Wide-field imaging

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Problems when mapping an extended source

- Non-coplanar baselines problem

  Solution: use appropriate algorithm if necessary
  mm-interferometers: problem can be forgotten

- The largest structures are filtered out due to the lack of the short spacings

  Solution: add the short spacings information

- The field of view is limited by the antenna primary beam width

  Solution: observe a mosaic = several adjacent overlapping fields

- Deconvolution algorithms are not very good at recovering small- *and* large-scale structures

  Solution: try SDI CLEAN, Multi-Scale CLEAN, Multi-Resolution CLEAN, ...

Short spacings

The short spacings problem

Missing short spacings :

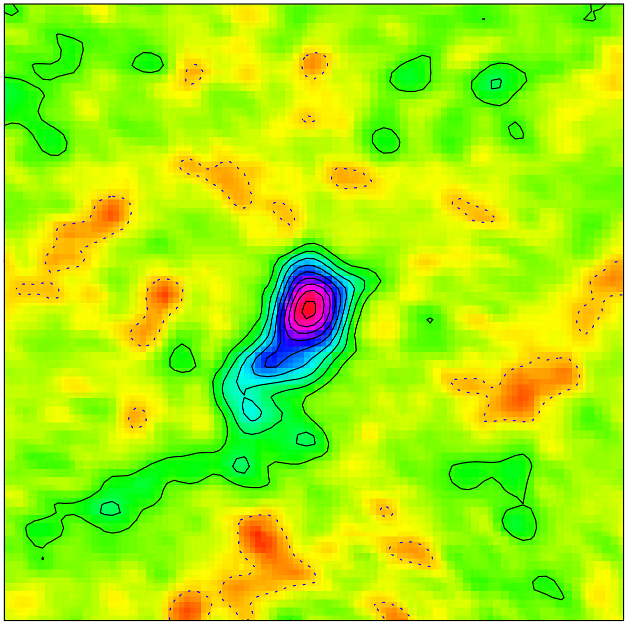
- Shortest baseline $B_{\text{min}} = 24$ m at Plateau de Bure
- Projection effects can reduce the minimal baseline (to the antenna diameter d in the best case)
- Deconvolution recovers some information (extrapolation in the uv plane)
- In any case: lack of the short spacings information

Consequence :

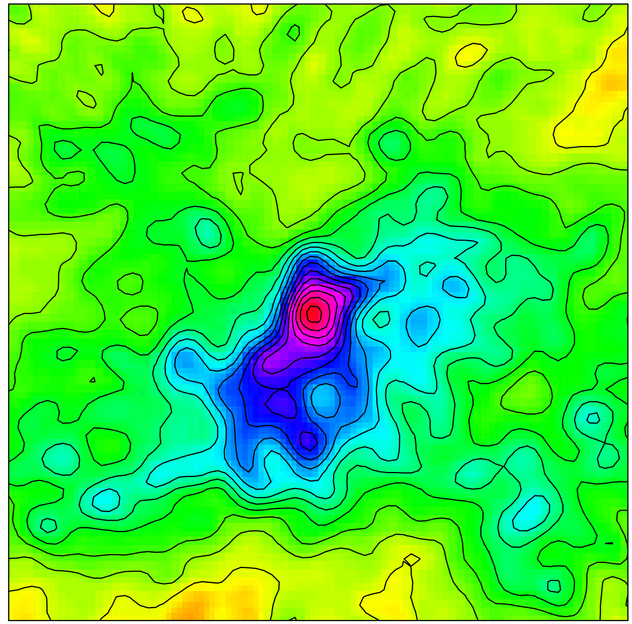
- The most extended structures are filtered out
- Maximal size is $\sim \lambda / B_{\text{min}}$
- The largest structures that can be mapped are $\sim 2/3$ of the primary beam

