Large-scale mapping

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Problems when mapping an extended source

- The largest structures are filtered out due to the lack of the short spacings

  Solution: add the **short spacing** information

- The field of view is limited by the antenna primary beam width

  Solution: observe a **mosaic** = several adjacent overlapping fields

- Deconvolution algorithms are not very good at recovering small- *and* large-scale structures

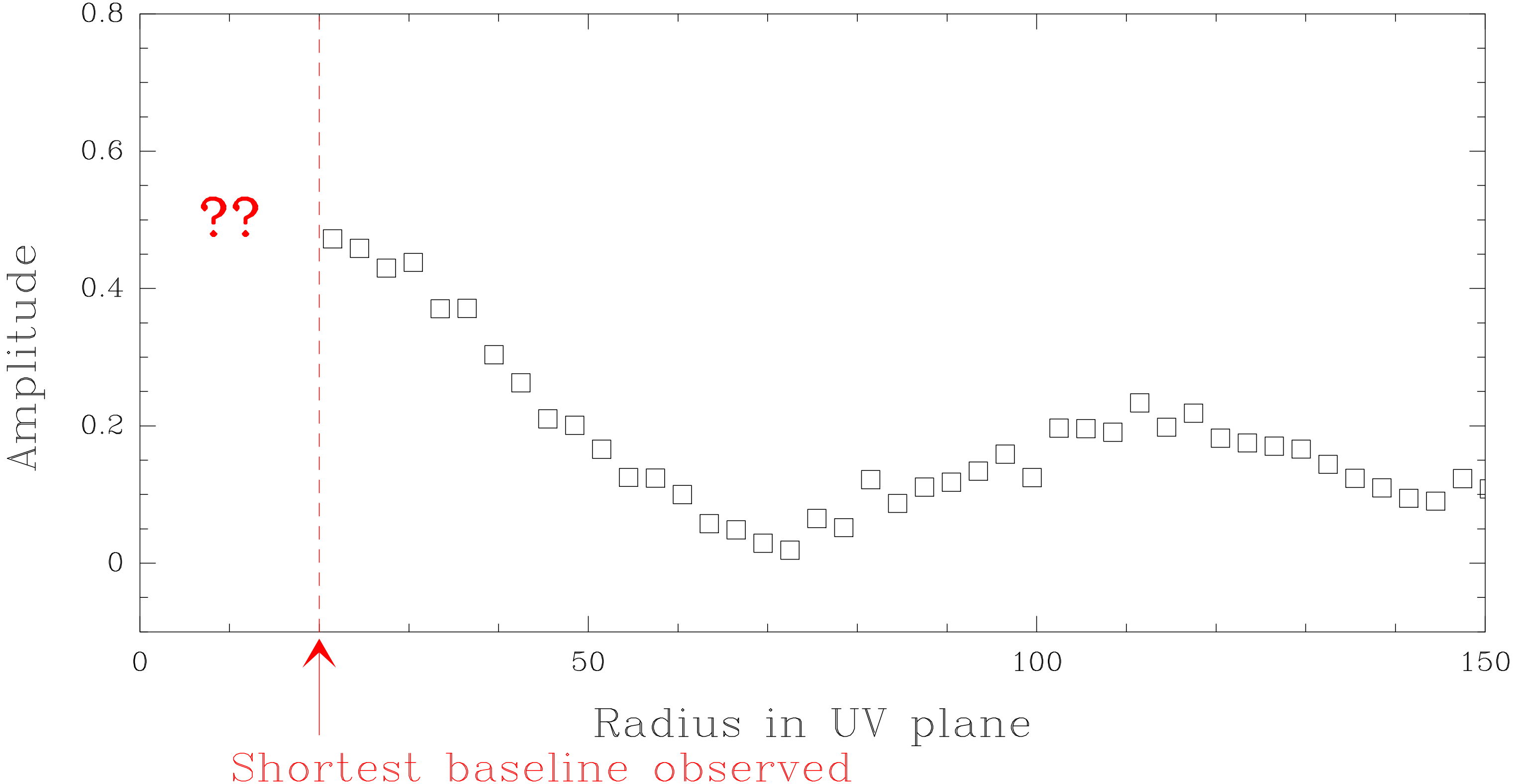
  Solution: try SDI CLEAN, Multi-Scale CLEAN, Multi-Resolution CLEAN, ...

- Non-coplanar baselines

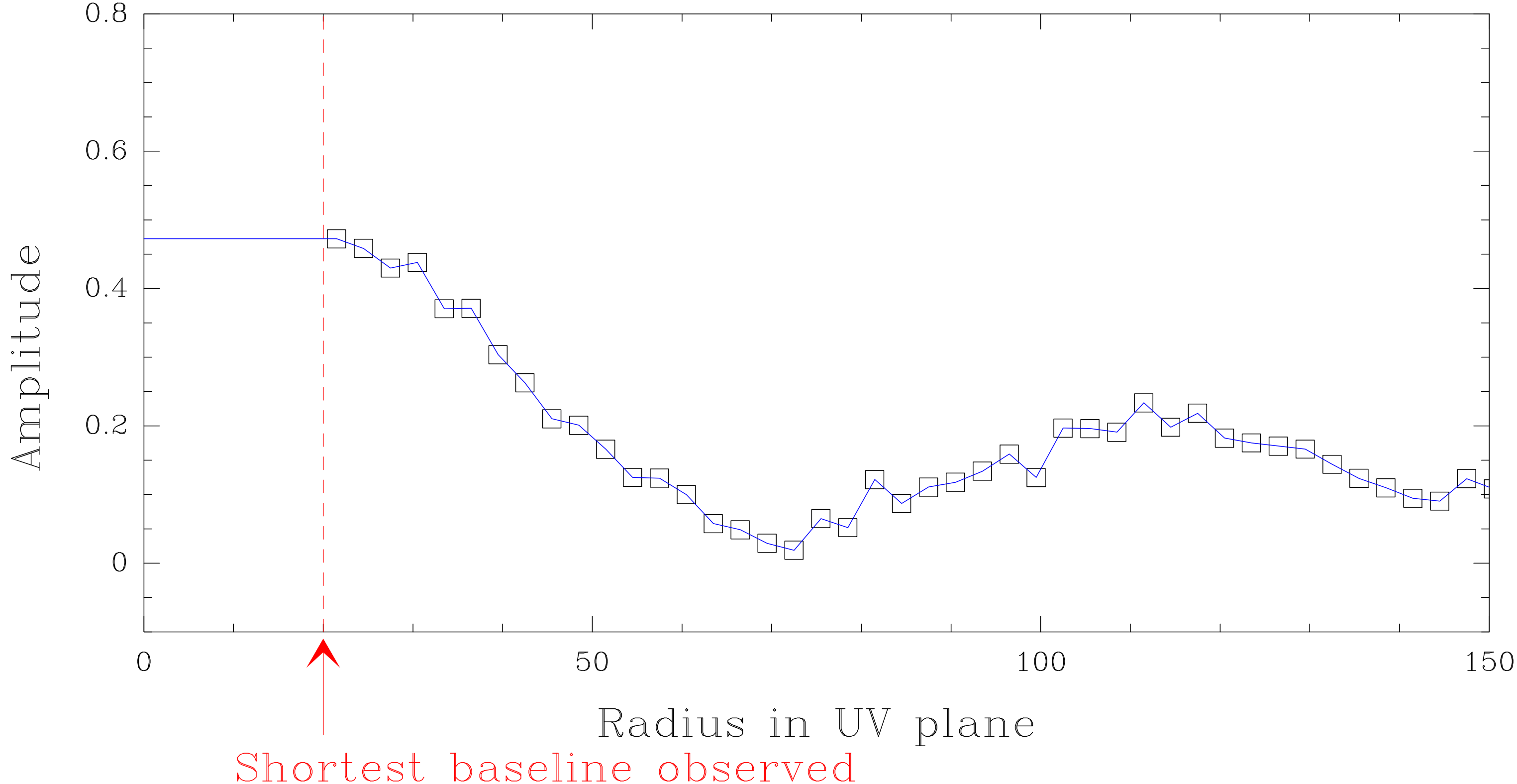
  Solution: use appropriate algorithm if necessary – not the case for mm-interferometers

