Deconvolution & Image analysis

Frédéric Gueth

IRAM Grenoble

Deconvolution techniques

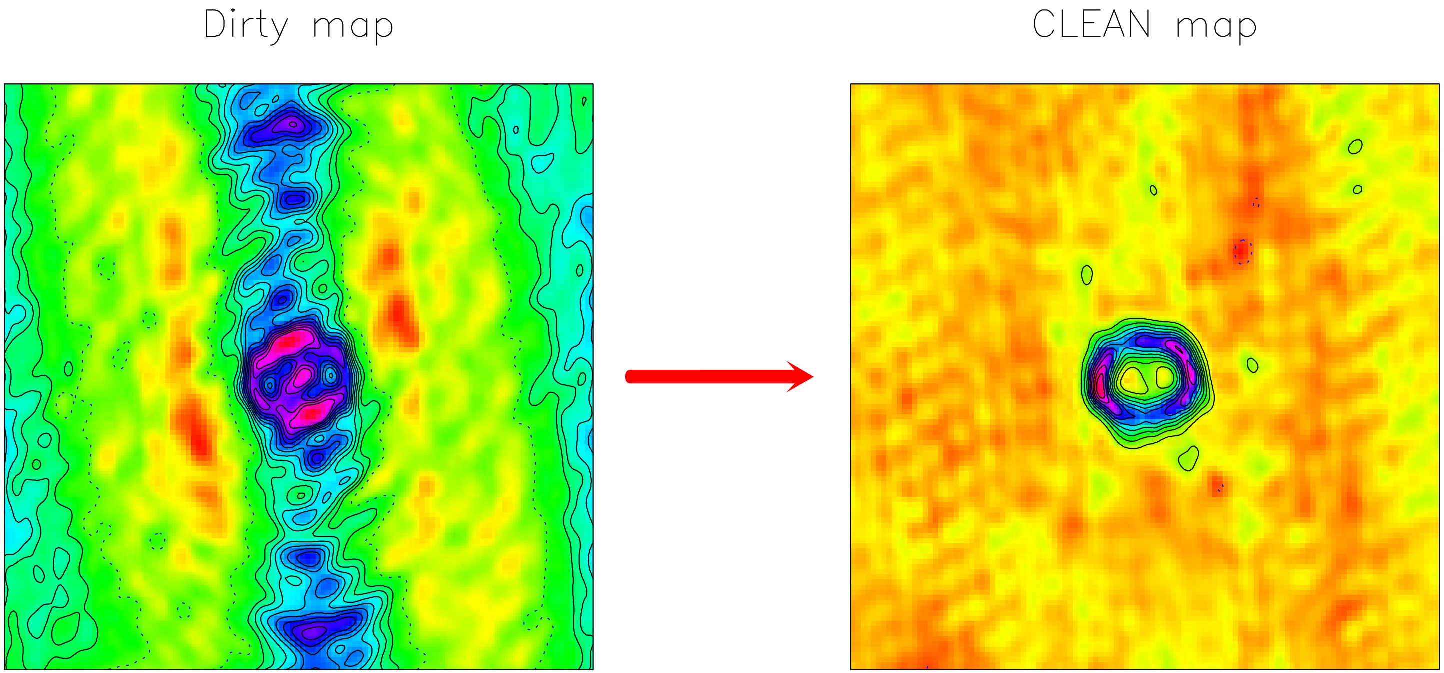
Measurement equation

$$F = D * (B \times I) + Noise$$

$$\begin{aligned} F &= \text{dirty map} = \text{FT of observed visibilities } V \\ D &= \text{dirty beam} = \text{FT of weighted uv coverage} \Longrightarrow \text{DECONVOLUTION} \\ B &= \text{primary beam} \\ I &= \text{sky brightness distribution} \end{aligned}$$

- An interferometer measures the product $B \times I$
- B has a finite support $\longrightarrow$ limits the size of the field of view
- Forget B in algorithms (except for mosaics — cf. dedicated lecture)
- But do not forget: the reconstructed image is $B \times I$

Deconvolution is REQUIRED



The trouble with deconvolution (I)

$$F = D * I + Noise$$

Deconvolution: F and D are known, find I

- Deconvolution is easy! Just compute FT(I)=FT(F)/FT(D)...
- This is not possible:
  - FT(D) is the weighted uv coverage
  - ... which is zero almost everywhere!
  - Need to use more complex, non-linear algorithms
- The trouble: no information is available for the uv points not sampled

The trouble with deconvolution (II)

$$F = D * I + Noise$$

Deconvolution: F and D are known, find I

- The measurement equation has an infinite number of solutions
- If I is a solution, then $I + Z$ is also a solution if $D * Z = 0$
- Just imagine Z so that $\mathrm{FT}(Z)$ is zero at the sampled uv points, and anything at all other points...

- The trouble: no information is available for the uv points not sampled
- Consequence: deconvolution can only recovers an image compatible with the data

The trouble with deconvolution (III)

- Deconvolution can only recovers an image compatible with the data
- Since there are many mathematical solutions, one has to choose one of them, with a reasonable behaviour
- In practice, deconvolution recovers an image compatible with a smoothed version of the source structure
- This is a regularization of a ill-defined mathematical problem
- Regularization can be *a priori* or *a posteriori*
  - CLEAN: *a posteriori*
  - MEM: *a priori* and *a posteriori*
  - WIPE: *a priori*

Maximum Entropy Method (MEM)

- Principle: assume the sky brightness is smooth and positive
- Method: maximize the "Entropy", e.g.

$$\mathcal{H} = -\sum_{ij} I_{ij} \log(I_{ij}/M_{ij}) \qquad M = \text{first guess image}$$

subject to the constraint that the χ^2

$$\chi^2 = \sum_r \frac{|V(u_r, v_r) - \mathrm{FT}(I)(u_r, v_r)|^2}{\sigma_r^2}$$

is equal to its expected value, the number of visibilities

MEM properties

- MEM finds a global minimum: the image can locally be poor
- MEM performs poorly with limited *uv* coverage $\longrightarrow$ NOT USED FOR PdBI data
- The MEM image has varying angular resolution ($\longrightarrow$ convolve it with a CLEAN beam
- The MEM image is biased toward the first-guess image

- Faster than CLEAN on large images ($> 1024 \times 1024$ pixels)
- Smoother images than CLEAN $\longrightarrow$ well adapted to large structures
- MEM is easier to generalize for complex problems, like mosaics with different primary beams

CLEAN

- Principle: assume the sky is empty, with some point sources

- Method: iteratively find the point sources

  (1) Locate the strongest emission
  (2) Assume a fraction γ (the loop gain) of it is a point source
  (4) Remove this source (incl. sidelobes) from the image
  (4) Iterate to (1) until the residual emission is weak enough
  (5) Convolve the accumulated list of point sources with a CLEAN beam
  (6) Add the residuals

The CLEAN algorithm (I)

- Measurement equation: $F = D * I + Noise$
- Iterate:

  (a) Find CLEAN component: position and intensity of the maximum of the residual $\longrightarrow$ this is the non-linear step in the algorithm

  (b) Remove corresponding point source: $F_{k+1} = F_k - \gamma D * \delta_k$
  γ is the loop gain: intensity = true source + sidelobes from all other sources

- The dirty map has been decomposed as: $F = D * \sum \gamma\delta_k + F_{k_{\max}}$
- If the residuals are weak enough (at the noise level), identify I and the sum of CLEAN components

The CLEAN algorithm (II)