Short spacings: example

Without short spacings



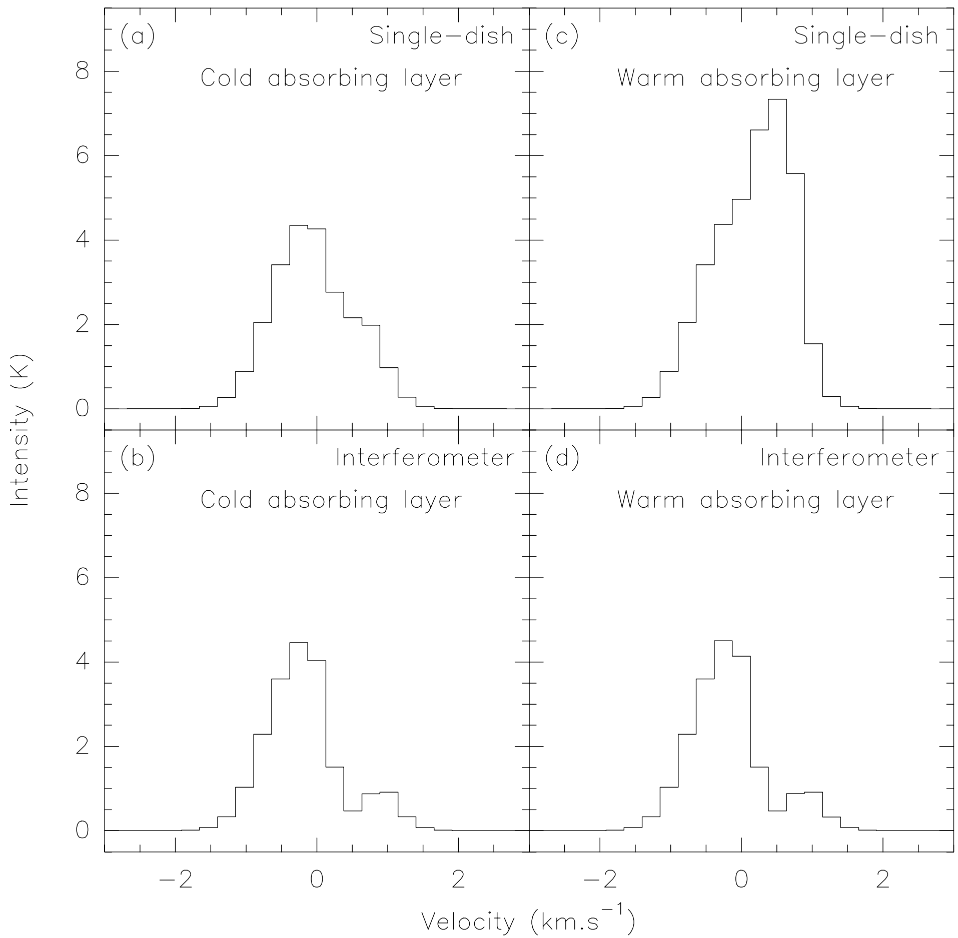
With short spacings



13CO(1–0) in the L1157 protostar

Short spacings: example

Lack of short spacings can introduce complex artifacts leading to wrong scientific interpretation



Obtaining short spacings

Interferometer vs. single-dish :

- An interferometer with smaller antennas (e.g. BIMA vs. PdBI) can provide short spacings information, but still with a central hole
- A single-dish of diameter D measures all spatial frequencies from 0 to D
- The zero spacing (= visibility at u=0 v=0 = total flux) can only be measured by a single-dish

Optimal solution: PdBI + 30–m

- $30 > 15 \longrightarrow$ uniformity, even overlap, in the uv plane
- Same calibration procedures
- Same softwares
- Same program committee

Short spacings from SD data

Bad solution :

- Combine the 30–m and the PdBI map in the image plane

Good solution :

- Combine the 30–m and the PdBI data before imaging and deconvolution ⟶ this drastically improves the deconvolution

Method :

1. Use the 30–m map to simulate what would have observed the PdBI, i.e. extract pseudo-visibilities
2. Merge with the interferometric visibilities
3. Process (gridding, FT, deconvolution) all data together

Extracting visibilities

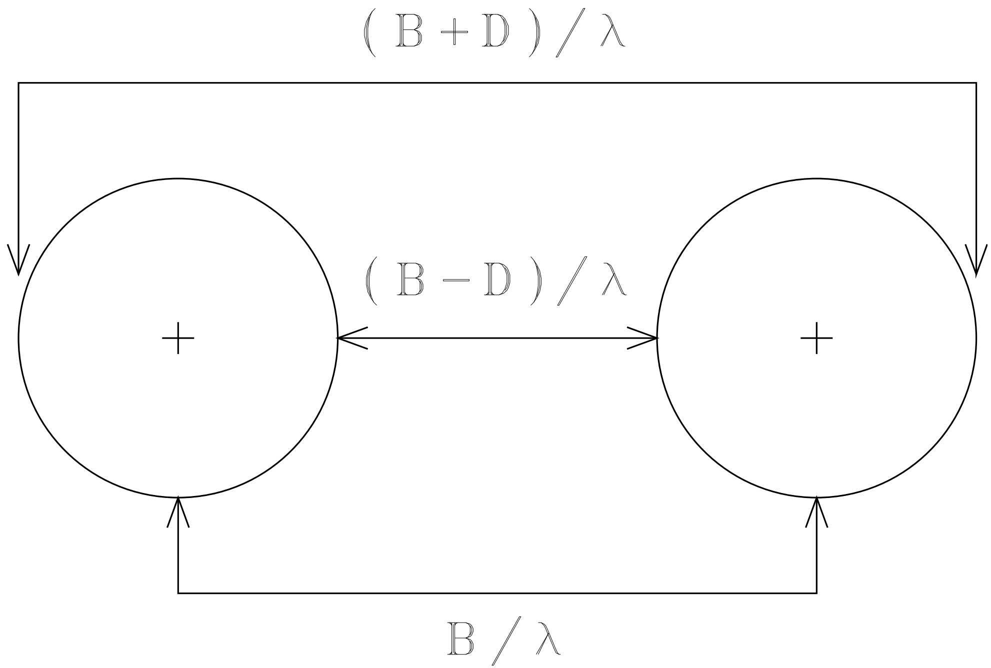
$$\text{SD map} = \text{SD beam} * \text{Sky}$$

$$\text{Int. map} = \text{Dirty beam} * (\text{Int beam} \times \text{Sky})$$

- Image plane   Gridding of the single-dish data
- Image plane   Extrapolation to zero outside the mapped region, with a function twice broader than the single-dish beam
- uv plane   Correction for single-dish beam and gridding function
- Image plane   Multiplication by interferometer primary beam
- uv plane   Extract visibilities up to $D - d$
- uv plane   Apply a weighting factor before merging with the interferometer data

