D FT}^{-1}\{S\}$;
- $I_{\text{meas}} = 2\text{D FT}^{-1}\{S.V\}$.

Fourier Transform Property #1:

$I_{\text{meas}} = B_{\text{dirty}} * \left\{B_{\text{primary}}.I_{\text{source}}\right\}$.

B_{dirty}: Point Spread Function (PSF) of the interferometer
(*i.e.* if the source is punctual, then $I_{\text{meas}} = I_{\text{tot}}.B_{\text{dirty}}$).

From Calibrated Visibilities to Images:
III. Why Deconvolving?



- Difficult to do science on dirty image.
- Deconvolution ⇒ a clean image compatible with the sky intensity distribution.

From Calibrated Visibilities to Images:
IV. Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	UV_STAT
⇓ Fourier Transform	UV_MAP
Dirty beam & image	
⇓ Deconvolution	CLEAN
Clean beam & image	
⇓ Image analysis	Your Job!
Physical information on your source	

Direct vs. Fast Fourier Transform

Direct FT:

- Advantage: Direct use of the irregular sampling;
- Inconvenient: Slow.

Fast FT:

- Inconvenient: Needs a regular sampling $\Rightarrow$ Gridding;
- Advantage: Quick for images of size $2^M \times 2^N$.

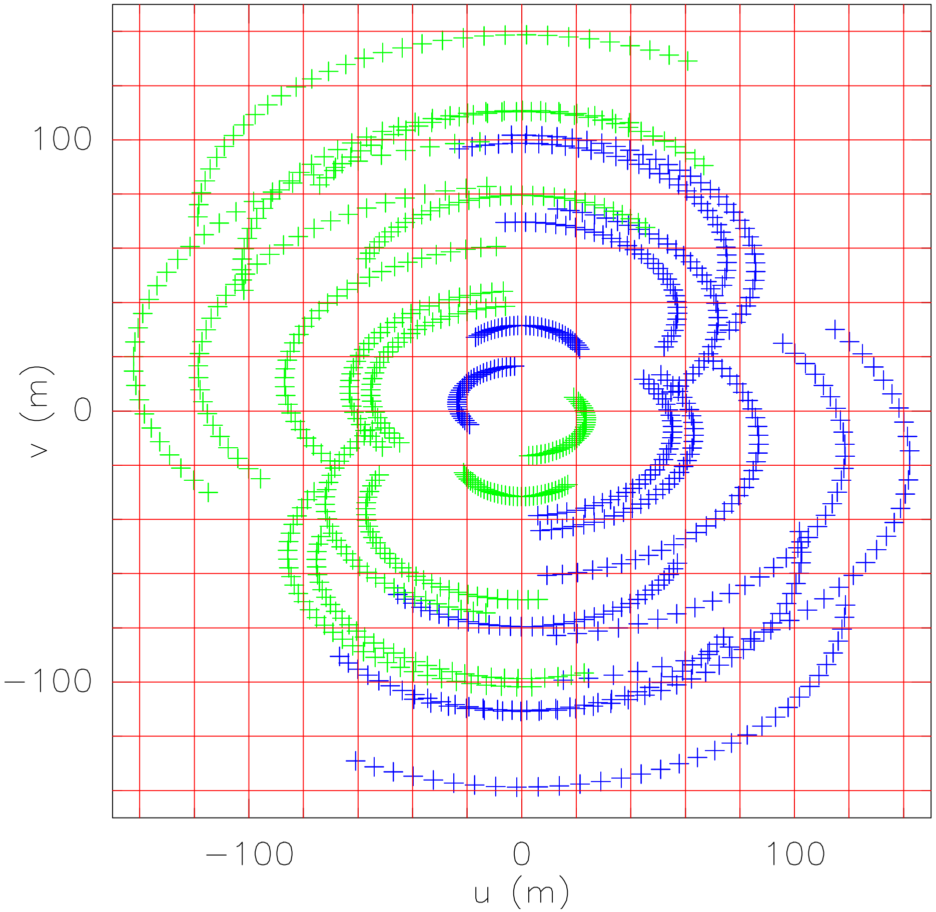
$\Rightarrow$ In practice, everybody use FFT.

From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	UV_STAT
⇓ Gridding & FFT	UV_MAP
Dirty beam & image	
⇓ Deconvolution	CLEAN
Clean beam & image	
⇓ Image analysis	Your Job!
Physical information on your source	

Gridding: I. Interpolation Scheme



Convolution because:

- Visibilities = noisy samples of a smooth function.
  $\Rightarrow$ Some smoothing is desirable.
- Nearby visibilities are not independent.
  – $V = $ 2D FT $\left\{B_{\text{primary}}.I_{\text{source}}\right\}$
  $= \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}}$;
  – FWHM(convolution kernel) $<$ FWHM($\tilde{B}_{\text{primary}}$)
  $\Rightarrow$ No real information lost.

Gridding: II. Convolution Equation is Kept Through Gridding

Demonstration:

- $I^{\text{grid}}_{\text{meas}} \overset{\text{2D FT}}{\rightleftharpoons} G * (S.V) \quad \Leftrightarrow \quad I^{\text{grid}}_{\text{meas}} = \tilde{G}.(\widetilde{S.V});$
- $B^{\text{grid}}_{\text{dirty}} \overset{\text{2D FT}}{\rightleftharpoons} G * S \quad \Leftrightarrow \quad B^{\text{grid}}_{\text{dirty}} = \tilde{G}.\tilde{S};$

$\Rightarrow$ $I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\}$

with $I_{\text{meas}} = I^{\text{grid}}_{\text{meas}}/\tilde{G}$

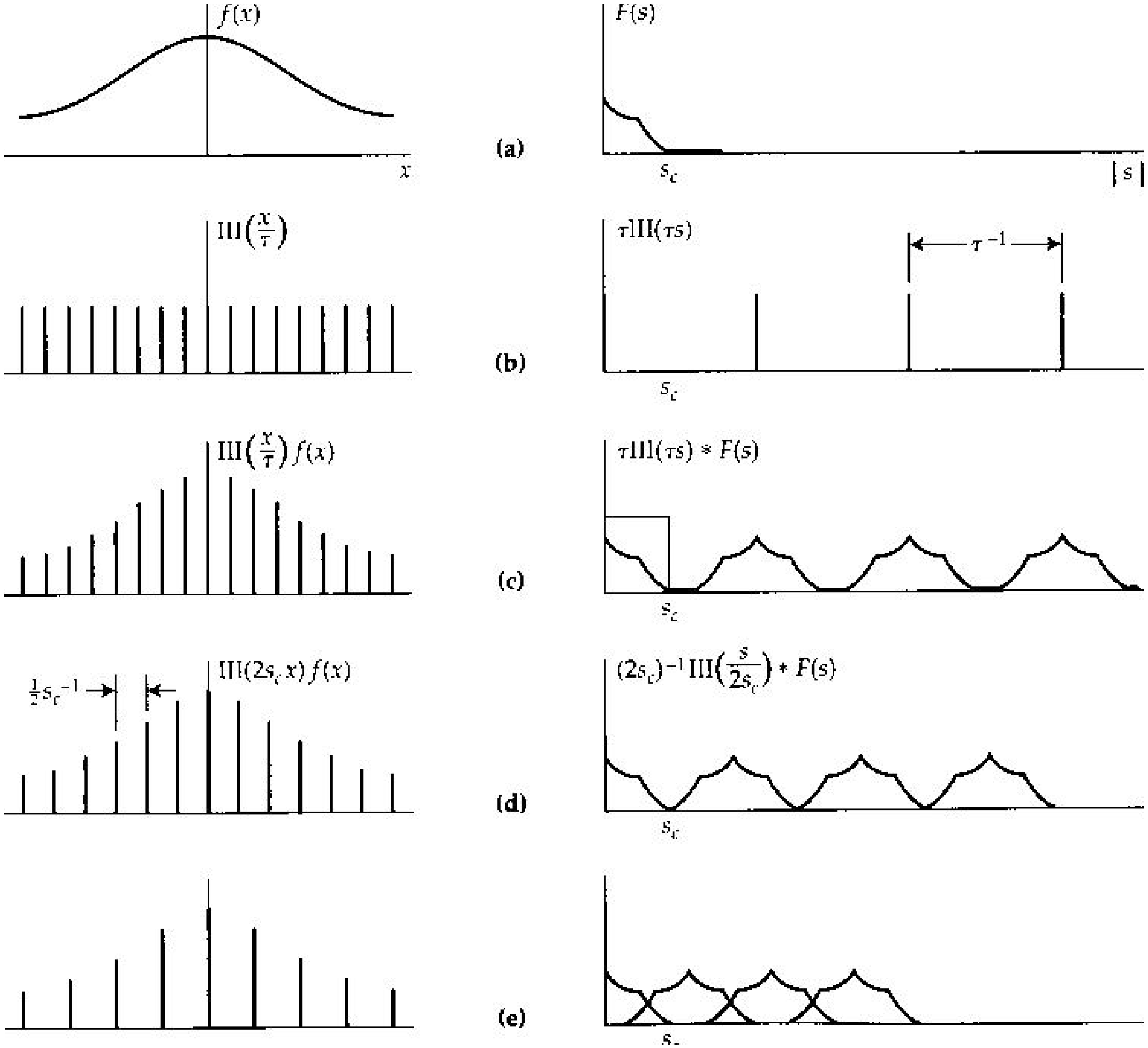
and $B_{\text{dirty}} = B^{\text{grid}}_{\text{dirty}}/\tilde{G}$.

Remark: Gridding may be hidden in equations but it is still there.

$\Rightarrow$ Artifacts due to gridding! (cf. next transparencies)

Gridding:
III. Effect of a Regular Sampling (Periodic Replication)

uv Plane | Image Plane



$B_{\text{primary}}.I_{\text{source}}$

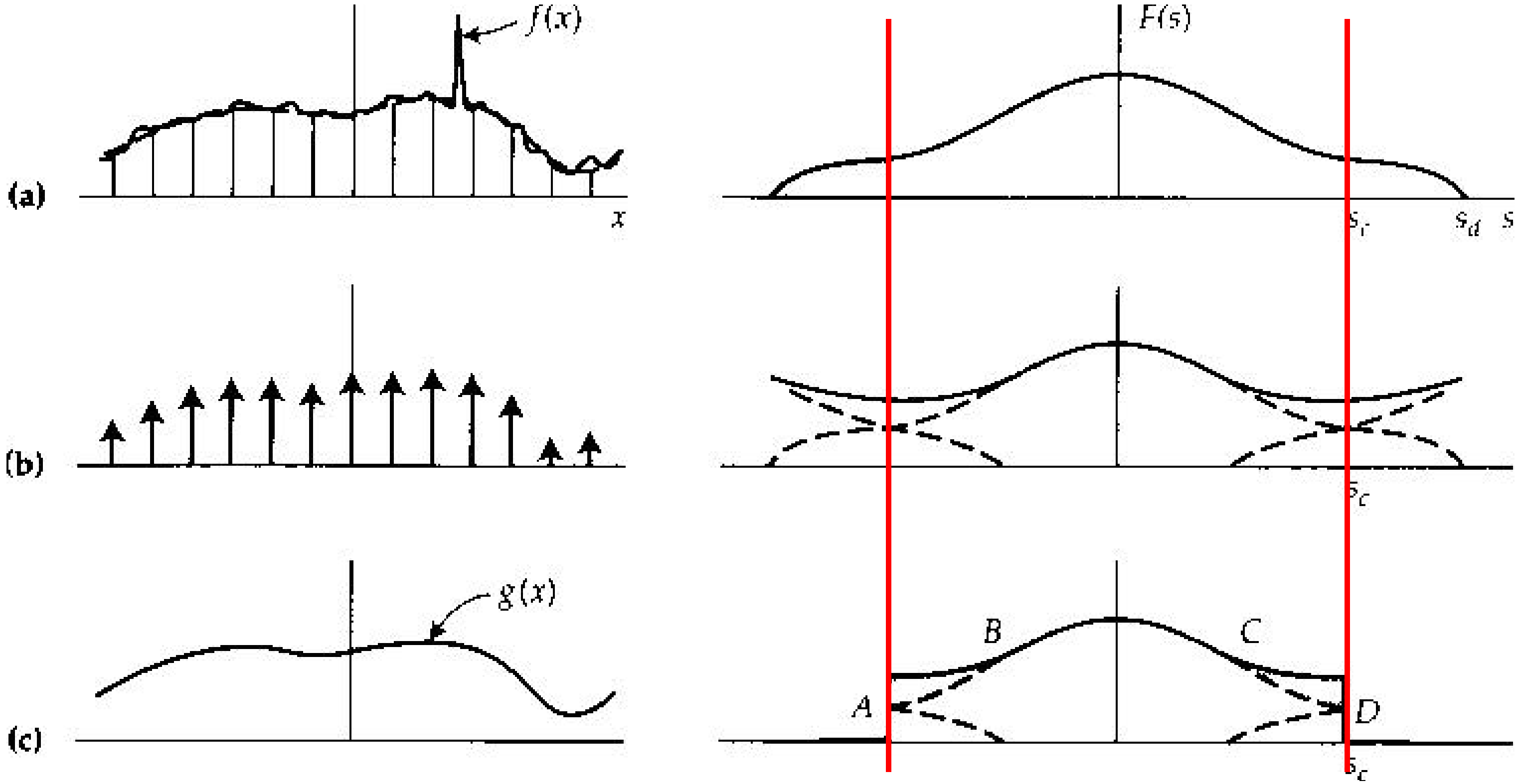
Regular Sampling function

Result for a fine sampling

Result for critical sampling (Nyquist's criterion)

Result for a coarse sampling

Gridding: III. Effect of a Regular Sampling (Aliasing)



Aliasing = Folding of intensity outside the image size into the image.

$\Rightarrow$ Image size must be large enough.

Gridding: IV. Pixel and Image Sizes

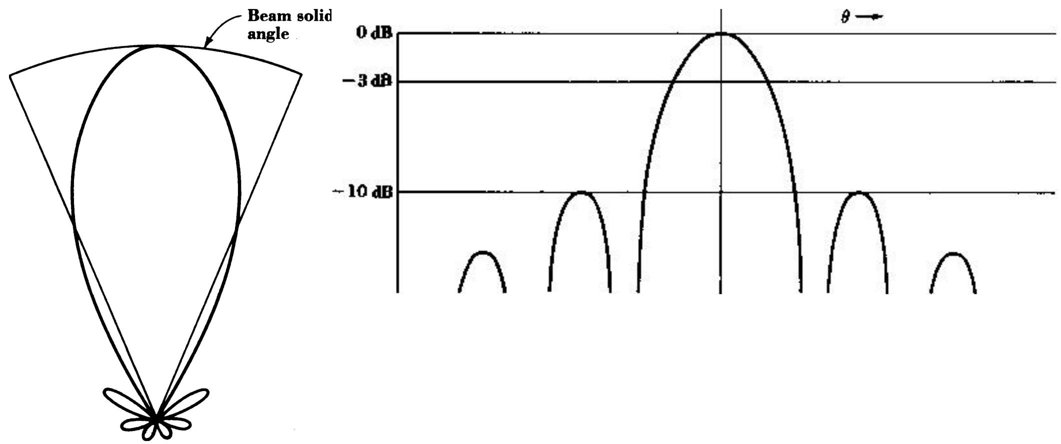
Pixel size: Between 1/3 and 1/4 of the synthesized beam size (*i.e.* more than the Nyquist's criterion in image plane to ease deconvolution).

Image size:

- $= uv$ plane sampling rate (FT property # 2);
- Natural resolution in the uv plane: $\tilde{B}_{\text{primary}}$ size;

$\Rightarrow$ At least twice the B_{primary} size (*i.e.* Nyquist's criterion in uv plane).

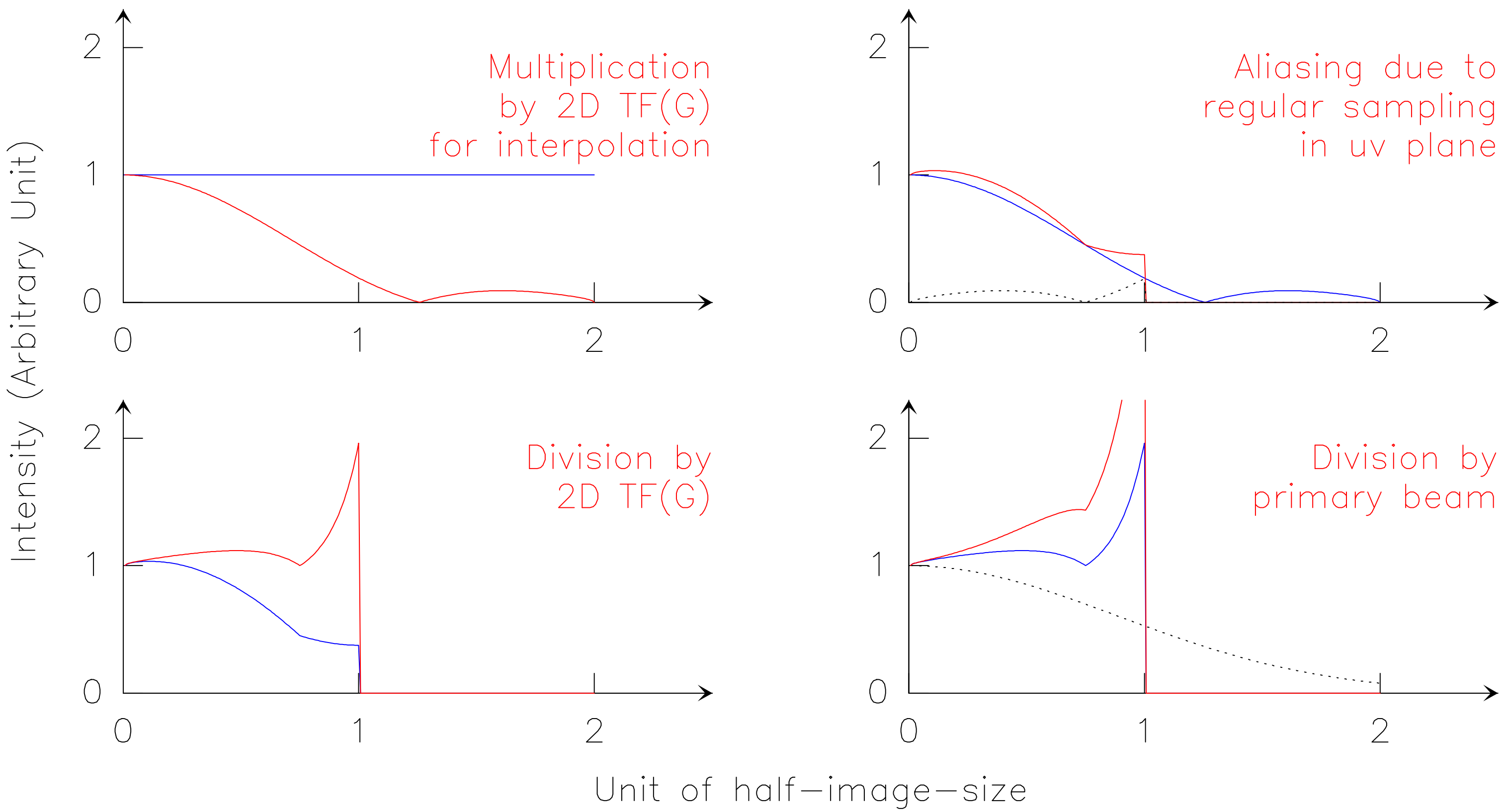
Gridding: V. Bright Sources in B_{primary} Sidelobes



Bright Sources in B_{primary} sidelobes
outside image size will be aliased into image.
⇒ Spurious source in your image!

Solution: Increase the image size.
(Be careful: only when needed for efficiency reasons!)

Gridding: VI. Noise Distribution



Gridding: VII. Choice of Gridding function

Gridding function must:

- Fall off quickly in image plane (to avoid noise aliasing);
- Fall off quickly in uv plane (to avoid too much smoothing).

$\Rightarrow$ Define a mathematical class of functions: Spheroidal functions.

GILDAS implementation: In UV_MAP

- Spheroidal functions = Default gridding function;
- Tabulated values are used for speed reasons.

Dirty Beam Shape and Image Quality

$$B_{\mathsf{dirty}} = \text{2D FT}^{-1}\{S\}.$$

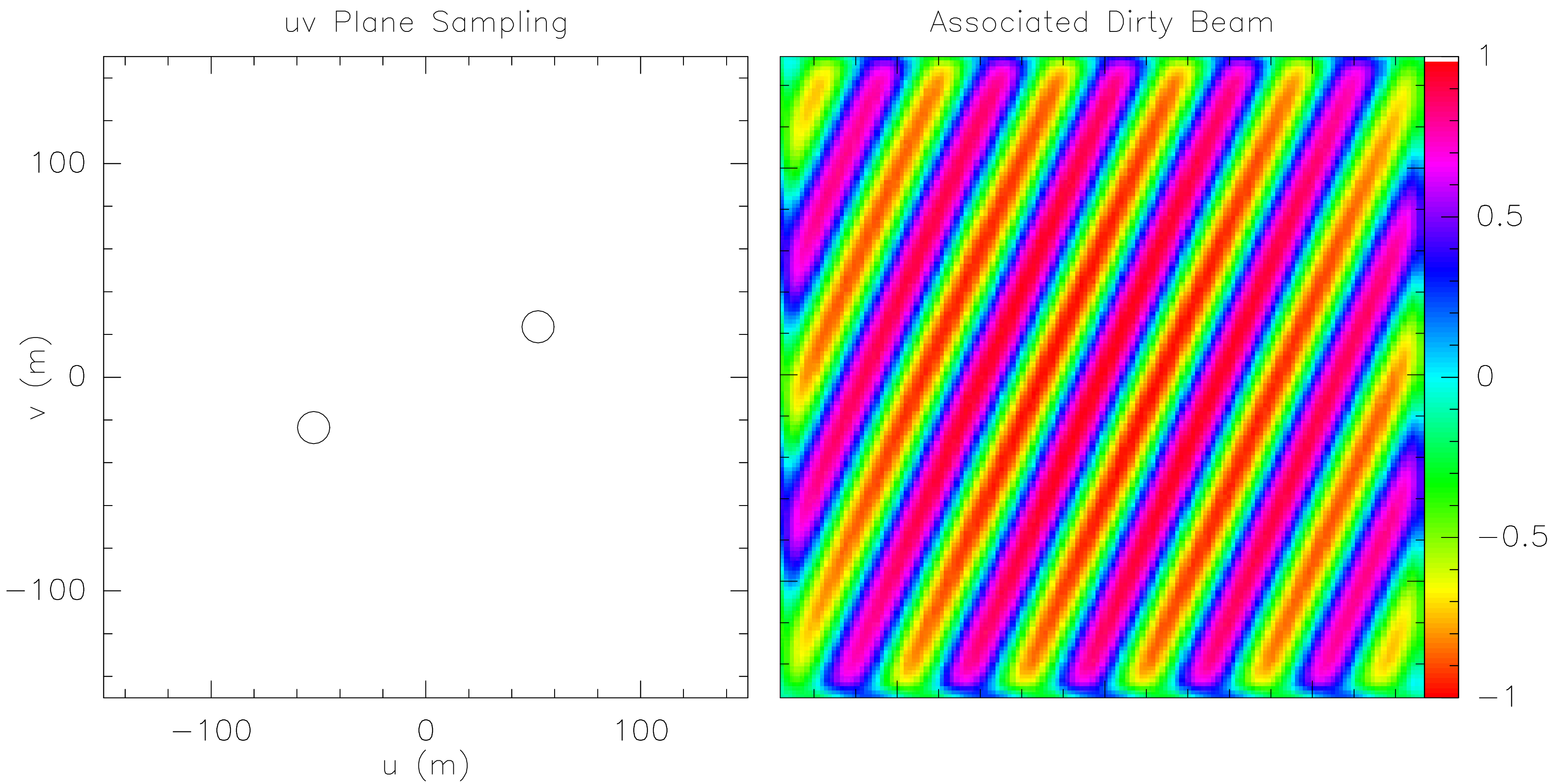
Importance of the Dirty Beam Shape:

- Deconvolving a dirty image is a delicate stage;
- The closest to a Gaussian B_{dirty} is, the easier the deconvolution;
- Extreme case:
  $B_{\mathsf{dirty}} = $ Gaussian $\Rightarrow$ No deconvolution needed at all!