- CLEAN map: $M = C * \sum \gamma\delta_k + F_{k_{\max}}$

  C = clean beam
  $F_{k_{\max}}$ = final residuals

- Convolution by CLEAN beam = regularization

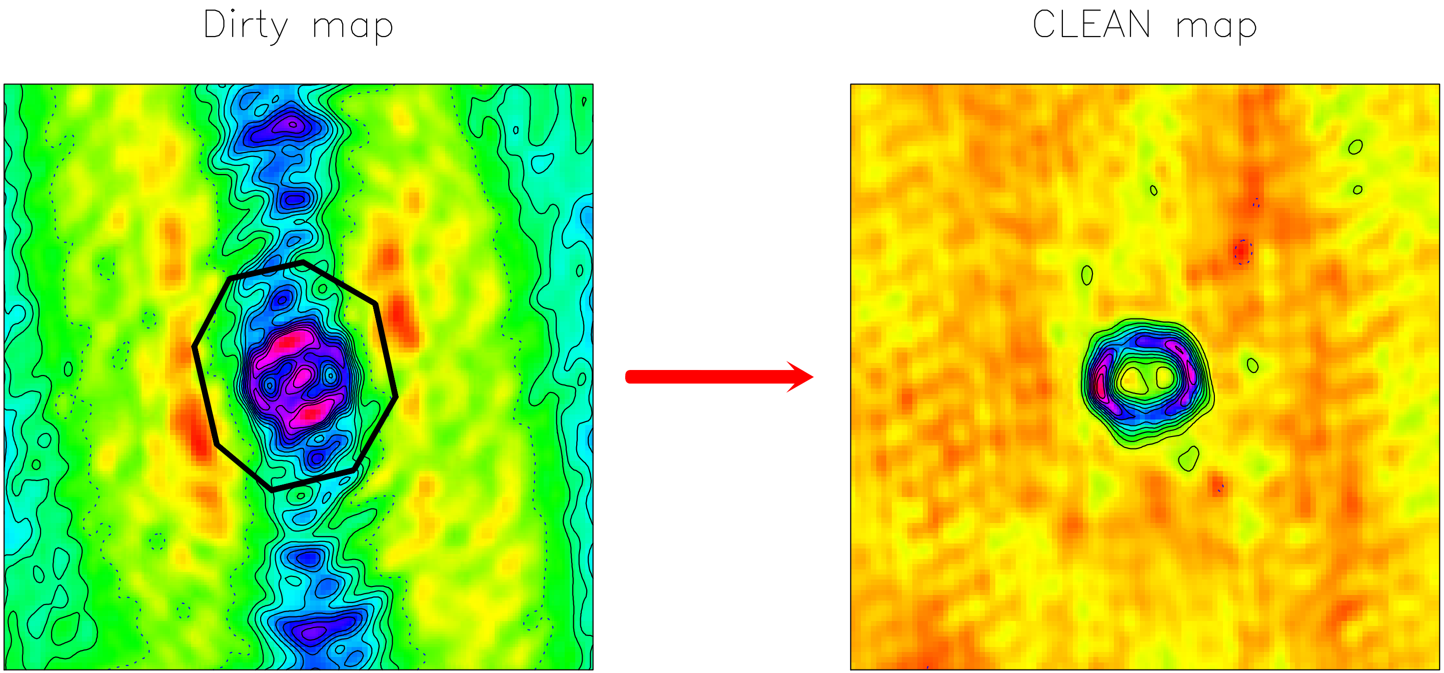
- Choice of the CLEAN beam
  - mathematically: free
  - physically: a gaussian which fits the main lobe of the dirty beam, in order to match the true angular resolution
  - using a gaussian smaller than the dirty beam is called super-resolution and must be avoided

CLEAN properties

- Negative components are allowed — and required because the sampled version of a positive function is not strictly positive
- CLEAN requires a beam twice larger than the image
- Otherwise: only the inner quarter of the image can be properly restored

- Support/Window
  - An appropriate support can (and should) be defined
  - It reduces the numbers of freedom, and thus helps CLEAN to converge
  - This is not cheating but giving some a-priori, usually obvious, information to the algorithm

Support for CLEAN



CLEAN interpolates in the uv plane

- CLEAN is a fitting procedure in the uv plane
  - try to fit a sum of 2D-sinusoidal functions to the visibilities
  - for each sinus, need to find amplitude and phase
  - to do that: find intensity and position of max. of the FT

- Consequence: CLEAN interpolates between measured uv samples

- CLEAN interpolates poorly in unconstrained regions of the uv plane $\longrightarrow$ this may lead to stripe formation

- CLEAN also extrapolates the unmeasured short spacings

CLEAN variants (I)

Hogbom: (Hogböm 1974)

- The basic CLEAN algorithm
- Robust, but slow

Clark: (Clark 1980)

- A minor-major cycle approach
  - Minor cycles: as `Hogbom` Clean, but only subtract a subset of the beam = main beam + large sidelobes
  - Major cycle: remove the cumulative list of point sources convolved by the true dirty beam, in one single operation (FFT)
- Much faster, but can be instable
- This is the default CLEAN for PdBI data

CLEAN variants (II)

MX: (Cotton& Schwab 1984)

- As the `Clark` method, but the major cycle subtraction is done in the ungridded *uv* data, which is then gridded again
- The most accurate method, since gridding only affects the residual
- Slow for large number of visibilities and small images, fast for small number of visibilities and large images.

SDI (Steer, Dewdney, Ito 1984)

- As the `Clark` method, but select a small area surrounding each peak, and do no deconvolution in the minor cycle.
- Better than `Hogbom` or `Clark` on extended structures (minimize stripes), but not on point sources.

CLEAN variants (III)

Multi-Resolution Clean (MRC): (Wakker & Schwarz 1988)

- CLEAN is not very good at recovering extended structures...
- Multi-Resolution approach
  - Separate the problem into a smoothed map and a difference map
  - Since the measurement equation is linear, both maps can be cleaned independently
  - The resulting maps are combined to produce the final CLEAN map
- The actual deconvolution of the smoothed and difference map can be done with any CLEAN algorithm $\longrightarrow$ in practice: `Clark`
- Very fast
- Not very useful for PdBI data, which have limited dynamics in the uv plane
- Not applicable to mosaics (measurement eq. is not linear) $\longrightarrow$ generalization possible (Gueth 1997)

CLEAN variants (IV)

Multi-Scale Clean (MULTI): (Cornwell 1998)

- CLEAN is not very good at recovering extended structures...
- Multi-Resolution approach
  - Produce several smoothed map, with different smoothing kernels
  - At each CLEAN iteration, select the component from the smoothed map which has the highest Signal-to-Noise ratio
  - Remove the corresponding source list for all smoothed maps
- Excellent results
- Slow (several CLEAN instead of one)

