Single-dish antenna at radio wavelength

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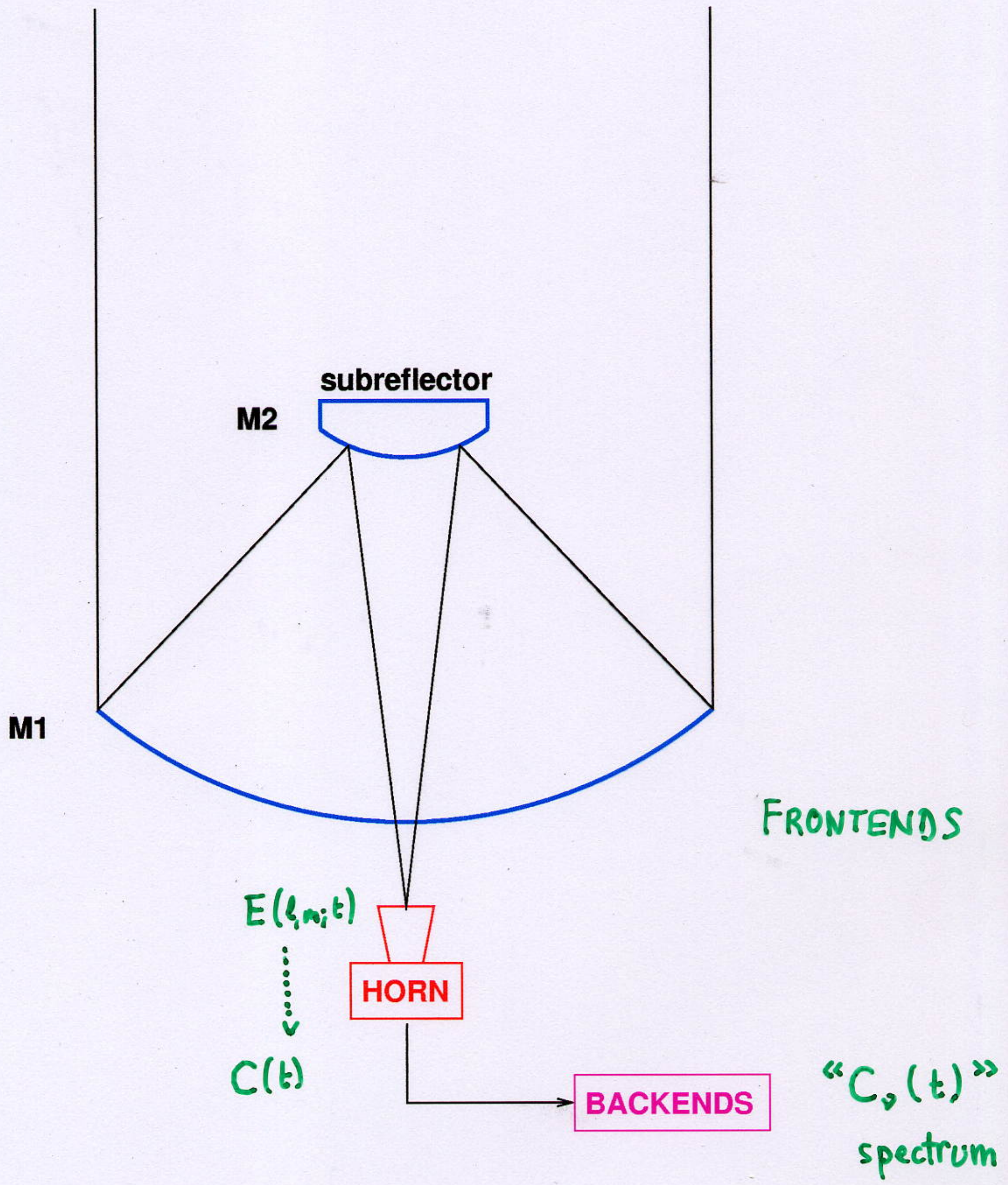
Outline

1. Perfect single-dish antenna
2. Real single-dish antenna
3. Temperature scales
4. Calibration
5. Summary

Characteristics of single-dish antenna



Why Cassegrain configuration ?

- Small $F/D \approx 0.3 - 0.4$ ratio:

  \+ focus (horns) close to reflector
  $\Rightarrow$ reduces mechanical load
  $\Rightarrow$ allows prime-focus antennas
  – decreases the effective area $(A_{\max})$
  – image distortion
  – small focal plane