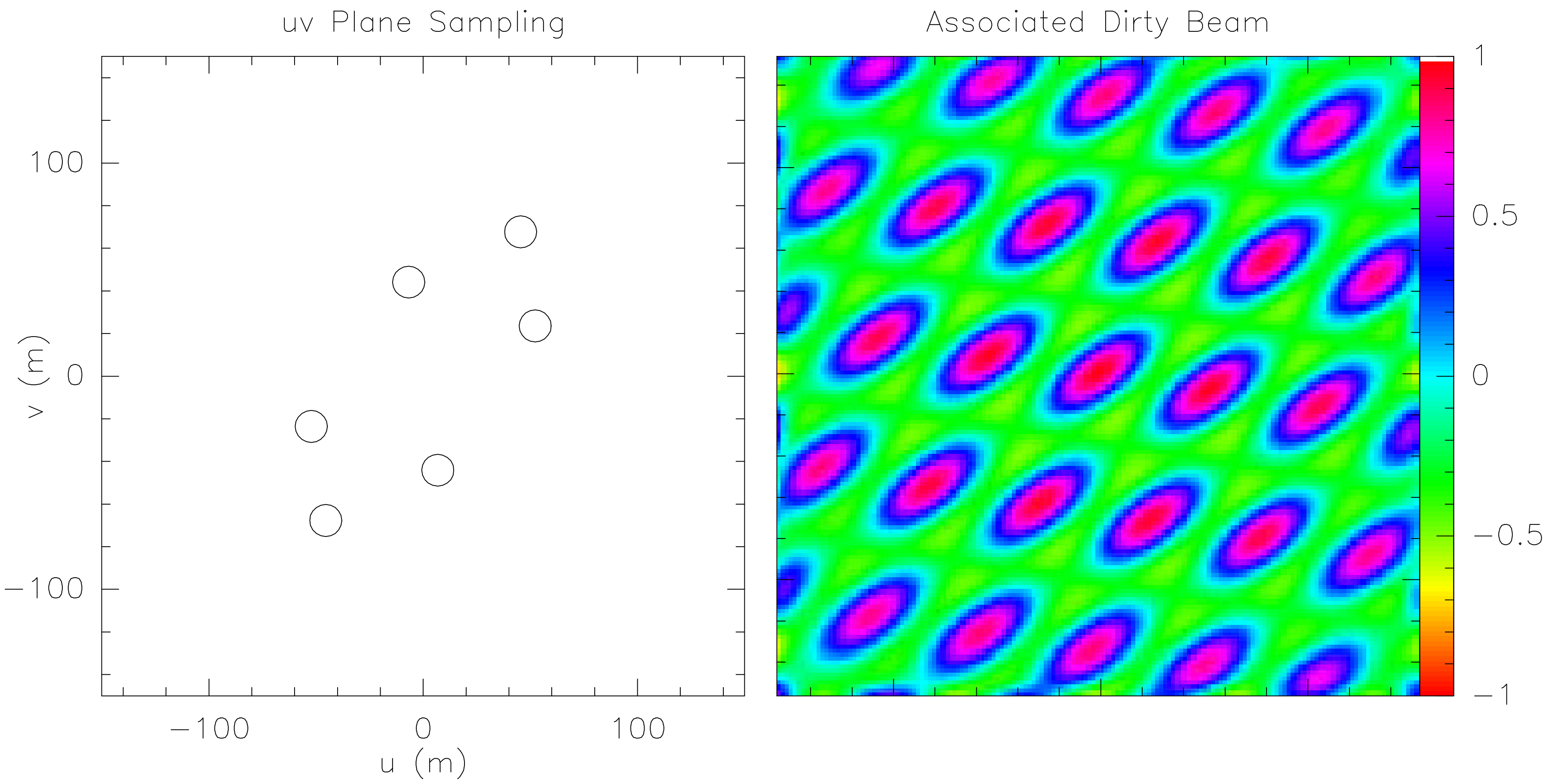
Ways to improve (at least change) B_{dirty} shape:

- Increase the number of antenna (costly).
- Change the antenna layout (technically difficult).
- Weight the irregular, limited sampling function S
  (the only thing you can do in practice).

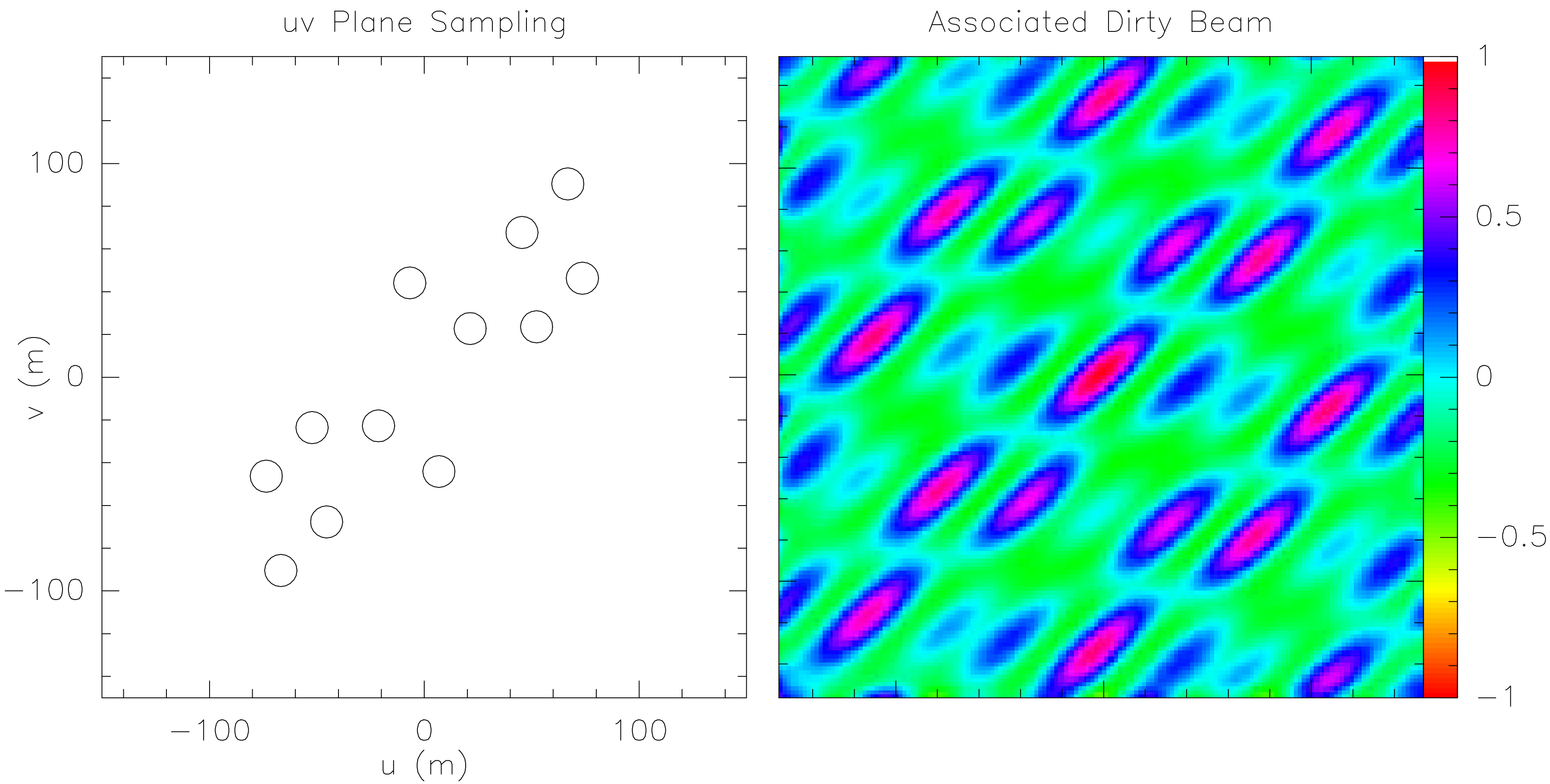
Dirty Beam Shape and Number of Antenna:
2 Antenna



Dirty Beam Shape and Number of Antenna:
3 Antenna

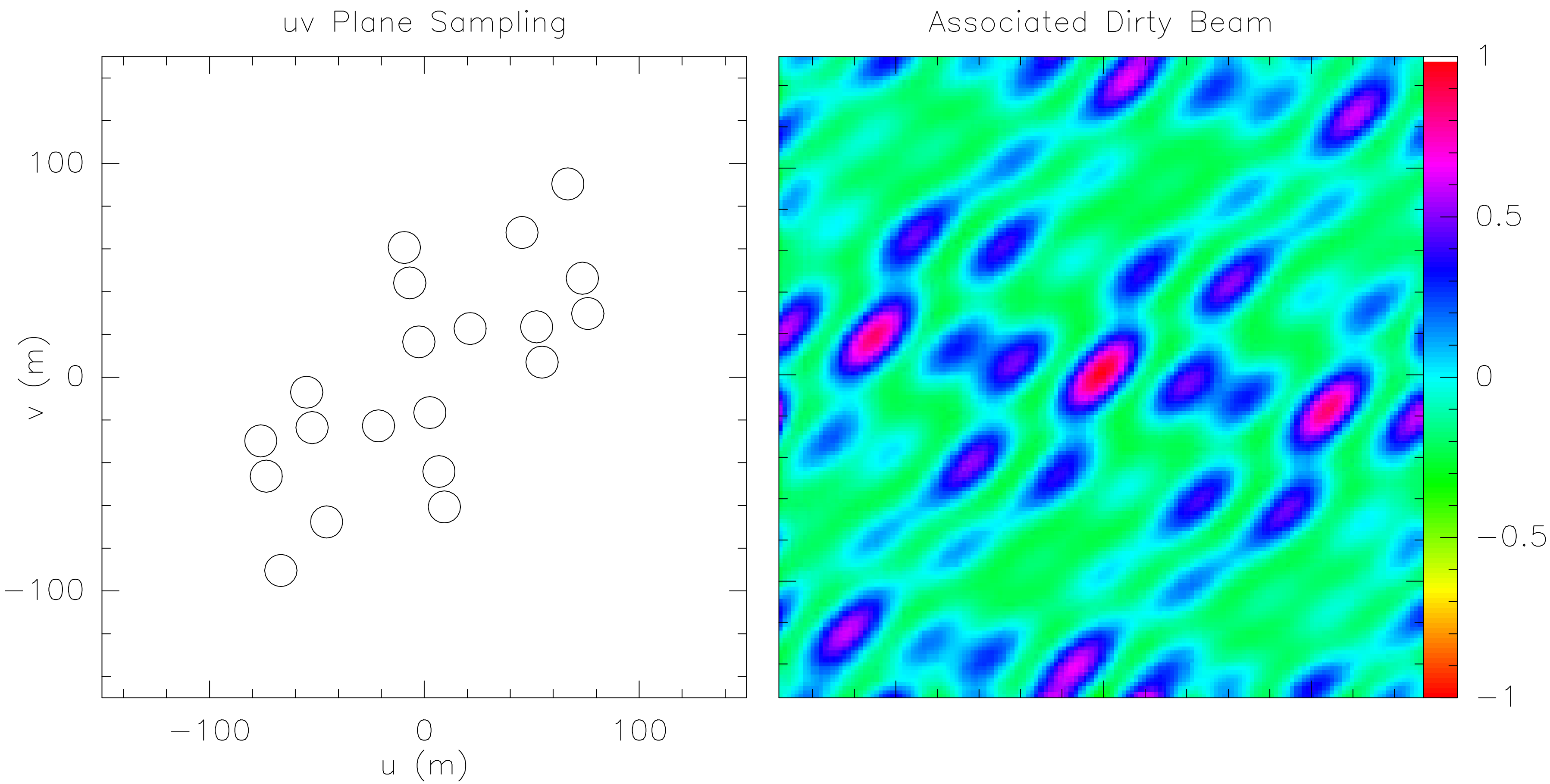


Dirty Beam Shape and Number of Antenna:
4 Antenna

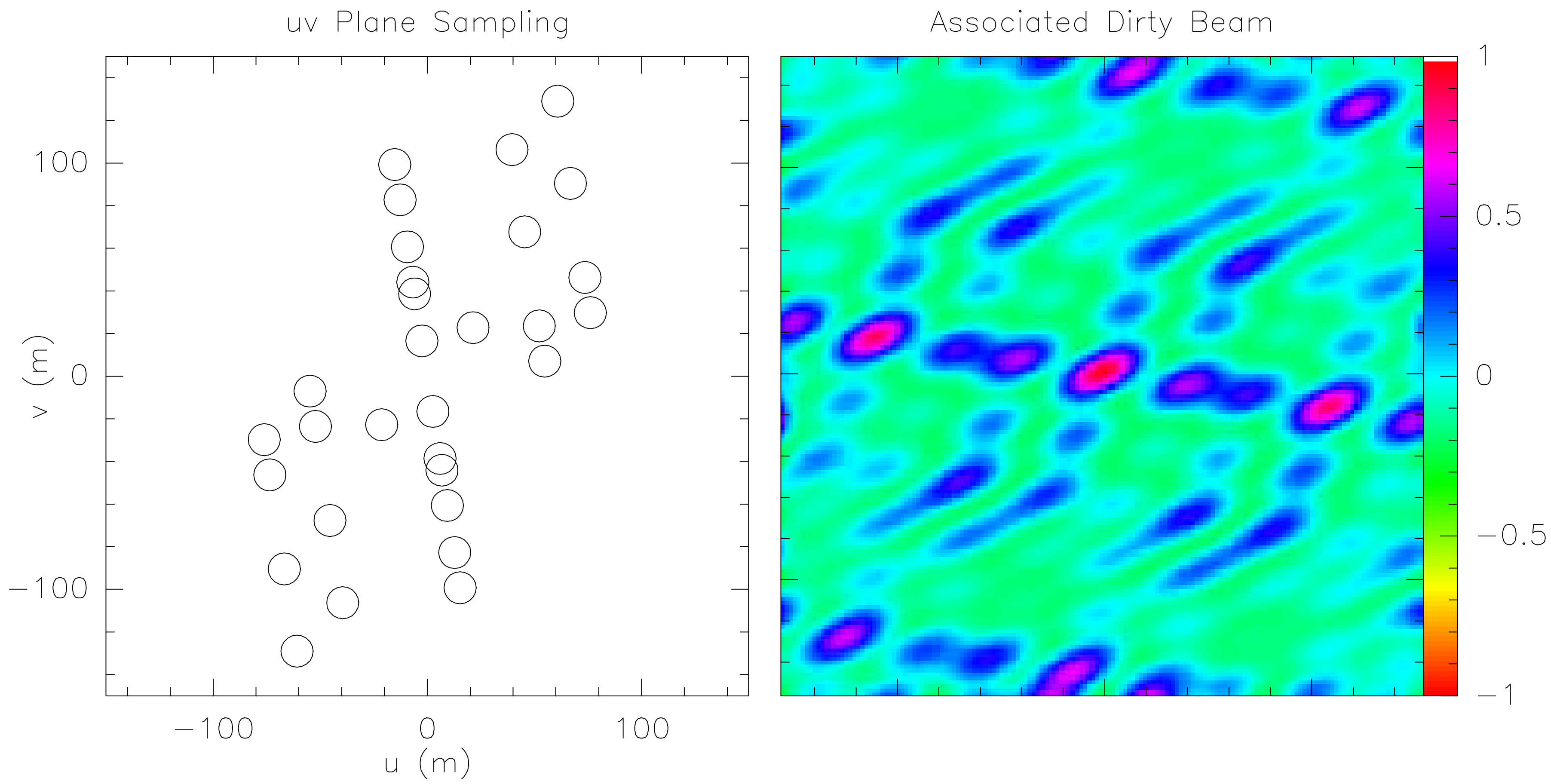


Dirty Beam Shape and Number of Antenna:

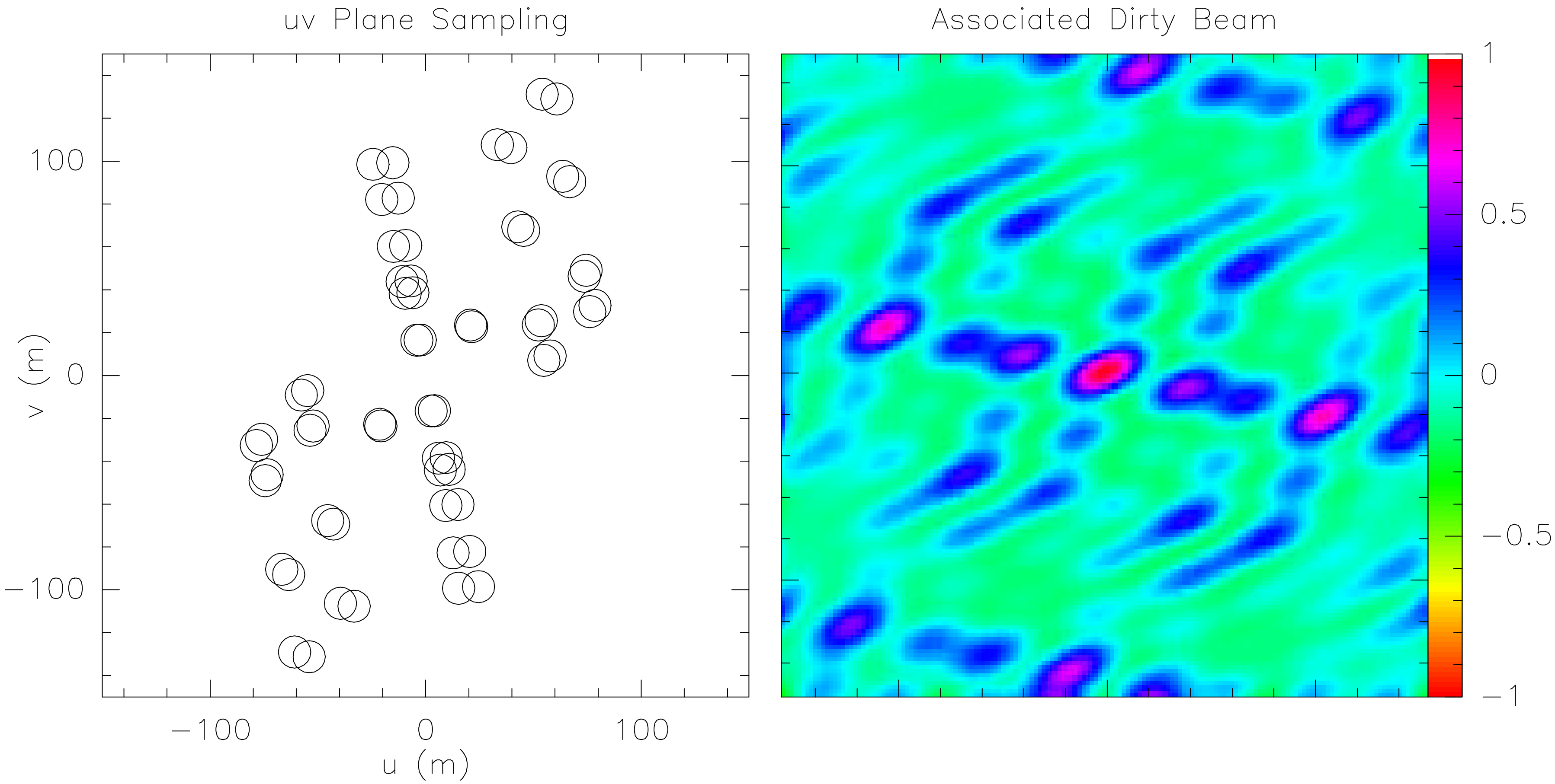
5 Antenna



Dirty Beam Shape and Number of Antenna:
6 Antenna



Dirty Beam Shape and Super Synthesis



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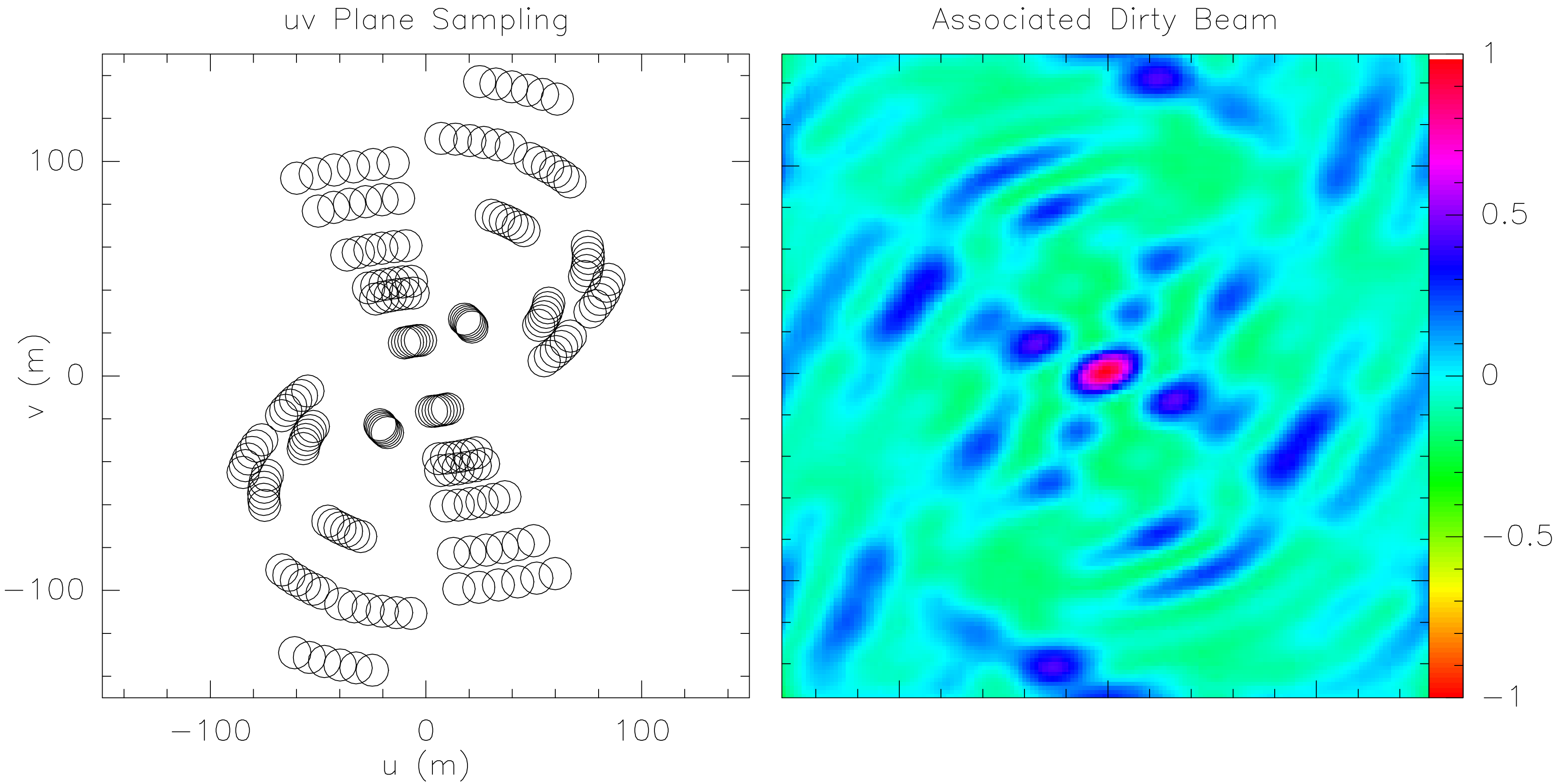