All CLEAN variants are available in GILDAS

- Cf. tutorial for demonstrations

GILDAS implementation

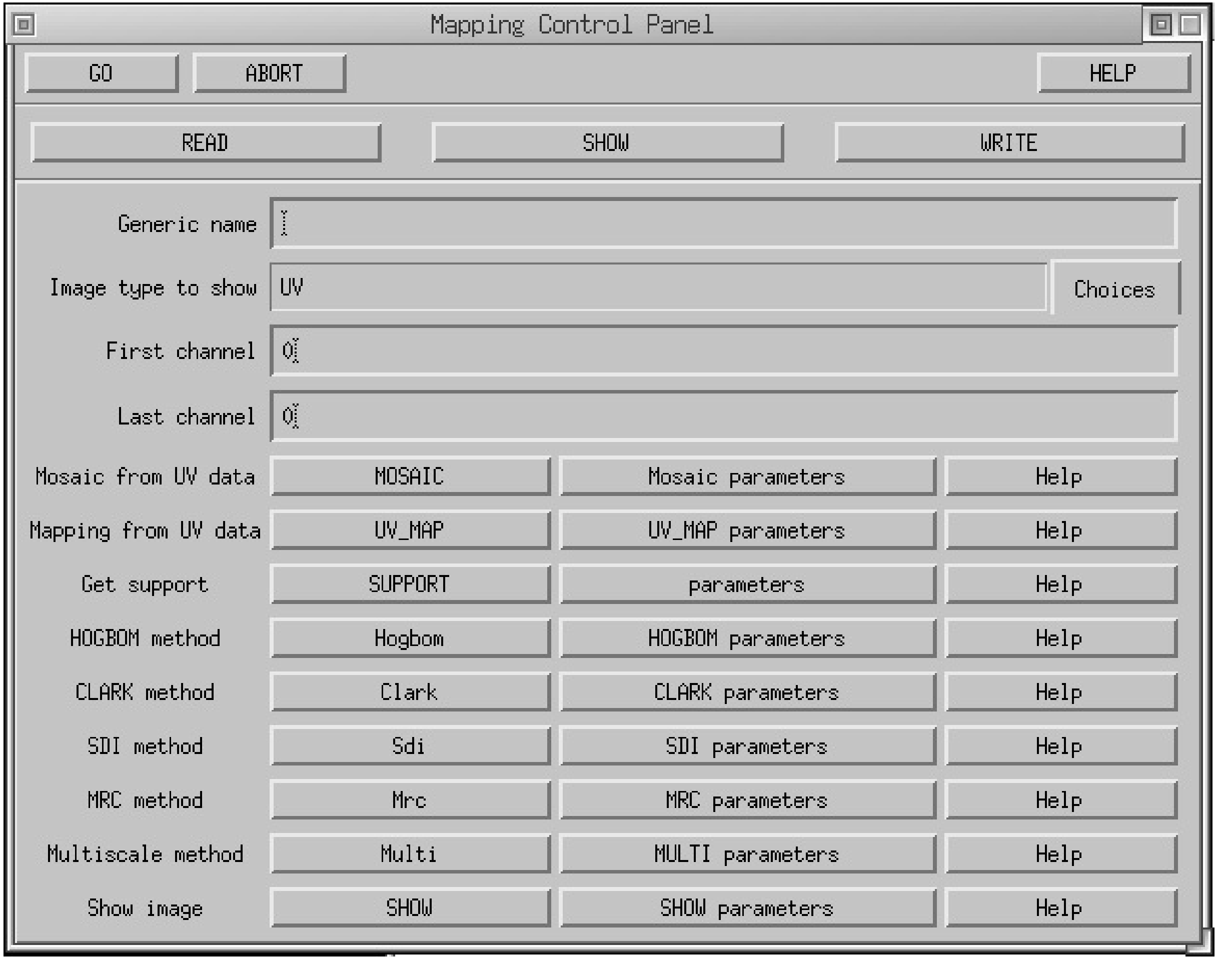
Non-interactive :

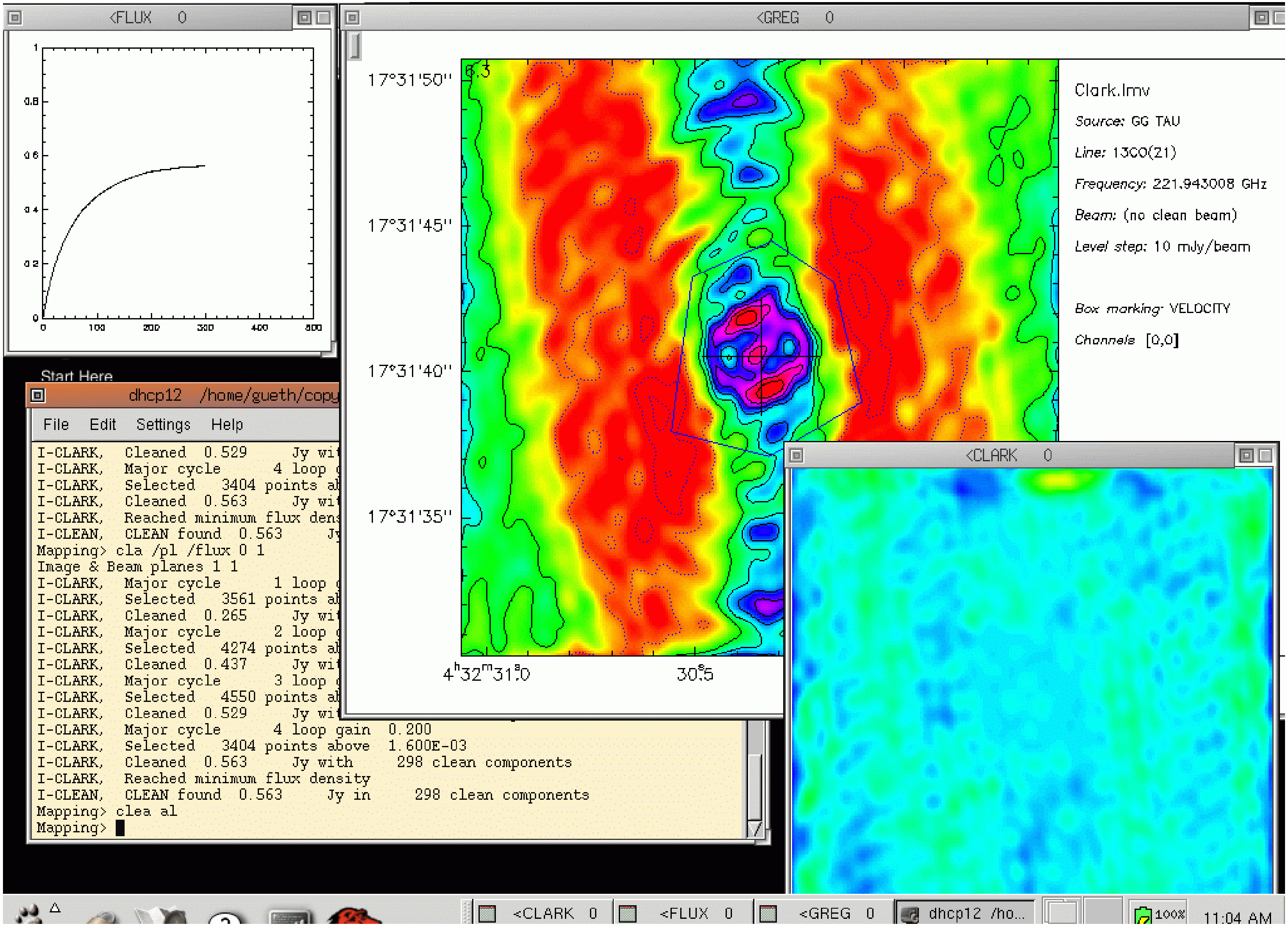
- Task `CLEAN` – with `HOGBOM`, `CLARK`, `MRC`, `MULTI`, `SDI` methods – and task `MX`
- Support is a simple box

Interactive: the MAPPING program

- Commands `HOGBOM`, `CLARK`, `MRC`, `MULTI`, `SDI`, `MX`
- The support can be defined with the cursor (polygon)
- The cumulative flux is plotted during the deconvolution
- The support and loop gain can be changed at each major cycle (`CLARK` and `SDI` methods)
- Also available: `WIPE` (for experts)
- Also available: mosaic deconvolution (cf. dedicated lecture)