Short spacings

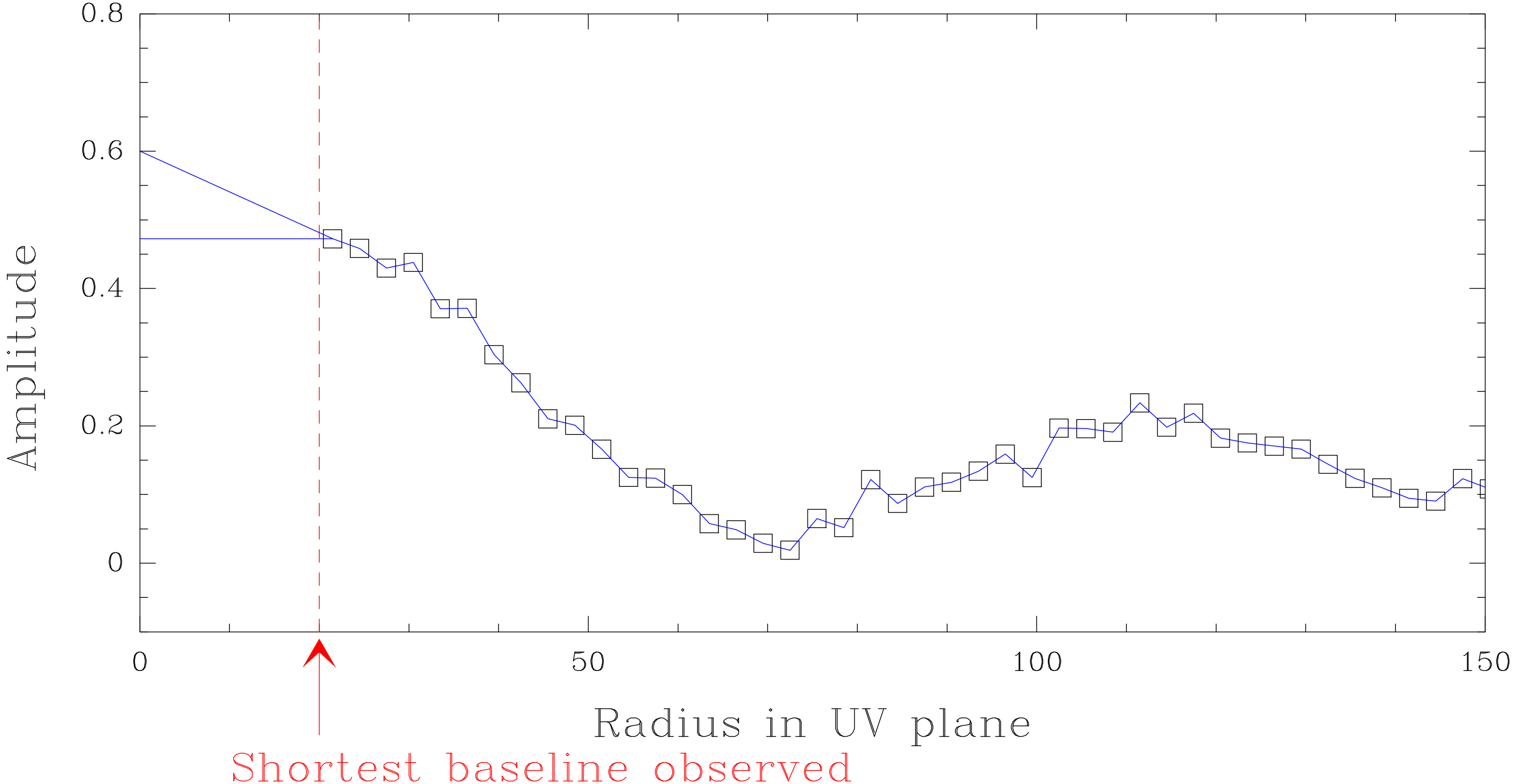
Lack of the short spacings



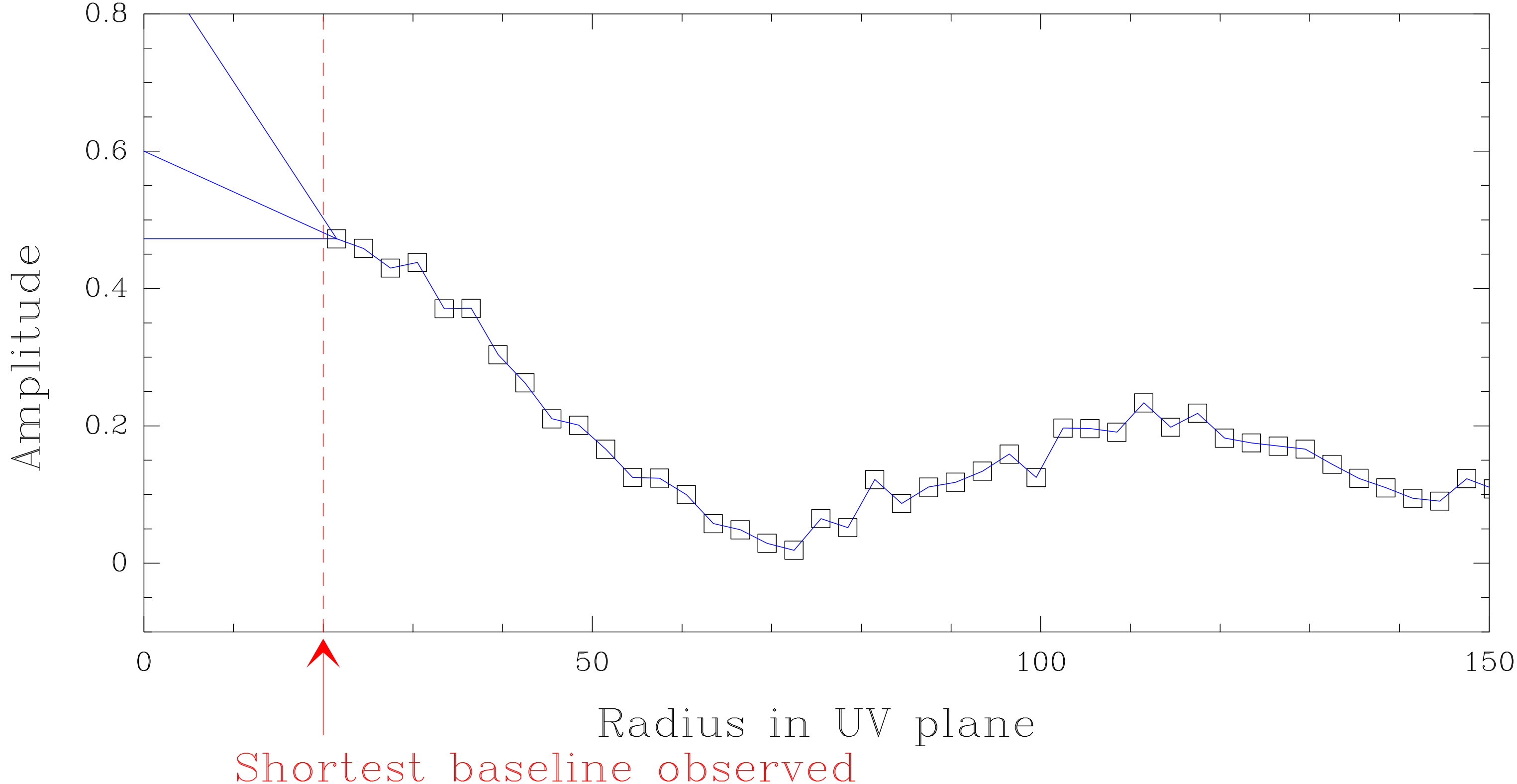
Lack of the short spacings



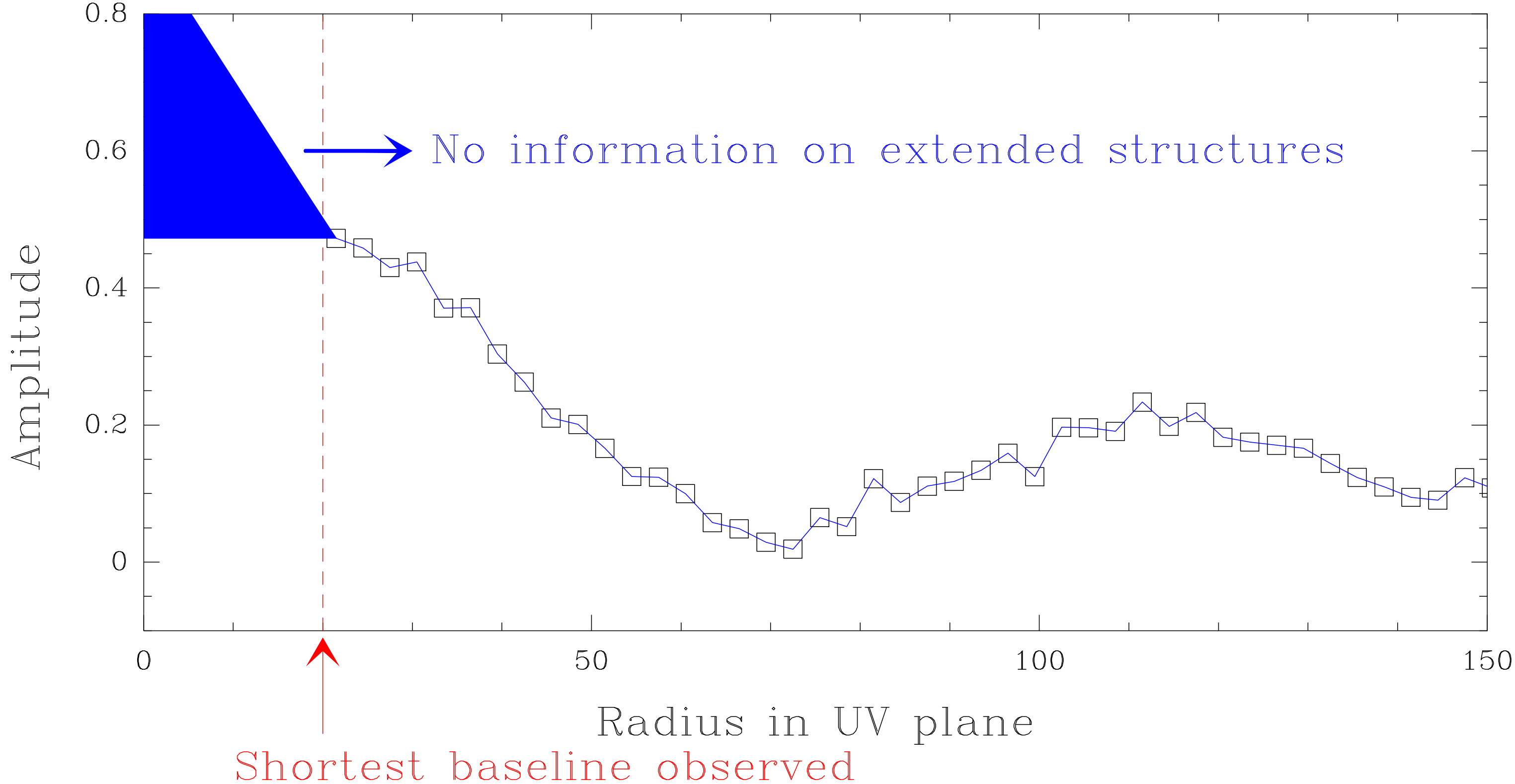
Lack of the short spacings



Lack of the short spacings



Lack of the short spacings



The short spacings problem

Missing short spacings :

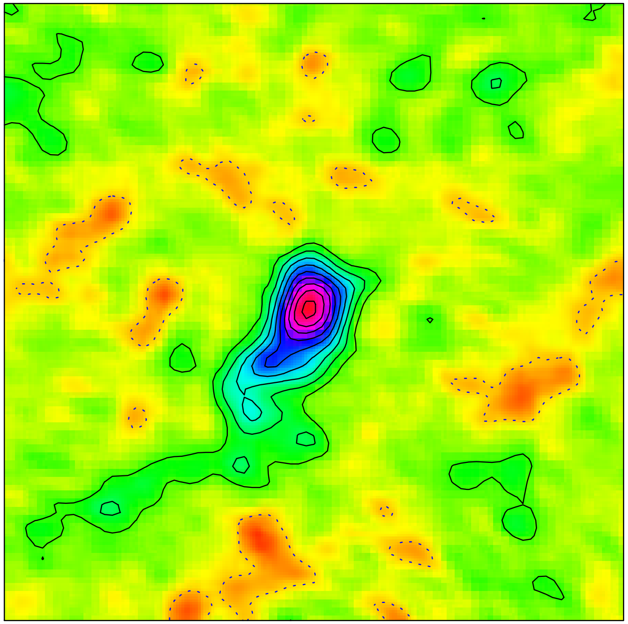
- Shortest baseline $B_{\text{min}} = 24$ m at Plateau de Bure
- Projection effects can reduce the minimal baseline – but baselines smaller than antenna diameter D can never be measured
- In any case: **lack of the short spacings information**

Consequence :

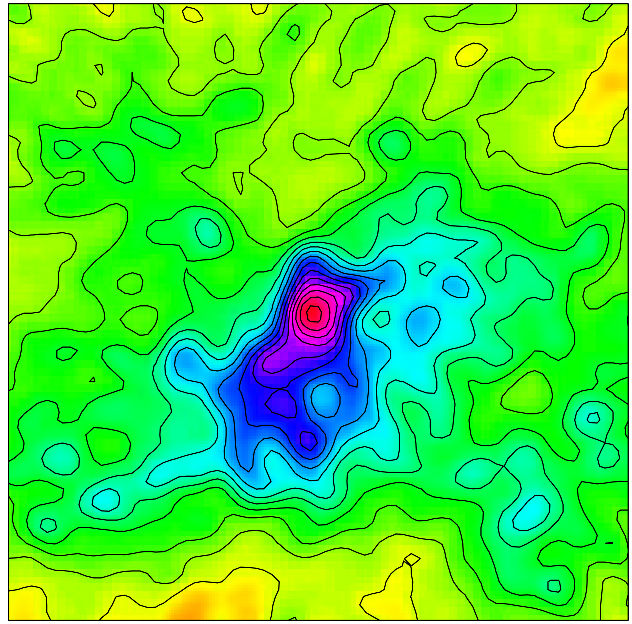
- **The most extended structures are filtered out**
- The largest structures that can be mapped are $\sim 2/3$ of the primary beam (field of view)
- Structures larger than $\sim 1/3$ of the primary beam may already be affected

Short spacings: example

Without short spacings



With short spacings

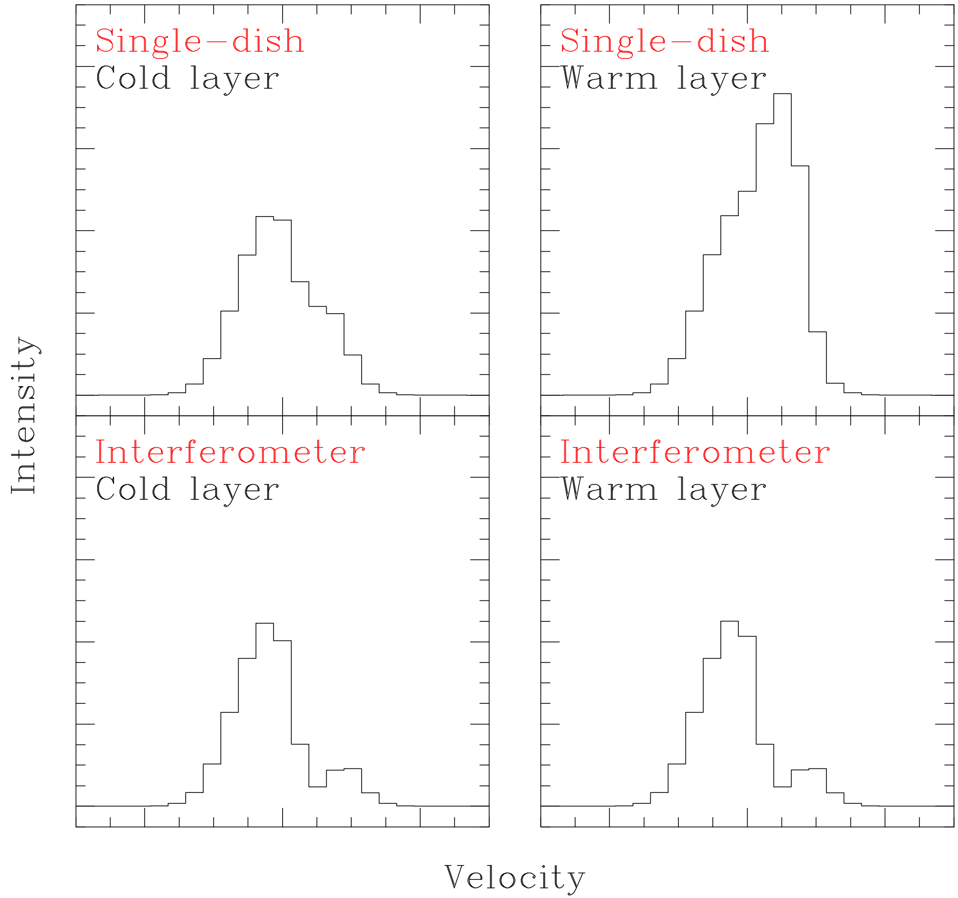


^{13}CO (1–0) in the L 1157 protostar (Gueth et al. 1997)

Short spacings: example

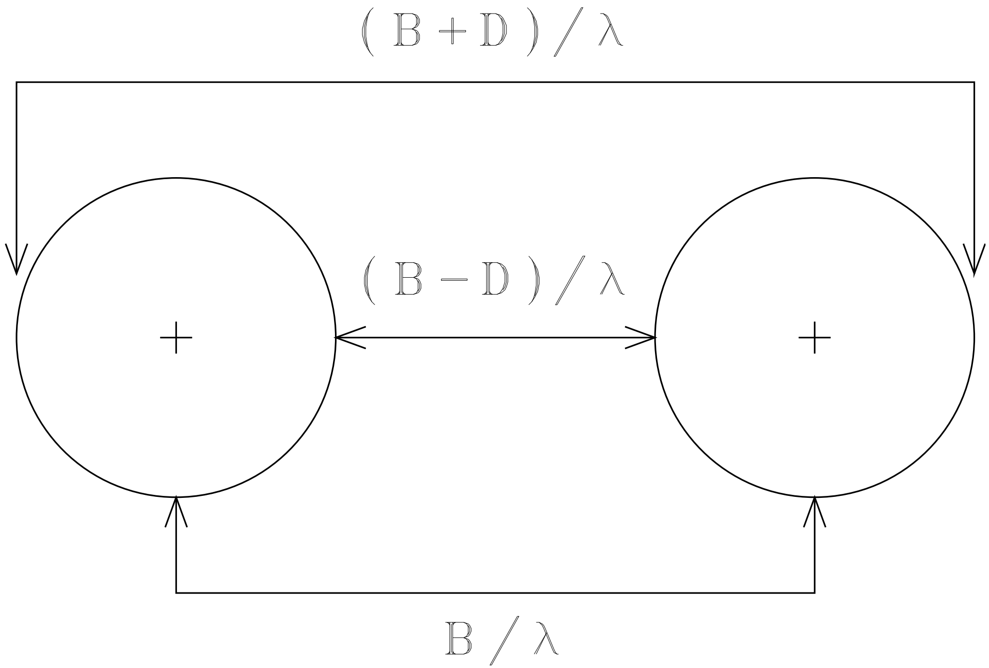
Simulations of small source + extended cold/warm layer

Lack of short spacings can introduce complex artifacts **leading to wrong scientific interpretation**