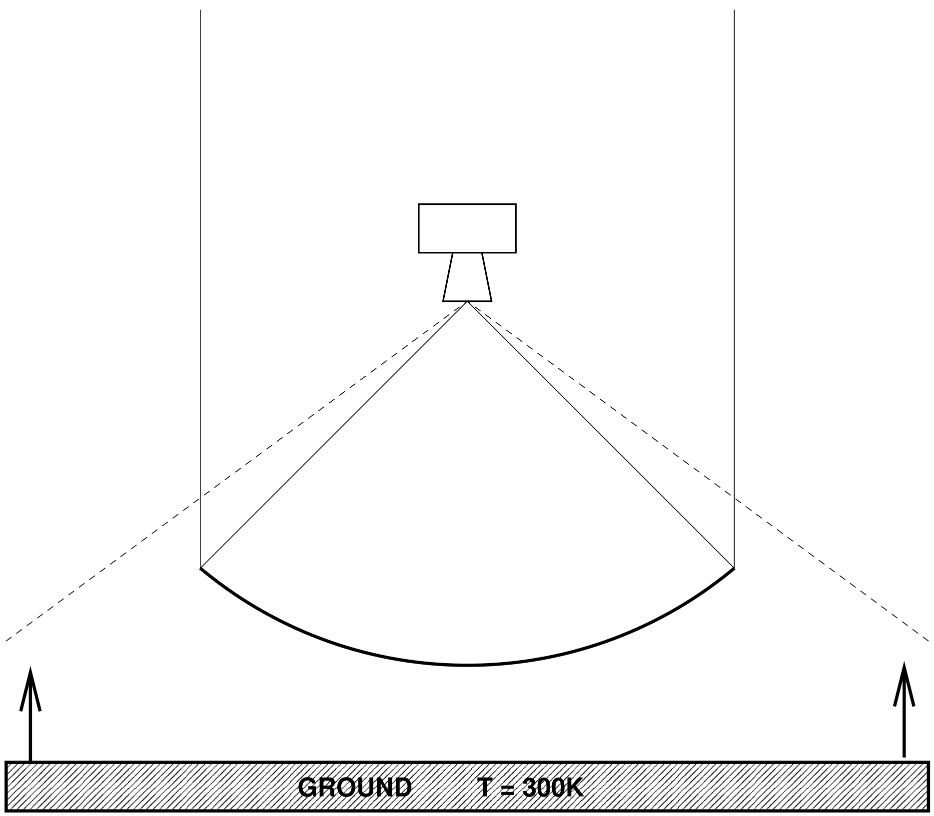
- prime-focus antenna $\Rightarrow$ spillover towards 300 K ground

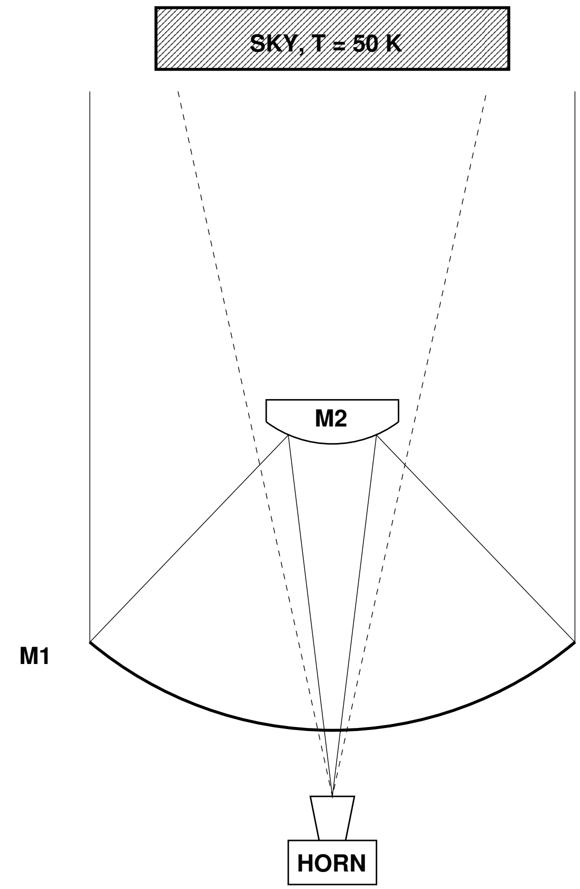
$\Rightarrow$ **Cassegrain configuration**

- Large effective $F_e/D = mF/D \approx 10$ ratio:

  \+ m = magnification ≈ 30 at 30-m
  \+ focal plane arrays (HERA & MAMBO at 30-m)
  \+ reduces mechanical deformation of beam pattern
  – large obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam

Spillover





Main single-dish antenna

Large aperture: f/D$\lesssim 1$

Institute	Diameter (m)	Frequency (GHz)	Wavelength (mm)	HPBW ($''$)	Latitude
Max-Planck	100	$0.09 - 1.15$	$3 - 300$	$11 - 680$	$+47^\circ$
IRAM	30	$80 - 280$	$1 - 3$	$9 - 30$	$+37^\circ$
JCMT	15	$210 - 710$	$0.2 - 2$	$8 - 20$	$+20^\circ$
APEX	12	$230 - 1200$	$0.3 - 1.3$	$6 - 30$	-22°
CSO	10.4	$230 - 810$	$0.4 - 1.3$	$10 - 30$	$+20^\circ$

Terminology

- Frontends:
  - Receivers (Rx): bandwidth ($\Delta\nu$), central frequency (ν_0)
  - $\Delta\nu \approx 1$ GHz, $\nu_0 \approx 100 - 800$ GHz
    $\Rightarrow \Delta\nu \ll \nu_0$, approximately monochromatic
- Backends:
  - spectrometers: filter banks (FB), autocorrelators (AC), acousto-optic (AOS), fourier transform (FTS)
  - spectral resolution $\delta\nu \approx 0.01 - 1$ MHz

$\Rightarrow$ $R = \nu_0/\delta\nu \approx 10^5 - 10^7$

Power received

What is the power received from a source of **flux density** S_ν $(\mathrm{W\,m^{-2}\,Hz^{-1}})$?