MAPPING window interface




<FLUX 0
<GREG 0
17°31'50''
17°31'45''
17°31'40''
17°31'35''
4h32m31s.0
30s.5
Clark.lmv
Source: GG TAU
Line: 13CO(21)
Frequency: 221.943008 GHz
Beam: (no clean beam)
Level step: 10 mJy/beam
Box marking: VELOCITY
Channels [0,0]
Start Here
dhcp12 /home/gueth/copy
File Edit Settings Help
I-CLARK, Cleaned 0.529 Jy wi
I-CLARK, Major cycle 4 loop
I-CLARK, Selected 3404 points al
I-CLARK, Cleaned 0.563 Jy wi
I-CLARK, Reached minimum flux dens
I-CLEAN, CLEAN found 0.563 Jy
Mapping> cla /pl /flux 0 1
Image & Beam planes 1 1
I-CLARK, Major cycle 1 loop
I-CLARK, Selected 3561 points al
I-CLARK, Cleaned 0.265 Jy wi
I-CLARK, Major cycle 2 loop
I-CLARK, Selected 4274 points al
I-CLARK, Cleaned 0.437 Jy wi
I-CLARK, Major cycle 3 loop
I-CLARK, Selected 4550 points al
I-CLARK, Cleaned 0.529 Jy wi
I-CLARK, Major cycle 4 loop gain 0.200
I-CLARK, Selected 3404 points above 1.600E-03
I-CLARK, Cleaned 0.563 Jy with 298 clean components
I-CLARK, Reached minimum flux density
I-CLEAN, CLEAN found 0.563 Jy in 298 clean components
Mapping> clea al
Mapping>
<CLARK 0
<CLARK 0
<FLUX 0
<GREG 0
dhcp12 /ho...
11:04 AM

GILDAS implementation

- `INPUT CLEAN` will give you the current parameters
- `GAIN` is the loop gain $\longrightarrow$ use 0.2
- `NITER` is the maximum number of iterations $\longrightarrow$ if 0, the software will try to guess
- `FRES` is the max. residual, expressed as fraction of peak intensity $\longrightarrow$ use 0.01 or 0.005
- `ARES` is the max. residual, in map units (Jy) $\longrightarrow$ use 0 to stop at the noise level
- `BLC` and `TRC` are the Bottom Left Corner and Top Right Corner of the support (non-interactive implementation)
- `MAJOR, MINOR, ANGLE` are the CLEAN beam parameters $\longrightarrow$ if 0, `CLEAN` will fit the dirty beam to determine the best values

Recommended practices

- No MEM for Plateau de Bure data (sidelobes are too high)
- Use `CLEAN` for Plateau de Bure data
- `CLARK` method is fast and good enough for most purposes
  - Use a large enough number of iterations
  - Set the support to surround the region with signal, but large enough to have some empty sky also
  - Clean down to the noise level unless a very strong source is present
  - If source is complex or uv coverage insufficient, use MAPPING interactive Cleaning, with a user defined polygon for the search area
- Possibly, use `MX` once a good solution has been found with `CLARK`
- Consult an expert until you become one too

Image analysis

After Deconvolution: image analysis

- The data analysis depends on the astronomical project
- mm interferometry involves handling and display of data cubes
- Only a few astronomy packages have been designed for this, among which GILDAS

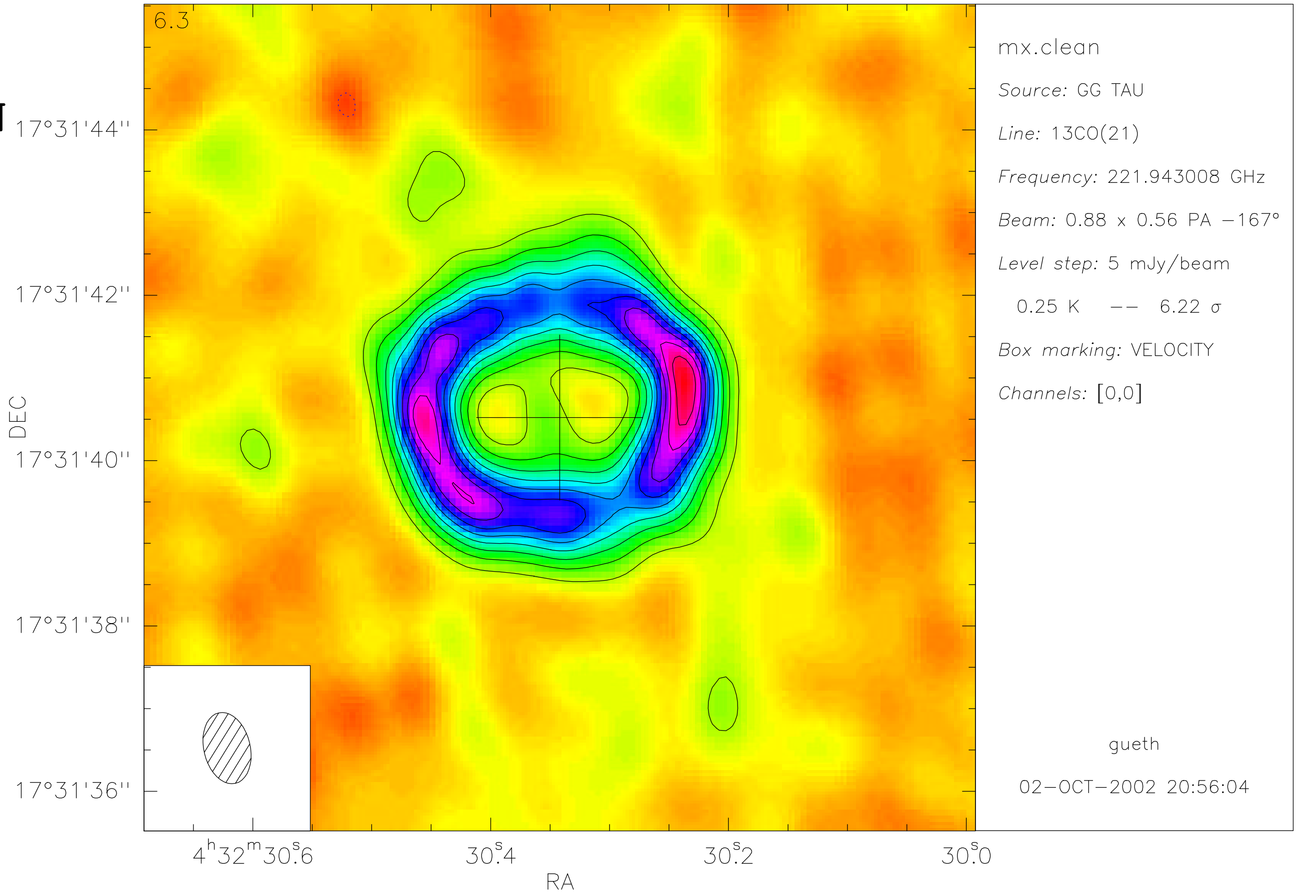
- Selected topics
  - Visualization
  - Map units: flux & brightness
  - Short spacings
  - Moments