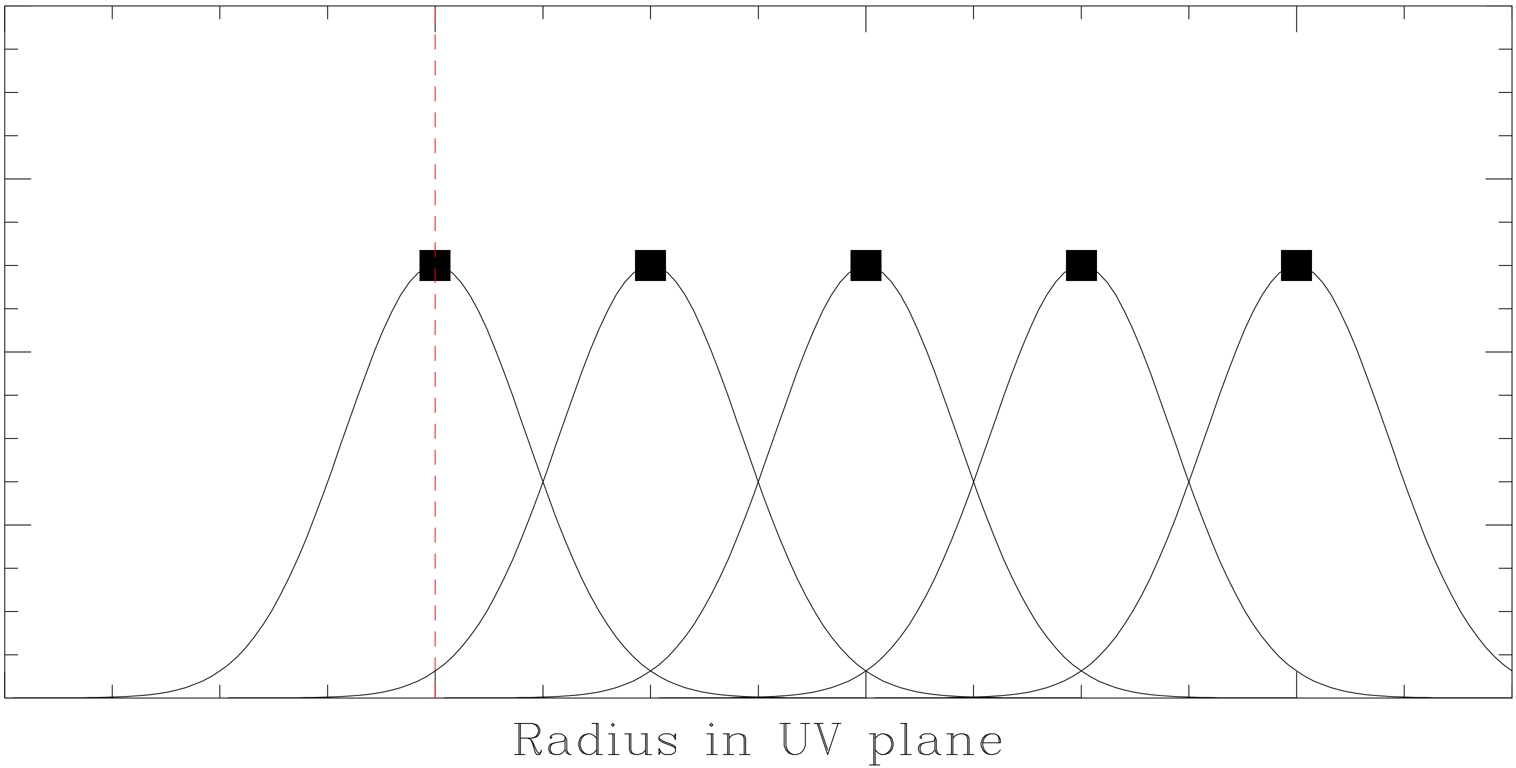
Spatial frequencies: measurements

- A single-dish of diameter D is sensitive to spatial frequencies from **0 to D**
- An interferometer baseline B is sensitive to spatial frequencies from $\mathbf{B - D}$ **to** $\mathbf{B + D}$



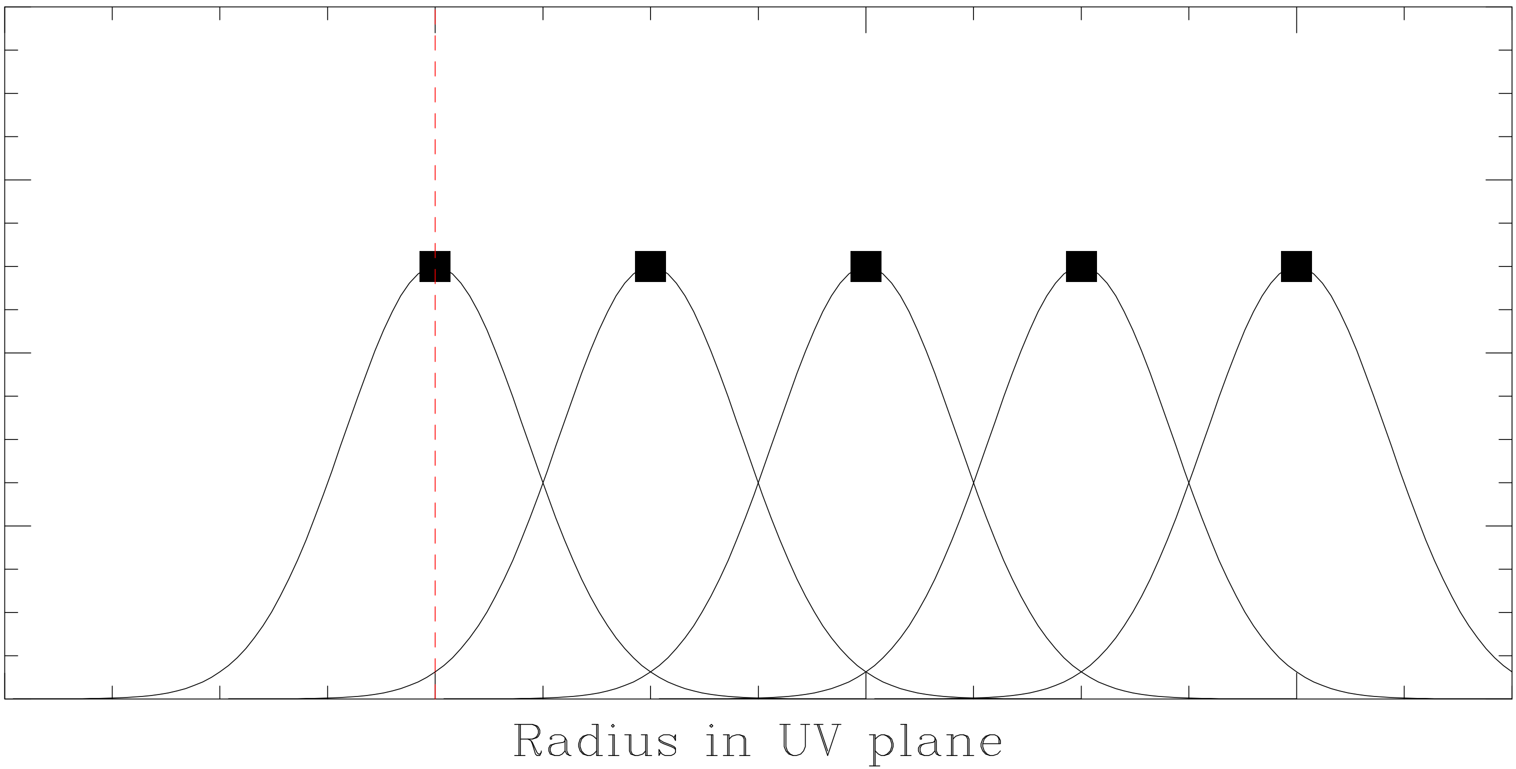
Spatial frequencies: measurements

An interferometer measures the **convolution** of the "true" visibility with the **antenna transfer function**



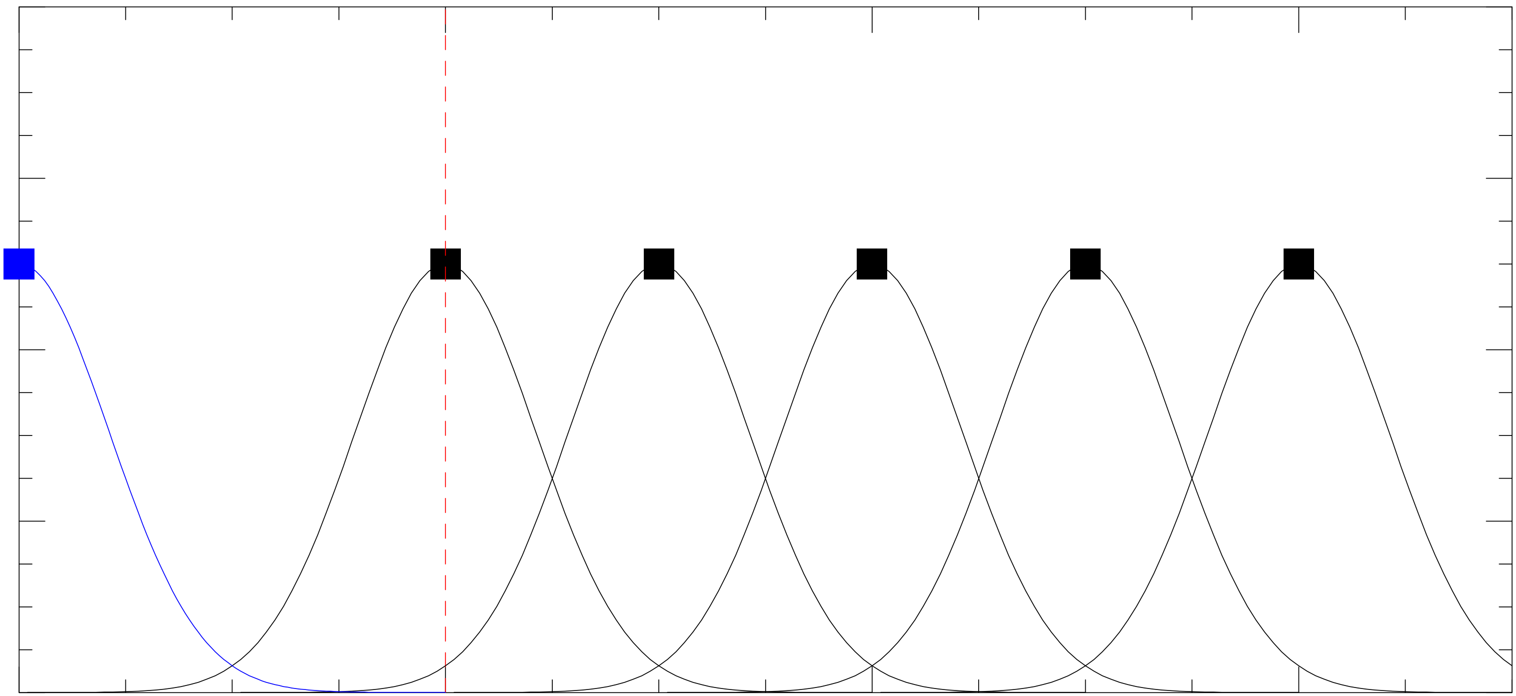
Obtaining short spacings

No short-spacings



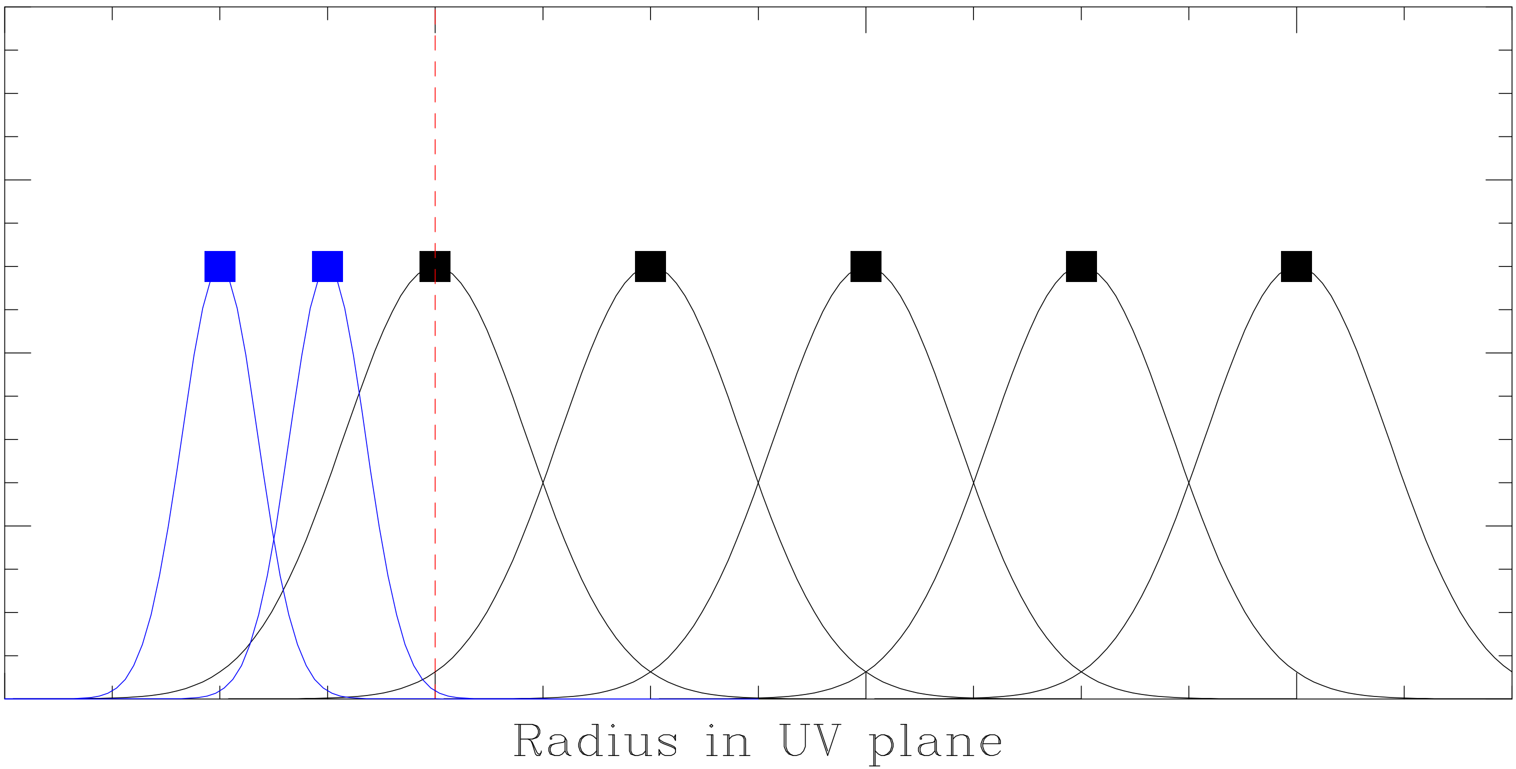
Obtaining short spacings

Single-dish measurement (same antenna diameter)