- S_ν measured in Jy: 1 Jy $= 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$
- Monochromatic power: p_ν (W $\mathrm{Hz^{-1}}$)

$$p_\nu = A \cdot S_\nu$$

- Power: $P_{\nu 0}$ (W)

$$P_{\nu 0} = A \cdot S_\nu \cdot \Delta\nu$$

- $A =$ effective area of the antenna,

$$A \leq A_{\mathrm{geom}}$$

Question: $A = ?$

Aperture – Beam pattern

- Diffraction theory:

$$E_{\mathrm{f-f}}(l,m) \propto \mathcal{F}[E_{\mathrm{ant}}(x,y)]$$

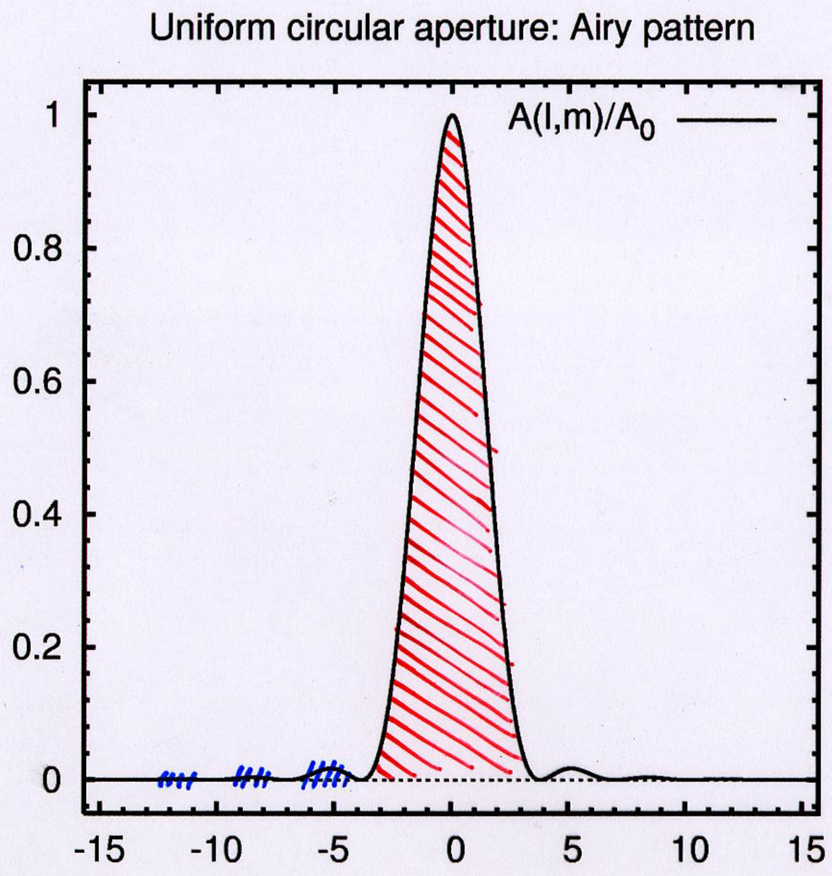
- $E_{\mathrm{ant}}(x,y)$
  - bounded on a finite domain $\Delta 1$
    $\Rightarrow$ $E_{\mathrm{f-f}}(l,m)$ also bounded on a finite domain $\Delta 2$ $(\Delta 1 \cdot \Delta 2 \sim 1)$
  - sharp cut of the antenna domain $\Rightarrow$ oscillations

- Antenna in emission:
  - pattern of the transmitted emission depends on the direction (l,m):

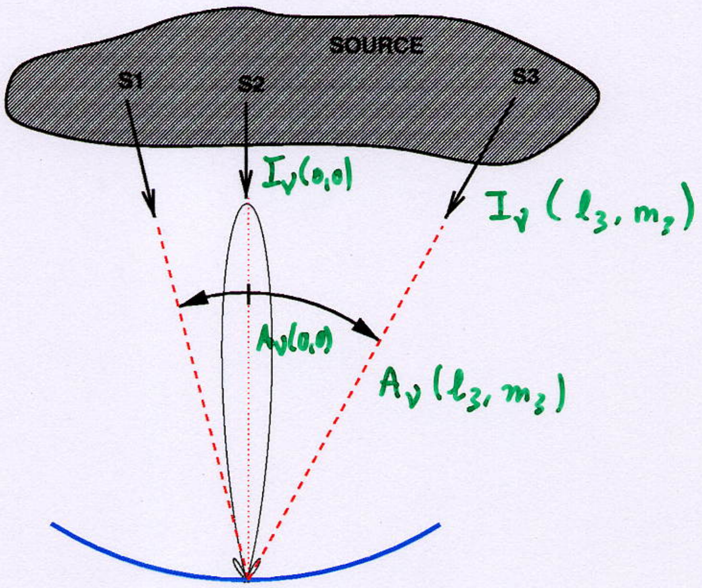
Power pattern	$\mathcal{P}(l,m)$	$\propto$	$\lvert E_{\mathrm{f-f}}(l,m)\rvert^2$
Effective area	$A(l,m)$	$=$	$A_{\mathrm{max}} \cdot \mathcal{P}(l,m)$