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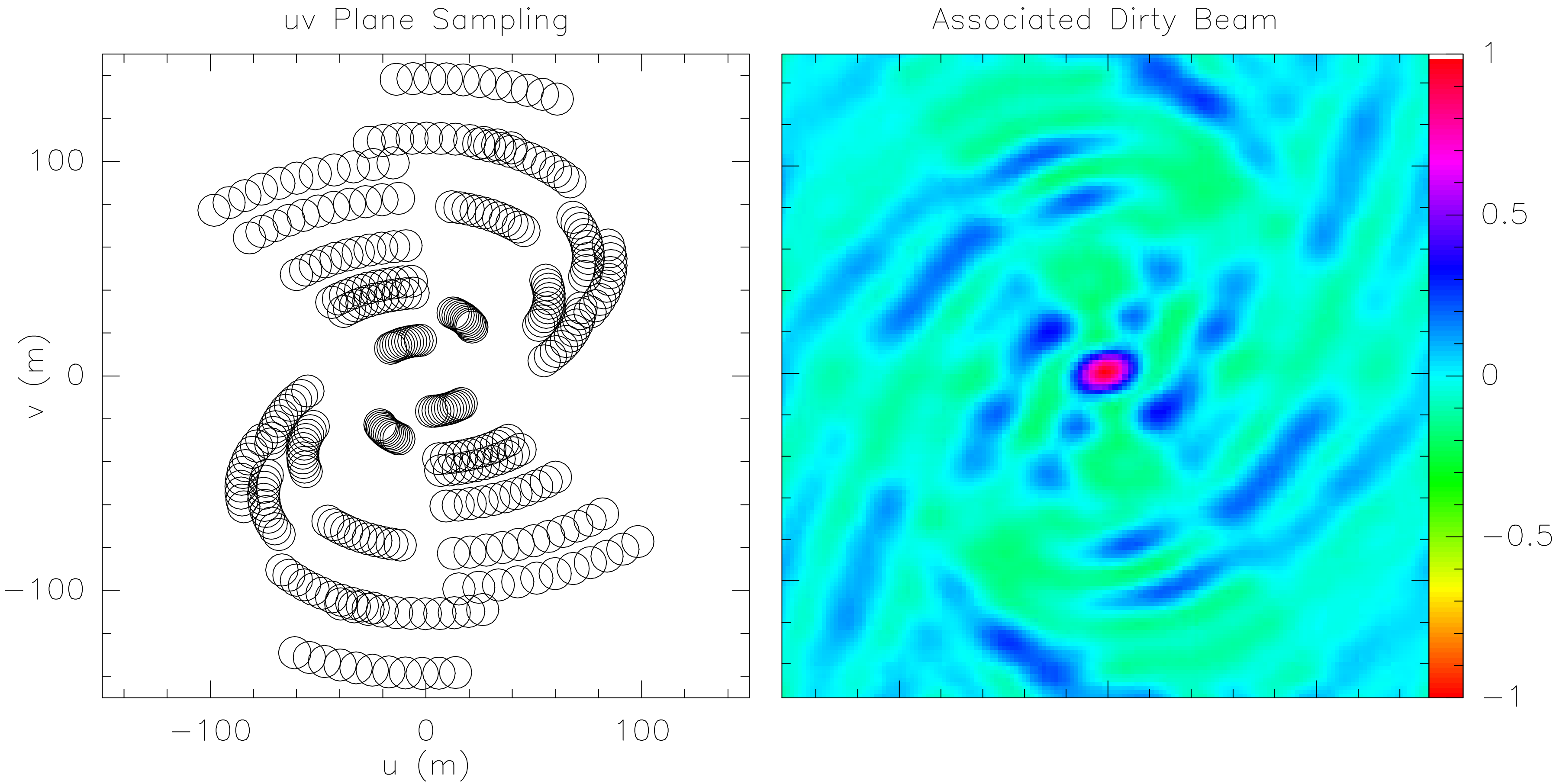

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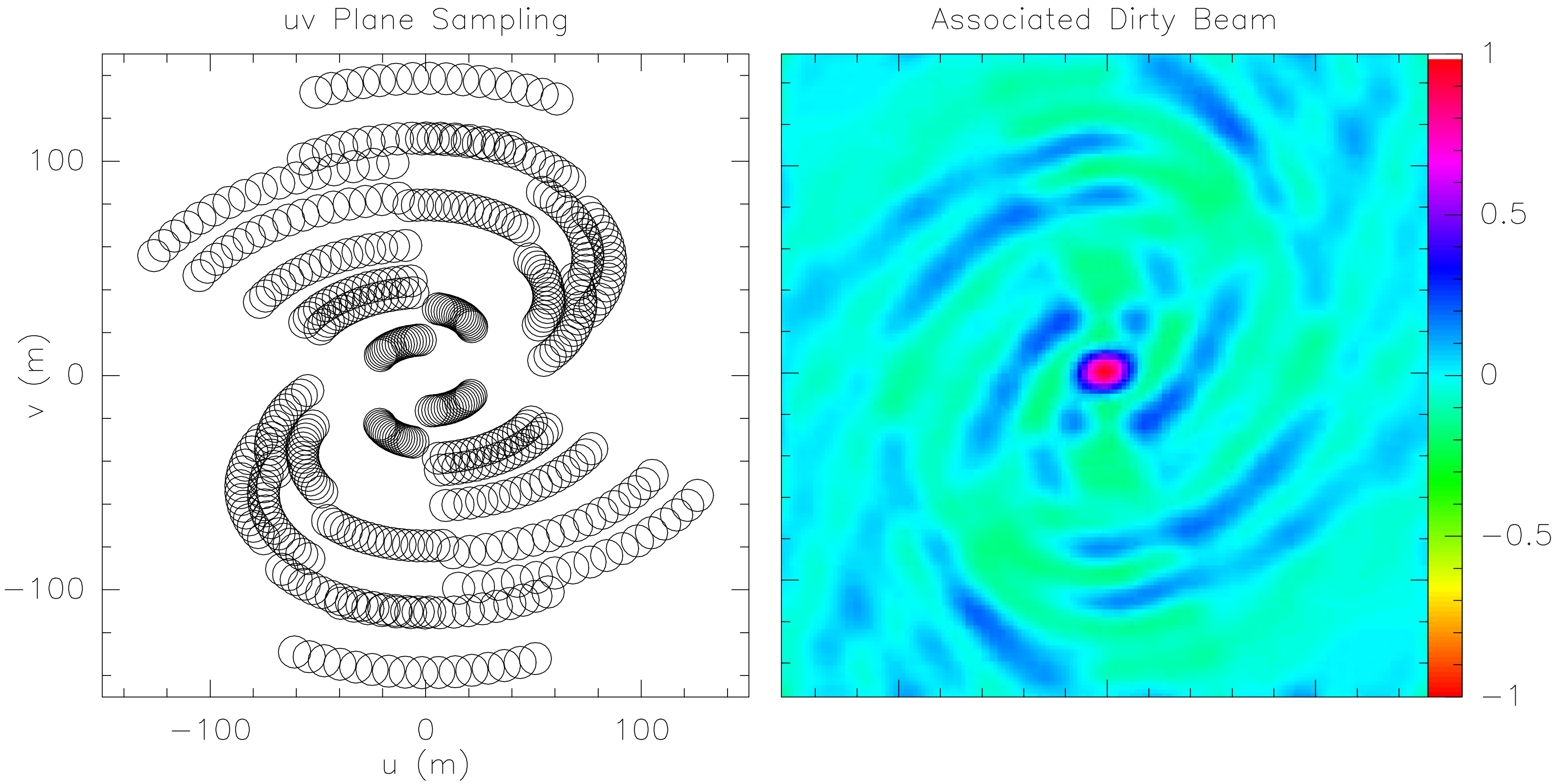

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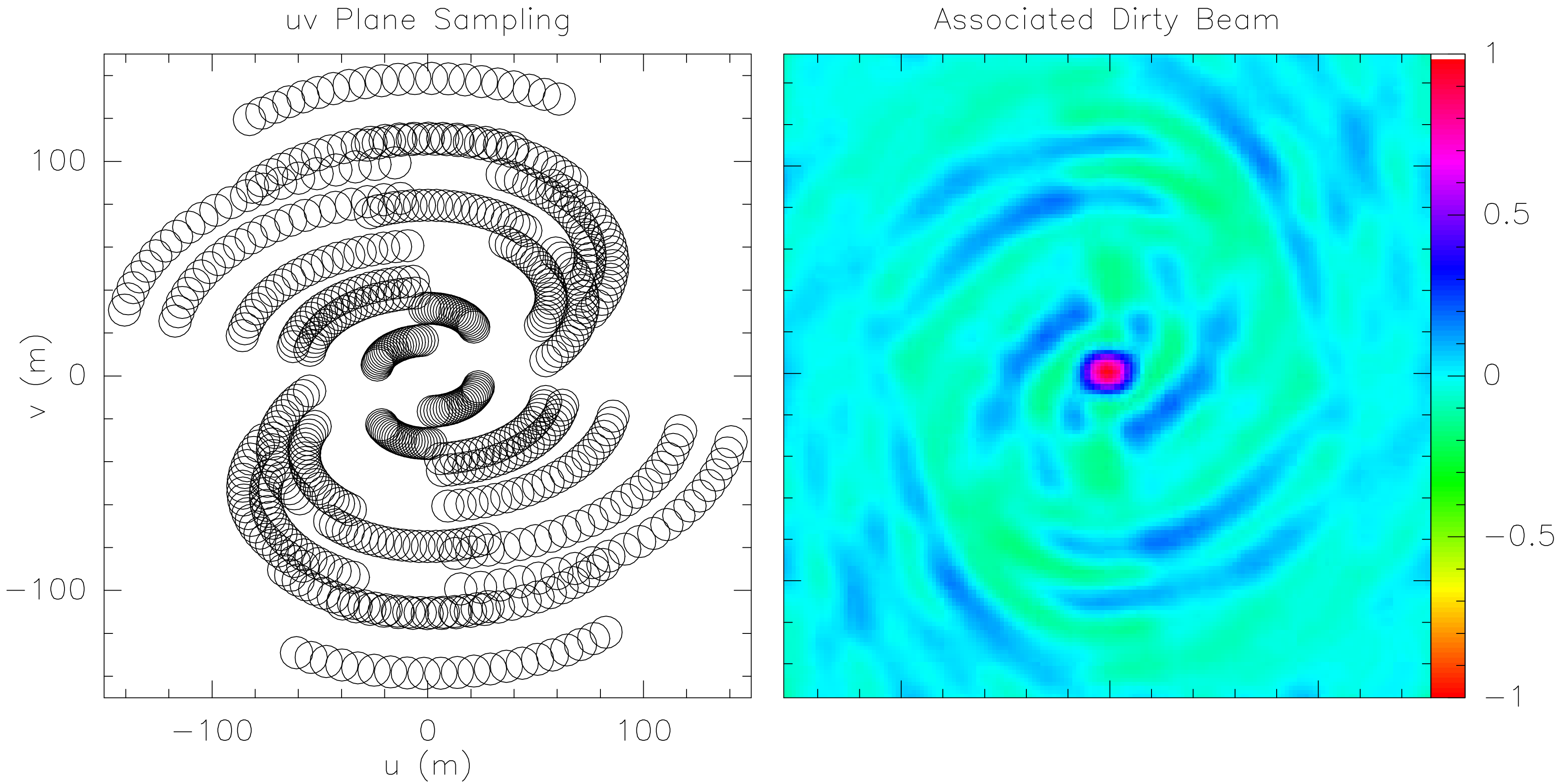

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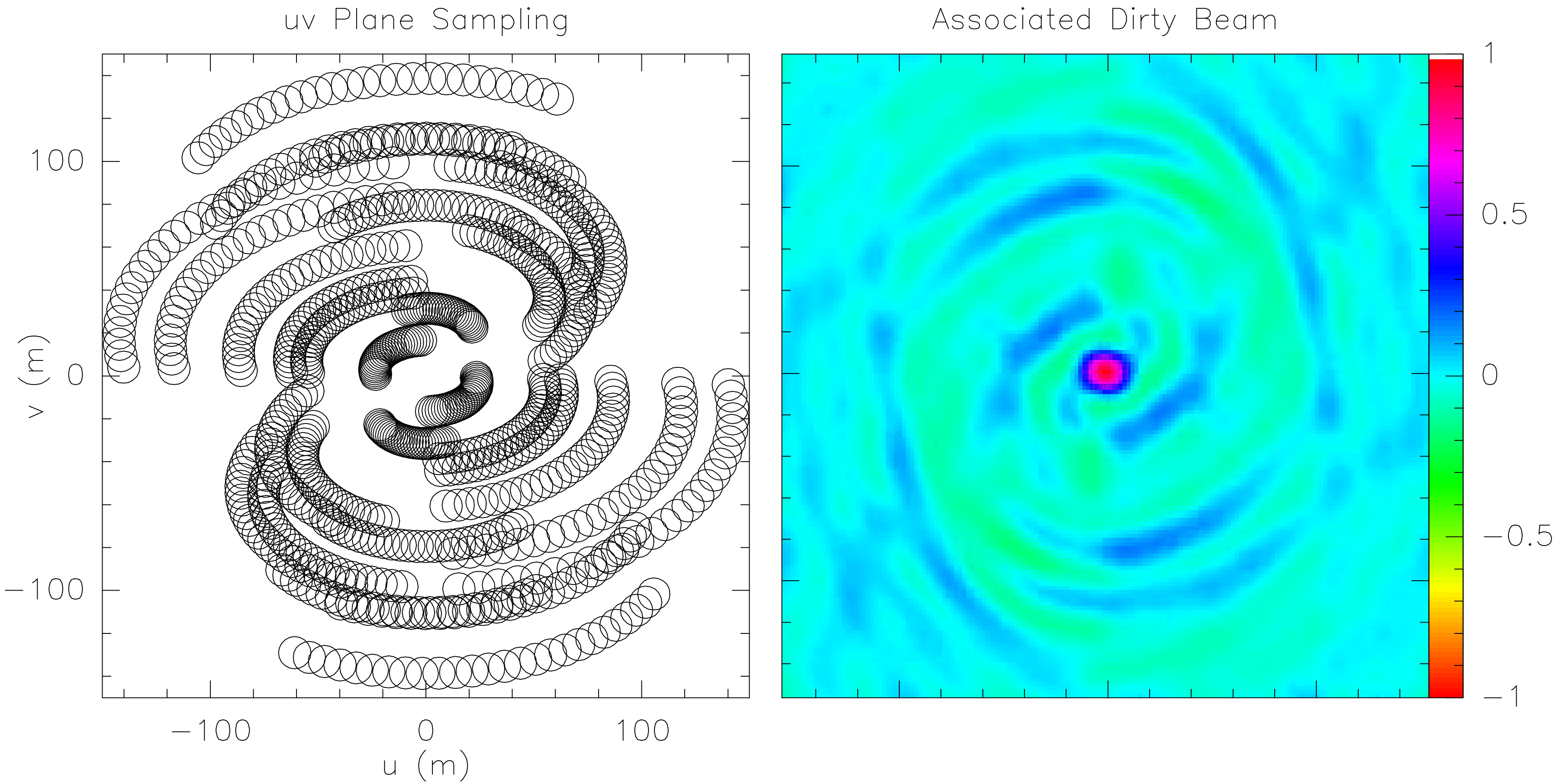

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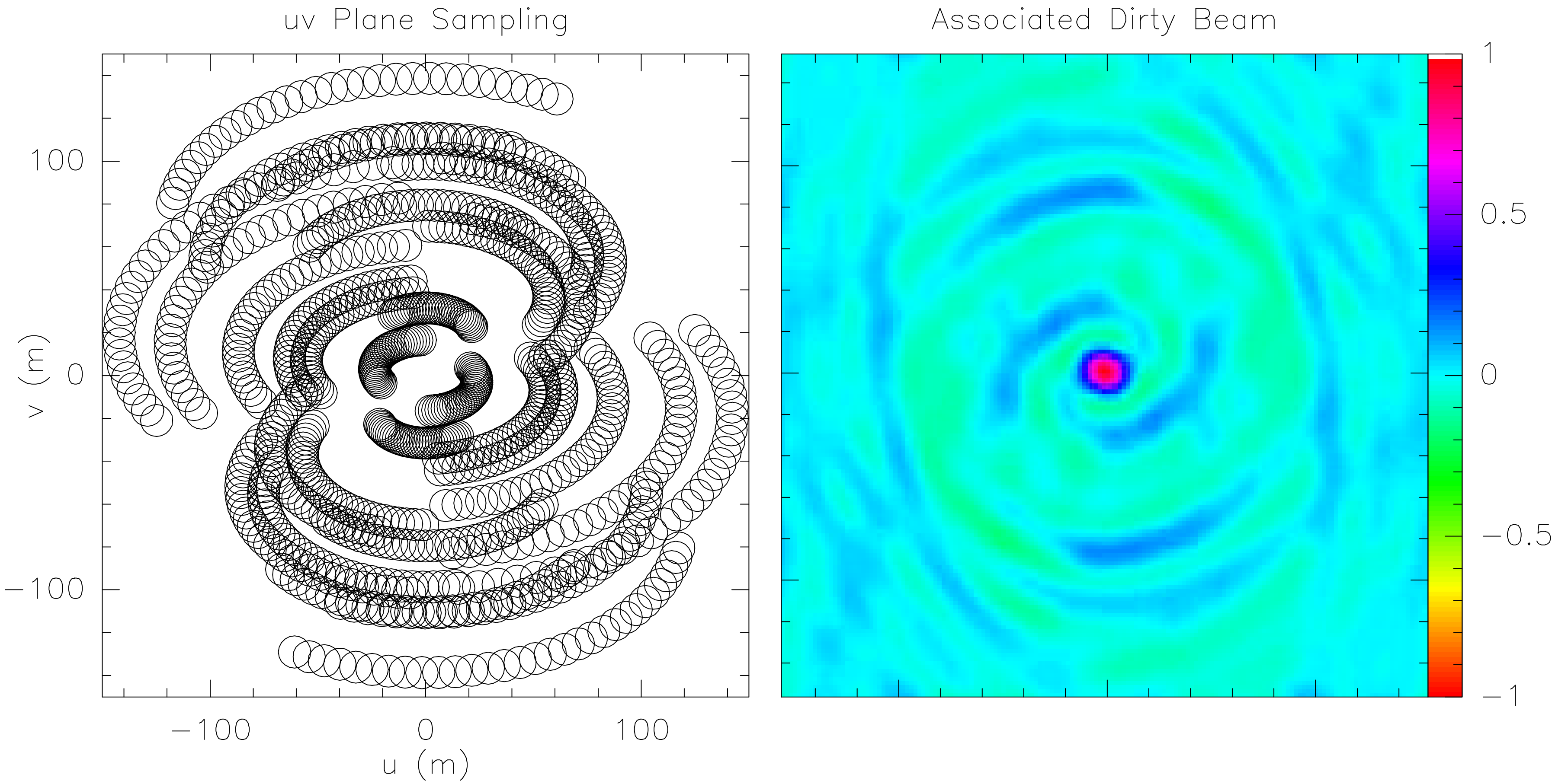

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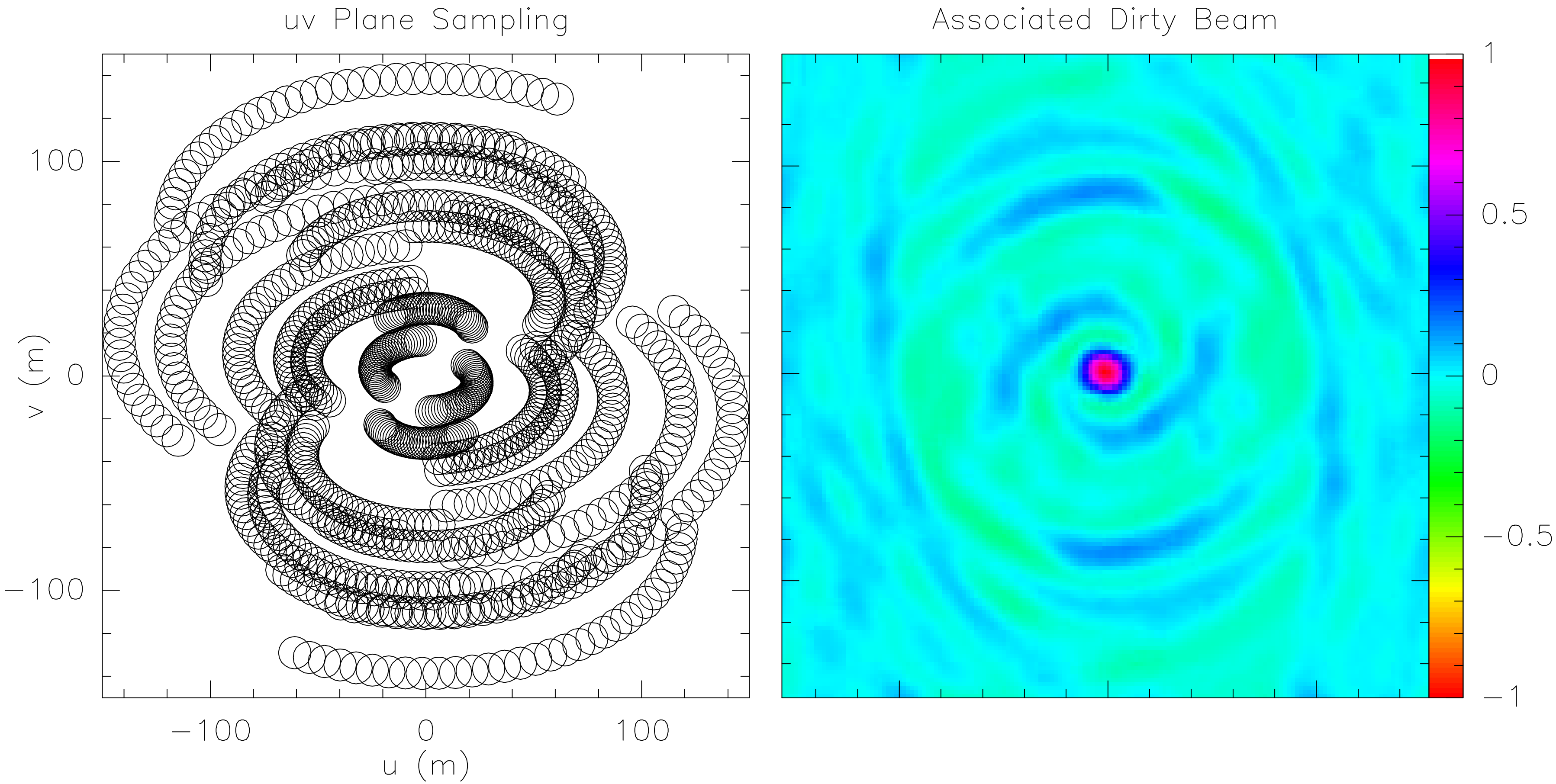


Dirty Beam Shape and Weighting

Natural Weighting: Default definition of the irregular sampling function at uv table creation.

- $S(u,v) = 1/\sigma^2$ at (u,v) points where visibilities are measured;
- $S(u,v) = 0$ elsewhere;

with $\sigma^2(u,v)$ the noise variance of the visibility.