The MAPPING software

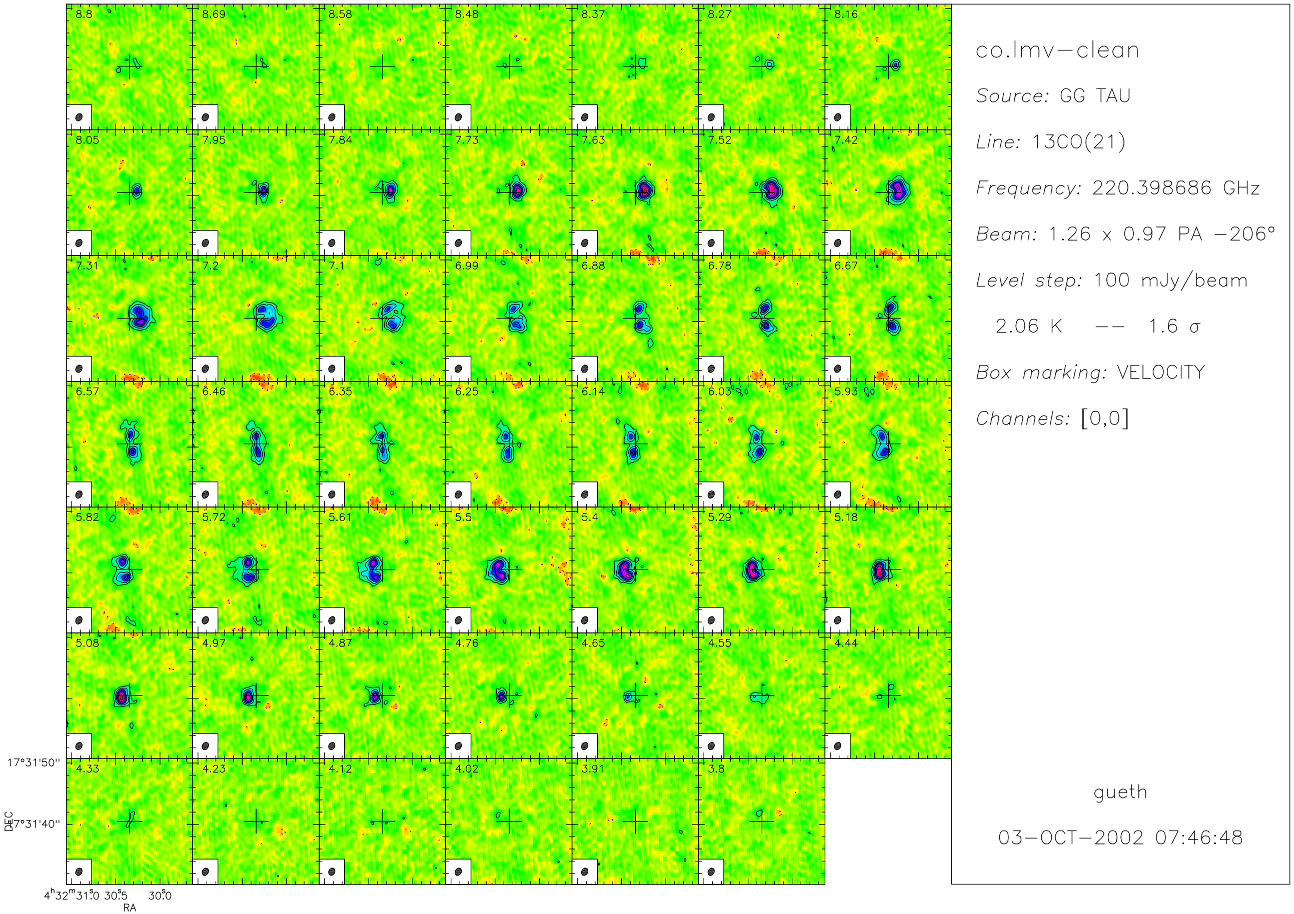
- Image processing tasks (`UV_MAP`, `CLEAN`, etc...),
- Interactive deconvolution tools
- Flexible controls for publication quality plots (`GREG`)
- Easy display of data cubes
  - `GO MAP`, for simple channel contour maps
  - `GO NICE`, with clean beam in addition
  - `GO BIT`, same with overlaid color bit map
  - `GO POS`, for Position-Velocity plots
  - `GO SPECTRE`, for maps of spectra
  - `GO VIEW`, for fast interactive display of data cubes

GO BIT

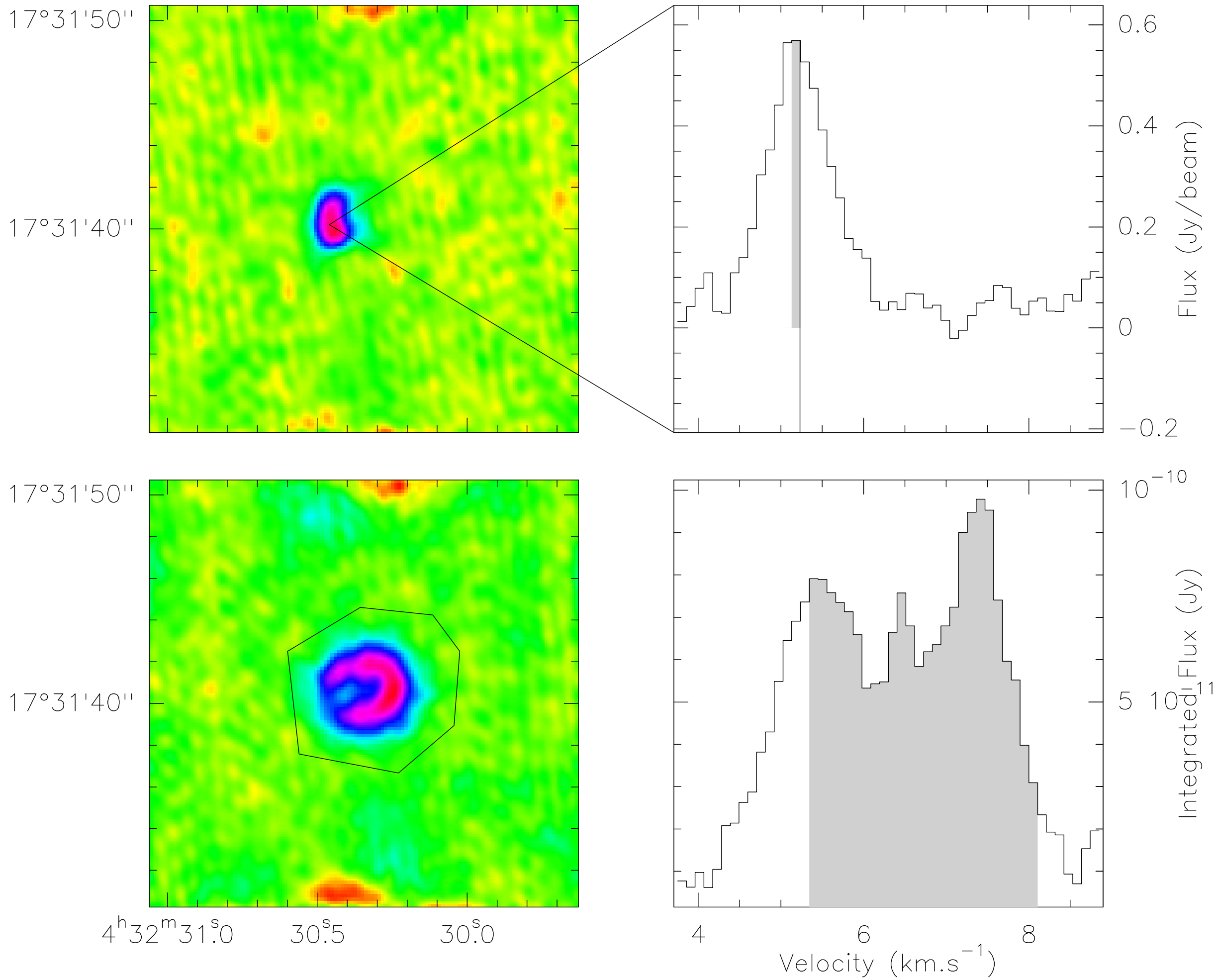
```
LET NAME MX
LET TYPE CLEAN
GO BIT
```



GO BIT – Spectral cube



GO VIEW



17°31'50"
17°31'40"
0.6
0.4
0.2
0
−0.2
Flux (Jy/beam)
17°31'50"
17°31'40"
10^{-10}
5 10^{-11}
Integrated Flux (Jy)
4h32m31.0s
30.5s
30.0s
4
6
8
Velocity (km.s^{-1})