  - example: circular aperture
    $\mathcal{P}(l,m) \propto$ Airy disk

Power pattern



Extended source

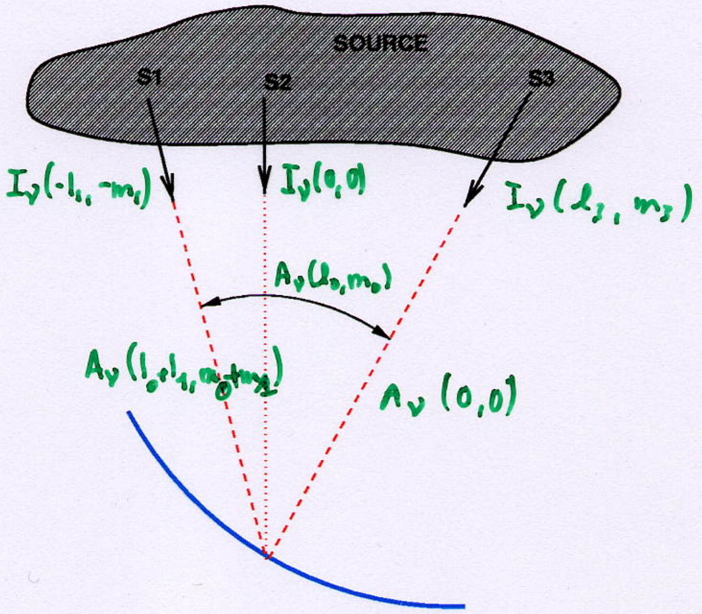


- point source: $S_\nu(\mathrm{W\,m^{-2}\,Hz^{-1}})$
  $\rightarrow$ extended source $I_\nu(\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}) = \mathrm{dS}_\nu/\mathrm{d}\Omega$

- from the direction $(l_i, m_i) \rightarrow \Omega_i = \Omega(l_i, m_i)$
$$\mathrm{d}p_\nu = A(\Omega_i) I_\nu(\Omega_i) \mathrm{d}\Omega_i$$

- incoherent emission
$$p_\nu = \int A(\Omega_i) I_\nu(\Omega_i) \mathrm{d}\Omega_i$$

Convolution



- antenna tilted w.r.t source towards (l_0, m_0)
- power received from S_i

$$\mathrm{d}p_\nu = A(l_i, m_i)\, I_\nu(l_0 - l_i, m_0 - m_i)\, \mathrm{d}l_i \mathrm{d}m_i$$

- convolution

$$p_\nu(l_0, m_0) = \iint A(l, m)\, I_\nu(l_0 - l, m_0 - m)\, \mathrm{d}l \mathrm{d}m$$

Beam pattern

- secondary lobes (due to finite surface antenna)
- surface irregularities

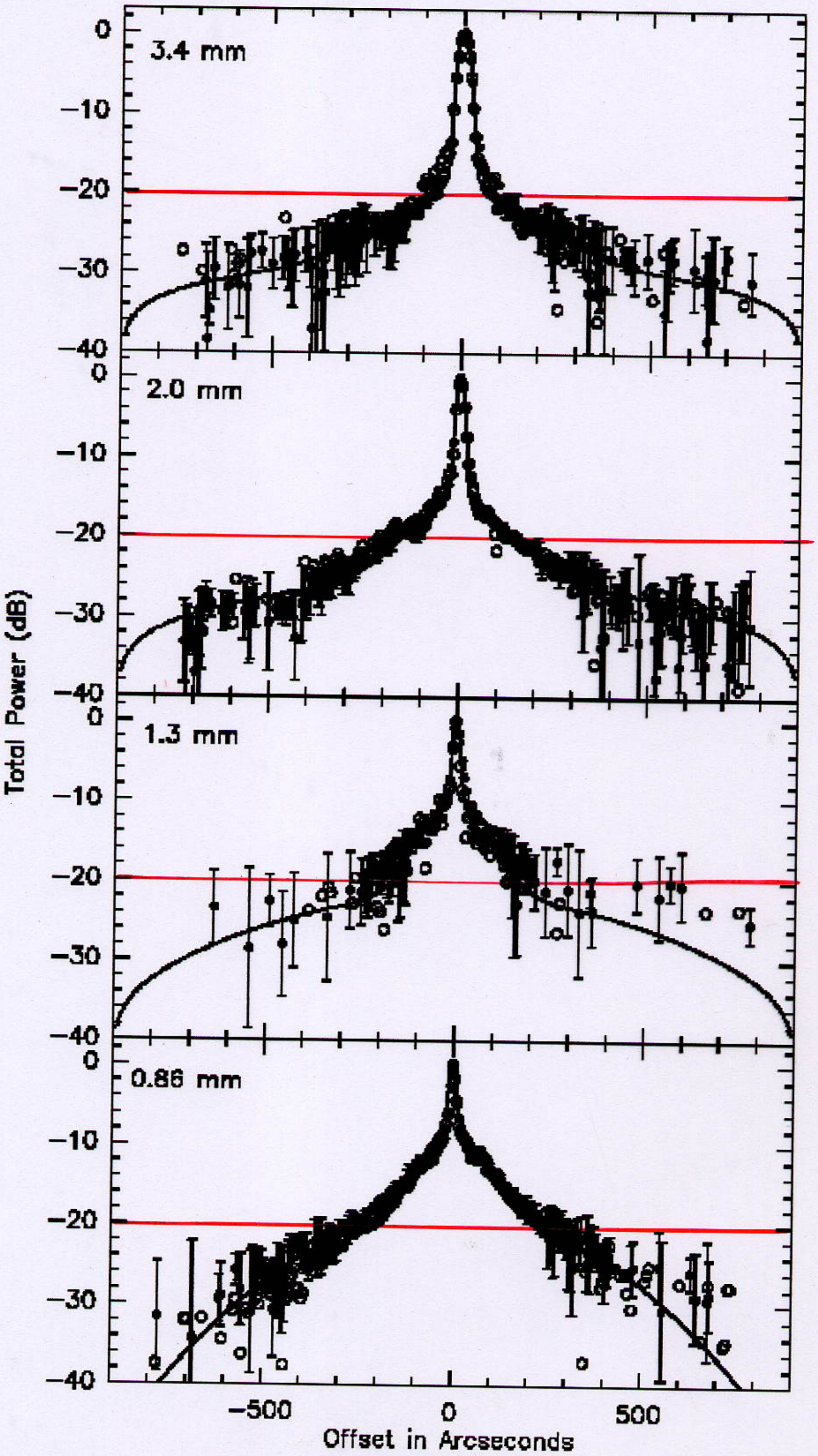
real beam =
main-beam + error-beam(s)