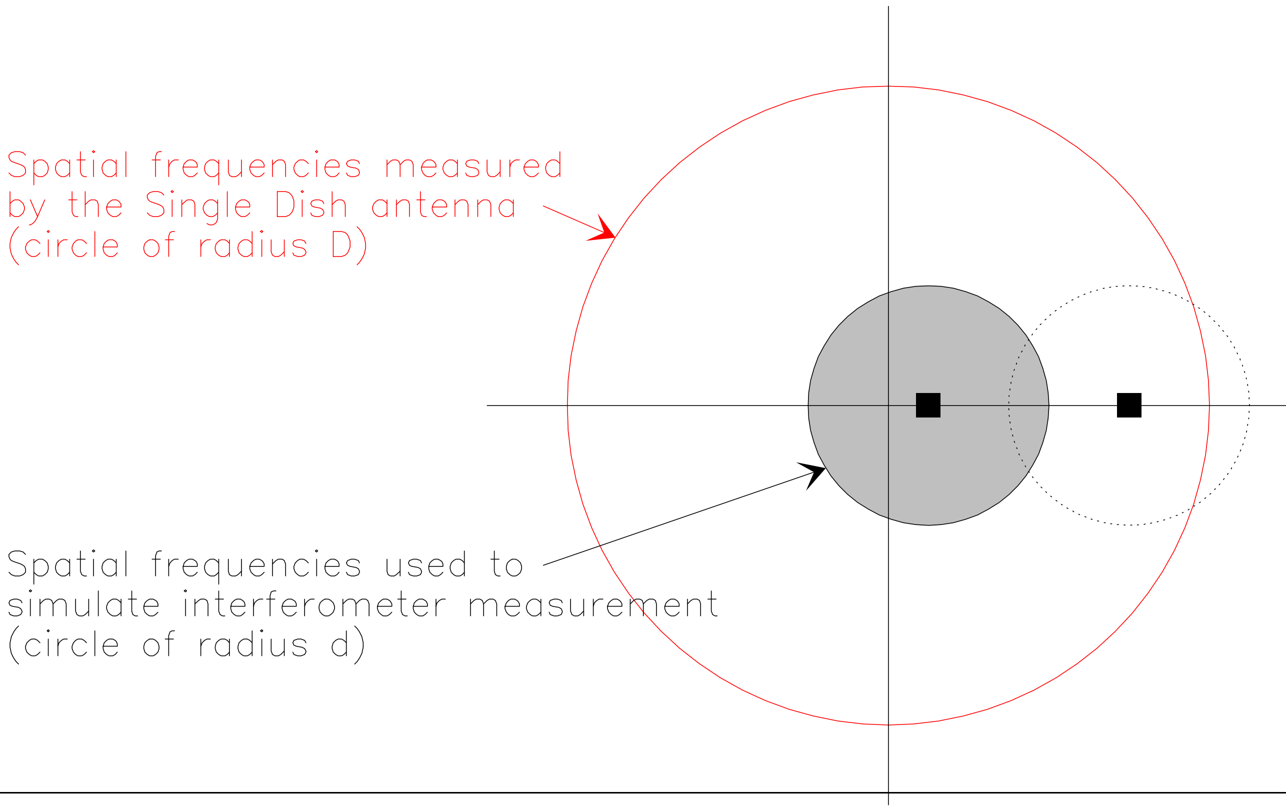
Spatial frequencies: measurements

- A single-dish of diameter D is sensitive to spatial frequencies from 0 to D
- An interferometer baseline B is sensitive to spatial frequencies from $B - d$ to $B + d$ (d = antenna diameter)



Spatial frequencies: what can be extracted from SD data

Single-dish data $\Longrightarrow$ interferometer pseudo-visibilities from 0 to $D - d$



Weighting factor

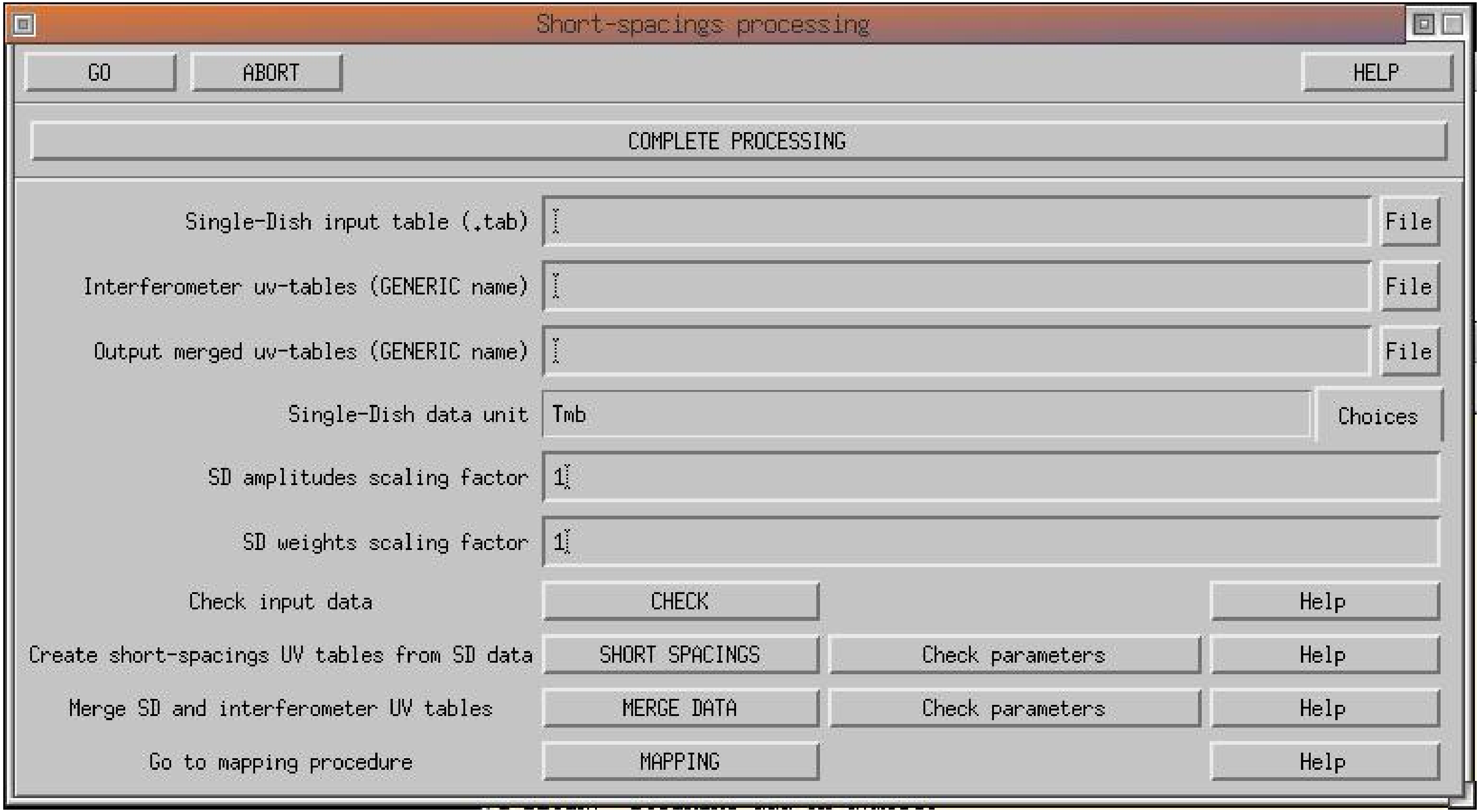
Weighting factor to SD data :