Map units

Unit of dirty map is ill defined

- Area of the dirty beam is 0 (if no zero-spacing)
- A single point source of 1 Jy appears with peak intensity of 1
- But several point sources are present, combination of positive or negative sidelobes from the other sources modify this result

Unit of clean map is better defined

- After deconvolution, the beam area is well defined
- The clean map unit is Jy per beam
- Problem: residuals have a different unit but are added to the CLEAN map $\longrightarrow$ no consequences in practice

Flux and Brightness

Integrated flux :

$$F = \frac{\sum_{ij} S_{ij}}{\Omega} \qquad \text{with} \qquad \Omega = \text{beam area} = \frac{\pi\theta_1\theta_2}{4\log 2}$$

Brightness temperature :

Conversion between flux in Jy/beam (S) and brightness temperature (T_B)

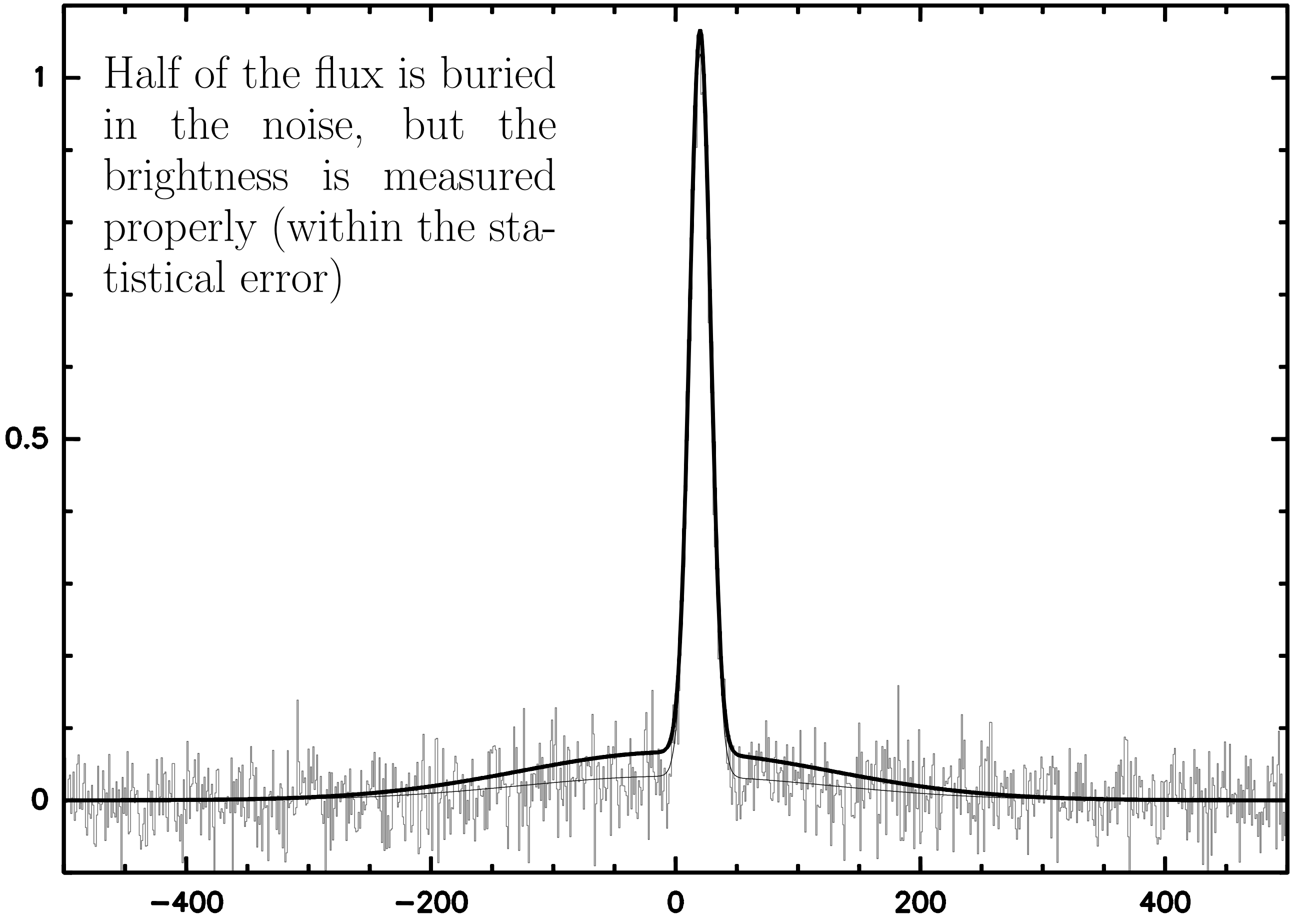
$$S = \frac{2k}{\lambda^2}\,\Omega\,T_B$$

This is a main-beam brightness temperature – it depends on the beam area

Caution :

Extended weak structures may contribute significantly to the integrated flux, without significant brightness

Flux vs. brightness



Accuracy of flux estimates

Seeing effects :

- Point source flux is underestimated
- Flux spread over the "seeing disk" $\longrightarrow$ total flux (nearly) preserved

Primary beam correction :

- The deconvolved image is $B \times I$, B is a gaussian
- Do not forget primary beam for a correct flux estimate!
- Unfortunately, noise is no longer uniform after primary beam correction...

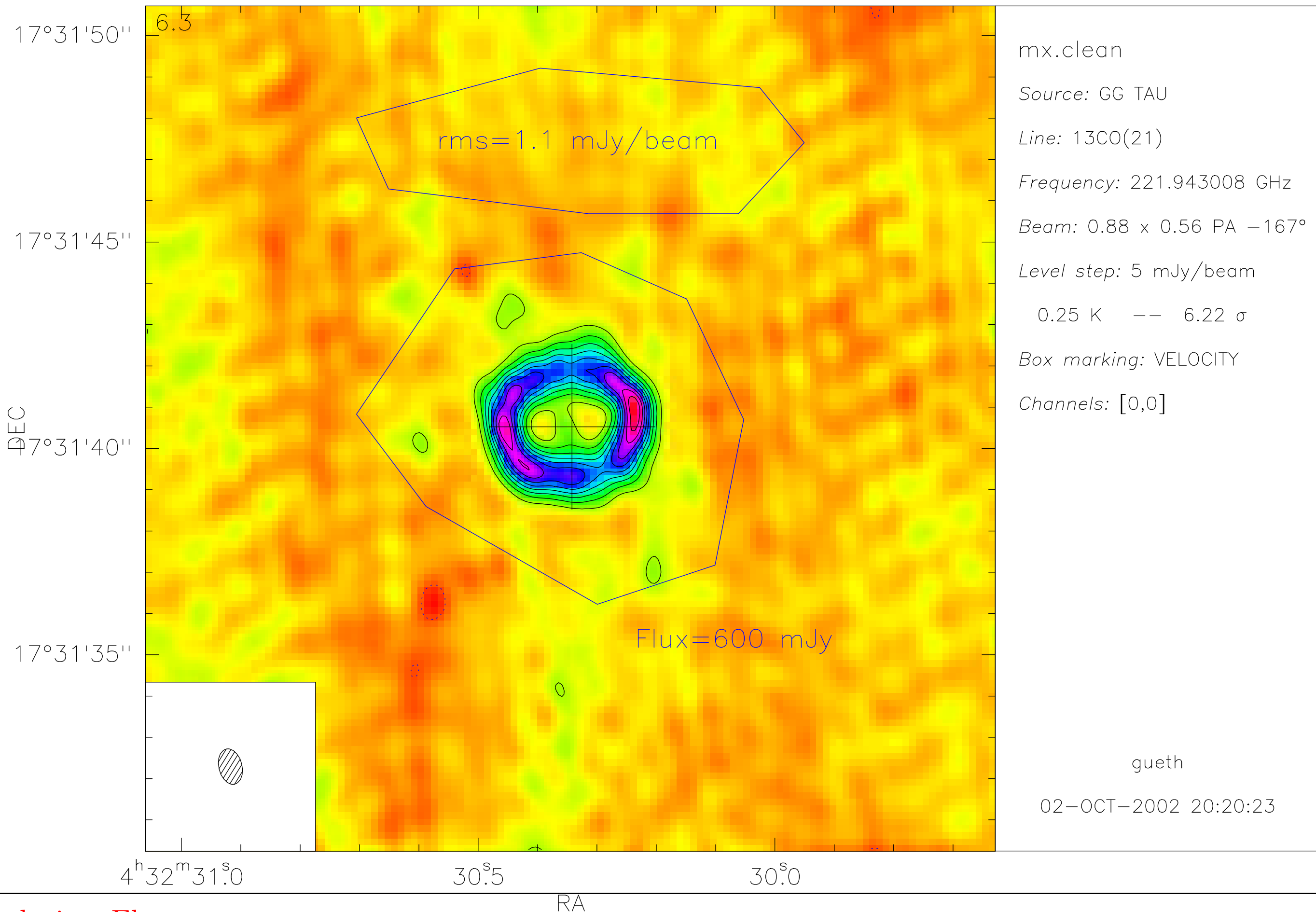
Flux calibration :