- Consequences:
  - main-beam collects less power
  - Surface irregularities: One typical length ⇒ one Gaussian error-beam

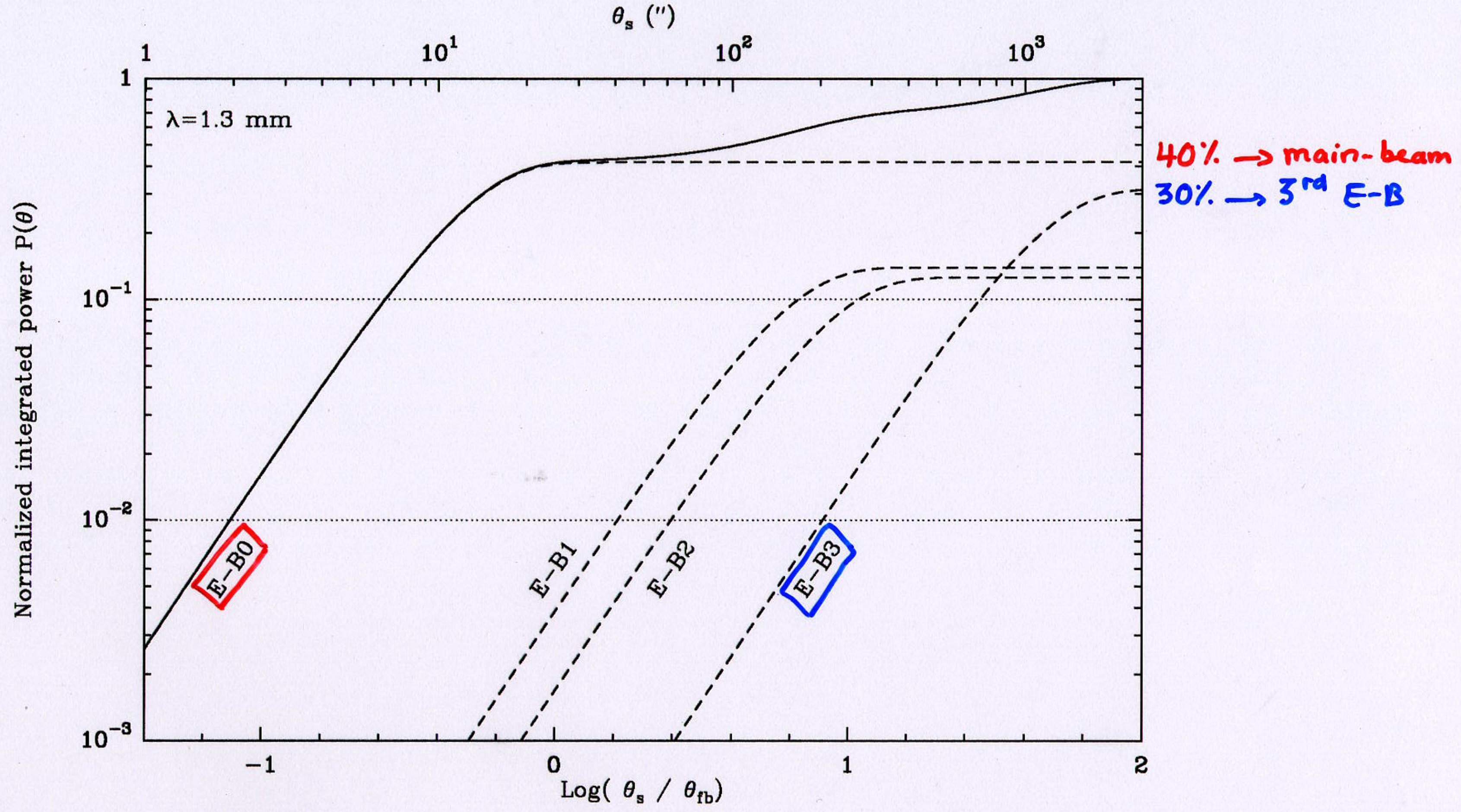
$$\ell \Longrightarrow \mathbf{FWHM} = \Theta_{\mathrm{EB}} \approx \frac{1}{2} \cdot \lambda/\ell$$

- Questions:
  - What power is collected in each beam ?
  - What are the FWHMs of the beams ?