Introduction of a weighting function $W(u,v)$:

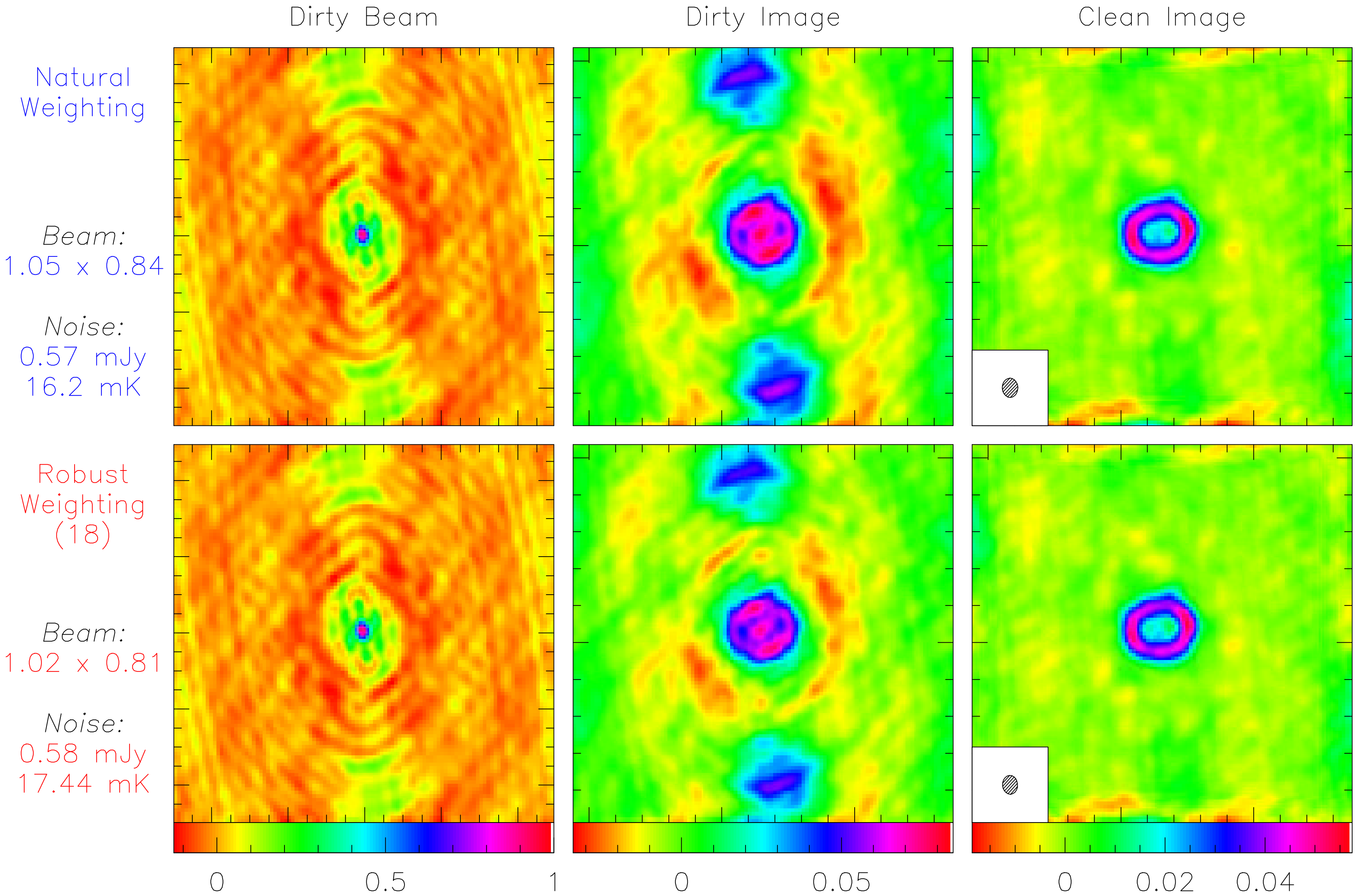
- $B_{\text{dirty}} = \text{2D FT}^{-1}\{W.S\}$;
- Robust weighting: W enhance the large baseline contribution;
- Tapering: W enhance the small baseline contribution.

Robust Weighting: I. Definition

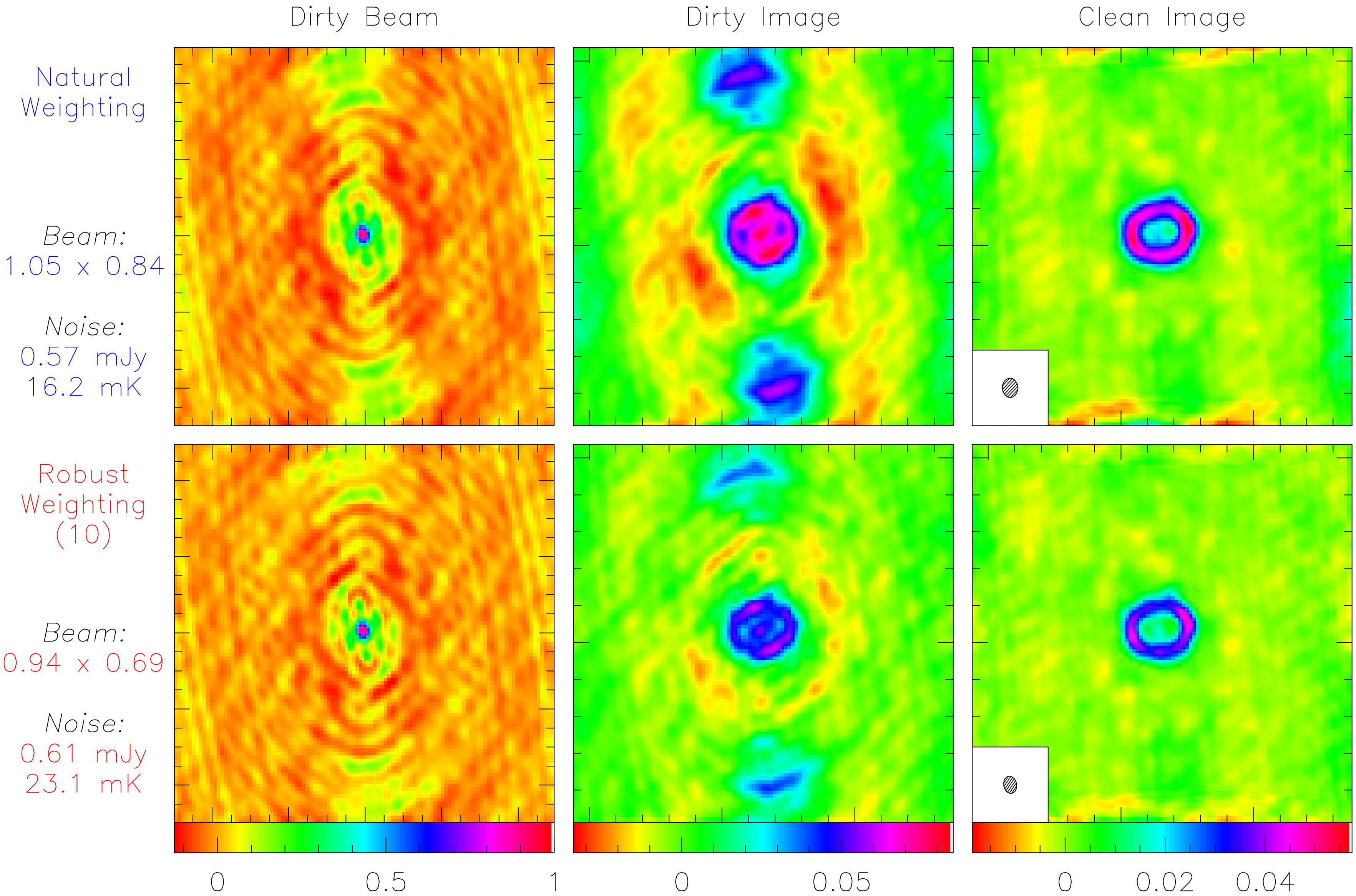
Definitions:

- $\text{Natural} = \sum_{(u,v)\in\text{Cell}} S;$
- $\sum_{(u,v)\in\text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \geq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is $0.5D$.

Robust Weighting: II. Examples



Robust Weighting: II. Examples



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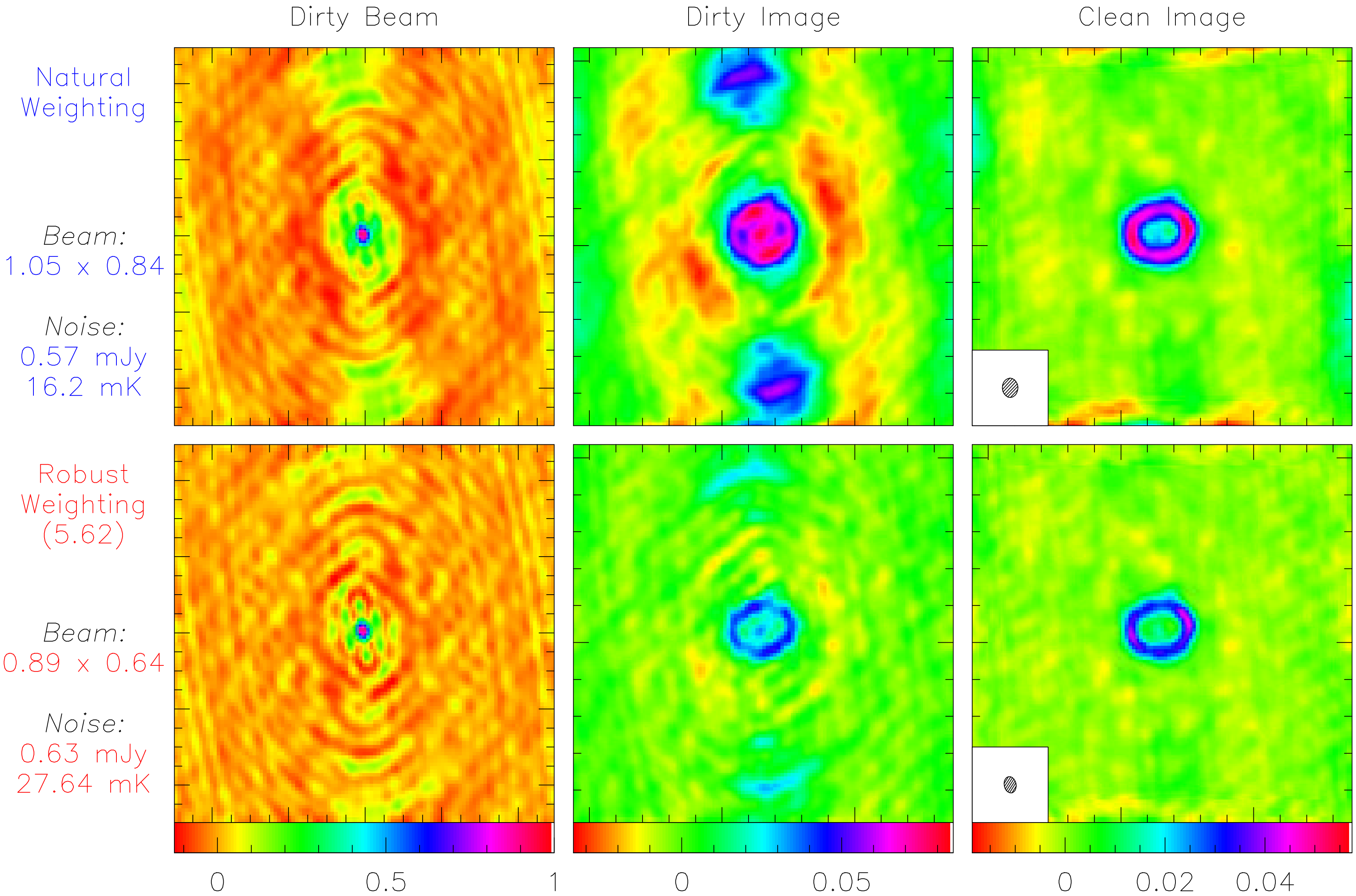
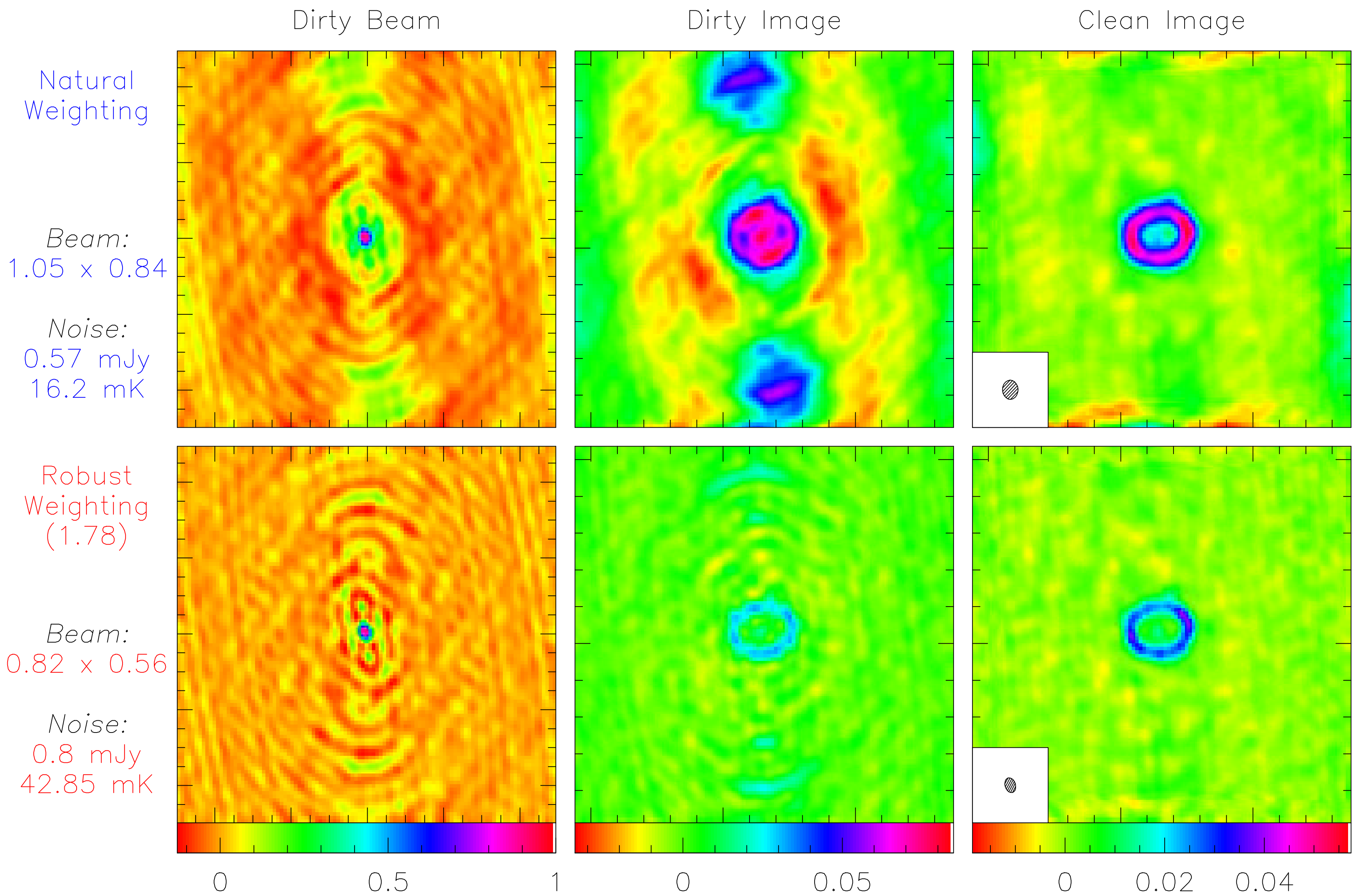


Robust Weighting: II. Examples



Robust Weighting: III. Definition and Properties

Definitions:

- Natural $= \sum_{(u,v)\in \text{Cell}} S$;
- $\sum_{(u,v)\in \text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \leq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is $0.5D$.

Properties:

- Increase the resolution;
- Lower the sidelobes;
- Degrade point source sensitivity.

Unfortunately: GILDAS implementation gives it the name of "uniform" weighting!

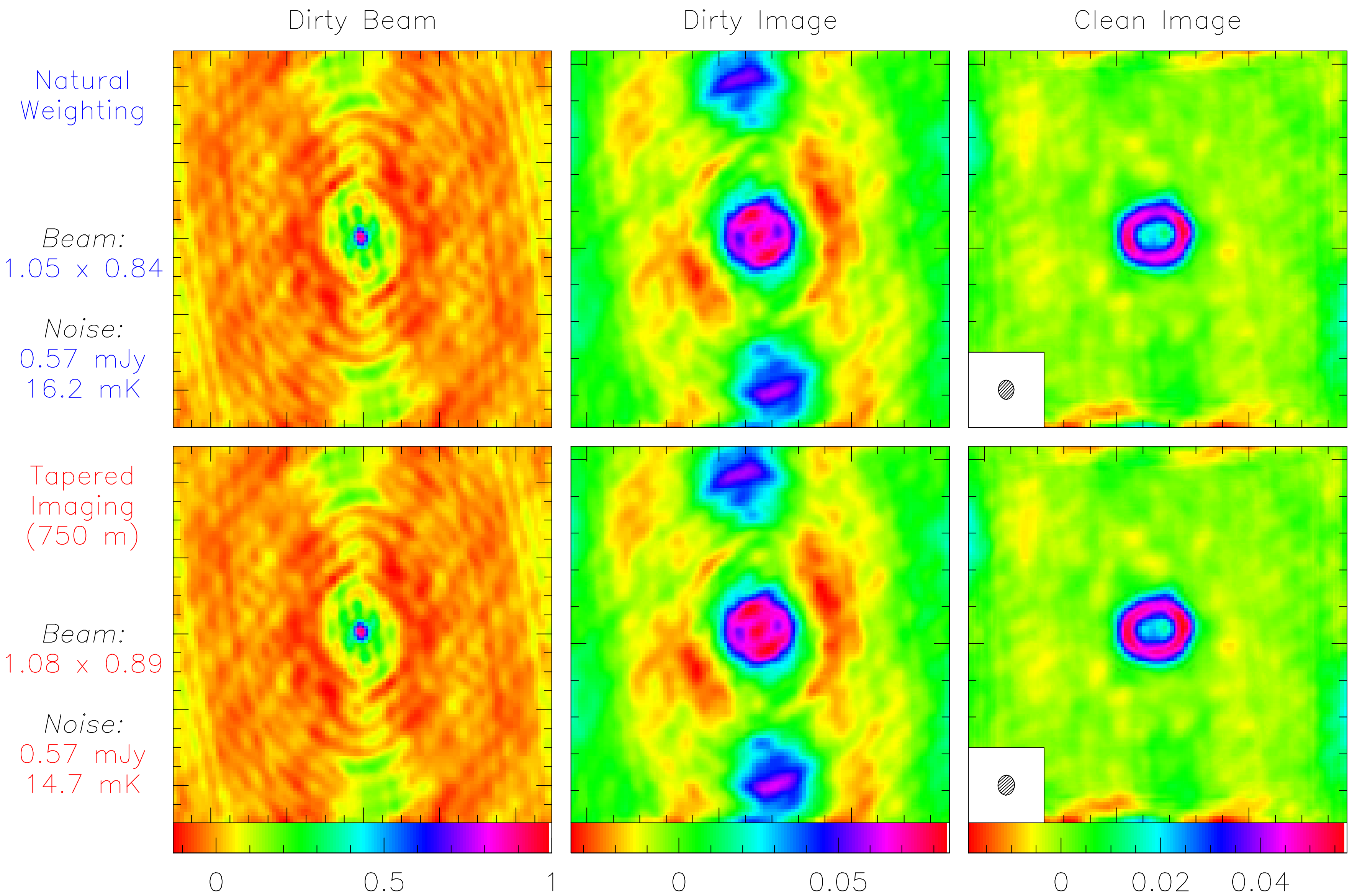
Tapering: I Definition

Definition:

- Apodization of the uv coverage in general by a Gaussian;
- $W = \exp\left\{-\frac{\left(u^2 + v^2\right)}{t^2}\right\}$ where t = tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Tapering: II. Examples



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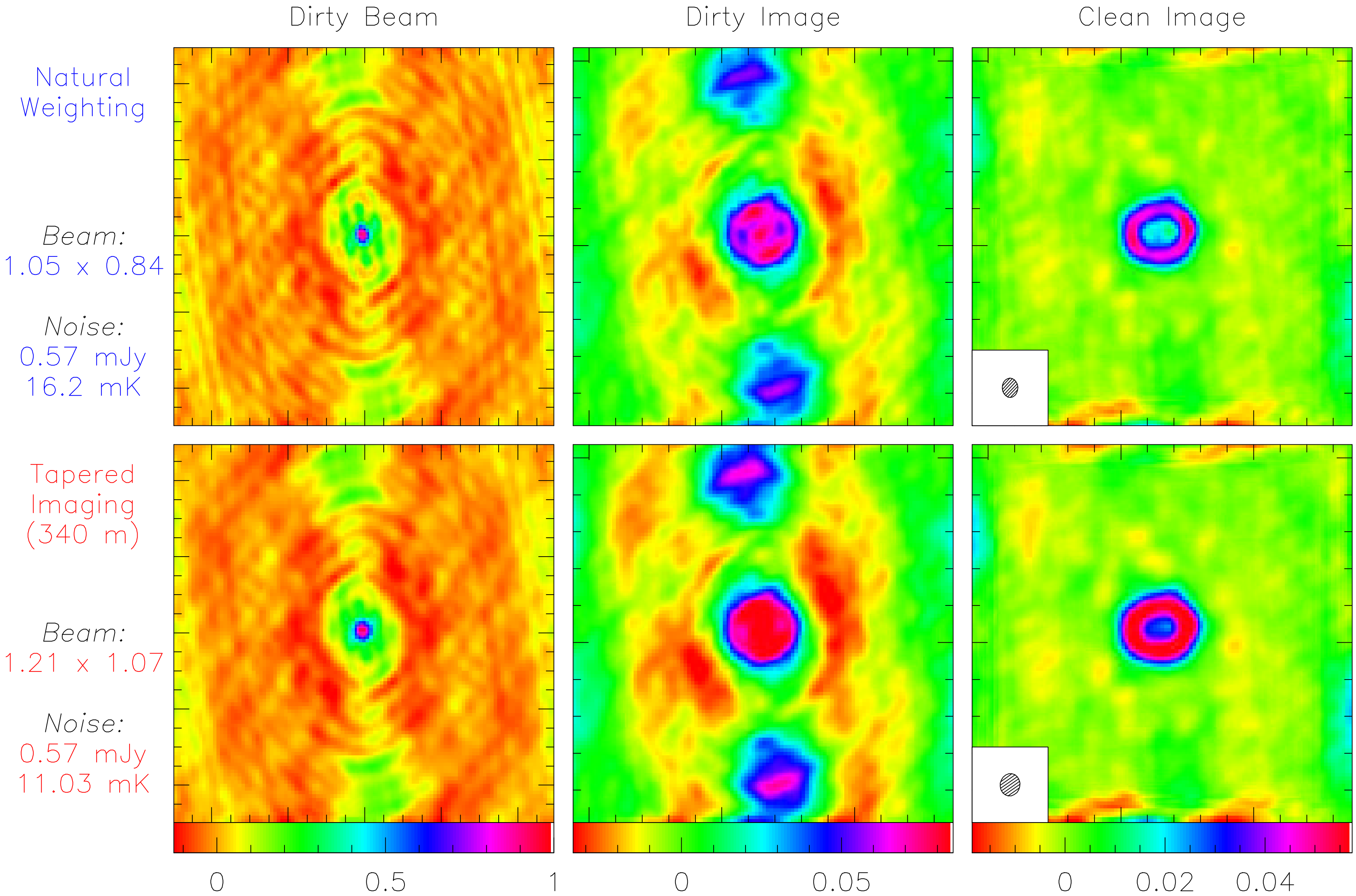