- Errors should include relative (calibrator) flux uncertainty
- Errors should include absolute flux scale uncertainty

GILDAS implementation: GO FLUX

- `GO MAP` and its variants automatically display the point source sensitivity as well as the Jy/K conversion factor
- `GO FLUX`
  - user-defined area (cursor)
  - gives intergrated flux
  - gives noise rms
- Map edges should not be included, since noise increases at map edges due to aliasing and gridding
- If primary beam correction has been applied, the noise is no longer uniform
- *uv* plane analysis is better for simple sources

GO FLUX



6.3
17°31'50''
17°31'45''
17°31'40''
17°31'35''
DEC
rms=1.1 mJy/beam
Flux=600 mJy
4h32m31.0s
30.5s
30.0s
RA
mx.clean
Source: GG TAU
Line: 13CO(21)
Frequency: 221.943008 GHz
Beam: 0.88 x 0.56 PA −167°
Level step: 5 mJy/beam
0.25 K −− 6.22 σ
Box marking: VELOCITY
Channels: [0,0]
gueth
02−OCT−2002 20:20:23

The short spacings problem

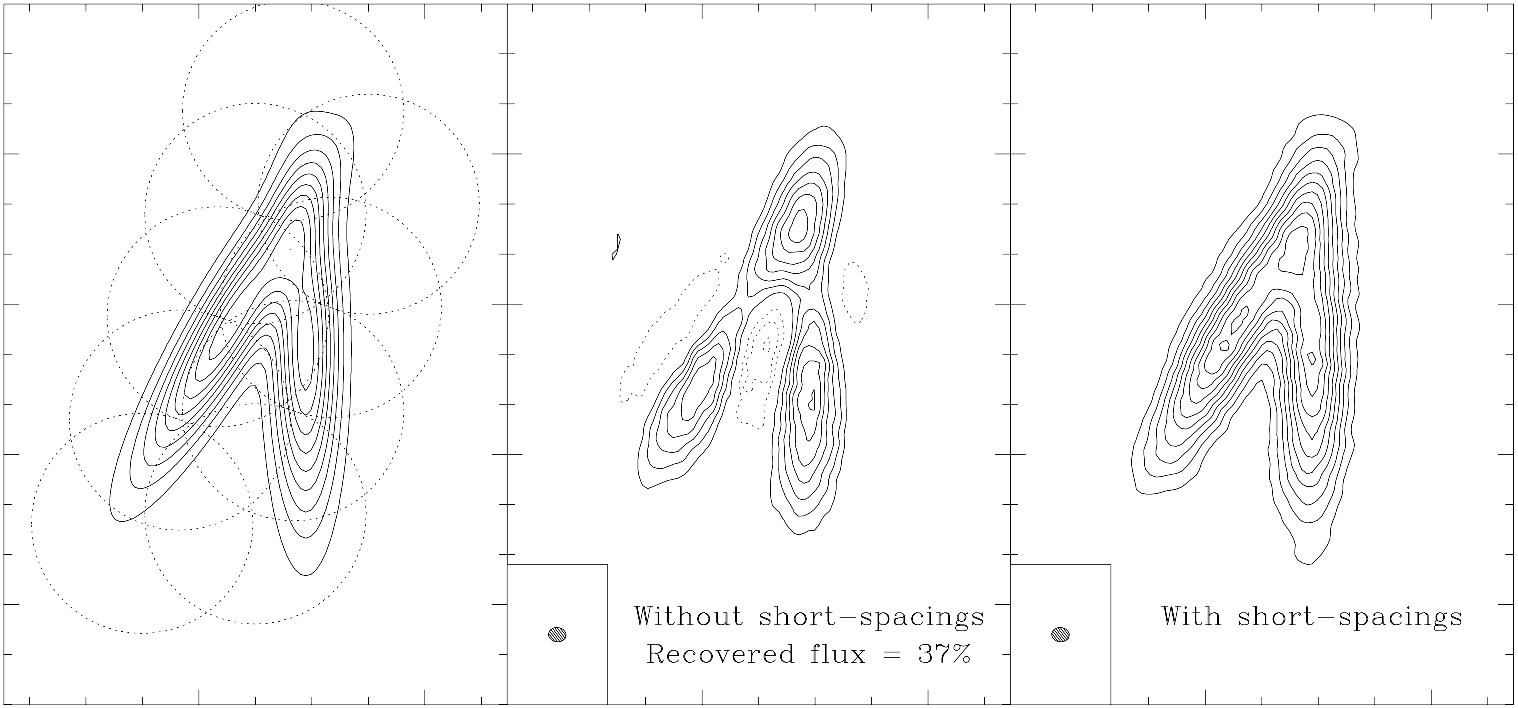
Missing short spacings :

- Smallest baseline $B_{\rm min} = 24$ m at Plateau de Bure
- Projection effects can reduce the minimal baseline (to the antenna diameter d in the best case)
- Deconvolution recovers some information (extrapolation in the uv plane)
- In any case: lack of the short spacings information

Consequence :

- The most extended structures are filtered out
- Maximal size is $\sim \lambda/B_{\rm min}$
- The largest structures that can be mapped are $\sim 2/3$ of the primary beam