IRAM 30-m antenna: 3 Error Beams



IRAM 30-m antenna: Error-Beams power



Brightness temperature

- T_{B} defined by

$$I_\nu = B_\nu(T_{\mathrm{B}}) \qquad (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$$

- radiation temperature, T_R, Rayleigh-Jeans approximation

$$I_\nu = \frac{2k\nu^2}{c^2} T_R \qquad (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$$

- relation $T_{\mathrm{B}} - T_R$

$$T_R = J_\nu(T_{\mathrm{B}}) = \frac{h\nu}{k} \frac{1}{\exp(h\nu/kT_{\mathrm{B}}) - 1}$$

- in the following

$$I_\nu(l,m) \rightarrow T_R(l,m)$$

- consequence: power $\propto T_R$

$$p_\nu(l_0, m_0) = \frac{2k}{\lambda^2} \iint_{4\pi} A(l,m)\, T_R(l_0 - l, m_0 - m)\, \mathrm{d}l\mathrm{d}m$$

Antenna temperature

- Johnson noise in terms of an equivalent temperature

average power transferred from a conductor to a line within $\delta\nu$

$$= k\,T\,\delta\nu$$

- Antenna temperature: antenna as a conductor

$$p_\nu = k\,T_A$$

$$[\mathrm{W}\cdot\mathrm{Hz}^{-1}] = [\mathrm{J}] = [\mathrm{J}\cdot\mathrm{K}^{-1}][\mathrm{K}]$$

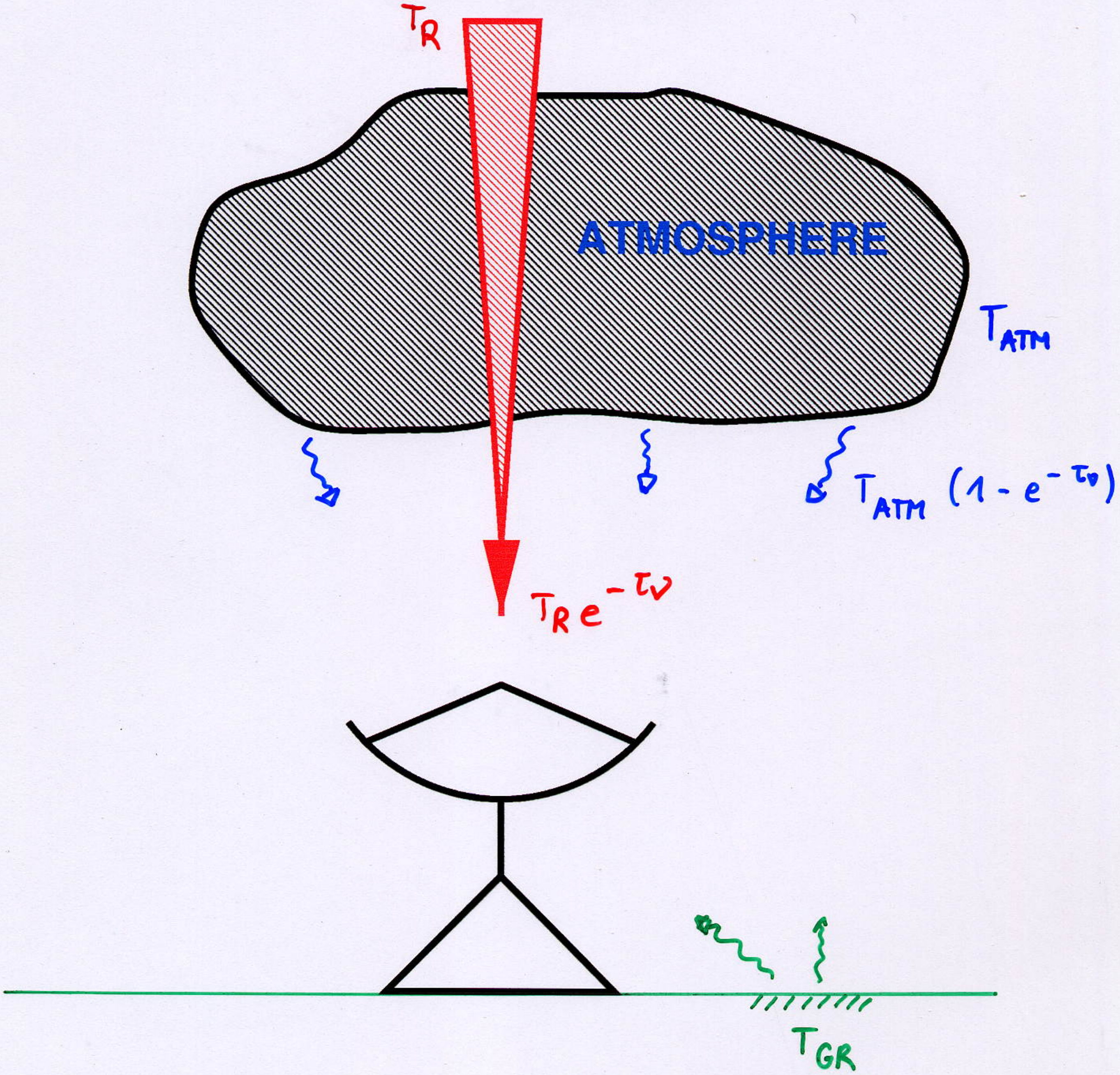
- Therefore

$$T_A(l,m) = \frac{2}{\lambda^2}\iint_{4\pi} A(l,m)\,T_R(l_0-l, m_0-m)\,\mathrm{d}l\mathrm{d}m$$

TEMPERATURES

$$1\,\mathrm{K} \Leftrightarrow 1.38\times10^{-23}\,\mathrm{W}\cdot\mathrm{Hz}^{-1}$$

Atmosphere



?

$$T = \alpha \left\{ T_R\, e^{-\tau_\nu} + (1-e^{-\tau_\nu})\, T_{ATM} \right\} + (1-\alpha)\, T_{GR}$$

T_{A}^{*} and T_{mb}

- T_{A}^{*}
  - takes into account **rear side-lobes**:

    FORWARD SIGNAL ONLY (2π sr)