- Produce different images and dirty beams
- Same result after deconvolution, if methods were perfect
- Methods are not perfect, noise $\longrightarrow$ weight to be optimized
- Usually, it is better to downweight the SD data (as compared to natural weight)

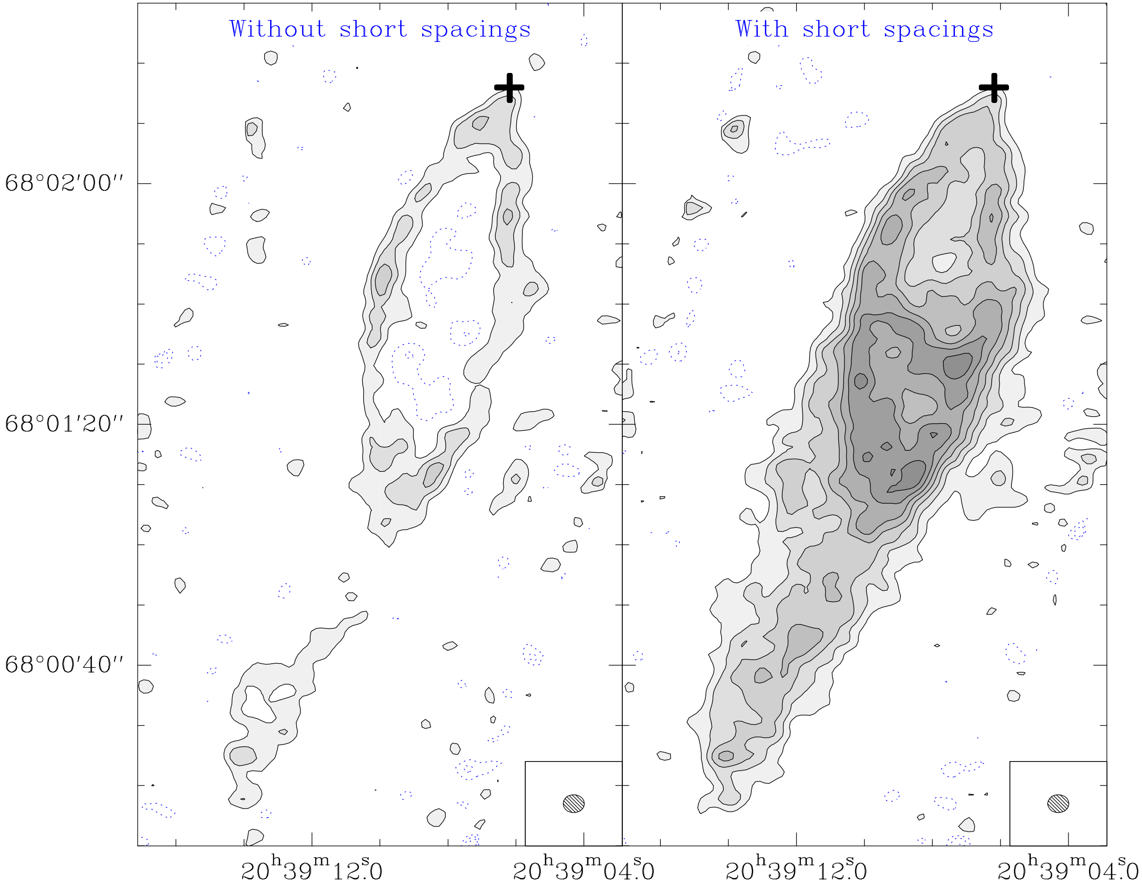
Optimal weight: proven good criteria

- Adjust the weights so that there is almost no negative sidelobes while keeping the highest angular resolution possible
- Adjust the weights so that the weight densities in 0–D and D–2D areas are equal $\longrightarrow$ mathematical criteria