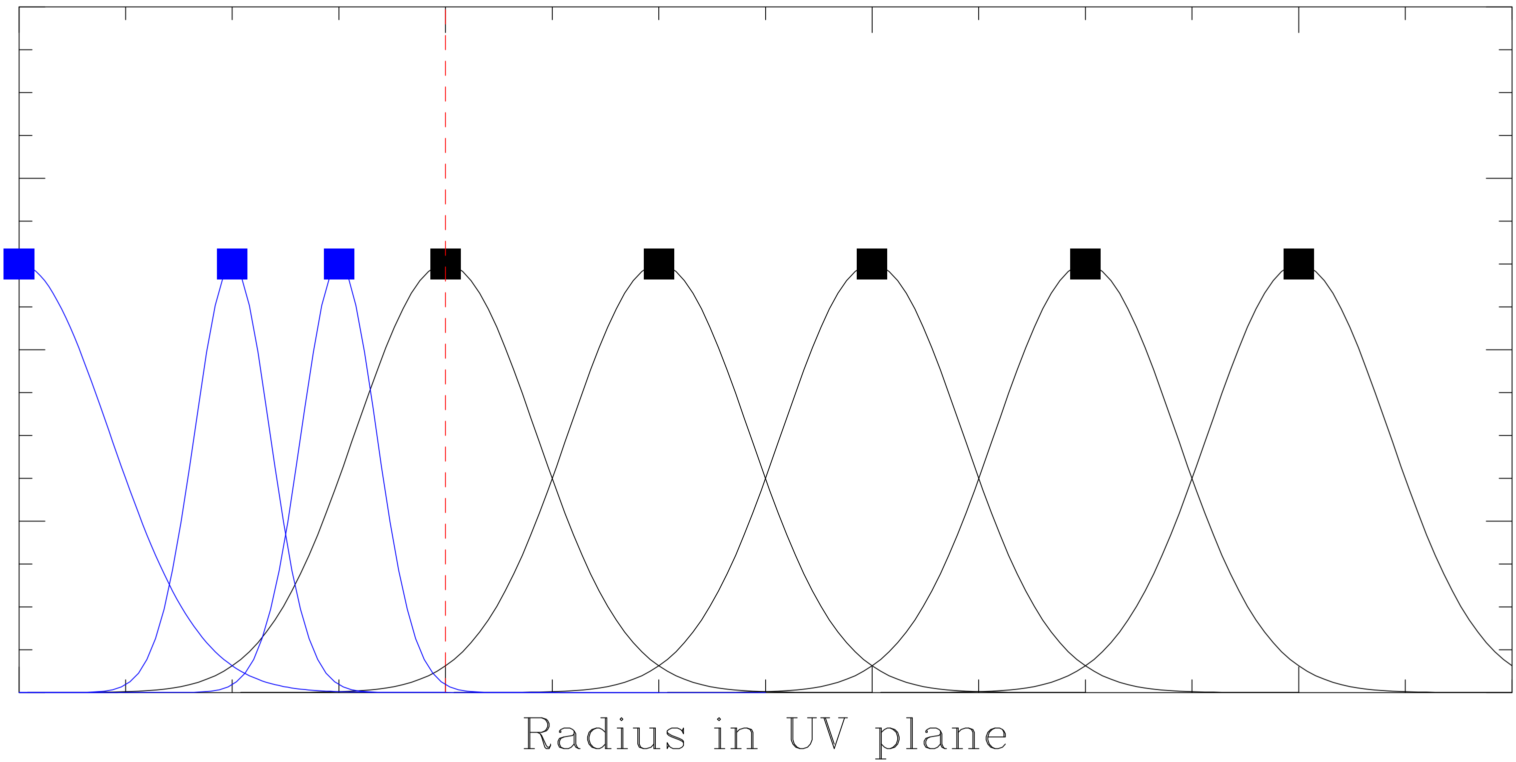
Obtaining short spacings

Interferometer with smaller antennas



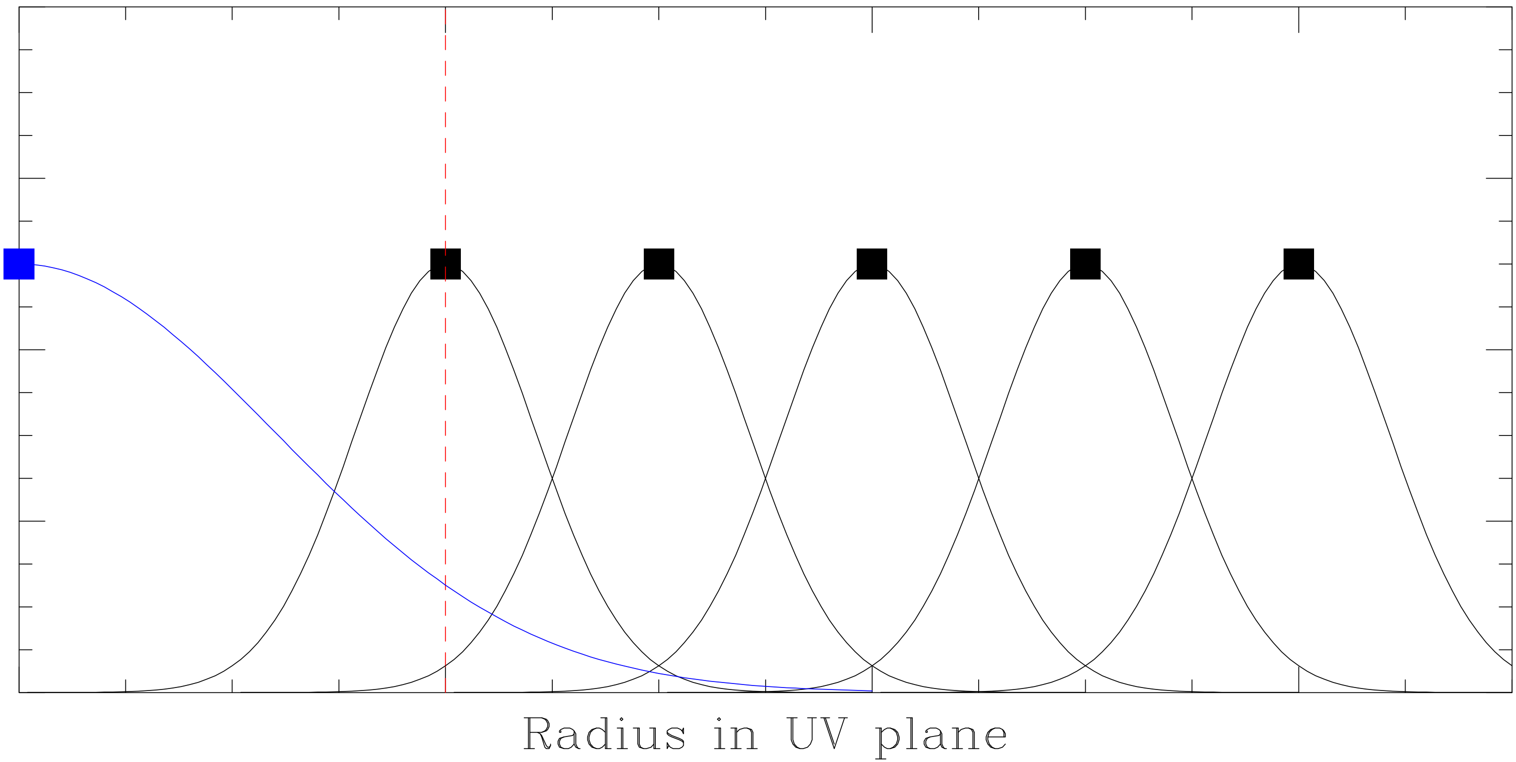
Obtaining short spacings

Small interferometer + Single-dish measurement

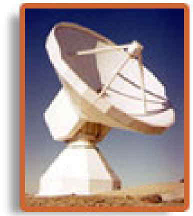


Obtaining short spacings

Single-dish measurement (larger antenna diameter)



IRAM PdBI + IRAM 30–m



- Get zero and short spacings
- Only two instruments to be merged
- Same calibration procedures
- Same software
- Same proposal

Short spacings from SD data

- Combine SD and Interferometric maps in the image plane
- Joint deconvolution (MEM or CLEAN)
- Hybridization: fill inner hole in uv plane with FT of single-dish image
- **Combine data in the uv plane before deconvolution**

  1. Use the 30–m map to simulate what would have observed the PdBI, i.e. extract "pseudo-visibilities"

  2. Merge with the interferometer visibilities

  3. Process (gridding, FT, deconvolution) all data together

  This **drastically improves the deconvolution**

Extracting visibilities

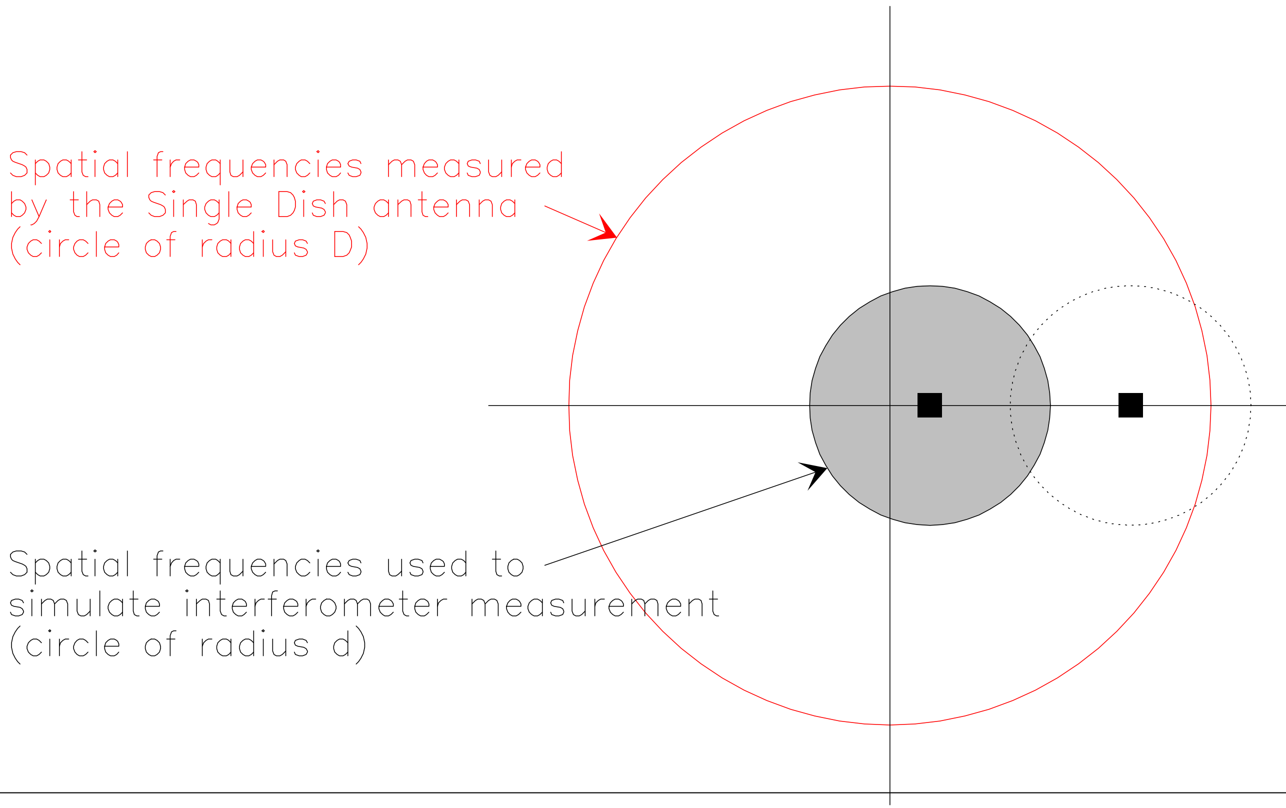
$$\text{SD map} = \mathbf{SD\ beam} * \mathbf{Sky}$$

$$\text{Int. map} = \text{Dirty beam} * (\mathbf{Int\ beam} \times \mathbf{Sky})$$

- Image plane — Gridding of the single-dish data
- Image plane — Extrapolation to zero outside the mapped region
- uv plane — Correction for single-dish beam and gridding function
- Image plane — Multiplication by interferometer primary beam
- uv plane — Extract visibilities up to $\mathbf{D_{SD} - D_{Int}}$
- uv plane — Apply a **weighting factor** before merging with the interferometer data

Spatial frequencies: what can be extracted from SD data

Single-dish data $\Longrightarrow$ pseudo-visibilities from $\mathbf{0\ to\ D_{SD} - D_{Int}}$