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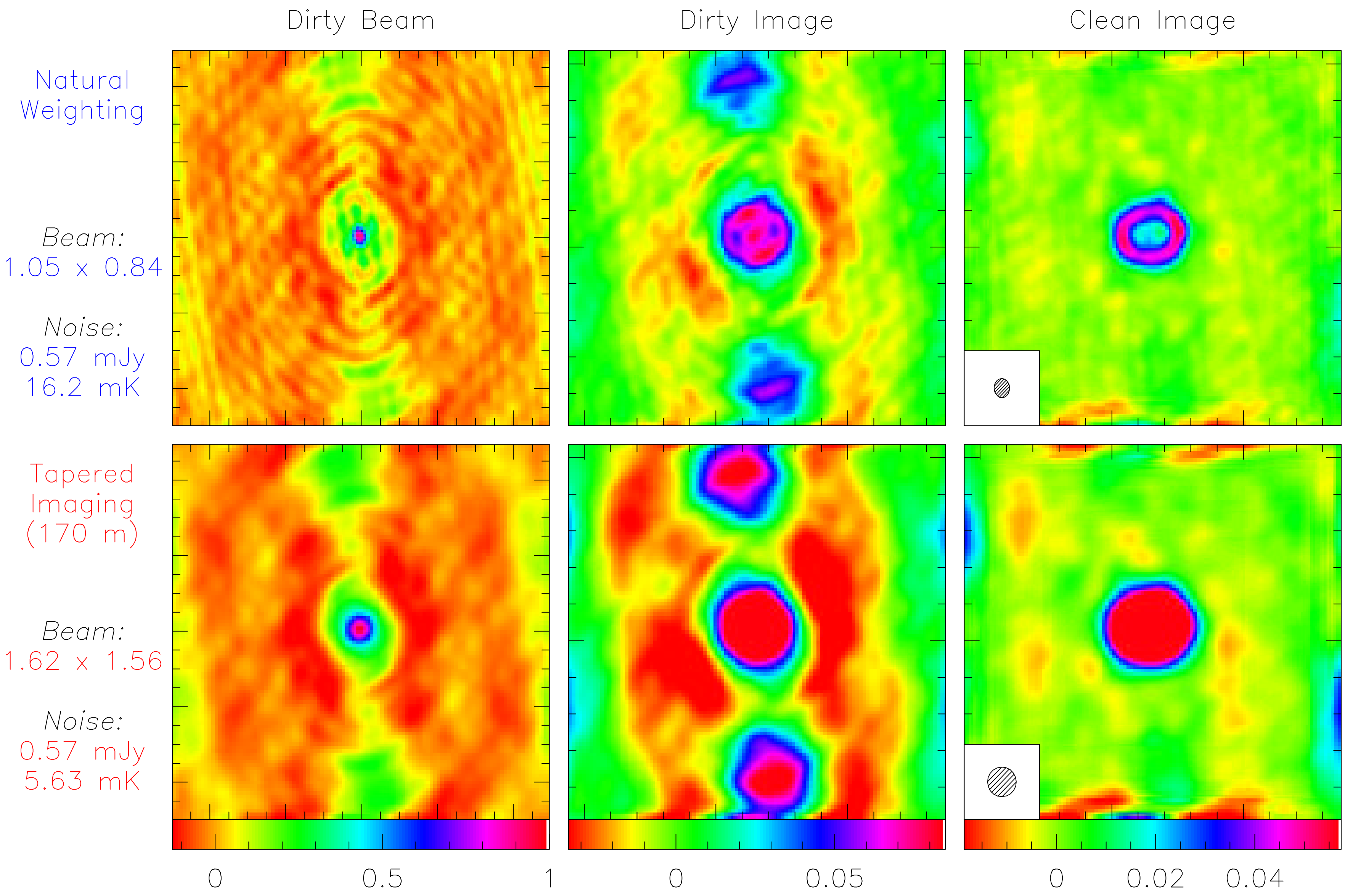

Tapering: III. Definition and Properties

Definition:

- Apodization of the uv coverage in general by a Gaussian;
- $W = \exp\left\{-\frac{\left(u^2 + v^2\right)}{t^2}\right\}$ where $t =$ tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Properties:

- Decrease the resolution;
- Degrade point source sensitivity;
- Increase sensitivity to "medium size" structures.

Inconvenient: Throw out some information.

$\Rightarrow$ To increase sensitivity to extended sources, use compact arrays not tapering.

Weighting and Tapering: Summary

	Robust	Natural	Tapering
Resolution	High	Medium	Low
Side Lobes	$\searrow$	Medium	?
Point Source Sensitivity	$\searrow$	Maximum	$\searrow$
Extended Source Sensitivity	$\searrow$	Medium	$\nearrow$

Non-circular tapering + Robust weighting:

Sometimes $\Rightarrow$ Better (*i.e.* smaller and more circular) beams.

GILDAS implementation:
"UV_STAT WEIGHT" or "UV_STAT TAPER"

Resolution, point/extended source sensitivity

as a function of

robust threshold or tapering distance.

From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

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Physical information on your source	

Deconvolution: I. Philosophy

$$I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\} + N.$$

Information lost:

- Irregular, incomplete sampling $\Rightarrow$ convolution by B_{dirty};
- Noise $\Rightarrow$ Low signal structures undetected.

$\Rightarrow$ Impossible to recover the intrinsic source structure!

Deconvolution goal: Finding an intensity distribution compatible with the intrinsic source one.

Deconvolution needs:

- Some *a priori* assumptions about the source intensity distribution;
- As much as possible knowledge of
  - B_{dirty} (OK in radioastronomy);
  - Noise properties.

The best solution: A Gaussian B_{dirty} $\Rightarrow$ No deconvolution needed!

Deconvolution: II. The Basic CLEAN Algorithm

a priori assumption: Source = Collection of point sources.

Idea: "Matching pursuit".

Algorithm:

1 Initialize
   - the residual map to the dirty map;
   - the Clean component list to an empty (NULL) value;

2 Identify pixel of $|I_{\text{max}}|$ in residual map as a point source;

3 Add $\gamma . I_{\text{max}}$ to clean component list;

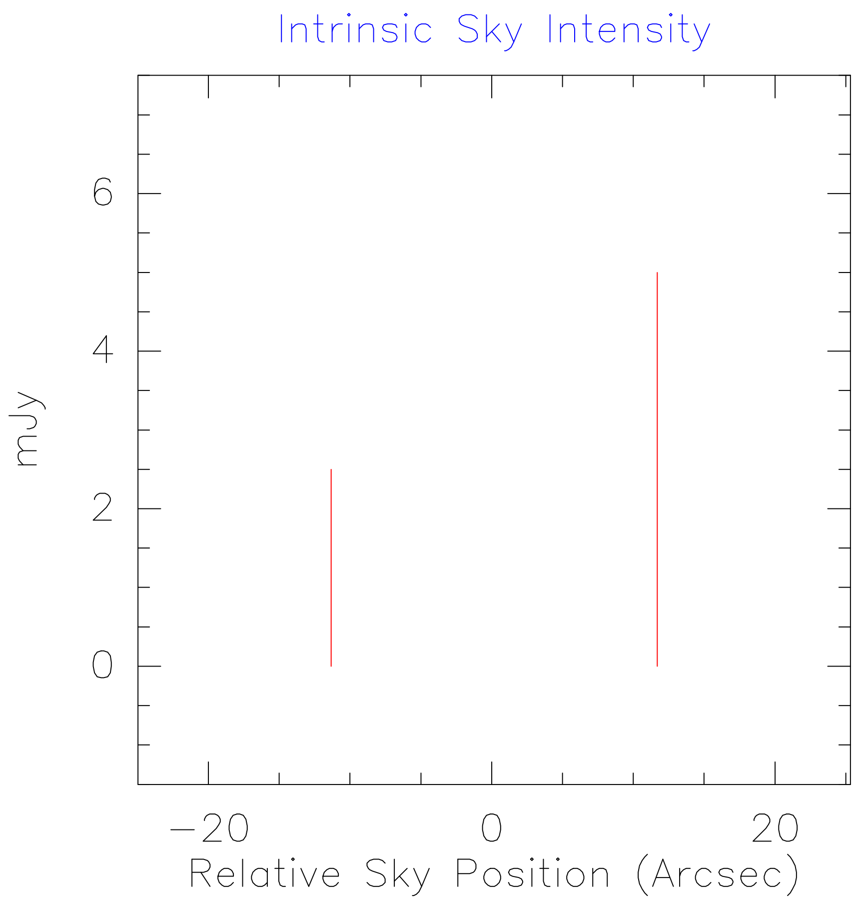
4 Subtract $\gamma . I_{\text{max}}$ from residual map;

5 Go back to point 2 while stopping criterion is not matched;

6 Convolution by Clean beam (*a posteriori* regularization);

7 Addition of residual map to permit:
   - Correction when cleaning is too superficial;
   - Noise estimation.

Deconvolution:
III. Illustration of the Basic Clean Algorithm



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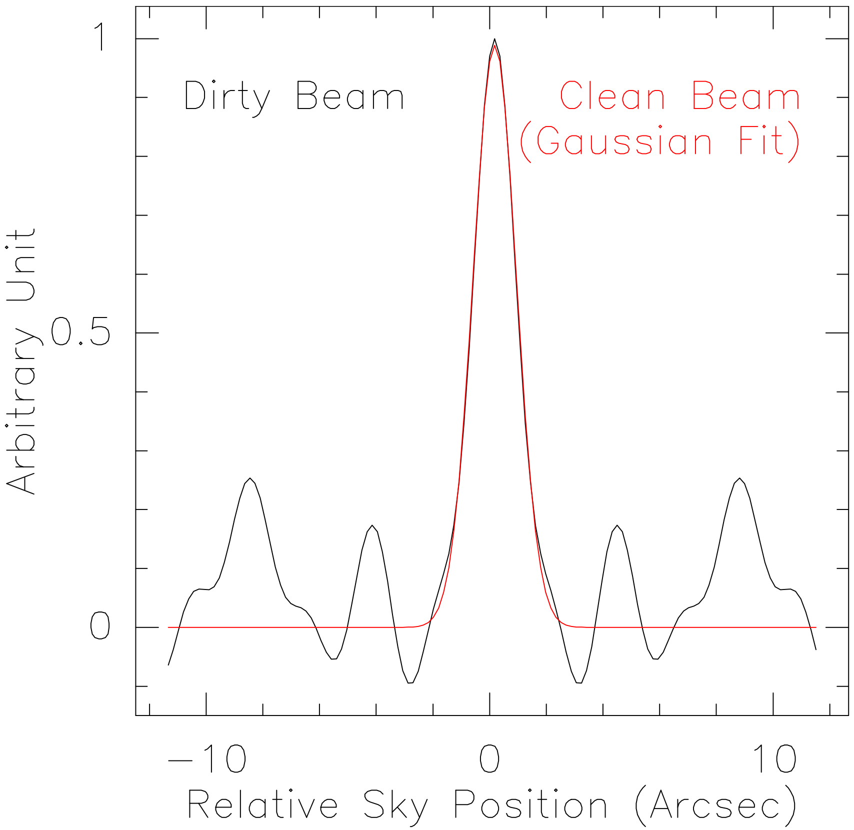

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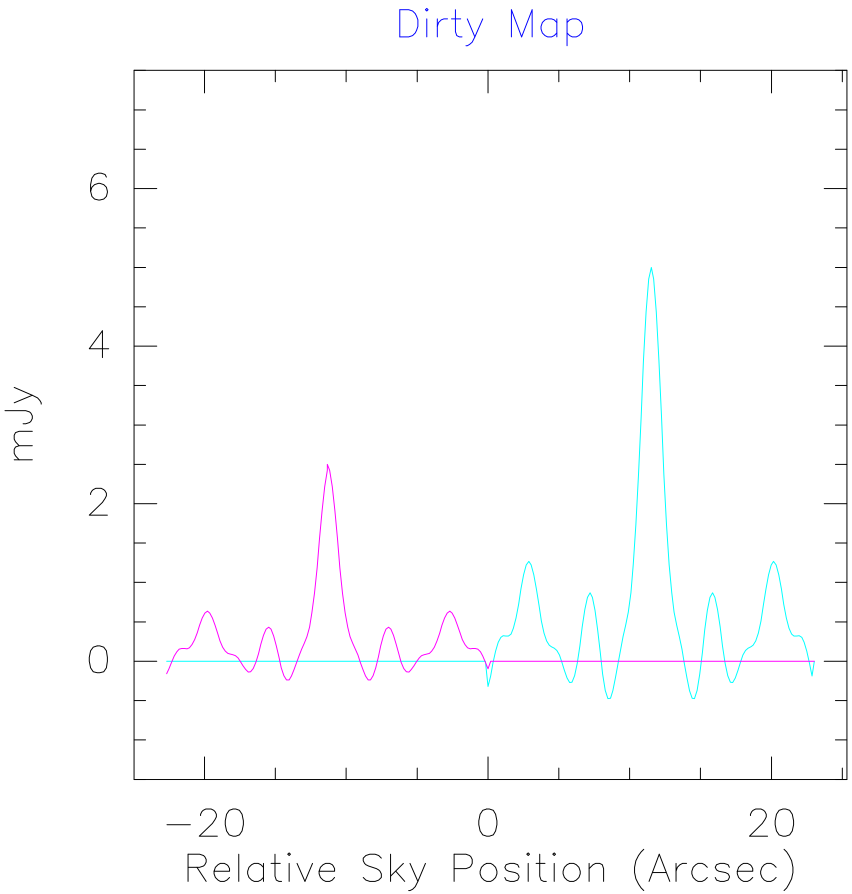

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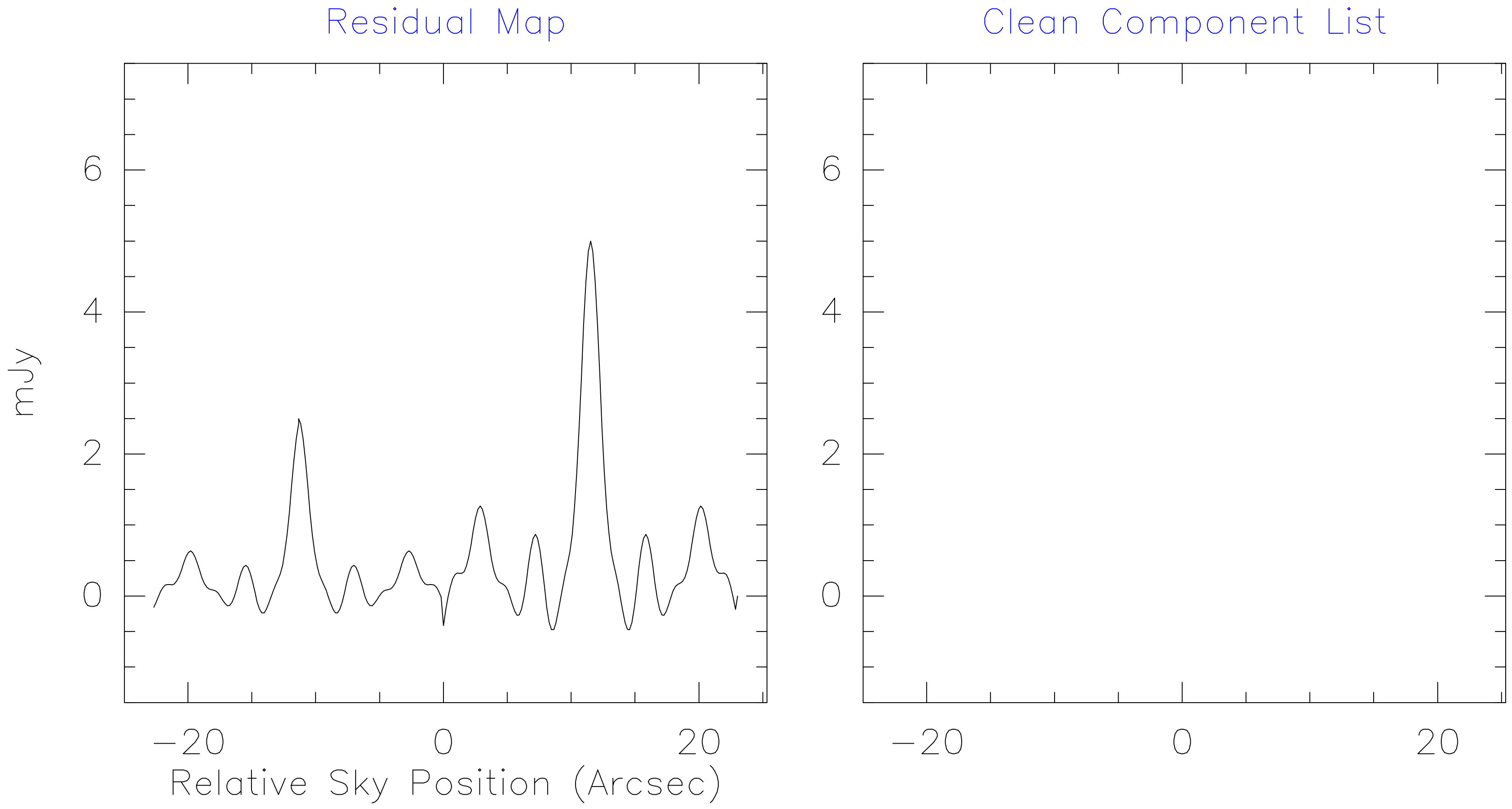

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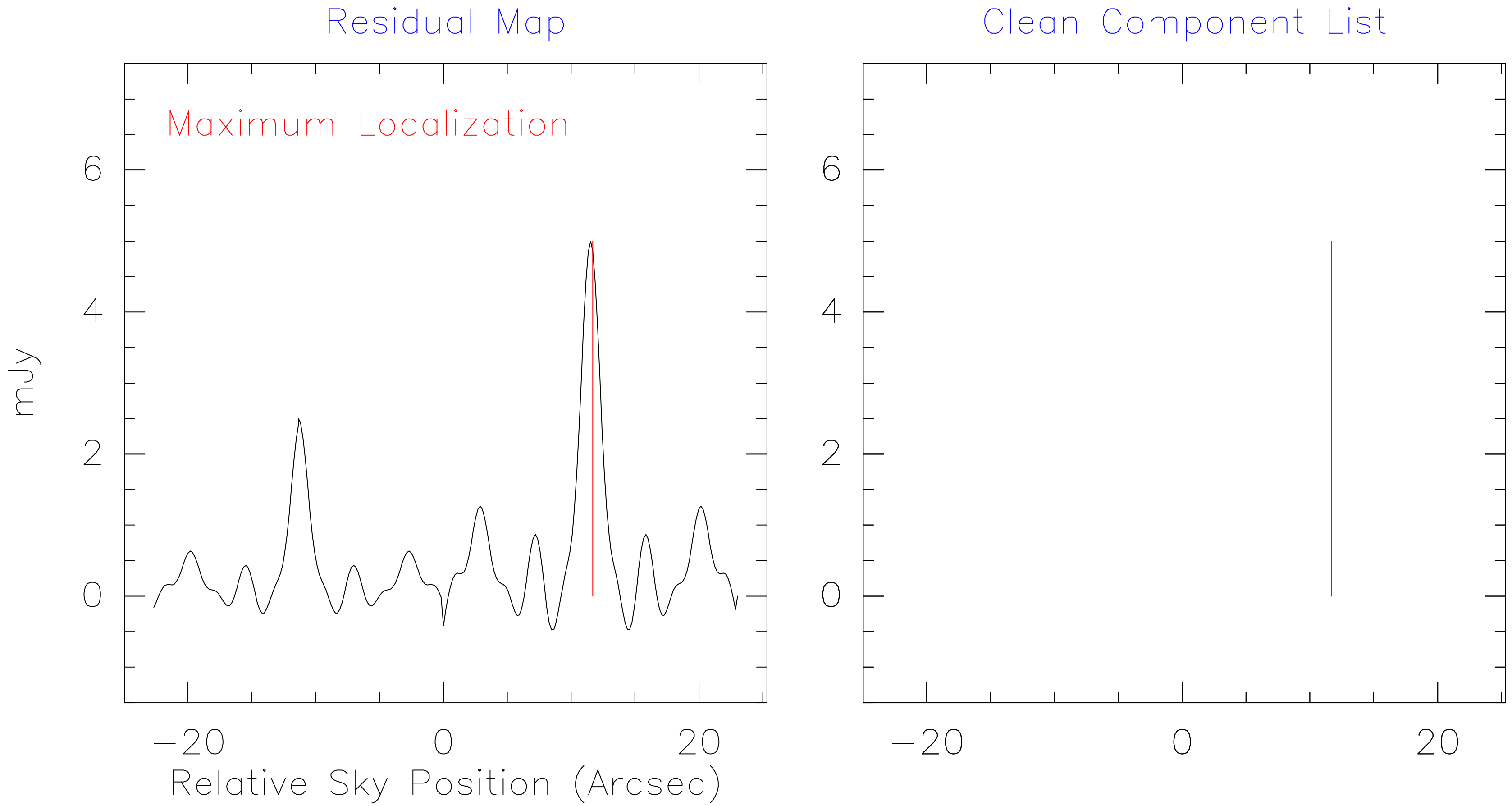

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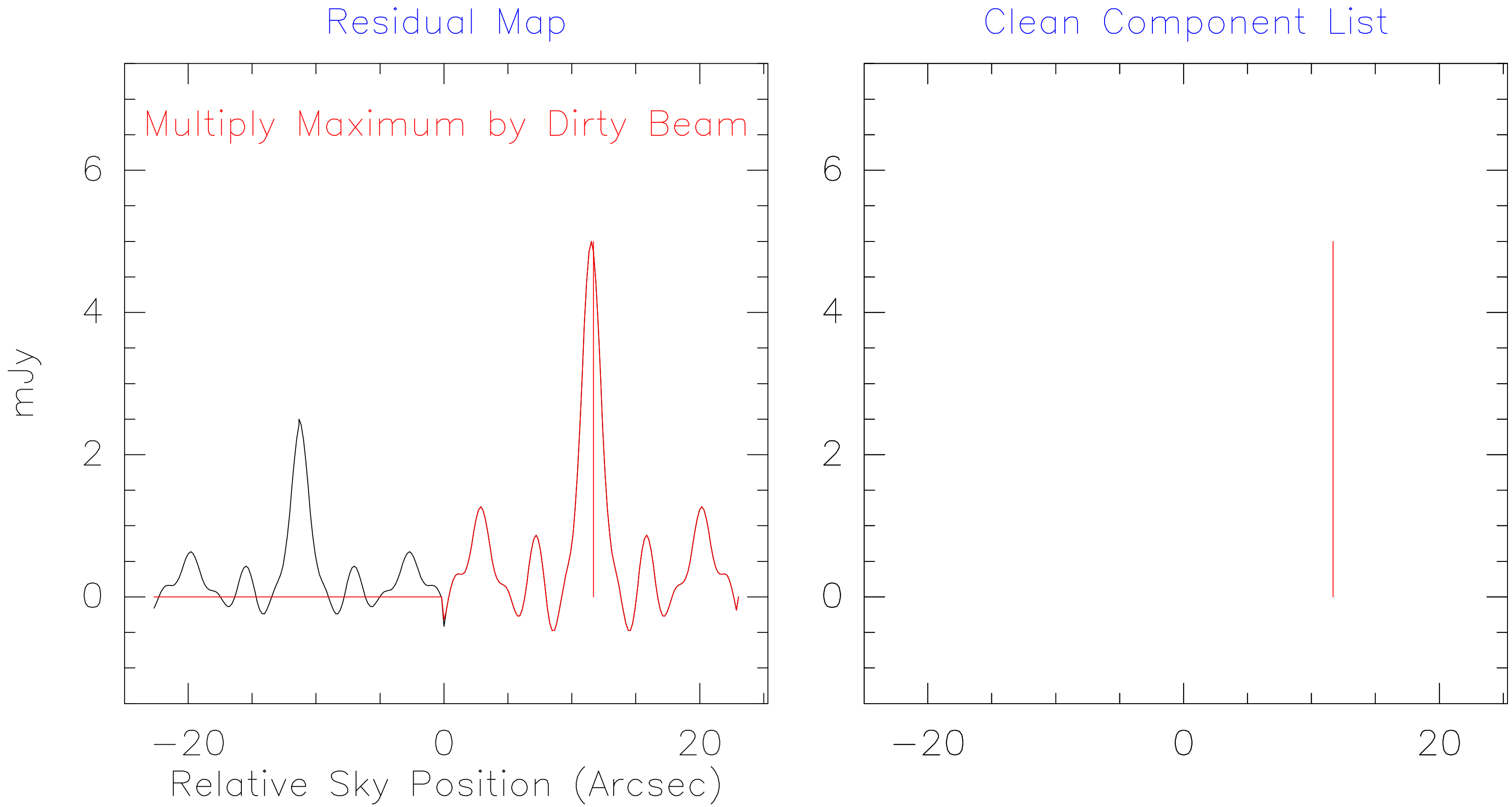