  - corrects for **atmospheric attenuation**

    $\times \exp(\tau_\nu)$

$$T_{\mathrm{A}}^{*}(\Omega_0) = T_R \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega_0 - \Omega) \mathrm{d}\Omega}{\mathcal{P}_{2\pi}}$$

$$\mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega) \mathrm{d}\Omega$$

- T_{mb}: Equivalent in **main-beam** instead of 2π

$$T_{\mathrm{mb}}(\Omega_0) = T_R \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega_0 - \Omega) \mathrm{d}\Omega}{\mathcal{P}_{\mathrm{mb}}}$$

$$\mathcal{P}_{\mathrm{mb}} = \int_{\Omega_{\mathrm{mb}}} \mathcal{P}(\Omega) \mathrm{d}\Omega$$

Temperature scales

Definitions

$$F_{\rm eff} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{4\pi}}$$

$$B_{\rm eff} = \frac{\mathcal{P}_{\rm mb}}{\mathcal{P}_{4\pi}}$$

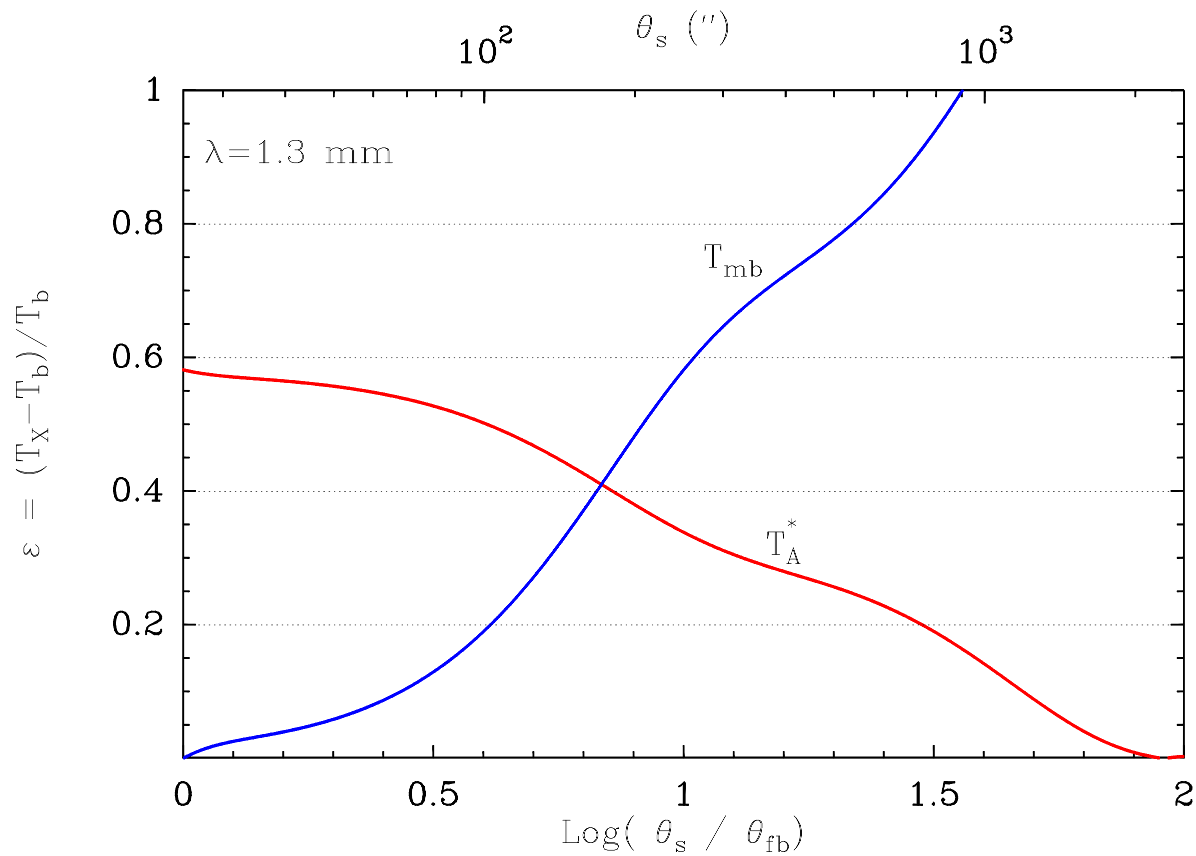
Consequences

$$T_{\rm mb} = \frac{F_{\rm eff}}{B_{\rm eff}}\, T_{\rm A}^{*} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{\rm mb}}\, T_{\rm A}^{*}$$

What you measure is T_A^* or T_{mb}

(usually $\neq T_R$)

Which temperature scale ?