GILDAS implementation: user interface



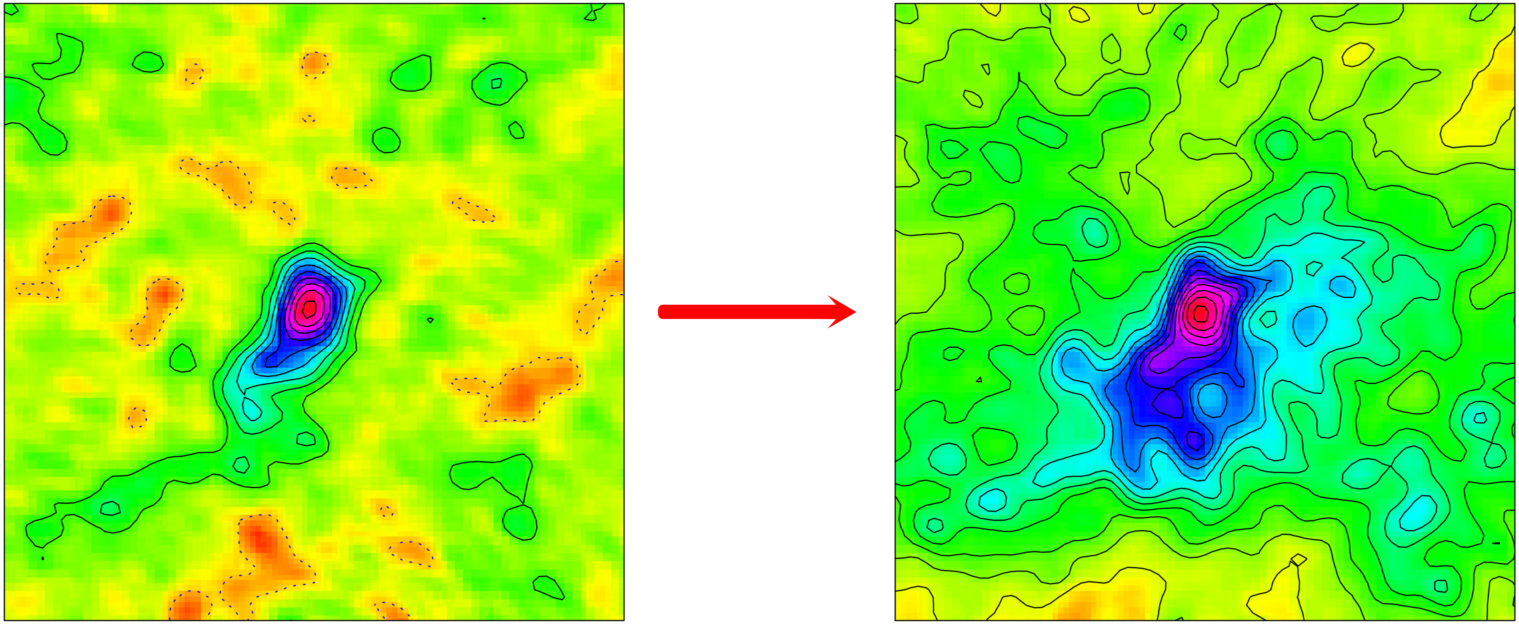
Example



Gueth et al. 1997

Example

Without short spacings With short spacings



13CO(1–0) in the L1157 protostar

Mosaics

Interferometer field of view

Measurement equation of an interferometric observation:

$$F = D * (B \times I) + N$$

$$\begin{array}{ccl} F & = & \text{dirty map} = \text{FT of observed visibilities} \\ D & = & \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ B & = & \text{primary beam} \\ I & = & \text{sky brightness distribution} \\ N & = & \text{noise distribution} \end{array}$$

- An interferometer measures the product $B \times I$
- B has a finite support $\longrightarrow$ limits the size of the field of view
- B is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

$$
\begin{array}{ccc}
\text{Aperture function} & \rightleftharpoons & \text{Voltage pattern} \\
\star \big\downarrow & & \big\downarrow |\cdot|^2 \\
\text{Transfert function } T(u,v) & \rightleftharpoons & \text{Power pattern } B(\ell, m) \\
 & & = \text{Primary beam}
\end{array}
$$

Gaussian illumination $\Longrightarrow$ to a good approximation, B is a Gaussian of $1.2\,\lambda/d$ FWHM

Plateau de Bure
$d = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	$58''$
100 GHz	3.0 mm	$50''$
115 GHz	2.6 mm	$43''$
215 GHz	1.4 mm	$23''$
230 GHz	1.3 mm	$22''$
245 GHz	1.2 mm	$20''$

Mosaicing with the PdBI

Mosaic :

- Fields spacing = half the primary beam FWHM i.e. one field each 11″ at 230 GHz
- Observations with two receivers: choice of the spacing for one frequency $\longrightarrow$ under- or oversampling for the other frequency
- Mosaic at 3 mm $\longrightarrow$ no mosaic at 1 mm

Observations :

- Fields are observed in a loop, each one during a few minutes $\longrightarrow$ similar atmospheric conditions (noise) and similar uv coverages (dirty beam, resolution) for all fields

Mosaicing with the PdBI

Size of the mosaic :

- Observing time to be minimized, uv coverage to be maximized $\longrightarrow$ maximal number of fields ~ 20

Calibration :

- Procedure identical with any other Plateau de Bure observations (only the calibrators are used)
- Produce one dirty map per field

Short spacings :