Weighting factor

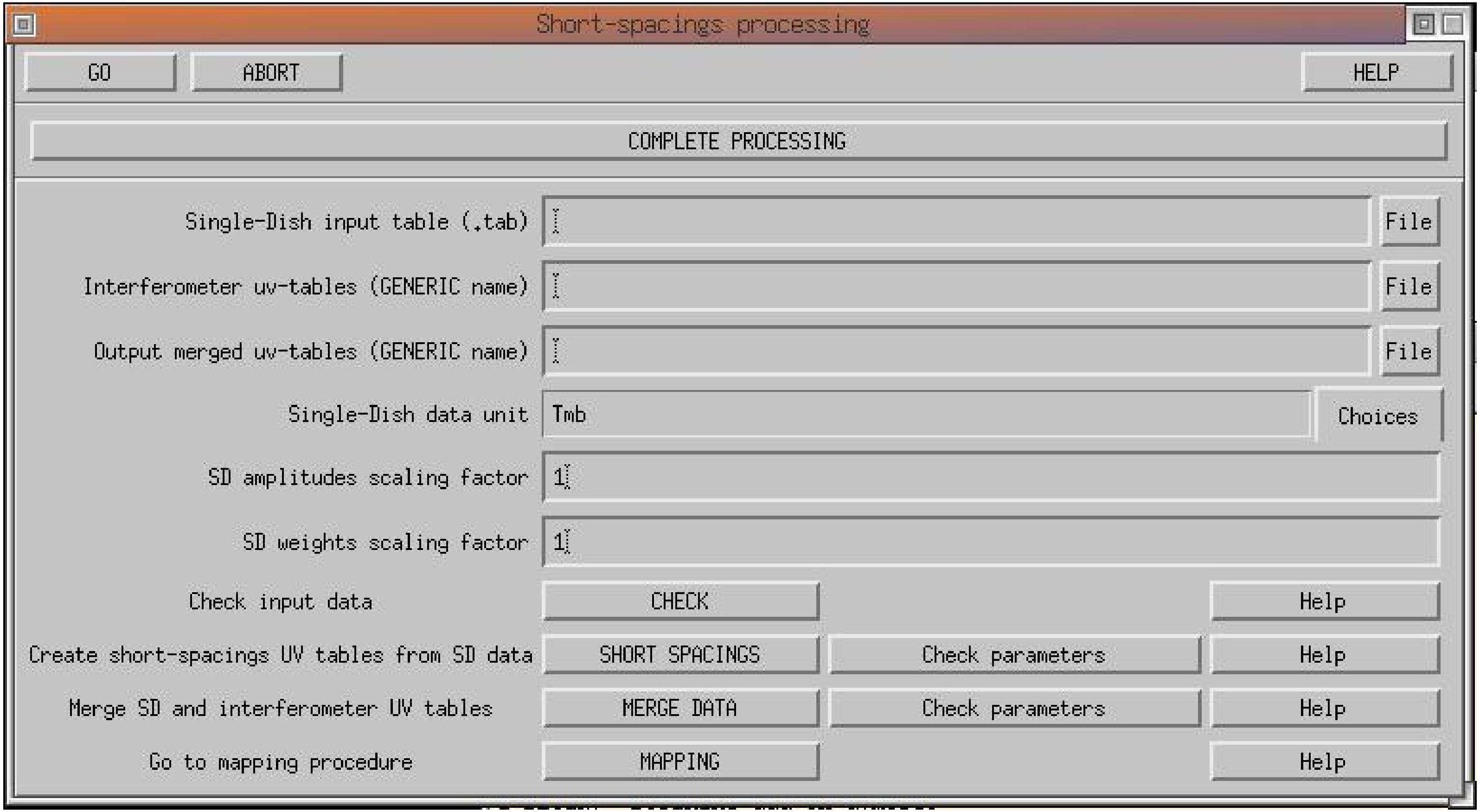
Weighting factor to SD data :

- Produce different images and dirty beams
- Same result after deconvolution, if methods were perfect
- Methods are not perfect, noise $\longrightarrow$ weight to be optimized
- Usually, it is better to **downweight the SD data** (as compared to natural weight)

Optimization :

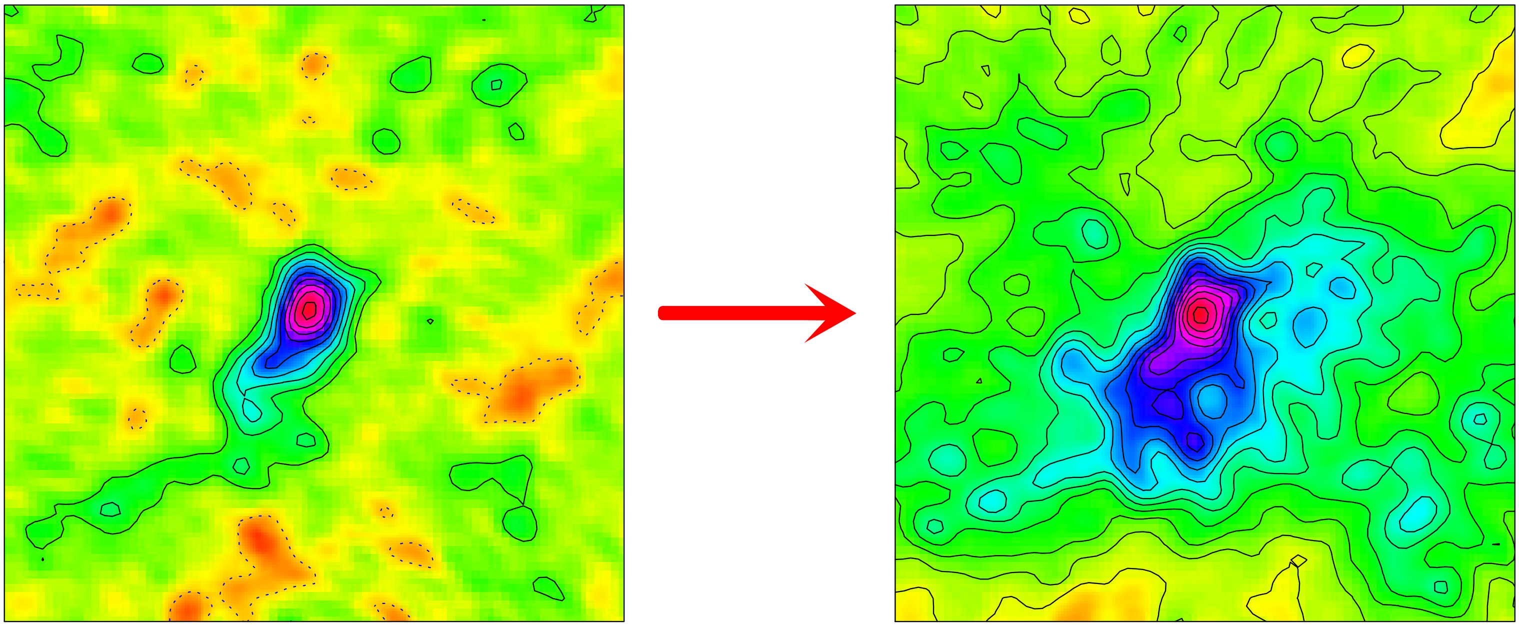
- Adjust the weights so that there is almost **no negative sidelobes** while keeping the highest angular resolution possible
- Adjust the weights so that the **weight densities in 0–D and D–2D** areas are equal $\longrightarrow$ mathematical criteria

GILDAS implementation: user interface



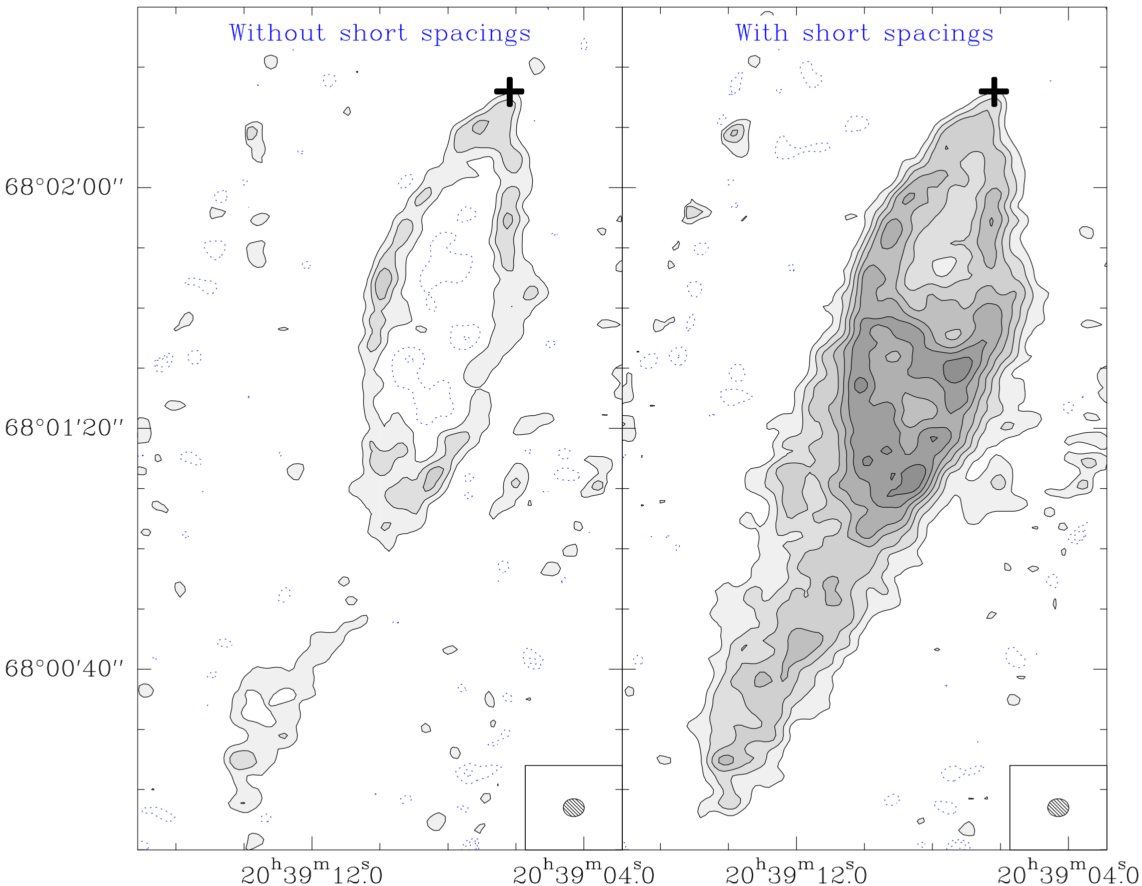
Short spacings: example

Without short spacings | With short spacings



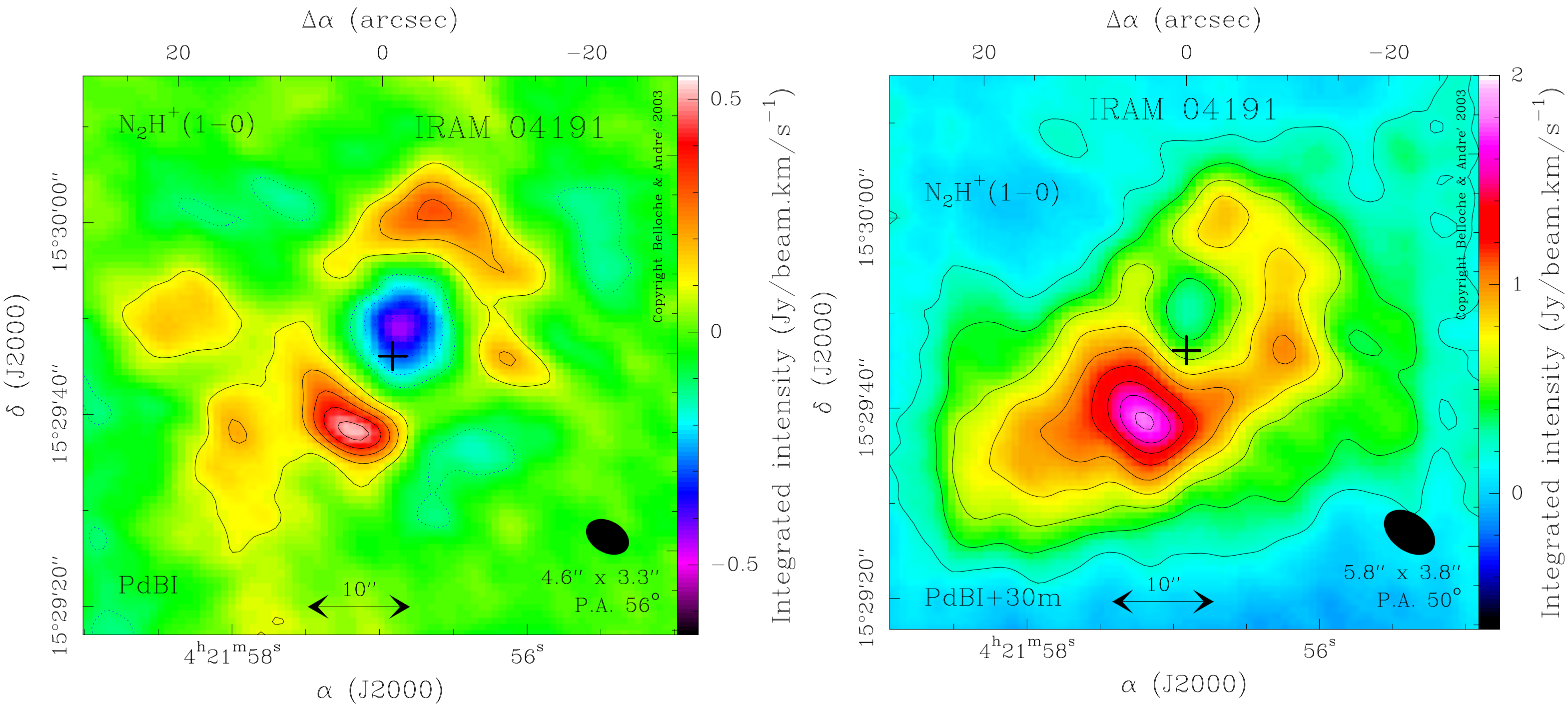
^{13}CO (1–0) in the L 1157 protostar (Gueth et al. 1997)

Short spacings: example



Gueth et al. 1996

Short spacings: example



N_2H^+ in the IRAM 04191 protostar (Belloche et al. 2004)

Mosaics

Interferometer field of view

Measurement equation of an interferometric observation:

$$\mathbf{F} = \mathbf{D} * (\mathbf{B} \times \mathbf{I}) + \mathbf{N}$$

$$\begin{array}{lcl} F & = & \text{dirty map} = \text{FT of observed visibilities} \\ D & = & \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ B & = & \text{primary beam} = \text{FT of transfer function} \\ I & = & \text{sky brightness distribution} = \text{FT of ``true'' visibilities} \\ N & = & \text{noise distribution} \end{array}$$

- **An interferometer measures the product $\mathbf{B} \times \mathbf{I}$**
- B has a finite support $\longrightarrow$ limits the size of the field of view
- $B \sim$ **Gaussian** $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

$$\begin{array}{ccc} \text{Aperture function} & \rightleftharpoons & \text{Voltage pattern} \\ \star \big\downarrow & & \big\downarrow |\cdot|^2 \\ \text{Transfer function } T(u,v) & \rightleftharpoons & \text{Power pattern } B(\ell,m) \\ & & = \textbf{Primary beam} \end{array}$$

Gaussian illumination $\Longrightarrow B \sim$ Gaussian of $\mathbf{1.2\,\lambda/D}$ FWHM

Plateau de Bure
$D = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	$58''$
100 GHz	3.0 mm	$50''$
115 GHz	2.6 mm	$43''$
215 GHz	1.4 mm	$23''$
230 GHz	1.3 mm	$22''$
245 GHz	1.2 mm	$20''$

Mosaicing with the PdBI

Mosaic :