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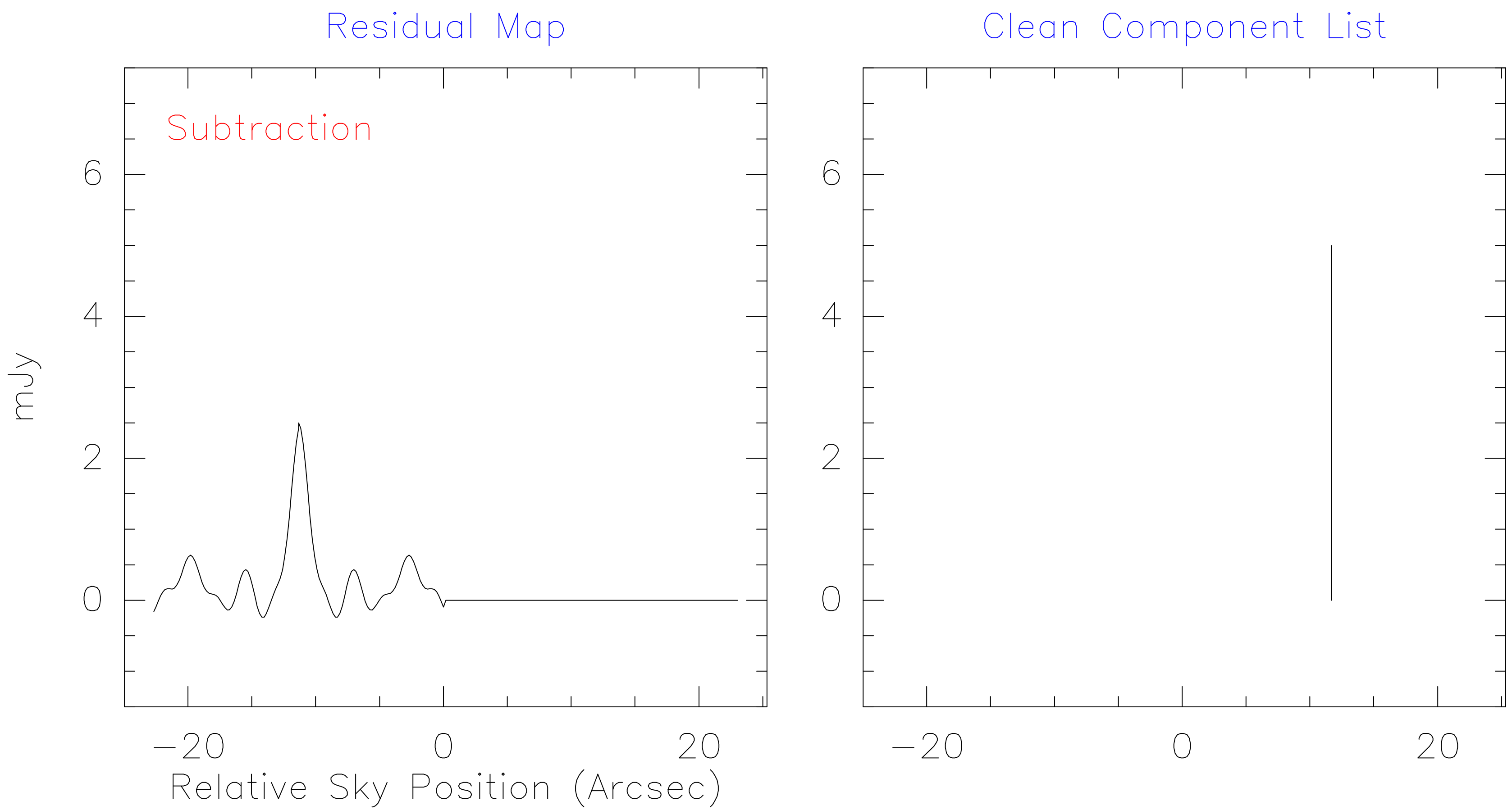

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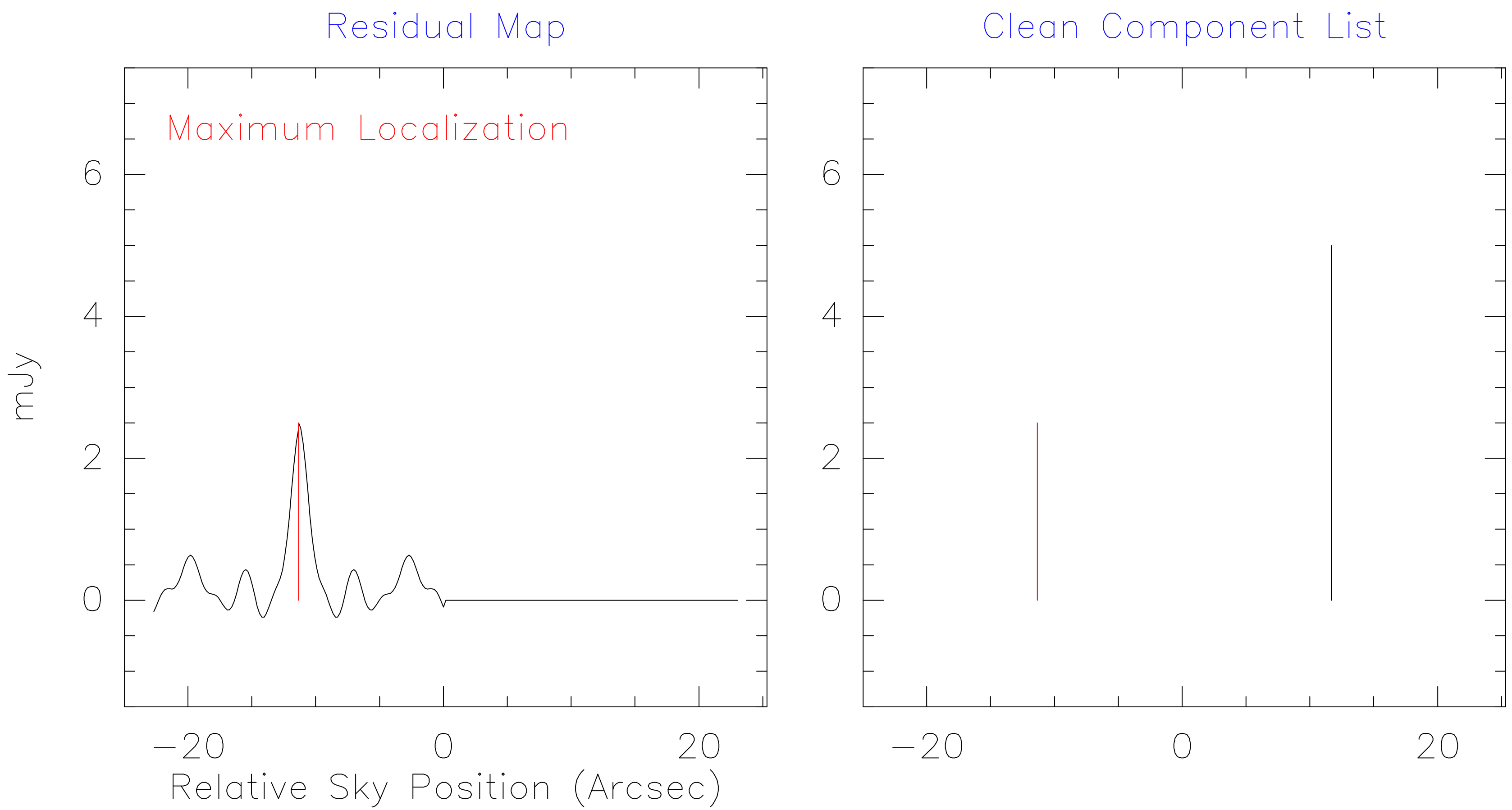

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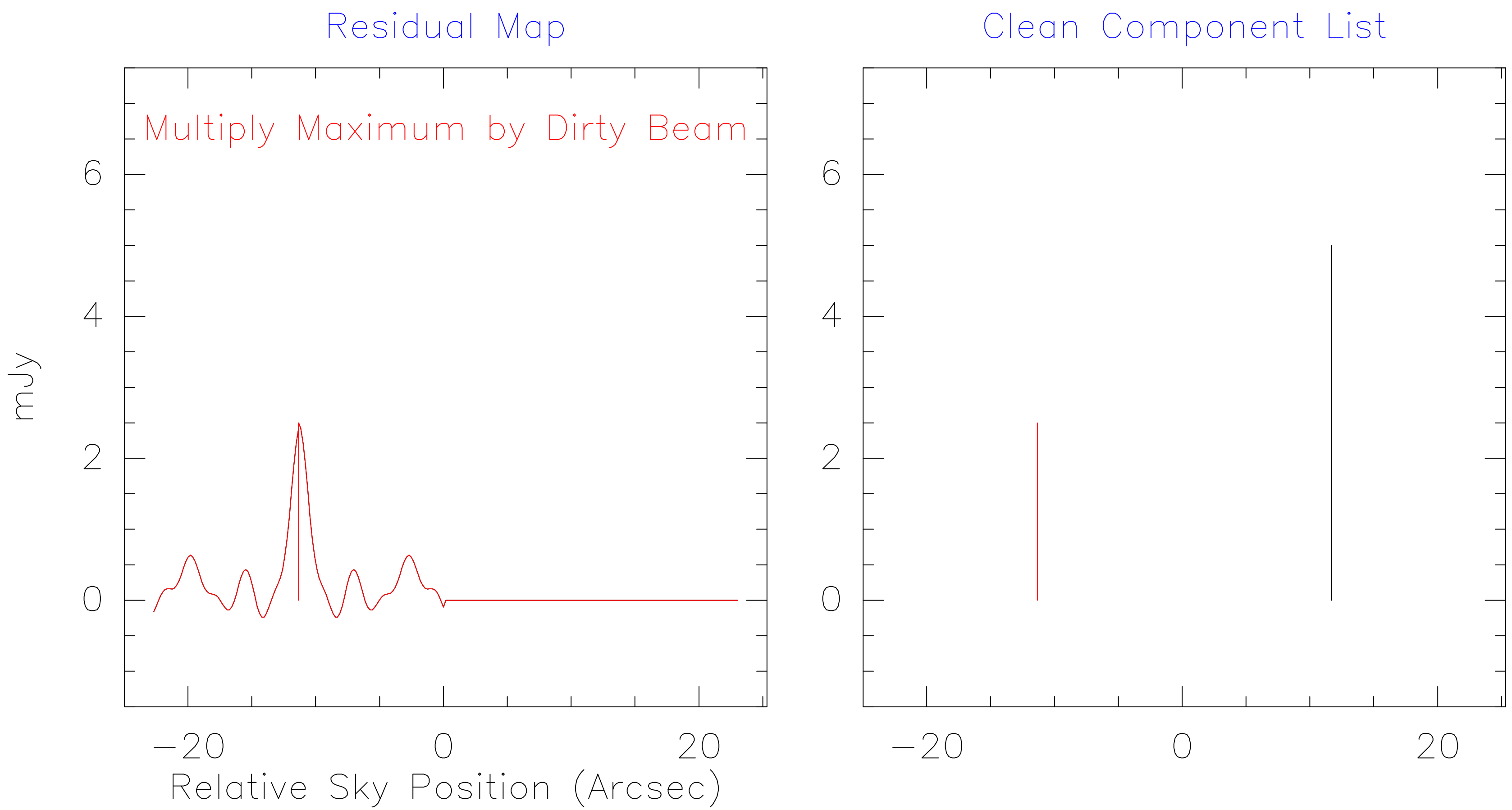

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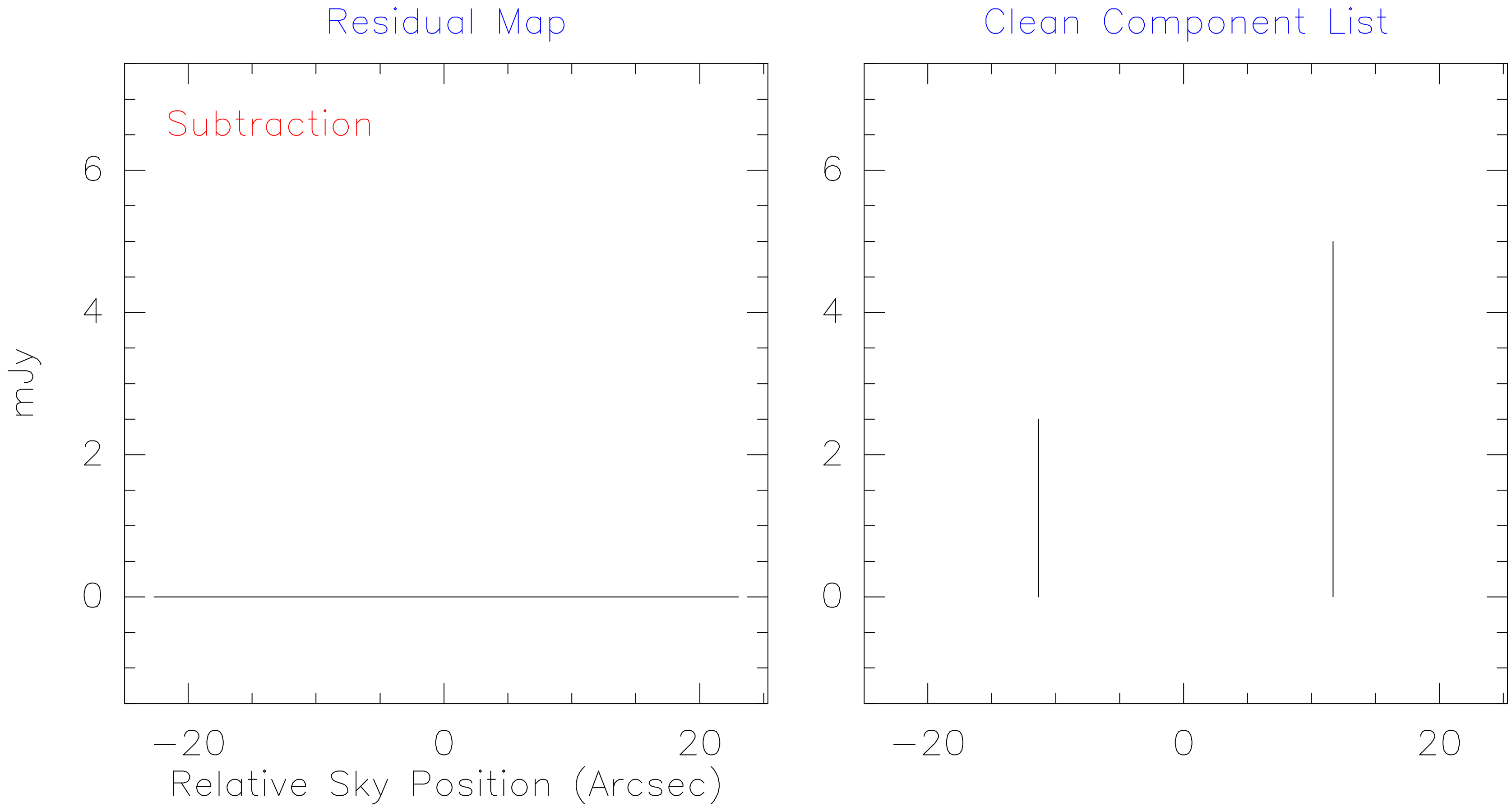

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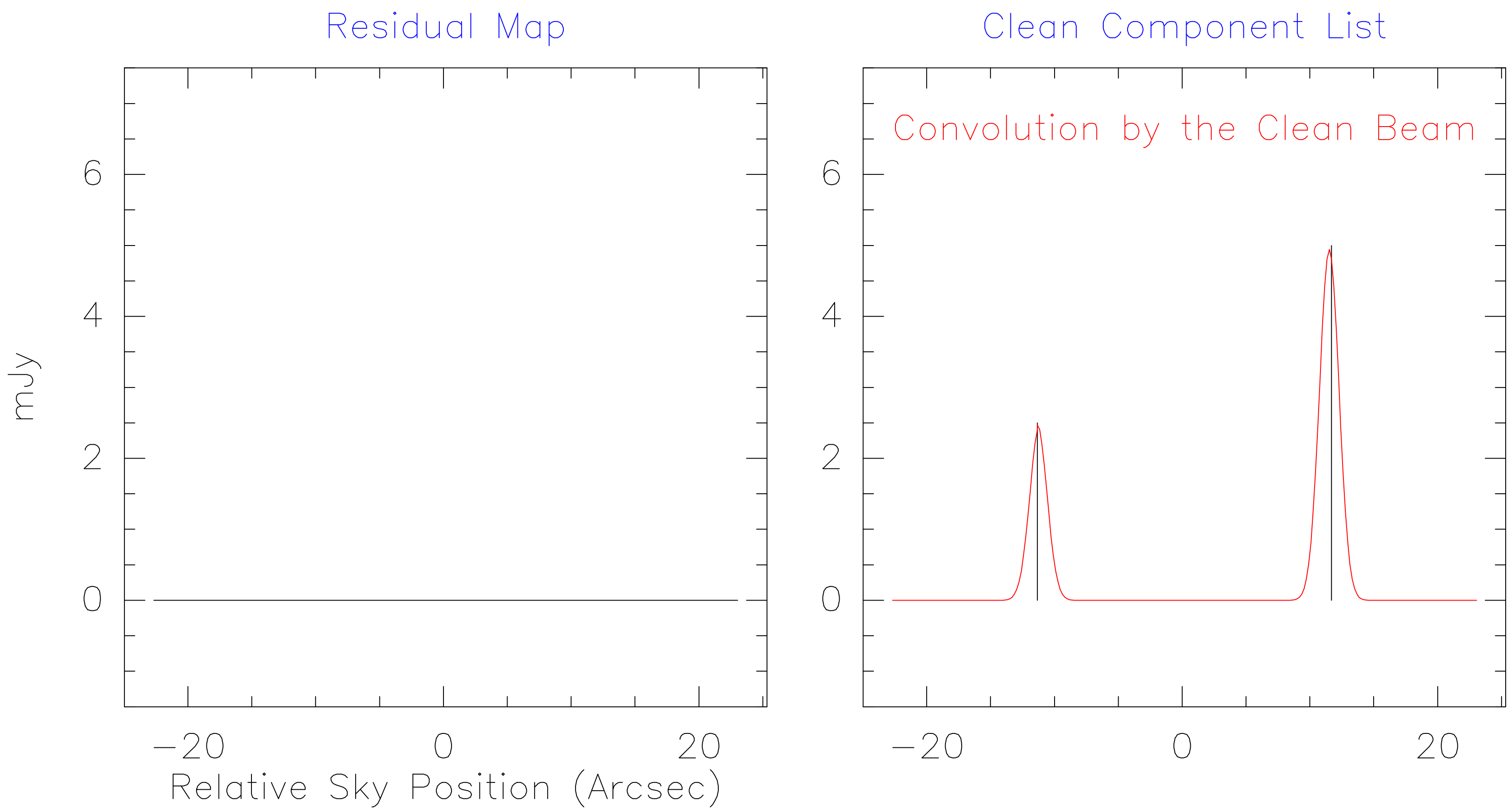

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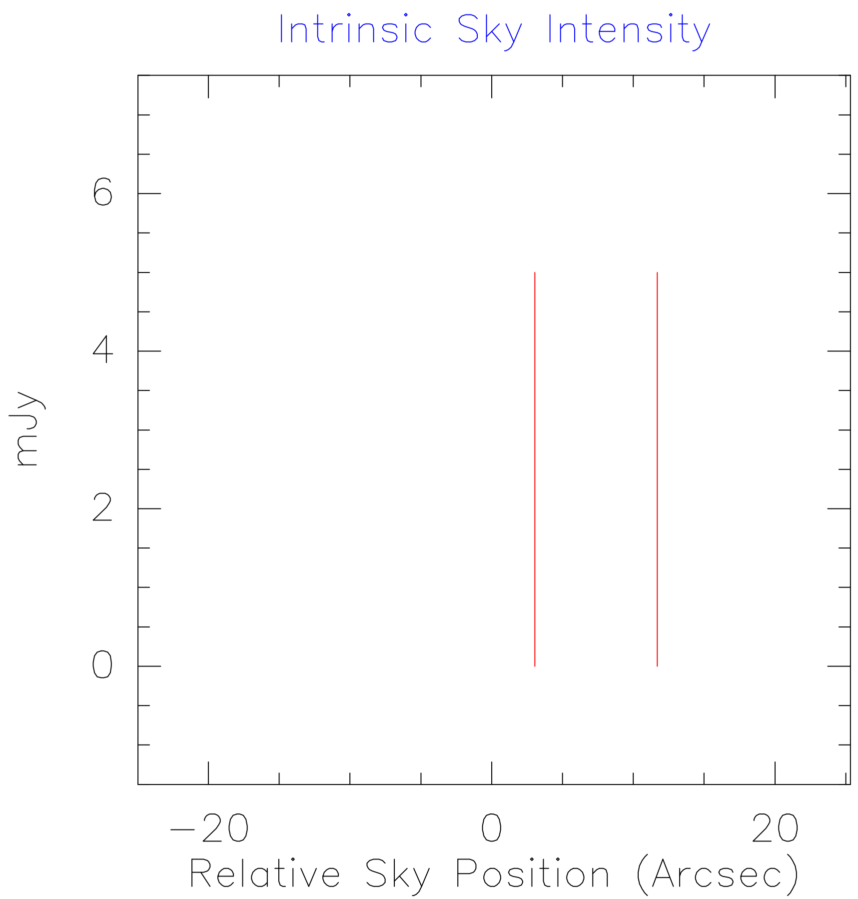