- Visibilities from 30–m data are computed and merged with Plateau de Bure data for each field $\longrightarrow$ process as a normal mosaic

Mosaic reconstruction

- Forgetting the effects of the dirty beam:

$$F_i = B_i \times I + N_i$$

- This is similar to several measurements of I, each one with a "weight" B_i
- Best estimate of I in least-square formalism (assuming same noise):

$$J = \frac{\sum_i B_i\,F_i}{\sum_i B_i^2}$$

- J is homogeneous to I, i.e. the mosaic is corrected for the primary beam attenuation

Mosaic deconvolution

- "Linear" mosaicing: deconvolution of each field, then mosaic reconstruction

  "Non-linear" mosaicing: mosaic reconstruction, then global deconvolution
- The two methods are not equivalent, because the deconvolution algorithms are (highly) non-linear
- Non-linear mosaicing gives better results
  - sidelobes removed in the whole map
  - better sensitivity
  - estimate of the missing spacings (Ekers & Rots's analysis)
- Plateau de Bure mosaics: non-linear joint deconvolution based on CLEAN

Noise distribution

In practice: *truncated* primary beam ($B_{\rm min} = 0.1 - 0.3$) to avoid noise propagation

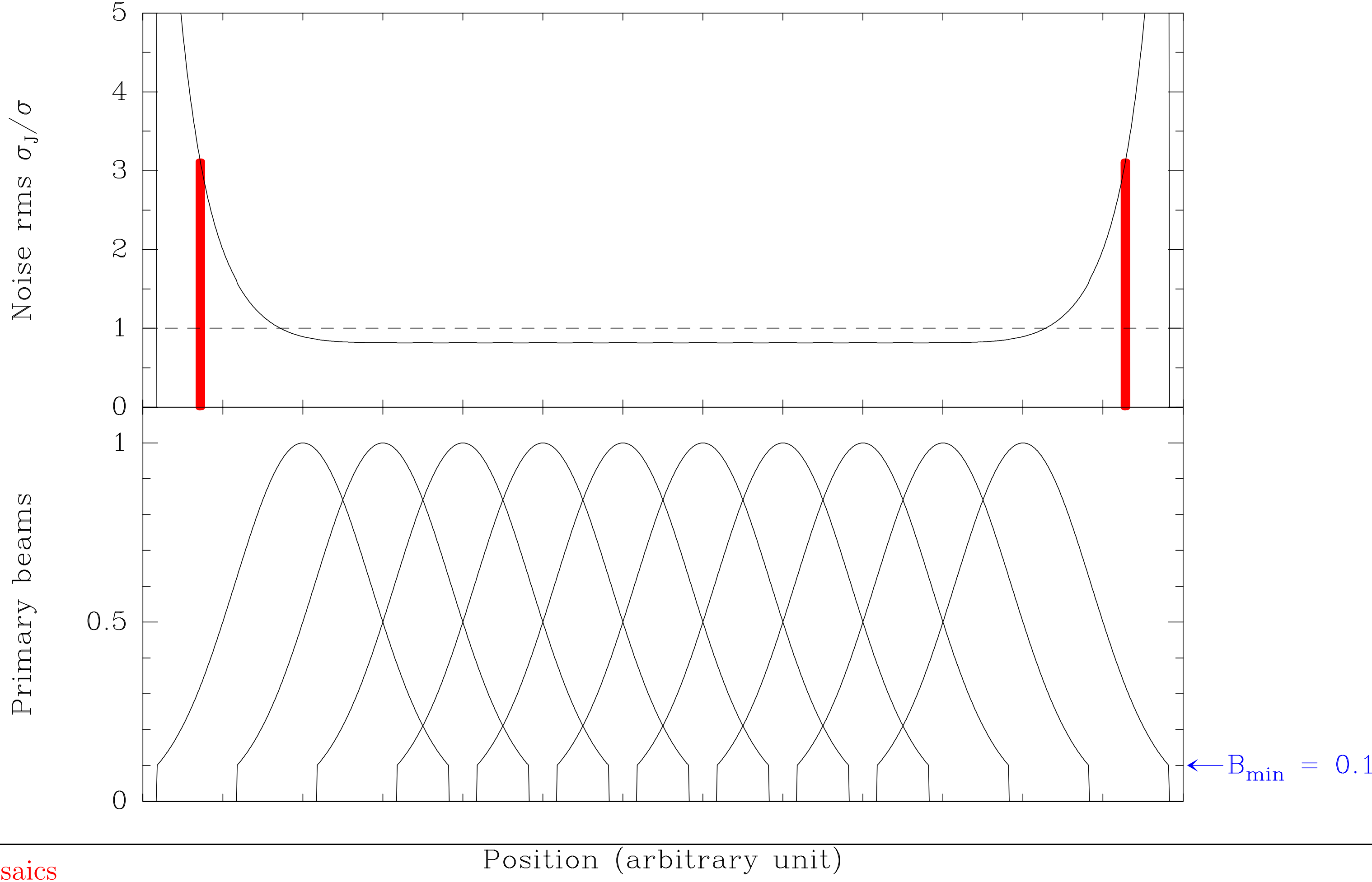
Mosaic: $$J = \frac{\sum B_i^t \, F_i}{\sum B_i^{t2}} = \frac{\sum B_i^t \left[D_i * (B_i \, I) + N_i \right]}{\sum B_i^{t2}}$$