- **Field spacing = half the primary beam FWHM** i.e. one field each 11$''$ at 230 GHz
- Observations with two receivers: choice of the spacing for one frequency $\longrightarrow$ under- or oversampling for the other frequency **NO LONGER VALID**
- Mosaic at 3 mm $\longrightarrow$ no mosaic at 1 mm **WITH NEW RECEIVERS**

Observations :

- **Fields are observed in a loop**, each one during a few minutes $\longrightarrow$ similar atmospheric conditions (noise) and similar uv coverages (dirty beam, resolution) for all fields

Mosaicing with the PdBI

Size of the mosaic :

- Observing time to be minimized, uv coverage to be maximized $\longrightarrow$ maximal number of fields ~ 20

Calibration :

- Procedure identical with any other Plateau de Bure observations (only the calibrators are used)
- Produce one dirty map per field

Short spacings :

- Visibilities from 30–m data are computed and merged with Plateau de Bure data **for each field** $\longrightarrow$ **process as a normal mosaic**

Mosaic reconstruction

- Forgetting the effects of the dirty beam:

$$F_i = B_i \times I + N_i$$

- This is similar to several measurements of I, each one with a "weight" B_i
- Best estimate of I in least-square formalism (assuming same noise):

$$\mathbf{J = \frac{\sum_i B_i\, F_i}{\sum_i B_i^2}}$$

- J is homogeneous to I, i.e. the mosaic is **corrected for the primary beam attenuation**

Noise distribution

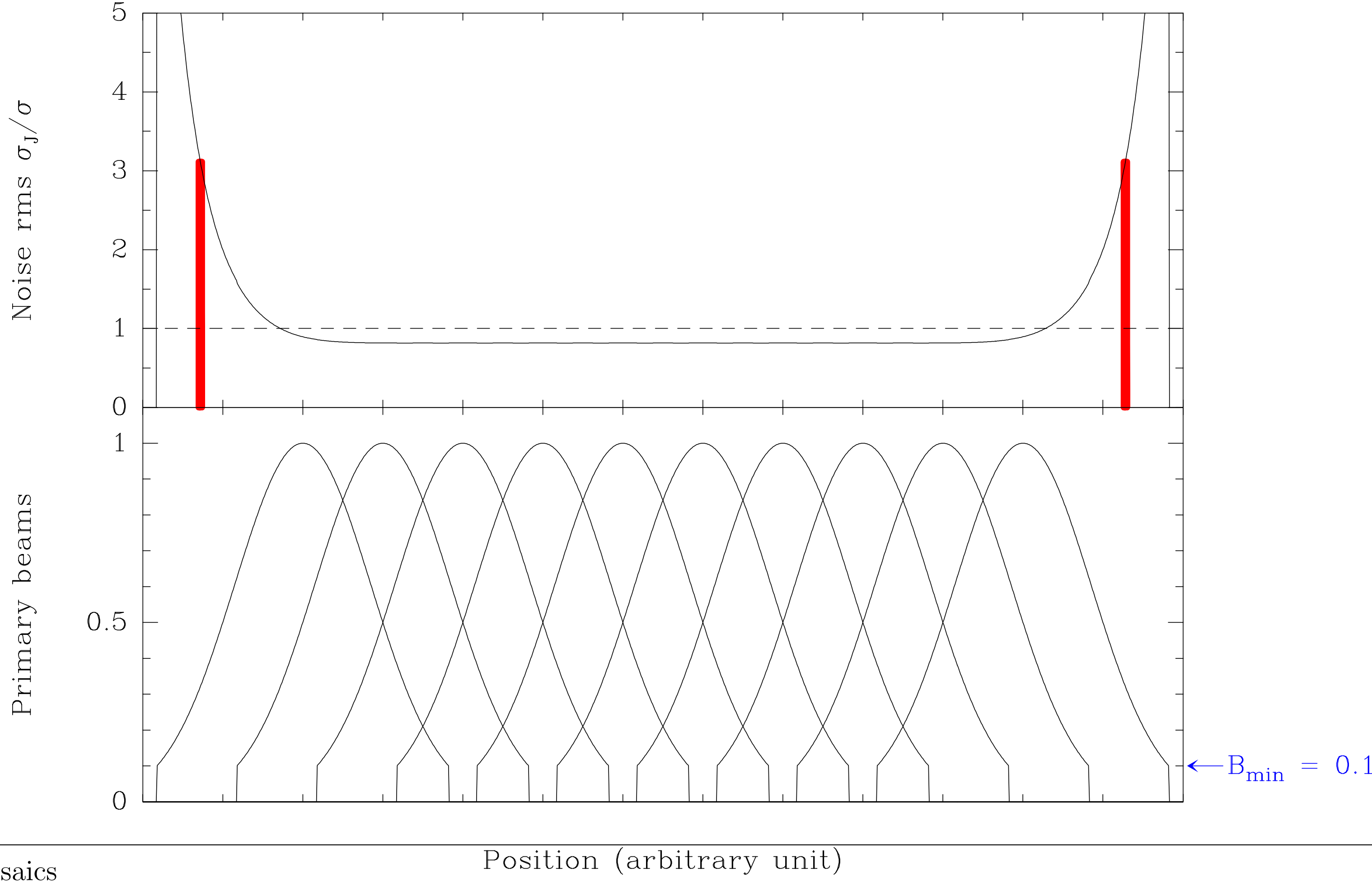
$$J = \frac{\sum_i B_i F_i}{\sum_i B_i^2} \quad \Longrightarrow \quad \sigma_J = \sigma \frac{1}{\sqrt{\sum B_i{}^2}}$$

The noise depends on the position and strongly increases at the edges of the field of view

In practice :

- Use **truncated primary beams** ($B_{\mathrm{min}} = 0.1 - 0.3$) to avoid noise propagation between adjacent fields
- **Truncate the mosaic**

Noise distribution



Mosaic deconvolution

- **Linear mosaicing:** deconvolution of each field, then mosaic reconstruction

  Non-linear mosaicing: mosaic reconstruction, then global deconvolution
- The two methods are not equivalent, because the deconvolution algorithms are (highly) non-linear
- **Non-linear mosaicing gives better results**
  - sidelobes removed in the whole map
  - better sensitivity
- Plateau de Bure mosaics: **non-linear joint deconvolution based on CLEAN**

Mosaic CLEAN

Signal-to-noise distribution :

$$\mathbf{H} = \frac{\mathbf{J}}{\sigma_{\mathbf{J}}} = \frac{1}{\sigma} \frac{\sum B_i^t \left[D_i * (B_i\, I) + N_i \right]}{\sqrt{\sum B_i^{t\,2}}}$$

Mosaic CLEAN :

- J has a non-uniform noise level
- It is safer to search for CLEAN components on H
- **Find positions of components on H**
- **Correct J**

Mosaic CLEAN

(1) Find `CLEAN` component: **position of the maximum of H and intensity of J** (even if it is not the maximum of J)

(2) Remove corresponding point source **from J and H**

$$J_{k+1} = J_k - \frac{\sum B_i^t \left[D_i * \left[B_i \, \delta_k \right] \right]}{\sum B_i^{t2}}$$

$$H_{k+1} = H_k - \frac{\sum B_i^t \left[D_i * \left[B_i \, \delta_k \right] \right]}{\sigma \sqrt{\sum B_i^{t2}}}$$

Mosaic CLEAN

(3) **Identify I and the sum of CLEAN components**

(4) Clean map:

$$M = C * \sum \delta_k + J_{k_{\max}}$$

C = clean beam
$J_{k\max}$ = final residuals

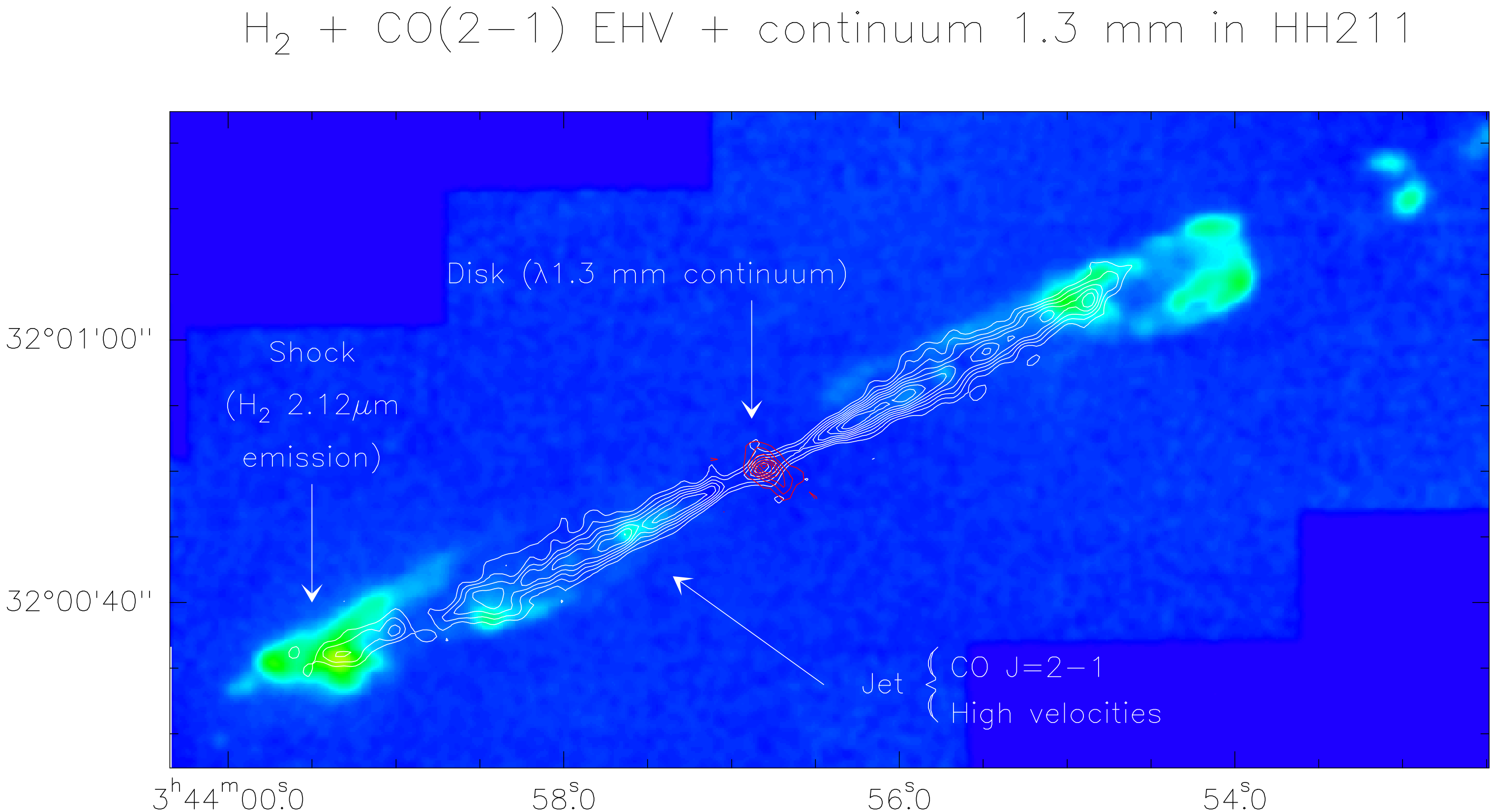
- **The algorithms CLARK, SDI, and MX can be adapted in a similar way: find position of CLEAN components on H, and correct J**
- This is not feasible for `MRC` – because this method relies on a linear measurement equation, which is not the case for mosaics

GILDAS implementation

The mosaicing algorithm is implemented in **MAPPING** for the `HOGBOM`, `CLARK`, and `SDI` methods.

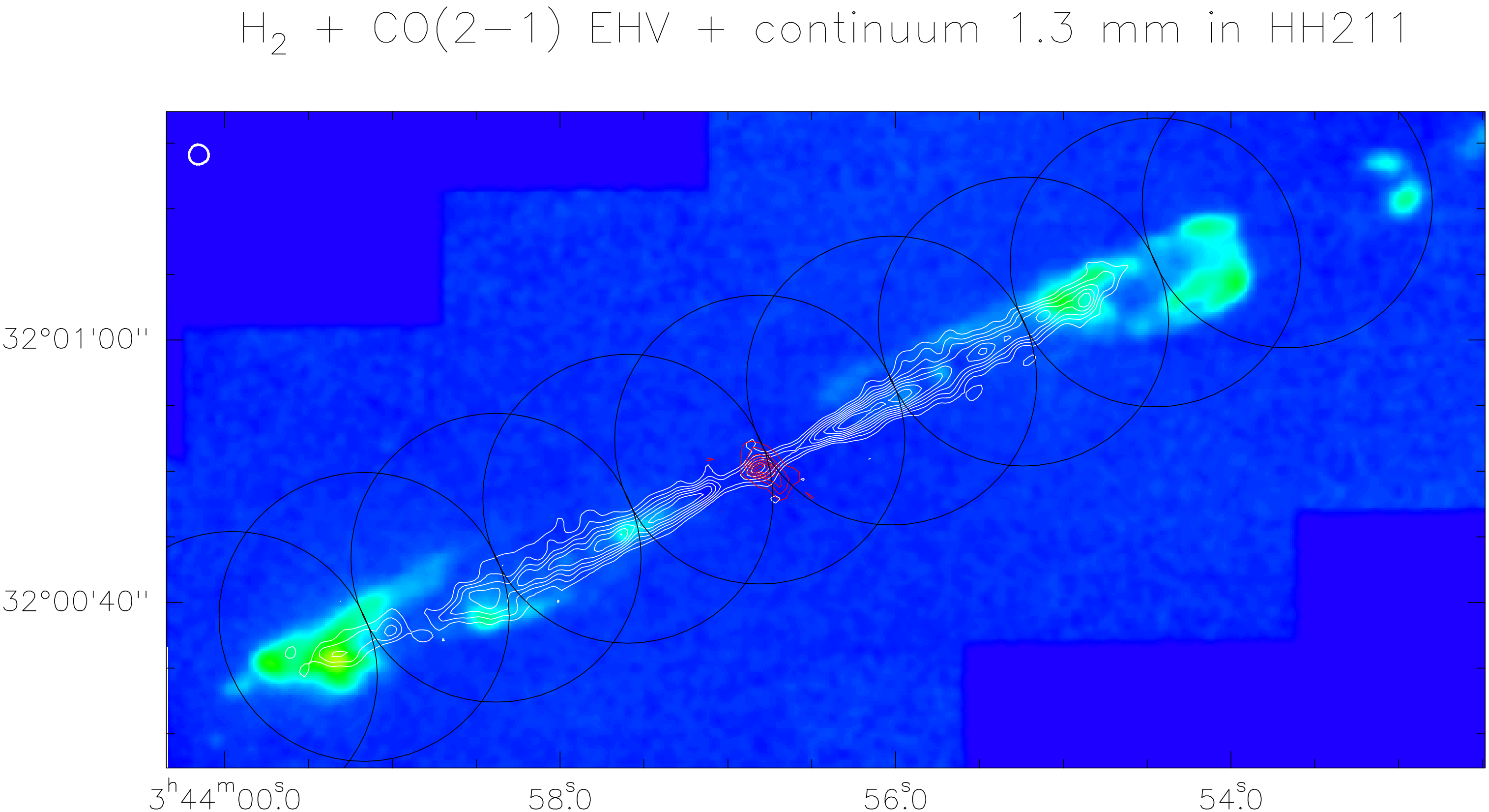
- Create a dirty map for each field, with the same phase center.
- Combine the fields to produce the dirty mosaic. Input parameters: primary beam width and truncation level ($B_{\min} \sim 0.1 - 0.3$).
- Mosaic mode switched on when loading a mosaic. Same parameters as normal deconvolution: windows, maximal number of iterations,...
- Clean beam is computed from the *first* field
- Mosaic has to be truncated at some value of σ_J. Default: truncation at $\sigma_J/\sigma = 1/\sqrt{B_{\min}}$.