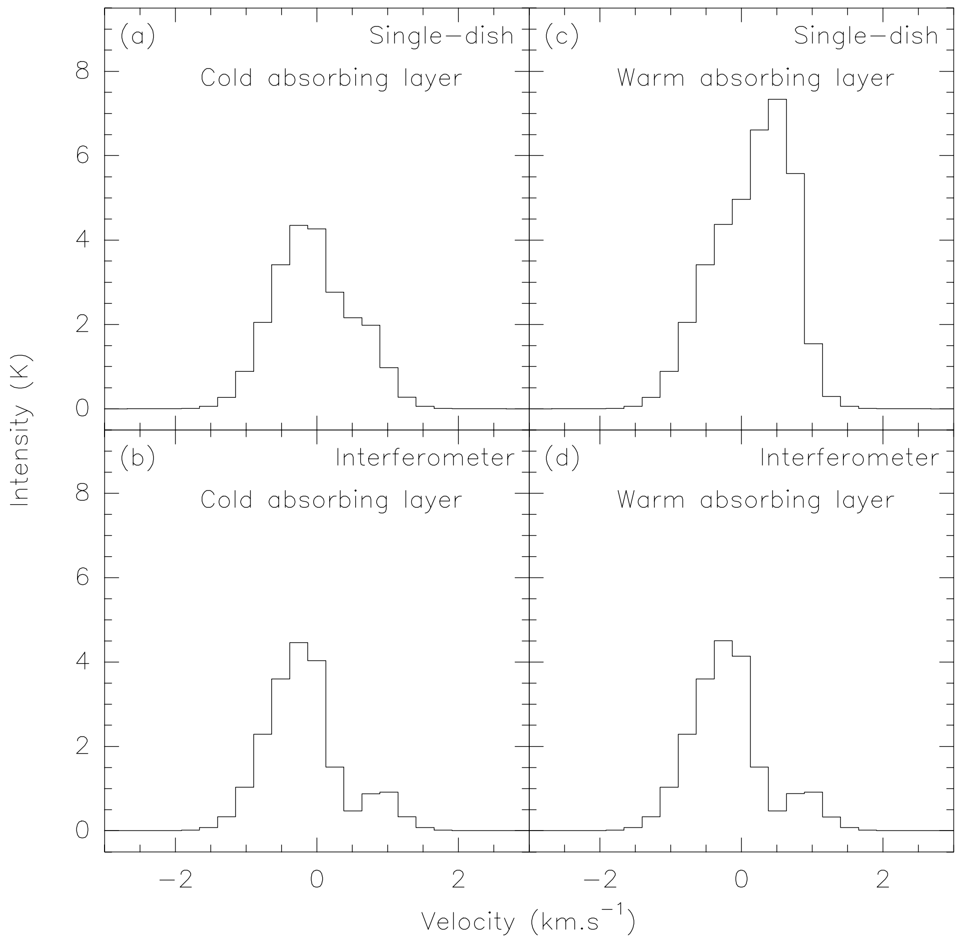
Example



Mapping an extended source requires the short spacings

Example

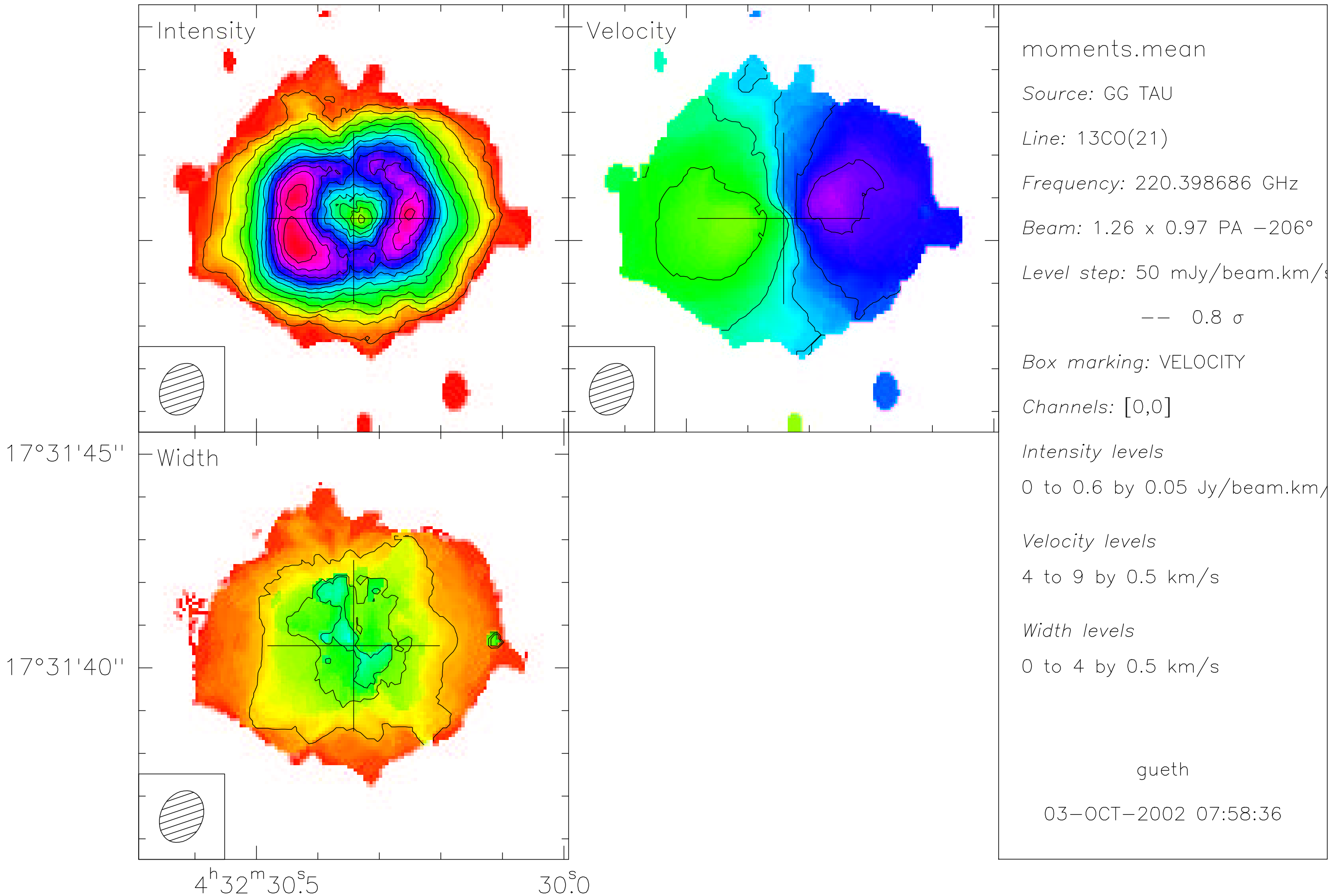
Lack of short spacings can introduce complex artifacts leading to wrong scientific interpretation



Spectral line image analysis

- GO MOMENTS computes the first three moments (relative to velocity) of a data cube
  - integrated intensity
  - mean velocity
  - line width
- Necessity of a window in velocity
- Necessity of a threshold in flux density
- GO VELOCITY displays the moment images

GO MOMENT + GO VELOCITY



Velocity integration vs. deconvolution

Sum + deconvolution or deconvolution + sum?

THERE IS NO ANSWER

- Deconvolution is a non-linear process
- Sum of deconvolutions $\neq$ deconvolution of a sum
- It is impossible to predict which method will give the best result
  - Summing the channel maps increases the signal-to-noise $\longrightarrow$ deconvolution easier
  - Summing the channel maps make the image more complex $\longrightarrow$ deconvolution more difficult
- It depends on the data...

Continuum Subtraction

- Continuum subtraction needed to compute moment maps...
- Caution: some astronomical input is needed: what about an optically thick line in front of a continuum source?
- In any case: it should be done in the uv plane in order to avoid the non linearity in the deconvolution (task `UV_SUBTRACT`)
- In some cases, a model may be better than the true measurements, because it is less noisy

Some useful tasks

- Most tasks operate on Ra-Dec-Velocity data cubes, unless specified
- MAP_SUM — Integrated flux in a velocity range
- MAP_AVER — Averaged flux in a velocity range
- SLICE — Position-Velocity image along a given line (no curved slice yet)
- MAP_INTER — Resampling along the third axis of a 3-D data cube
- COMBINE — Combine two data cubes (or a data cube and a 2-D image) to produce another one; can be used to subtract continuum, compute spectral index, ...
- MOMENTS — Compute image moments (integrated flux, velocity, line width)
- SPECTRUM_SUM — Computes integrated spectrum over a polygon-defined area
- SPECTRUM — Extract spectrum at a given absolute position
- PRIMARY — Correct an image for the primary beam attenuation
- ...