Noise distribution: $$N = \frac{\sum B_i^t \, N_i}{\sum B_i^{t2}}$$

Noise rms: $$\sigma_J = \sigma \frac{1}{\sqrt{\sum B_i^{t2}}}$$

The noise depends on the position and strongly increases at the edges of the field of view

Noise distribution



Mosaic CLEAN

Signal-to-noise distribution:

$$H = \frac{J}{\sigma_J} = \frac{1}{\sigma} \frac{\sum B_i^t \left[D_i * (B_i\, I) + N_i \right]}{\sqrt{\sum B_i^{t\,2}}}$$

Mosaic CLEAN :

- J has a non-uniform noise level
- It is safer to search for CLEAN components on H
- Find positions of components on H
- Correct J

Mosaic CLEAN

(1) Find `CLEAN` component: position of the maximum of H and intensity of J (even if it is not the maximum of J)

(2) Remove corresponding point source from J and H

$$J_{k+1} = J_k - \frac{\sum B_i^t \left[D_i * \left[B_i \, \delta_k \right] \right]}{\sum B_i^{t\,2}}$$

$$H_{k+1} = H_k - \frac{\sum B_i^t \left[D_i * \left[B_i \, \delta_k \right] \right]}{\sigma \sqrt{\sum B_i^{t\,2}}}$$

Mosaic CLEAN

(3) Identify I and the sum of CLEAN components

(4) Clean map:

$$M = C * \sum \delta_k + J_{k_{\max}}$$

C = clean beam
$J_{k_{\max}}$ = final residuals

- The algorithms `Clark`, `SDI`, and `MX` can be adapted in a similar way: find position of CLEAN components on H, and correct J
- This is not feasible for `MRC` – because this method relies on a linear measurement equation, which is not the case for mosaics

GILDAS implementation

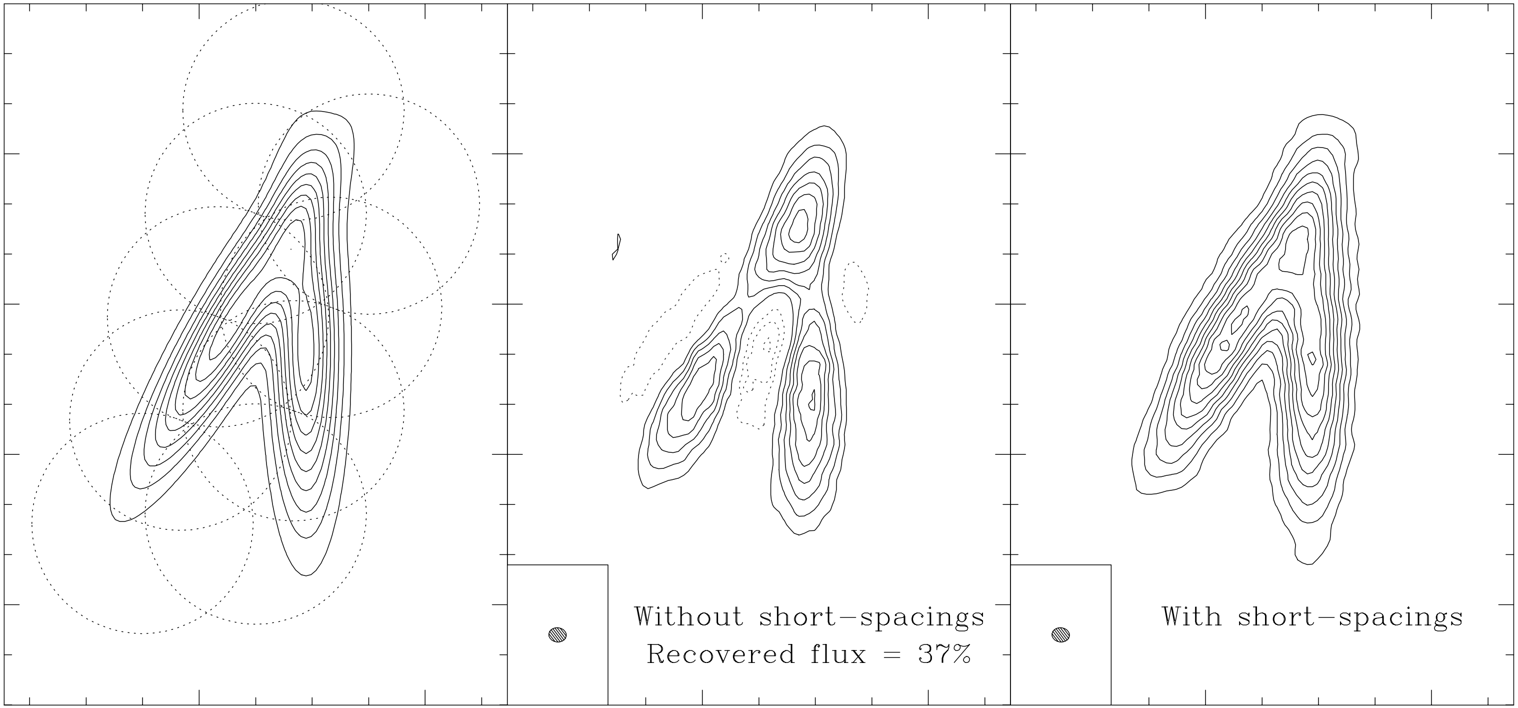
- Create a dirty map for each field (task `UV_MAP`), with the *same* phase center (`UV_SHIFT = YES`)
- Task `MAKE_MOSAIC` combines the fields to produce the dirty mosaic. Input parameters: primary beam width and truncation level ($B_{\mathrm{min}} \sim 0.1 - 0.3$)
- Deconvolution has to be done with `MAPPING`. Implemented deconvolution methods: `HOGBOM`, `CLARK`, `SDI`
- Mosaic mode switched on when loading a mosaic. Same parameters as normal deconvolution: windows, maximal number of iterations,...
- Clean beam is computed from the *first* field
- Mosaic has to be truncated at some value of σ_J. Default: truncation at $\sigma_J/\sigma = 1/\sqrt{B_{\mathrm{min}}}$