Source size	Temperature scales
$\Omega_S = 2\pi$	$T_R = T_A^*$
$\Omega_S = \Omega_{mb}$	$T_R = T_{mb}$
$2\pi < \Omega_S$	$T_R < T_A^*$
$\Omega_{mb} < \Omega_S < 2\pi$	$T_A^* < T_R < T_{mb}$
$\Omega_{mb} > \Omega_S$	$T_{mb} < T_R$

Goal of the calibration

- Atmosphere: opacity τ_ν
- Antenna-sky coupling: $F_{\rm eff}$
- Output at backends: "counts"

Question: **counts $\longrightarrow$ Temperature ?**

$$C = \chi T \Longrightarrow \chi = ?$$

$$
\begin{aligned}
C_{\rm sou} &= \chi \left\{ T_{\rm rec} + F_{\rm eff} e^{-\tau_\nu} T_{\rm sou} + T_{\rm emi} \right\} \\
T_{\rm emi} &= F_{\rm eff}(1 - e^{-\tau_\nu}) T_{\rm atm} + (1 - F_{\rm eff}) T_{\rm gr}
\end{aligned}
$$

$\Rightarrow$ How many unknowns ? **4 unknowns**

$$\boxed{\{\chi,\ \tau_\nu,\ \mathbf{T}_{\rm sou},\ \mathbf{T}_{\rm rec}\}}$$

4 unknowns $\Rightarrow$ 4 equations $\Rightarrow$ 4 measurements:

$$\boxed{T_{\rm sou},\ T_{\rm atm},\ T_{\rm hot}\ \text{ and } T_{\rm col}}$$

"Chopper Wheel"

$$\begin{aligned} C_{\rm sou} &= \chi \left\{ T_{\rm rec} + T_{\rm emi} + F_{\rm eff} e^{-\tau_\nu} T_{\rm sou} \right\} \\ C_{\rm atm} &= \chi \left\{ T_{\rm rec} + T_{\rm emi} \right\} \\ C_{\rm hot} &= \chi \left\{ T_{\rm rec} + T_{\rm hot} \right\} \\ C_{\rm col} &= \chi \left\{ T_{\rm rec} + T_{\rm col} \right\} \end{aligned}$$

Making differences

$$\begin{aligned} C_{\rm sou} - C_{\rm atm} &= \chi F_{\rm eff} e^{-\tau_\nu} T_{\rm sou} \\ C_{\rm hot} - C_{\rm atm} &= \chi (T_{\rm hot} - T_{\rm emi}) \end{aligned}$$

Definition of $T_{\rm cal}$:

$$T_{\rm sou} = \frac{C_{\rm sou} - C_{\rm atm}}{C_{\rm hot} - C_{\rm atm}} \, T_{\rm cal}$$

$$\Rightarrow T_{\rm cal} = (T_{\rm hot} - T_{\rm emi}) \, \frac{e^{\tau_\nu}}{F_{\rm eff}}$$

Outputs of calibration procedure: $T_{\rm rec}$

Hot & cold loads $\longrightarrow T_{\rm rec}$:

$$
\begin{aligned}
Y &= \frac{C_{\rm hot}}{C_{\rm col}} \\
T_{\rm rec} &= \frac{T_{\rm hot} - Y T_{\rm col}}{Y - 1}
\end{aligned}
$$

Outputs of calibration procedure: $T_{\rm cal}$

Rewrite $T_{\rm emi}$

$$T_{\rm emi} = T_{\rm gr} + F_{\rm eff}(T_{\rm atm} - T_{\rm gr}) - F_{\rm eff} e^{-\tau_\nu} T_{\rm atm}$$

$$\begin{aligned} C_{\rm hot} - C_{\rm atm} &= \chi\{(T_{\rm hot} - T_{\rm gr}) + F_{\rm eff}(T_{\rm gr} - T_{\rm atm}) \\ &+ F_{\rm eff} e^{-\tau_\nu} T_{\rm atm}\} \end{aligned}$$

- Assume $T_{\rm hot} = T_{\rm atm} = T_{\rm gr} \Rightarrow \{\chi, \tau_\nu\} \rightarrow \{\chi e^{-\tau_\nu}\}$ $\Rightarrow$ 3 unknowns $\Rightarrow$ *e.g.* don't need to solve for τ_ν (Penzias & Burrus 1973)

$$\boxed{T_{\rm cal} = T_{\rm atm}}$$

- General case: different $T_{\rm atm}$, $T_{\rm hot}$ and $T_{\rm gr}$ $\Rightarrow$ solve for the 4 unknowns

Outputs of calibration procedure: $T_{\rm sys}$

System temperature: describes the noise including all sources from the sky down to the backends

$$\sigma_T = \frac{\kappa \cdot T_{\rm sys}}{\sqrt{\delta\nu\,\Delta t}}$$

- κ depends on the observing mode: ON-OFF $t_{\tt ON} = t_{\tt OFF} \Rightarrow \kappa = \sqrt{2}$
- δ_ν: spectral resolution
- Δt: integration time ($t_{\tt ON}$=$t_{\tt OFF}$)

From $T_{\rm mb}$ to I_ν

How to convert the temperatures into $\rm W\,m^{-2}\,Hz^{-1}$?

$$S_\nu = \frac{2k}{\lambda^2}\int_{\Omega_r} T_{\rm mb}\,{\rm d}\Omega$$

If source & lobe are Gaussians:

$$\mathrm{HPBW} = \theta_{\rm r} = \sqrt{{\theta_{\rm mb}}^2 + \theta_{\rm s}^2}$$

$$\frac{\rm S_\nu}{\rm Jy} = 7\left(\frac{\lambda}{\rm mm}\right)^{-2}\left(\frac{\theta_{\rm r}}{10''}\right)^2\left(\frac{\rm T_{mb}}{\rm K}\right)$$

(1 Jy $= 10^{-26}\ \rm W\,m^{-2}\,Hz^{-1}$)

Summary

- antenna moves w.r.t. source: $\mathbf{B} * \mathbf{I}$
- interferometry sensitive to $\mathbf{B} \times \mathbf{I}$
- **amplitude calibration:**
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- lobe = main-lobe + error-lobes (*e.g.* as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the **temperature scale** to use (T_A^*, T_{mb},...)