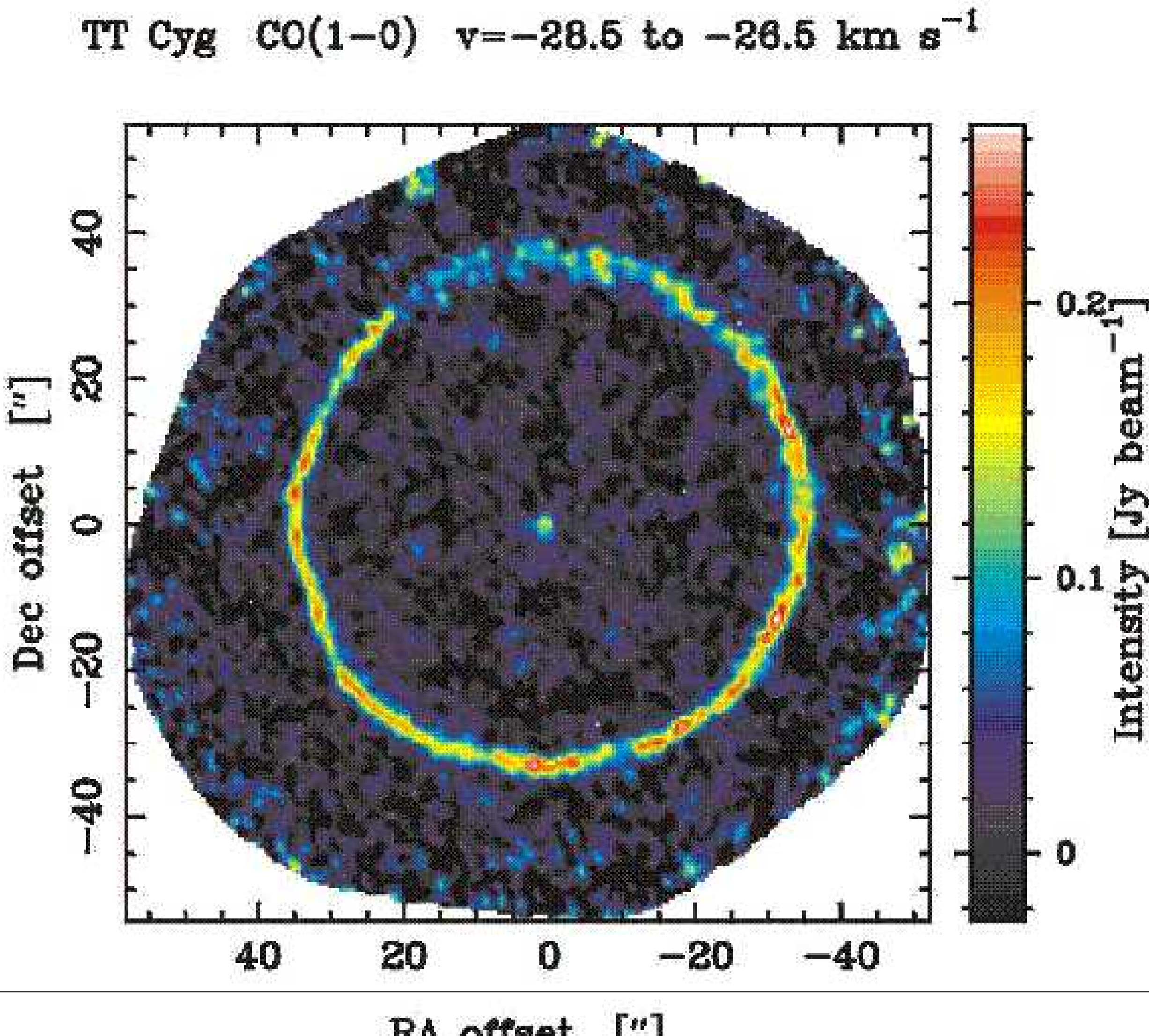
Mosaics: example



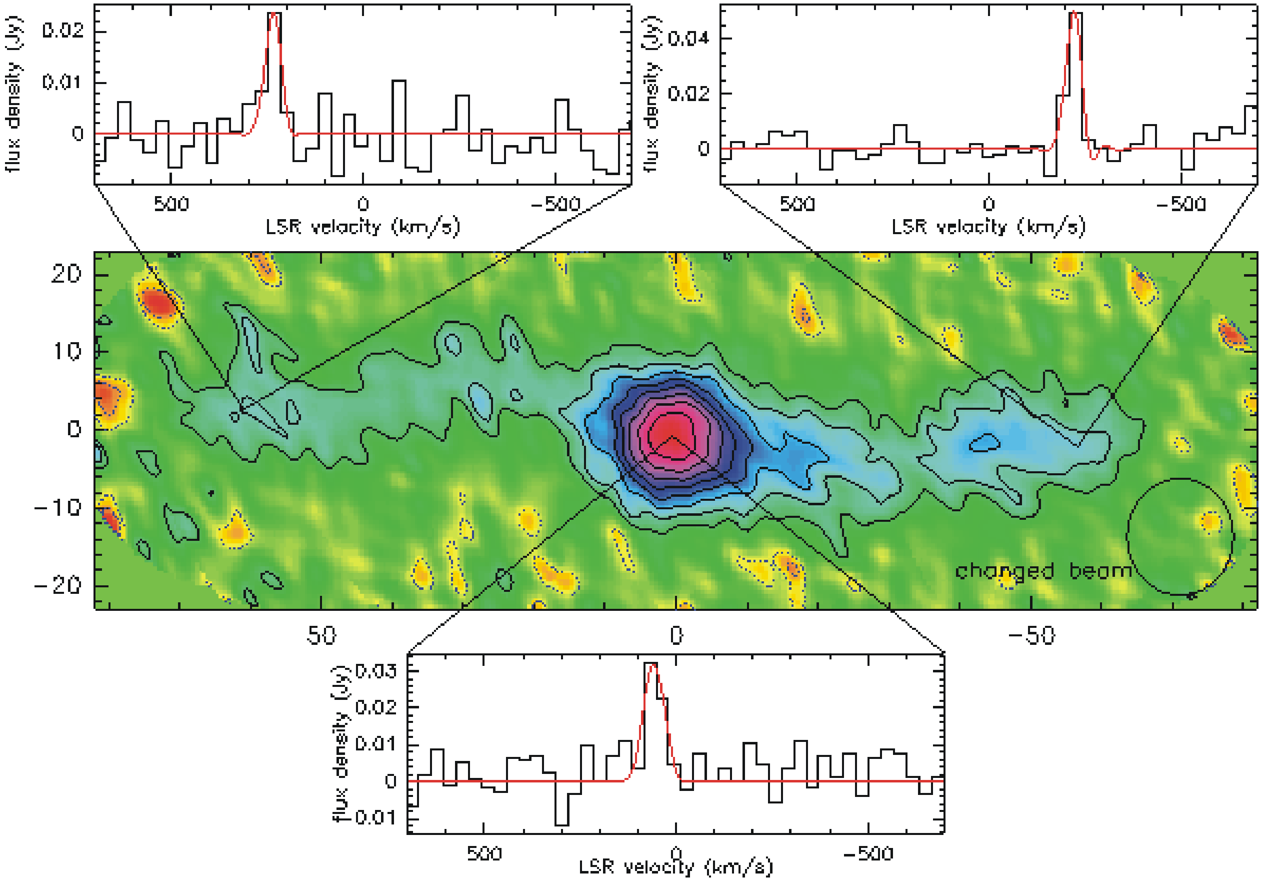
(Gueth & Guilloteau 1999)

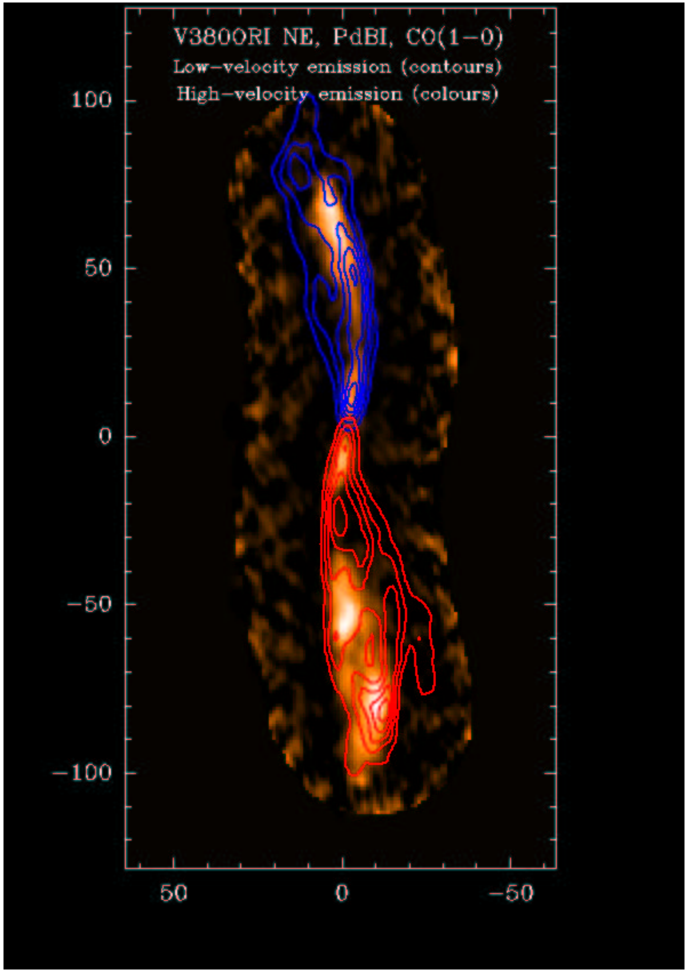
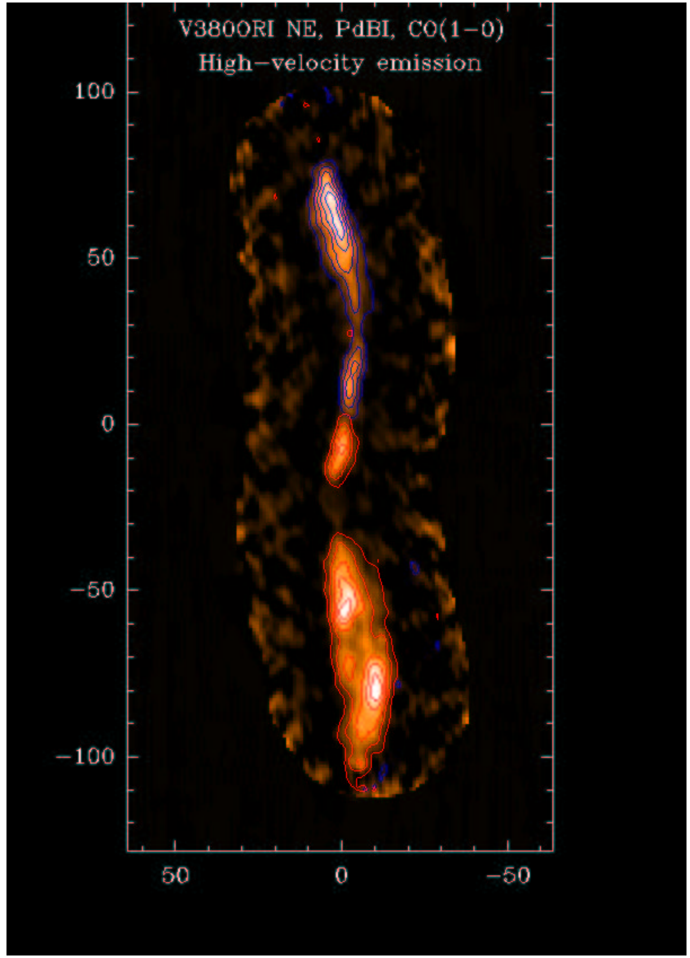
Mosaics: example