Mosaics and short spacings

Mosaics and short spacings

- Effect of missing short spacings more severe on mosaics than on single-field images:
  - Extended structures are filtered out in each field
  - Lack of information on an intermediate scale as compared to the mosaic size
  - Possible artefact: extended structures split in several parts
  - In most cases cases, adding the short spacings is required

Mosaic and short spacings



Mosaics and short spacings

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  - Extended structures are filtered out in each field
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  - In most cases cases, adding the short spacings is required

- However, mosaics are able to recover part of the short spacings information (Ekers & Rots's analysis)

Image formation in a mosaic

Ekers & Rots's analysis: ideal "on-the-fly" mosaic: (u, v) fixed, (ℓ_p, m_p) continuously modified, visibility $V_{\rm mes}$ monitored

- Phase center = Pointing center = $(0, 0)$

$$V_{\rm mes}(u, v) = [{\rm FT}(B \times I)](u, v) = \iint_{-\infty}^{+\infty} B(\ell, m)\, I(\ell, m)\, {\rm e}^{-2i\pi(u\ell + vm)}\, d\ell\, dm$$

- Phase center $(0, 0) \neq$ Pointing center (ℓ_p, m_p)

$$V_{\rm mes}(u, v, \ell_p, m_p) \; = \iint_{-\infty}^{+\infty} B(\ell - \ell_p, m - m_p)\, \underbrace{I(\ell, m)\, {\rm e}^{-2i\pi(u\ell + vm)}}_{\mathcal{F}(u, v, \ell, m)}\, d\ell\, dm$$

Image formation in a mosaic

- $V_{\rm mes}$ can be written as a convolution product:

$$V_{\rm mes}(u, v, \ell_p, m_p) = B(\ell_p, m_p) * \mathcal{F}(u, v, \ell_p, m_p)$$

- Fourier transform of $V_{\rm mes}$ with respect to (ℓ_p, m_p):

$$\left[{\rm FT_p}(V_{\rm mes})\right](u_p, v_p) = T(u_p, v_p)\, V(u_p + u, v_p + v)$$

  - $T = {\rm FT}(B) =$ transfer function $T(u_p, v_p) = 0$ if $\sqrt{u_p^2 + v_p^2} > d$
  - $V =$ "true" visibility $= {\rm FT}(I)$
  - $\mathcal{F} = I \times$(phase term) $\Rightarrow {\rm FT}(\mathcal{F}) =$ V at a shifted point

Image formation in a mosaic

- For $\sqrt{u_p^2 + v_p^2} < d$:

$$V(u_p + u, v_p + v) = \frac{[\mathrm{FT_p}(V_{\mathrm{mes}})]\,(u_p, v_p)}{T(u_p, v_p)}$$

- The measurements were done at (u, v), but the "true" visibility can be recovered in a disk of radius d, centered in (u, v)

- Redundancy of the adjacent pointings allows to estimate the source visibility at points which were not sampled!

Interpretation

- Baseline B, antenna diameter $d \Longrightarrow$ interferometer sensitive to all spatial frequencies from $B - d$ to $B + d \Longrightarrow$ an interferometer measures a local average of the "true" visibilities

- Measured visibilities: $V_{\text{mes}} = \text{FT}(B \times I) = T * V$

- Pointing center $\neq$ Phase center: phase gradient across the antenna aperture

$$V_{\text{mes}}(u, v) = \left[T(u, v)\,\mathrm{e}^{-2i\pi(u\ell_p + vm_p)}\right] * V(u, v)$$

- Combination of measurements at different (ℓ_p, m_p) should allow to derive $V \longrightarrow$ recovery algorithm is a Fourier transform

Consequence: short spacings

- Mosaicing can recover information in a disk of radius d around each sample in the uv plane

- Minimal baseline $B_{\text{min}} \longrightarrow$ Recovery down to $B_{\text{min}} - d$

- Mosaics are able to recover part of the short spacings information

- In practice:
  - Noisy data, rapidly decreasing function $T \longrightarrow$ expect only gain of d/2
  - Analysis not used: instead, direct reconstruction of the mosaic + deconvolution $\longrightarrow$ more complex properties

Consequence: image quality

- Mosaicing can recover information in a disk of radius d around each sample in the uv plane! Mosaicing can recover part of the short spacings information!

- The resulting image should be wonderful! NO!

- The image quality is not drastically improved in a mosaic because of additional information being recovered. The "equivalent" uv coverage is denser, but the region to be imaged is larger.

Consequence: fields spacing

- In practice: not on-the-fly measurements, but sampling of the pointing positions

- Primary beam is a Gaussian (of $1.2\,\lambda/d$ FWHM) $\longrightarrow$ large overlap between the adjacent fields is needed

- Ekers and Rots's analysis includes Fourier transform on a support which extends up to $\pm d/\lambda$

  $\Longrightarrow$ same information can be recovered with pointing centers separated by $\lambda/2\,d$

  $\Longrightarrow$ optimal separation between pointing centers = half the primary beam FWHM