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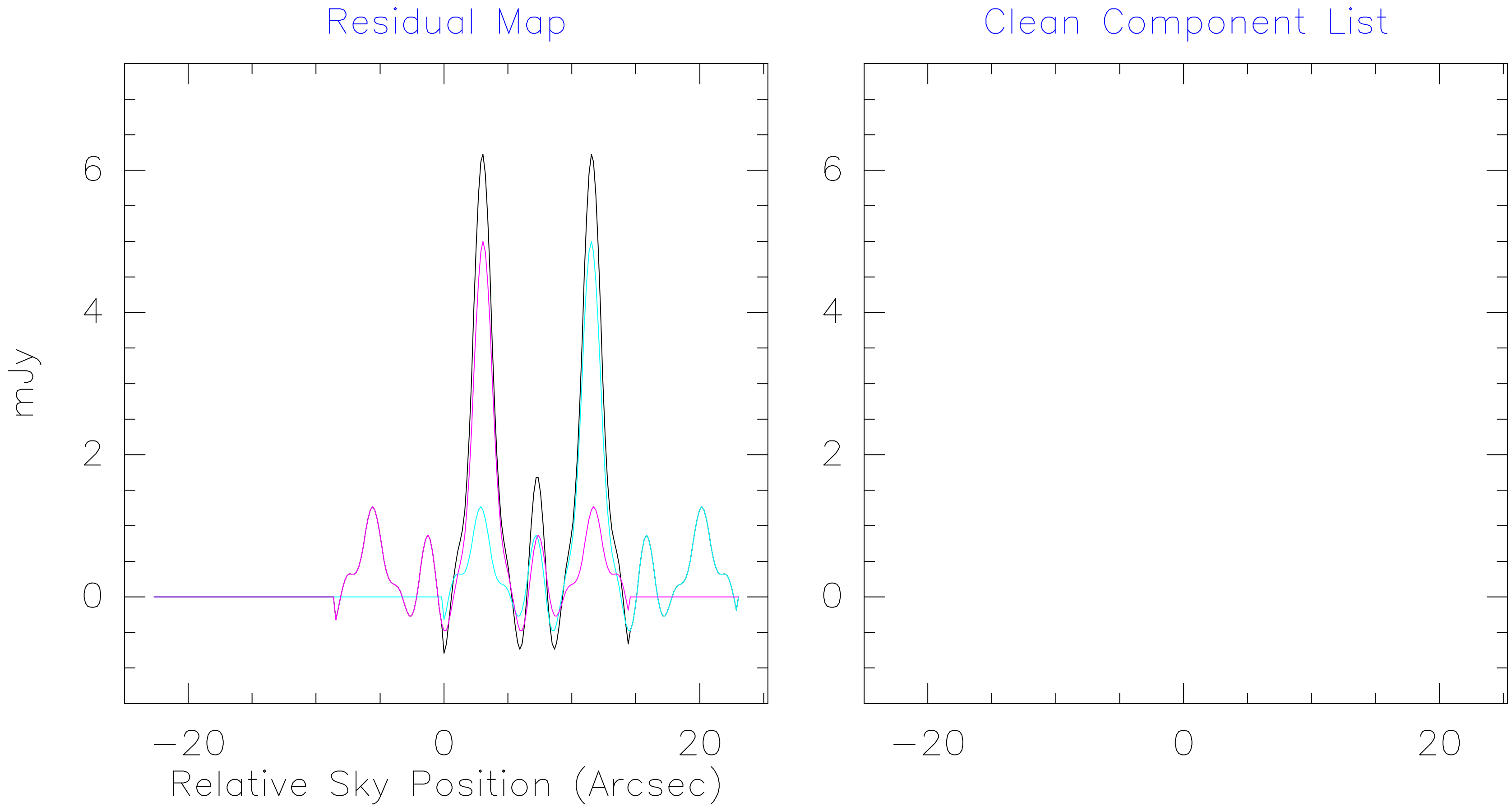

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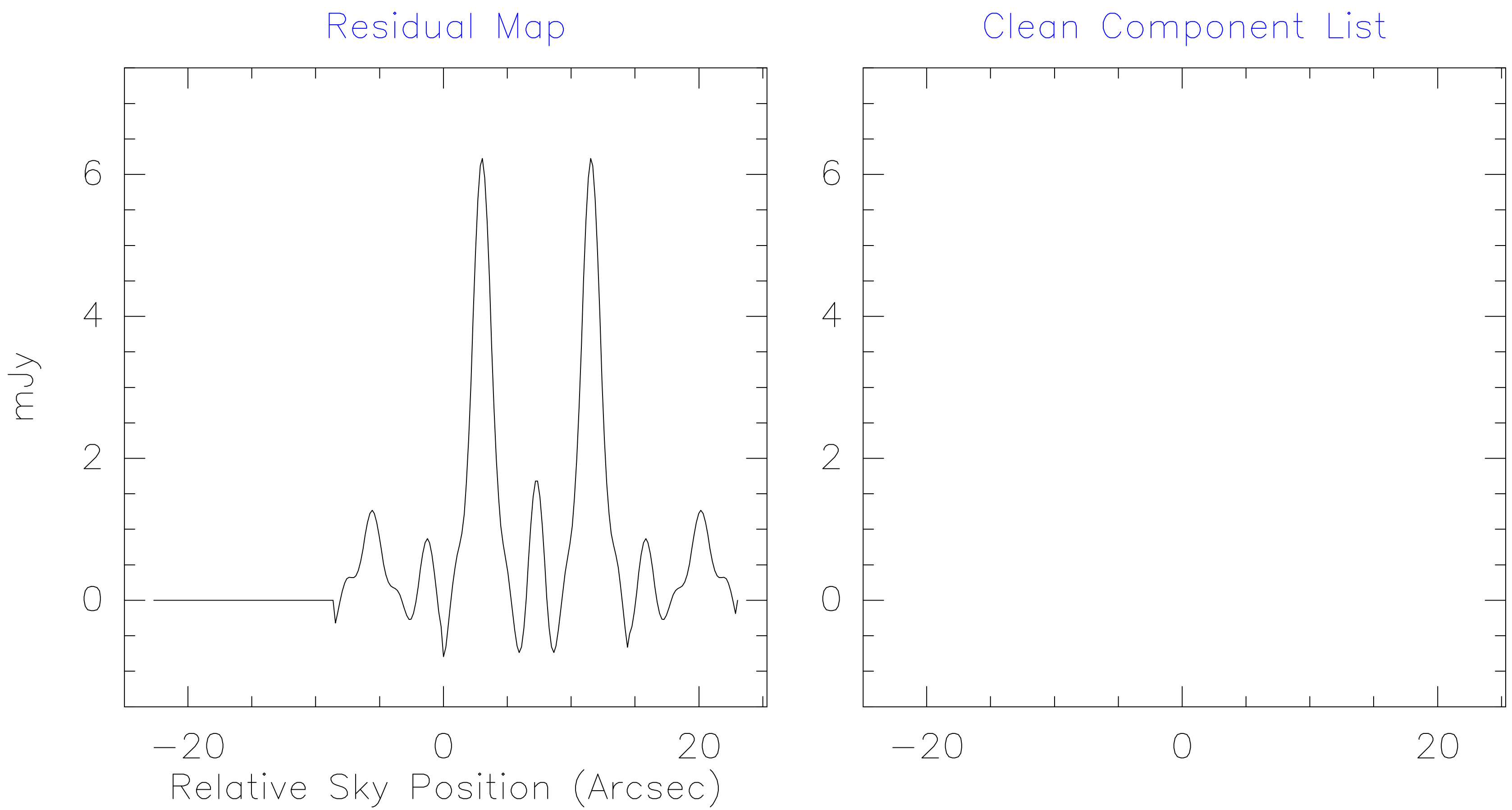
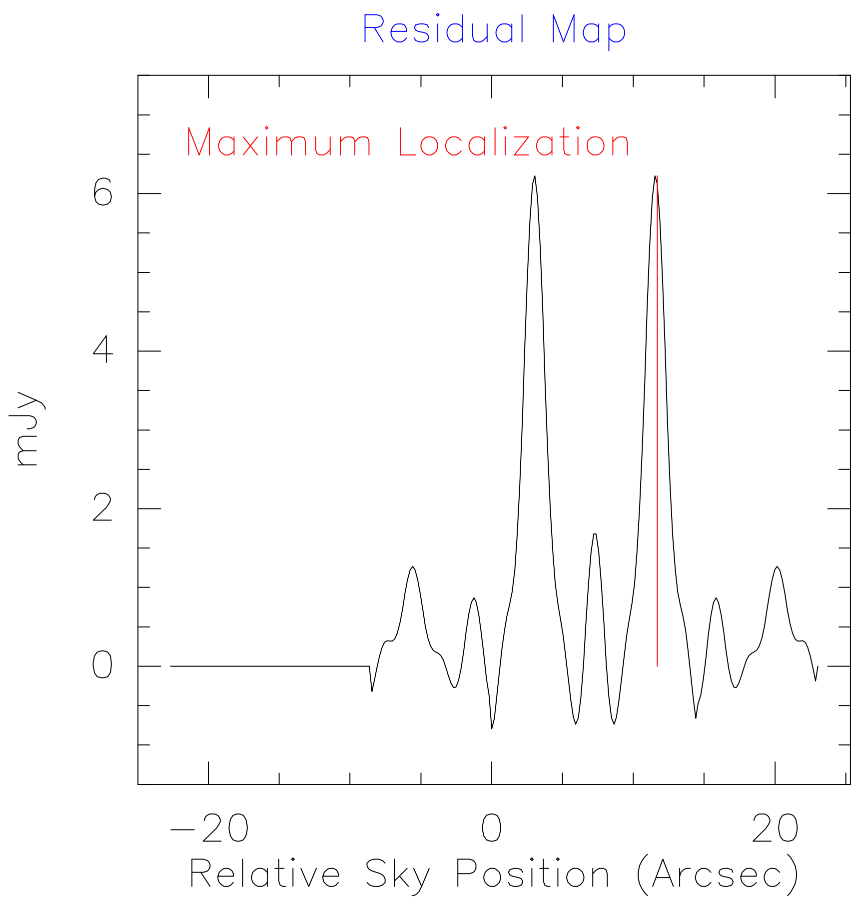
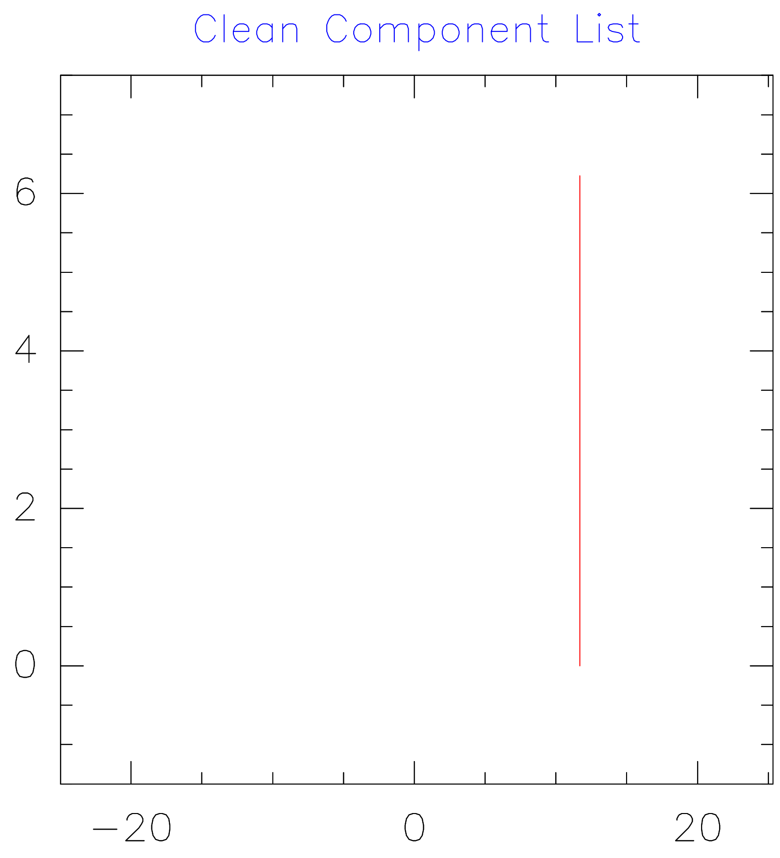
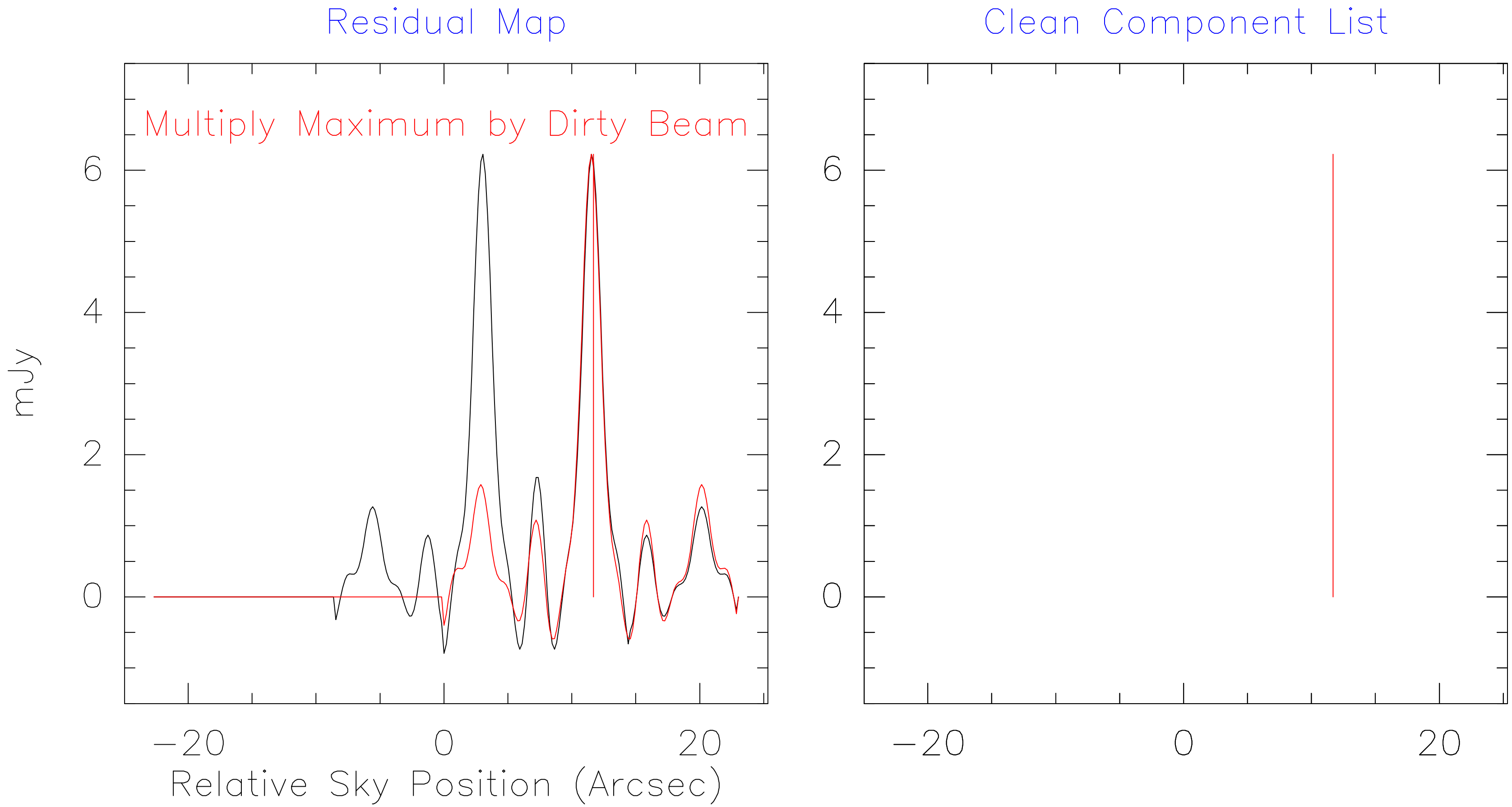


Deconvolution:
III. Illustration of the Basic Clean Algorithm

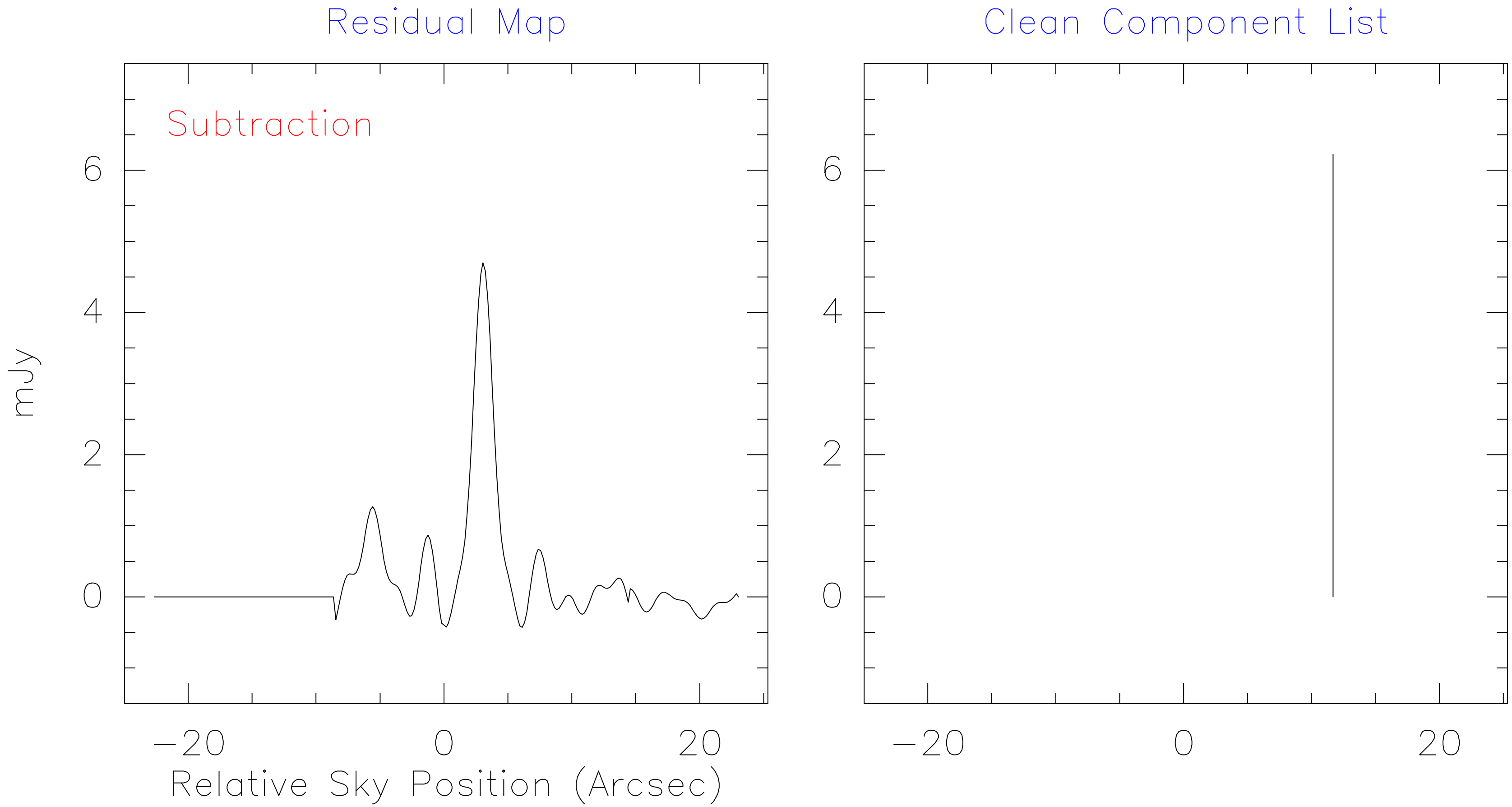




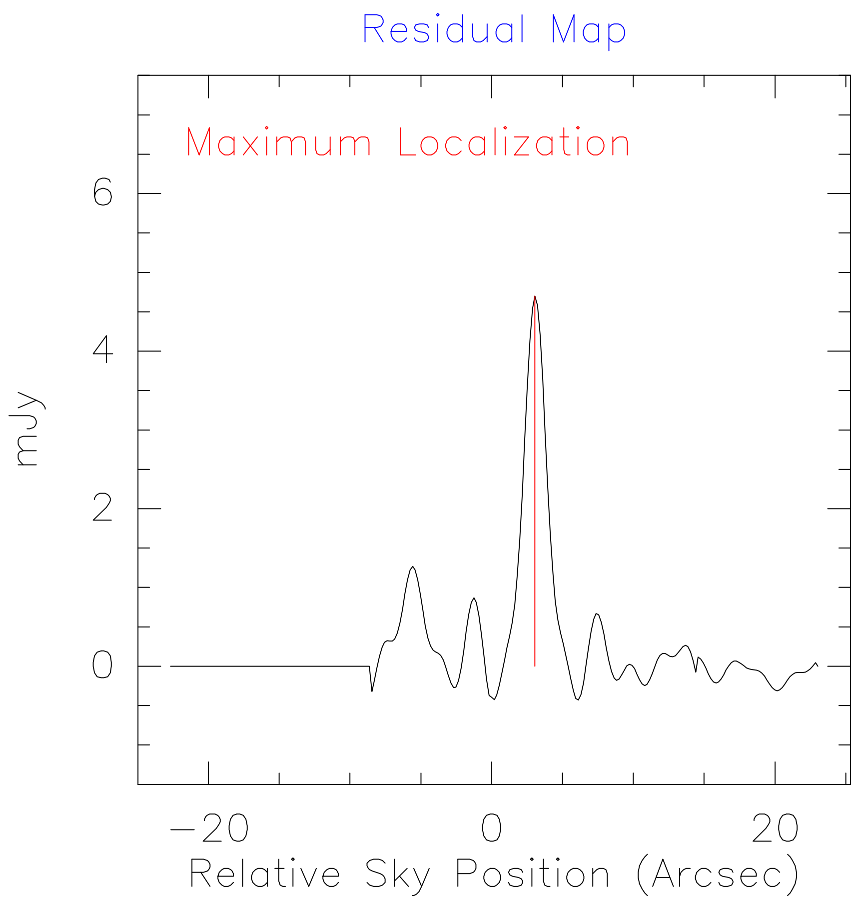
Deconvolution:
III. Illustration of the Basic Clean Algorithm