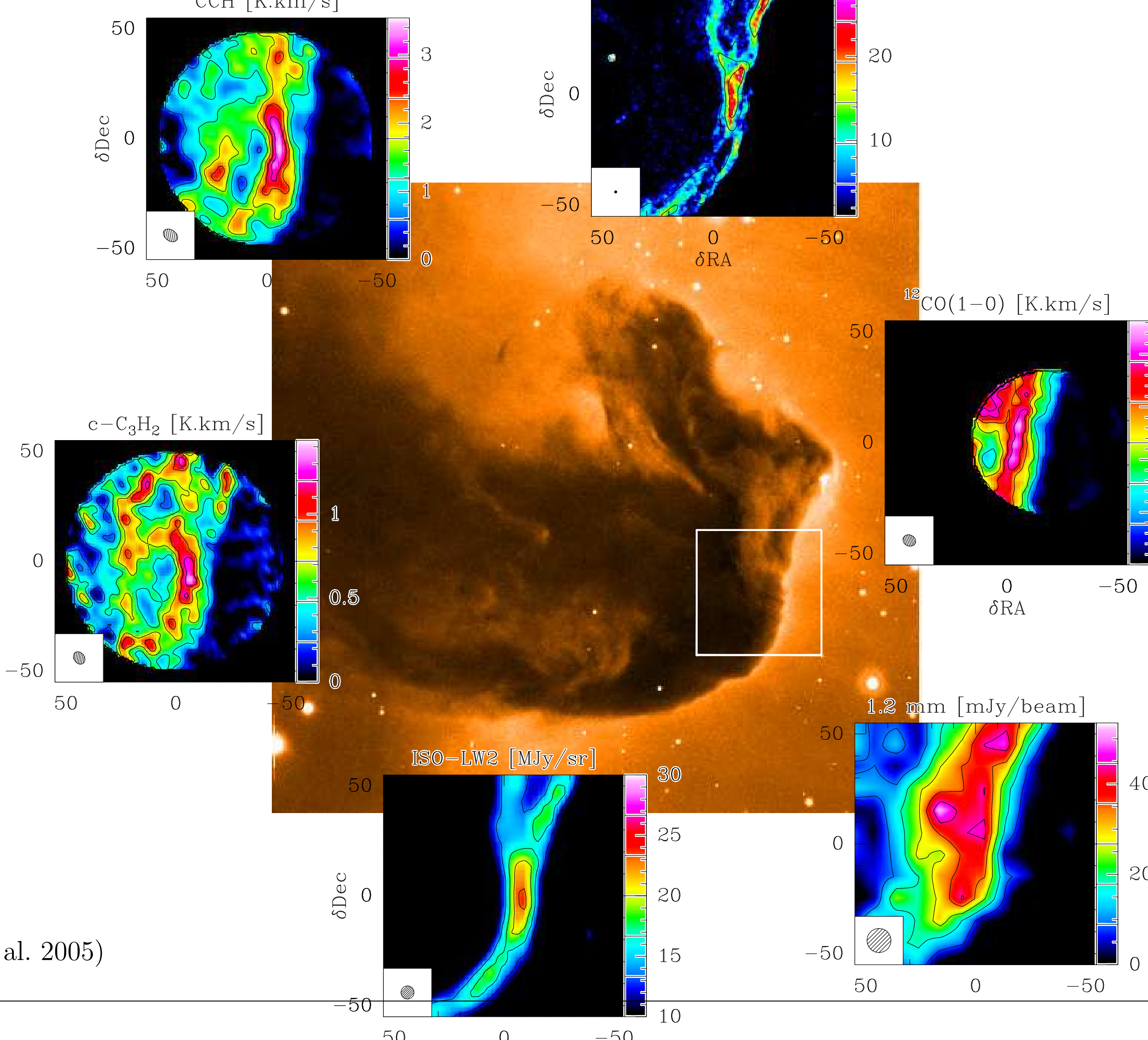



TT Cyg CO(1−0) v=−28.5 to −26.5 km s⁻¹
Dec offset ["]
RA offset ["]
Intensity [Jy beam⁻¹]
40
20
0
−20
−40
0.2
0.1
0

CO in the warped galaxy NGC 3718




V380ORI NE, PdBI, CO(1−0)
Low−velocity emission (contours)
High−velocity emission (colours)
100
50
0
−50
−100
50
0
−50

V380ORI NE, PdBI, CO(1−0)
High−velocity emission
100
50
0
−50
−100
50
0
−50



(Pety et al. 2005)

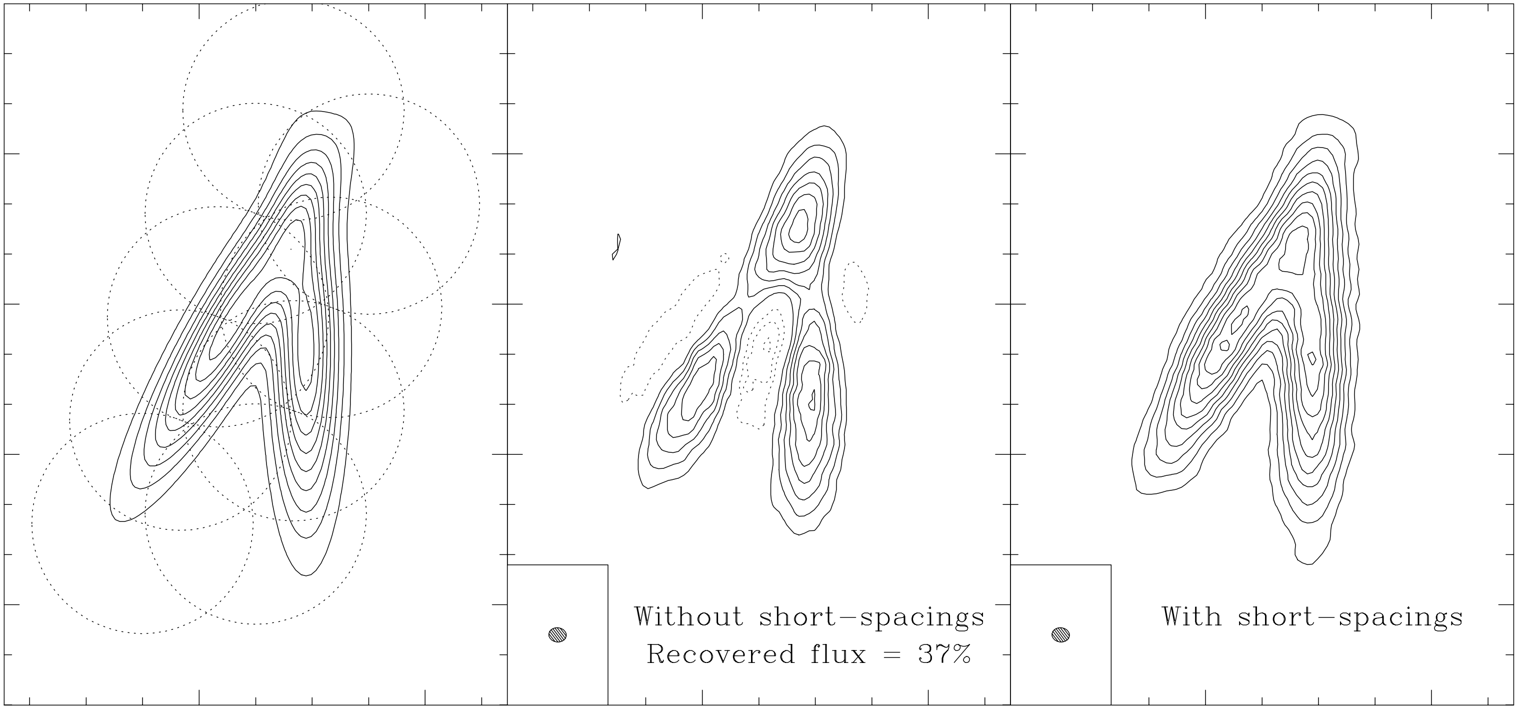
Mosaics and short spacings

Mosaics and short spacings

Effect of missing short spacings more severe on mosaics than on single-field images:

- Extended structures are filtered out in each field
- Lack of information on an **intermediate scale** as compared to the mosaic size
- Possible artefact: extended structures split in several parts
- **In most cases cases, adding the short spacings is required**

Mosaics and short spacings



Mosaics and short spacings

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- Lack of information on an **intermediate scale** as compared to the mosaic size
- Possible artefact: extended structures split in several parts
- **In most cases cases, adding the short spacings is required**

However, **mosaics are able to recover part of the short spacings information**

Image formation in a mosaic

Ekers & Rots's analysis: ideal "on-the-fly" mosaic: (u, v) fixed, (ℓ_p, m_p) continuously modified, visibility $V_{\rm mes}$ monitored

- Phase center = Pointing center = $(0, 0)$

$$V_{\rm mes}(u, v) = [\mathrm{FT}(B \times I)](u, v) = \iint_{-\infty}^{+\infty} B(\ell, m)\, I(\ell, m)\, \mathrm{e}^{-2i\pi(u\ell+vm)}\, d\ell\, dm$$

- Phase center $(0, 0) \neq$ Pointing center (ℓ_p, m_p)

$$V_{\rm mes}(u, v, \ell_p, m_p) = \iint_{-\infty}^{+\infty} B(\ell - \ell_p, m - m_p)\, \underbrace{I(\ell, m)\, \mathrm{e}^{-2i\pi(u\ell+vm)}}_{\mathcal{F}(u,v,\ell,m)}\, d\ell\, dm$$

Image formation in a mosaic

- $V_{\rm mes}$ can be written as a convolution product:

$$V_{\rm mes}(u,v,\ell_p,m_p) = B(\ell_p,m_p) * \mathcal{F}(u,v,\ell_p,m_p)$$

- Fourier transform of $V_{\rm mes}$ with respect to (ℓ_p,m_p):

$$\left[{\rm FT_p}(V_{\rm mes})\right](u_p,v_p) = T(u_p,v_p)\,V(u_p+u,v_p+v)$$

  - $T = {\rm FT}(B) =$ transfer function $T(u_p,v_p) = 0$ if $\sqrt{u_p^2+v_p^2} > d$
  - V = "true" visibility = ${\rm FT}(I)$
  - $\mathcal{F} = I \times$(phase term) $\Rightarrow$ ${\rm FT}(\mathcal{F})$ = V at a shifted point

Image formation in a mosaic

- **Ideal "on-the-fly" mosaic: (u, v) fixed, (ℓ_p, m_p) continuously modified, visibility V_{mes} monitored**

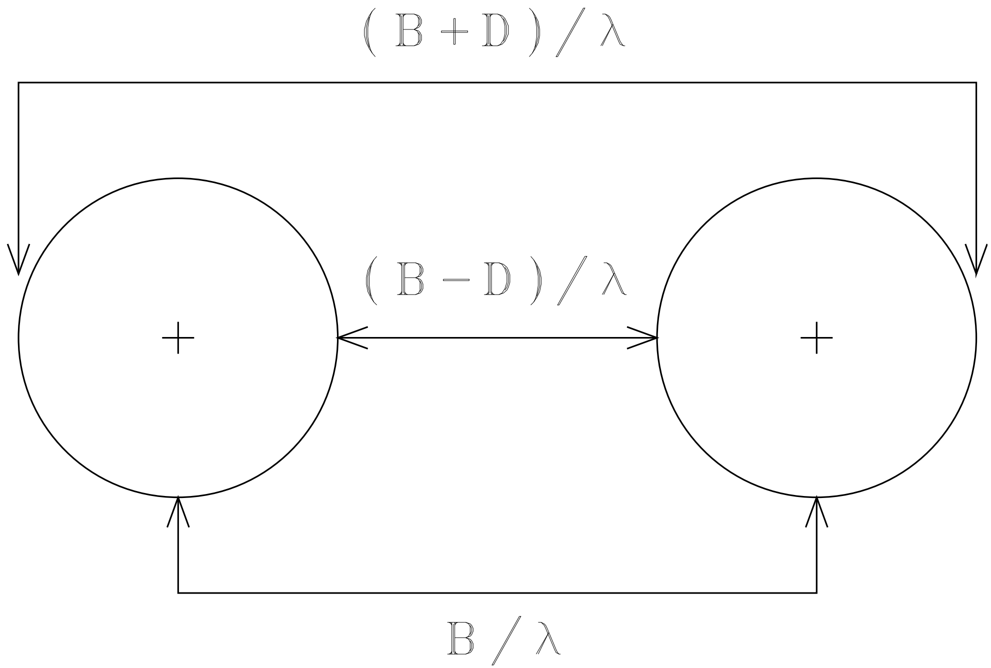
- For $\sqrt{u_p^2 + v_p^2} < d$: $V(u_p + u, v_p + v) = \frac{[\text{FT}_\text{p}(V_{\text{mes}})]\,(u_p, v_p)}{T(u_p, v_p)}$

- **The measurements were done at (u, v), but the "true" visibility can be recovered in a disk of radius d, centered in (u, v)**

- Redundancy of the adjacent pointings allows to estimate the source visibility at points which were not sampled!

Interpretation

- An interferometer is sensitive to all spatial frequencies from **B–D** to **B+D** $\Longrightarrow$ it measures a **local average** of the "true" visibilities



Interpretation

- An interferometer is sensitive to all spatial frequencies from **B–D** to **B+D** $\Longrightarrow$ it measures a **local average** of the "true" visibilities
- Measured visibilities: $V_{\rm mes} = {\rm FT}(B \times I) = \mathbf{T} * \mathbf{V}$ where T is the transfert function of the antenna
- Pointing center $(\ell_p, m_p) \neq$ Phase center: phase gradient across the antenna aperture

$$V_{\rm mes}(u,v) = \left[T(u,v)\,{\rm e}^{-2i\pi(u\ell_p+vm_p)}\right] * V(u,v)$$

- **Combination of measurements at different (ℓ_p, m_p) should allow to derive V**
- The recovery algorithm is a simple Fourier Transform

Consequences: short spacings

- Mosaicing can recover information in a disk of radius D around each sample in the uv plane
- Minimal baseline $B_{\rm min} \longrightarrow$ **Recovery down to $\mathbf{B_{min} - D}$**
- **Mosaics are able to recover part of the short spacing information**
- In practice:
  - Noisy data, rapidly decreasing function $T \longrightarrow$ expect only gain of **$\mathbf{D/2}$**
  - Direct analysis not used: instead, direct reconstruction of the mosaic + deconvolution $\longrightarrow$ more complex properties

Consequences: image quality

- Mosaicing can recover information in a disk of radius D around each sample in the uv plane! Mosaicing can recover part of the short spacings information!

- The resulting image should be wonderful! **NO!**

- The image quality is not drastically improved in a mosaic because of additional information being recovered. **The "equivalent" uv coverage is denser, but the region to be imaged is larger.**

Consequences: field spacing

- In practice: not on-the-fly measurements, but sampling of the pointing positions
- Primary beam is a Gaussian (of $1.2\,\lambda/D$ FWHM) $\longrightarrow$ large overlap between the adjacent fields is needed
- Previous analysis includes Fourier transform on a support which extends up to $\pm D/\lambda$

$\Longrightarrow$ same information can be recovered with pointing centers separated by $\lambda/2\,d$

$\Longrightarrow$ **optimal separation between pointing centers = half the primary beam FWHM**

Conclusions

- Mosaicing is a **standard observing mode** at Plateau de Bure
- Adding short spacings from the IRAM 30–m is an **(almost) standard procedure**