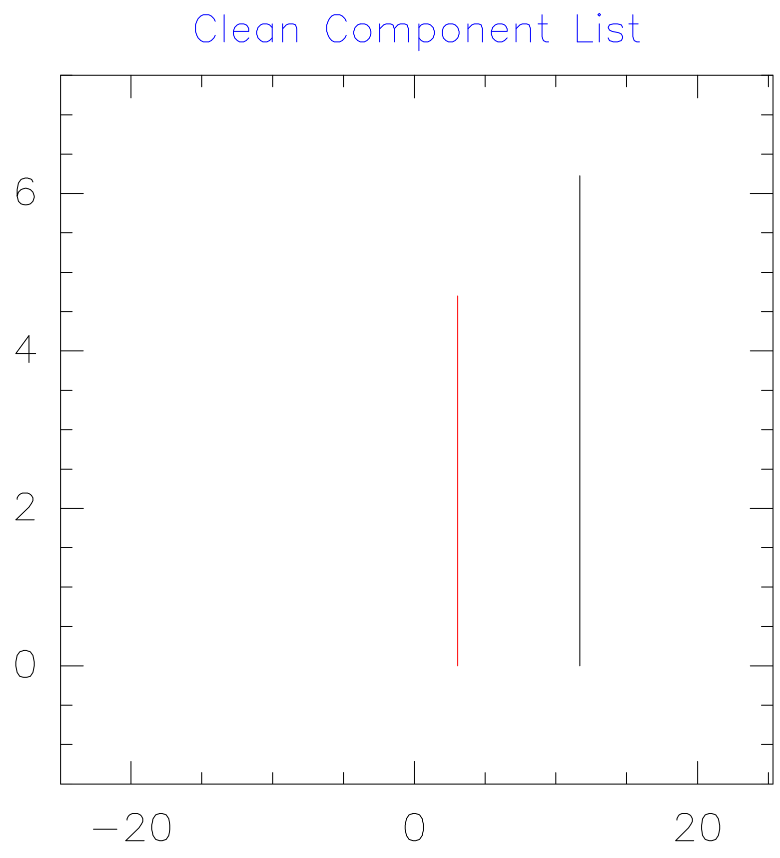


Deconvolution:
III. Illustration of the Basic Clean Algorithm

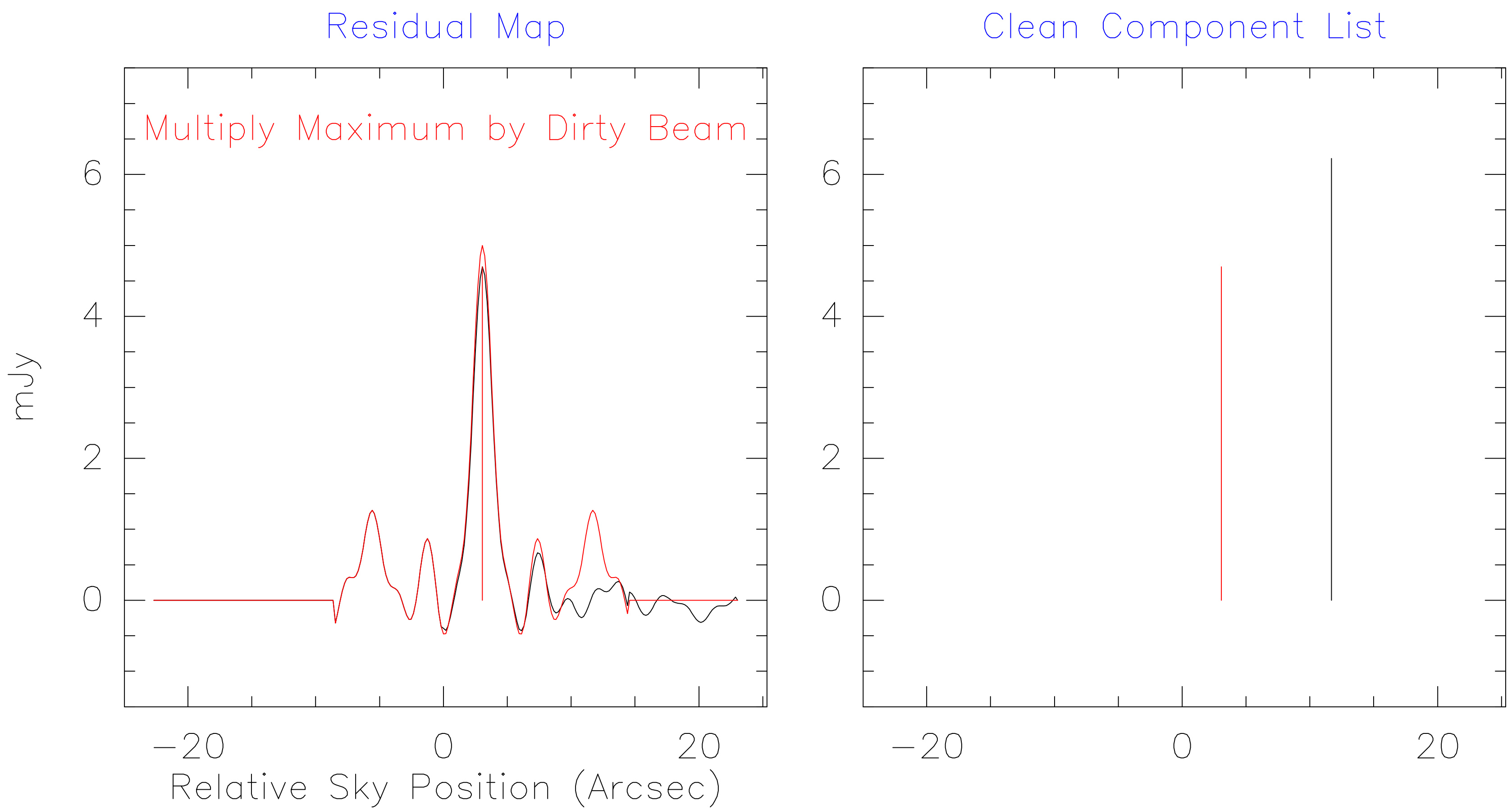


Deconvolution:
III. Illustration of the Basic Clean Algorithm

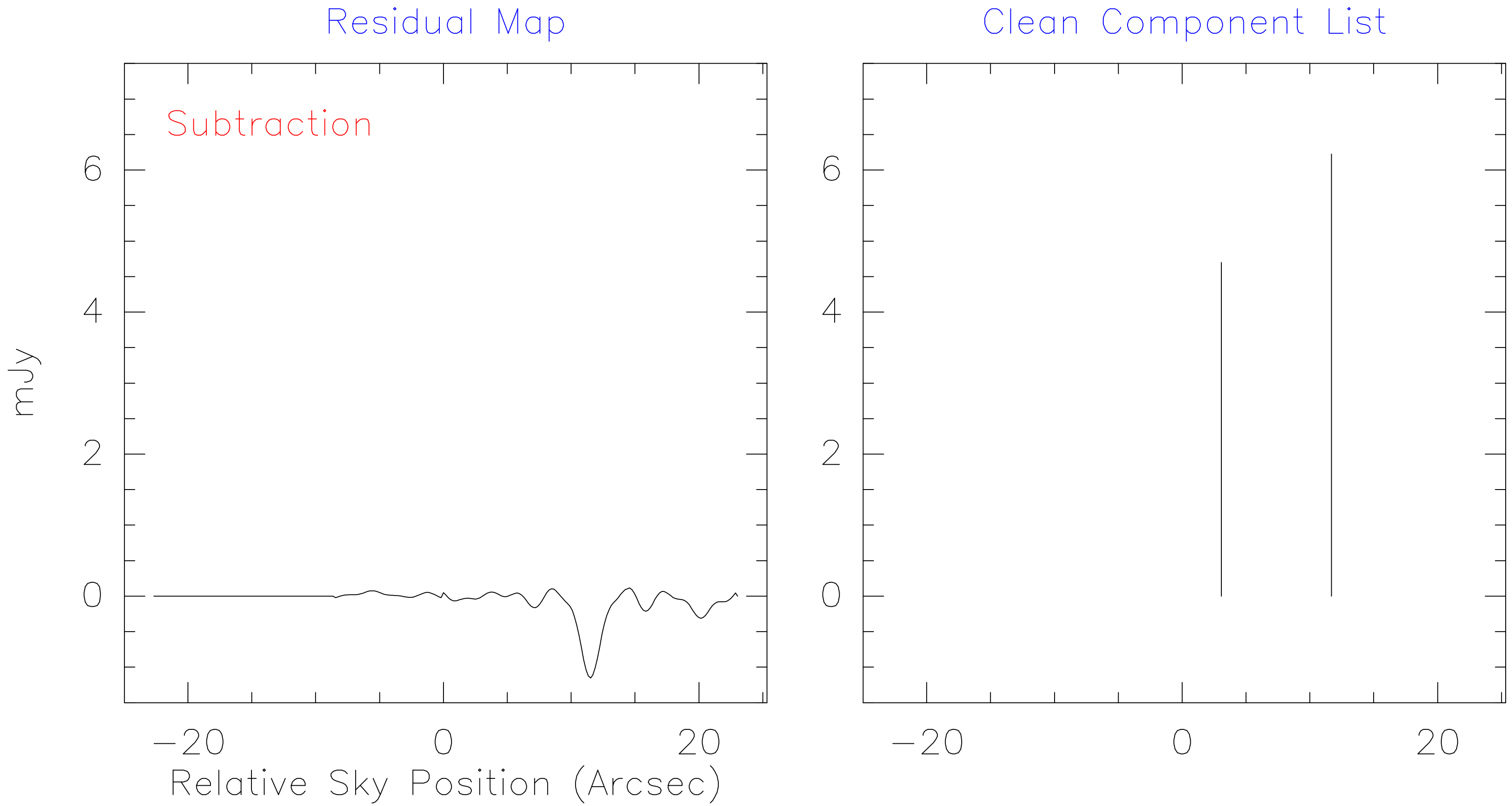




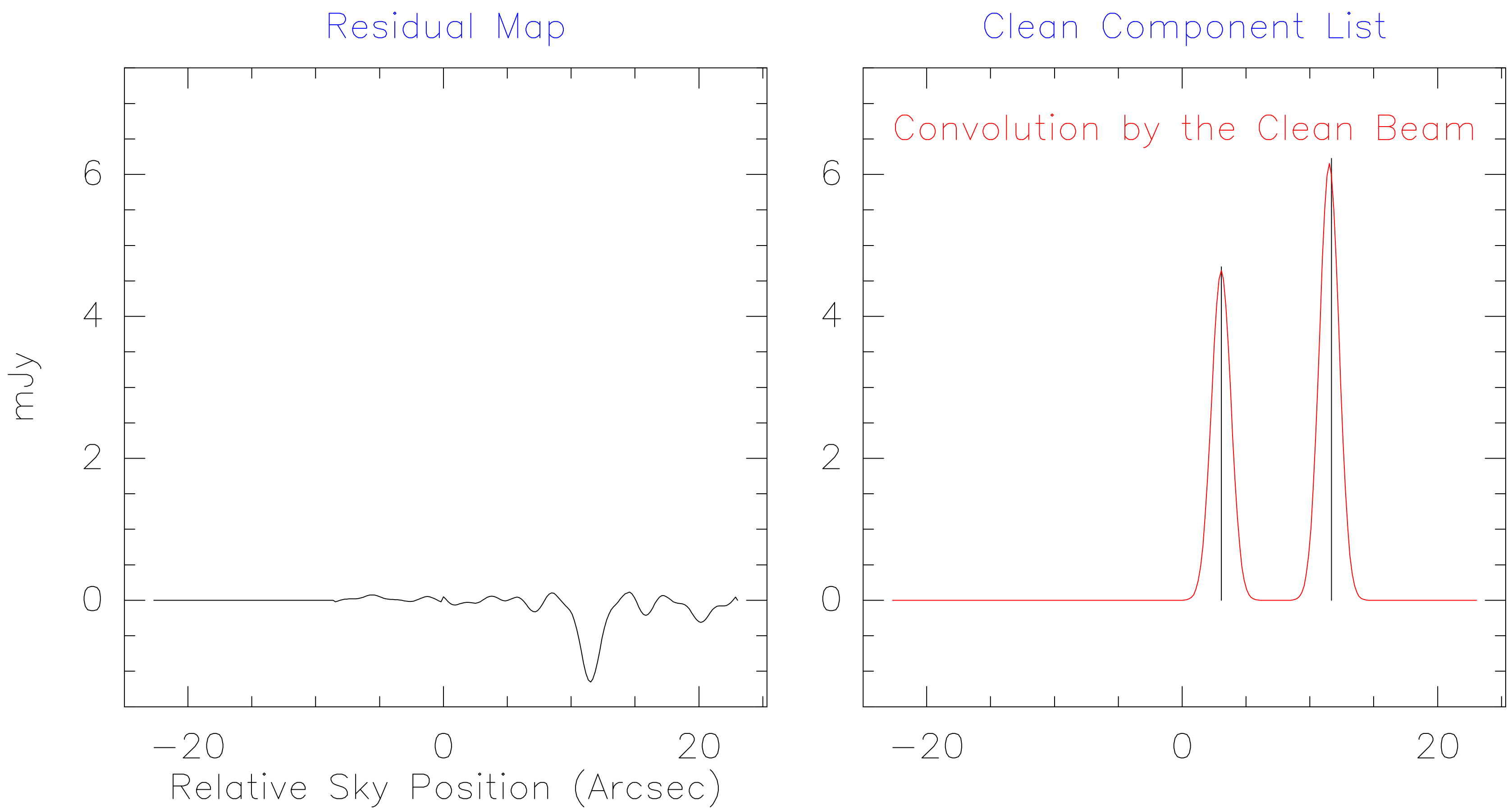
Deconvolution:
III. Illustration of the Basic Clean Algorithm



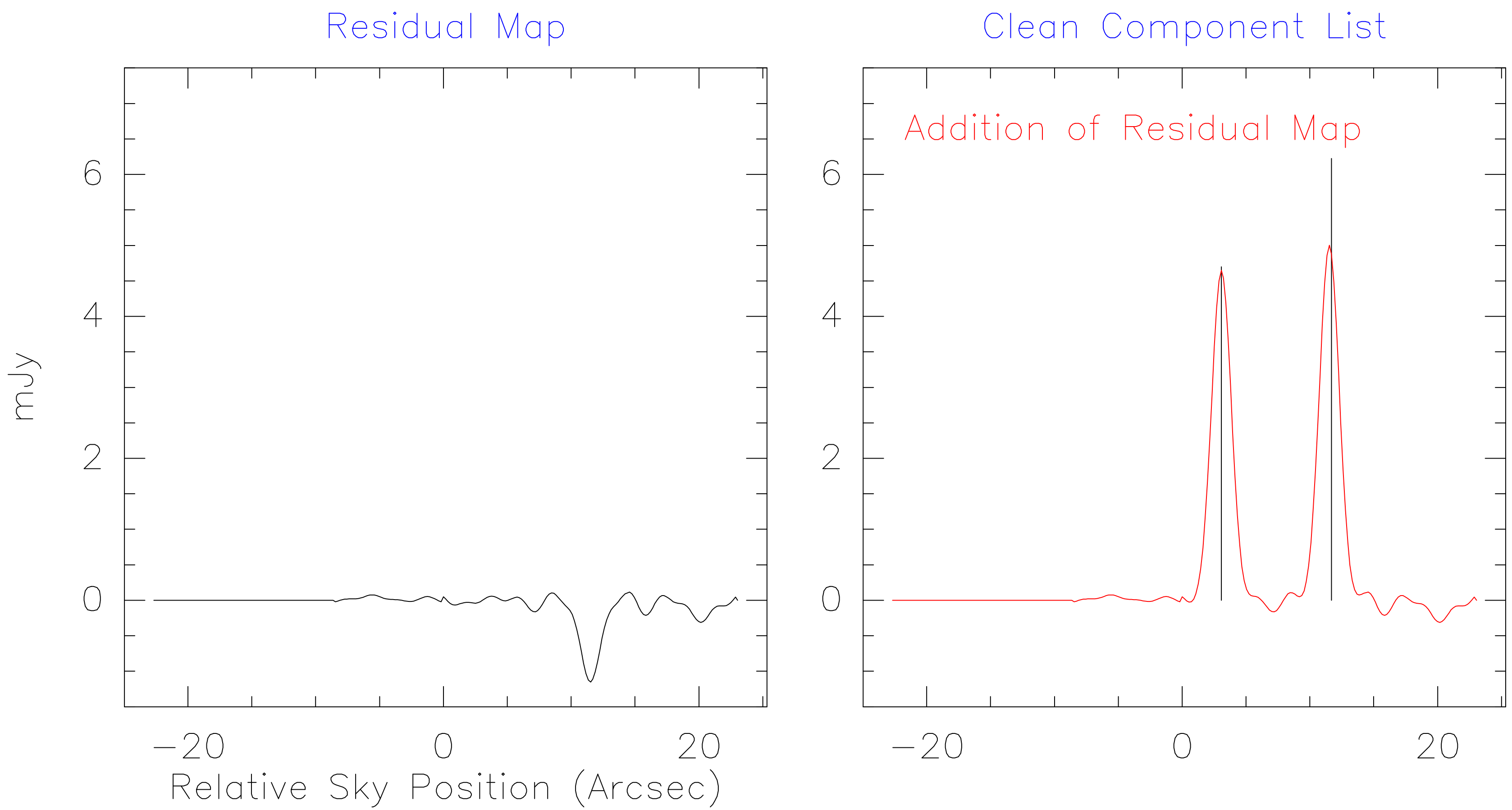
Deconvolution:
III. Illustration of the Basic Clean Algorithm



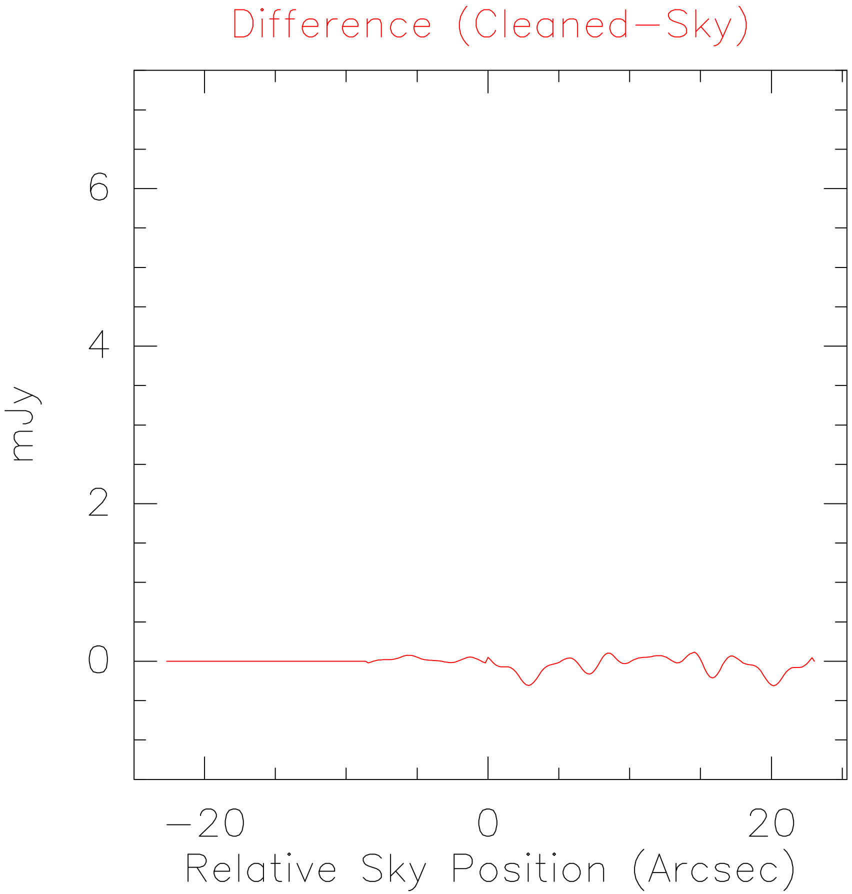
Deconvolution:
III. Illustration of the Basic Clean Algorithm



Deconvolution:
III. Illustration of the Basic Clean Algorithm



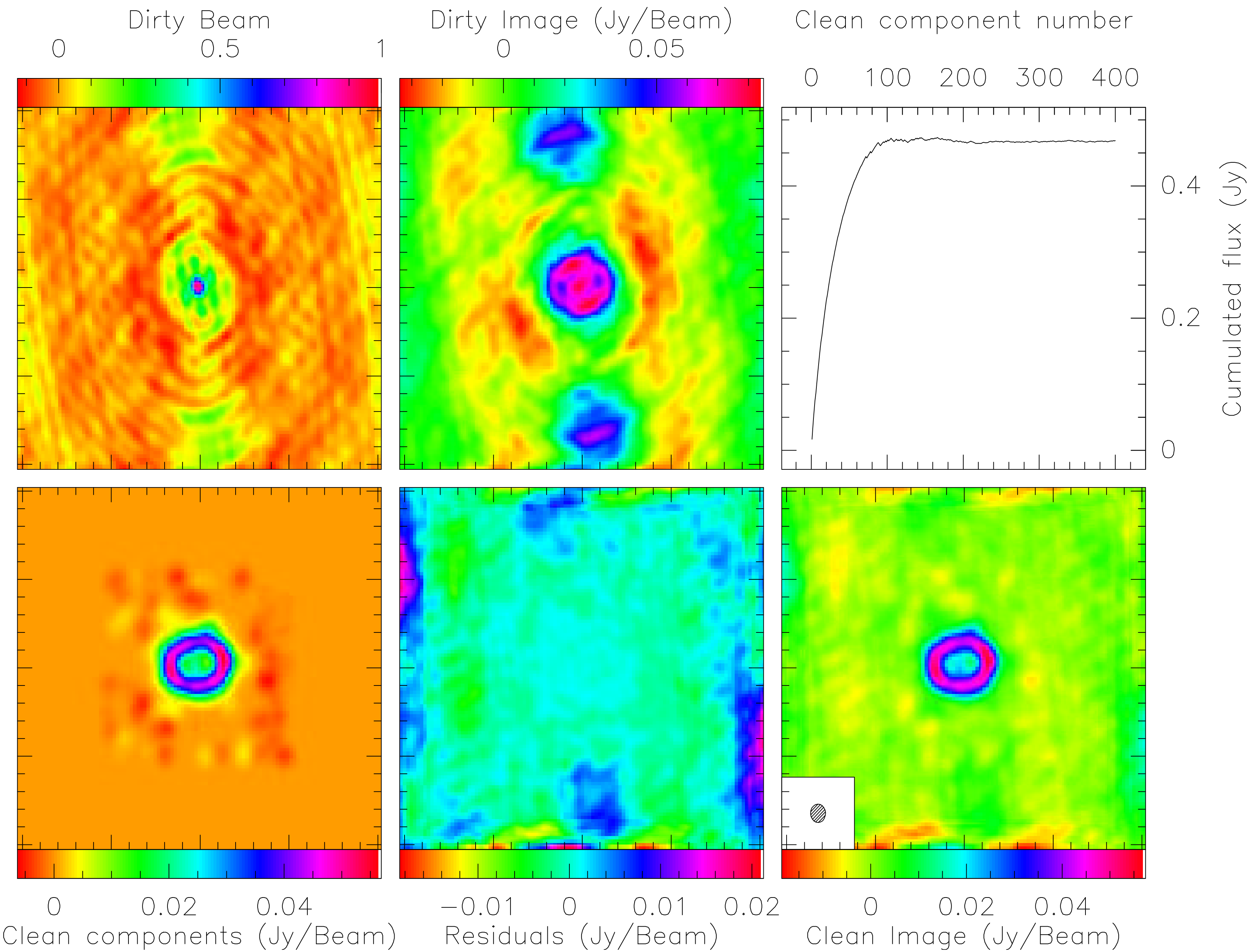
Deconvolution:
III. Illustration of the Basic Clean Algorithm



Deconvolution:
IV. Little secrets of the Basic CLEAN algorithm

- Stopping criterions:
  - Total number of Clean components;
  - $|I_{\text{max}}| <$ fraction of noise (when noise limited);
  - $|I_{\text{max}}| <$ fraction of dirty map max (when dynamic limited).
- Clean beam:
  - In general, Gaussian;
  - Size should match the synthesized beam size (else flux density estimates will be incorrect): sometimes difficult;
- Others:
  - Good results when $\gamma \sim 0.1 - 0.3$ (Loop gain);
  - Needs negative clean components;
  - Only the inner quarter of the dirty image is correctly cleaned;
  - Too deep Cleaning $\Rightarrow$ Divergence.

Deconvolution: V A True Example



Conclusion

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	UV_STAT
⇓ Fourier Transform	UV_MAP
Dirty beam & image	
⇓ Deconvolution	CLEAN
Clean beam & image	
⇓ Image analysis	Your Job!
Physical information on your source	

There are tools to help you in the image analysis:
"go bit", "go noise", "go view", "go moment"...
(cf. Lecture by F. Gueth).

Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_1 \text{and} F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_2), \text{ then } F_1.F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F}_1 * \tilde{F}_2.$$

2 Sampling size $\overset{\text{FT}}{\rightleftharpoons}$ Image size.

3 Bandwidth size $\overset{\text{FT}}{\rightleftharpoons}$ Pixel size.

4 Finite support $\overset{\text{FT}}{\rightleftharpoons}$ Infinite support.

Photographic Credits

- R. N. Bracewell, “The Fourier Transform and its Applications”.
- J. D. Kraus, “Radio Astronomy”.