Single−dish antenna at radio wavelengths

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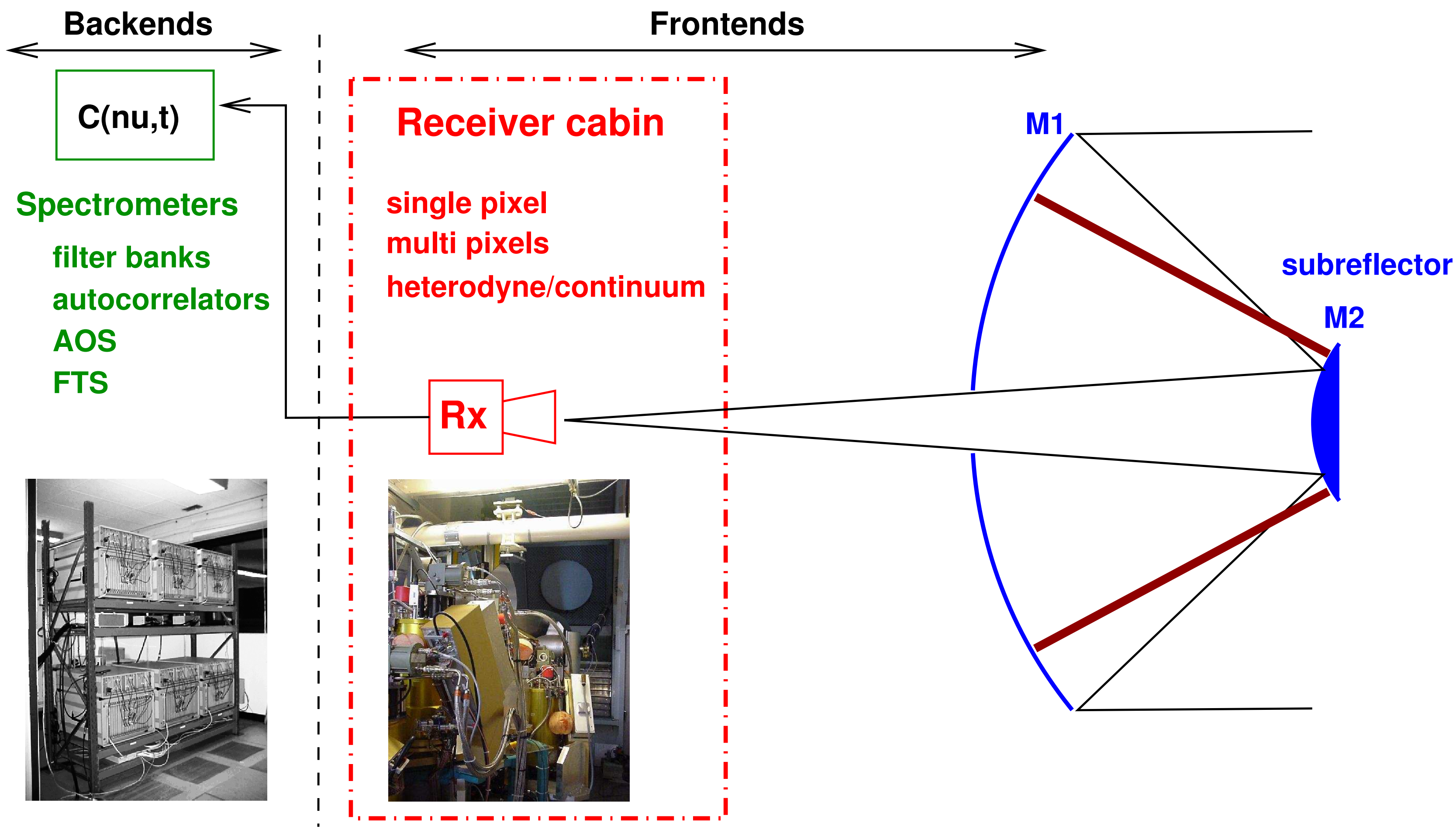
IRAM interferometry school 2006

Characteristics of a Cassegrain single-dish antenna



Why Cassegrain configuration ?

We want a large F/D

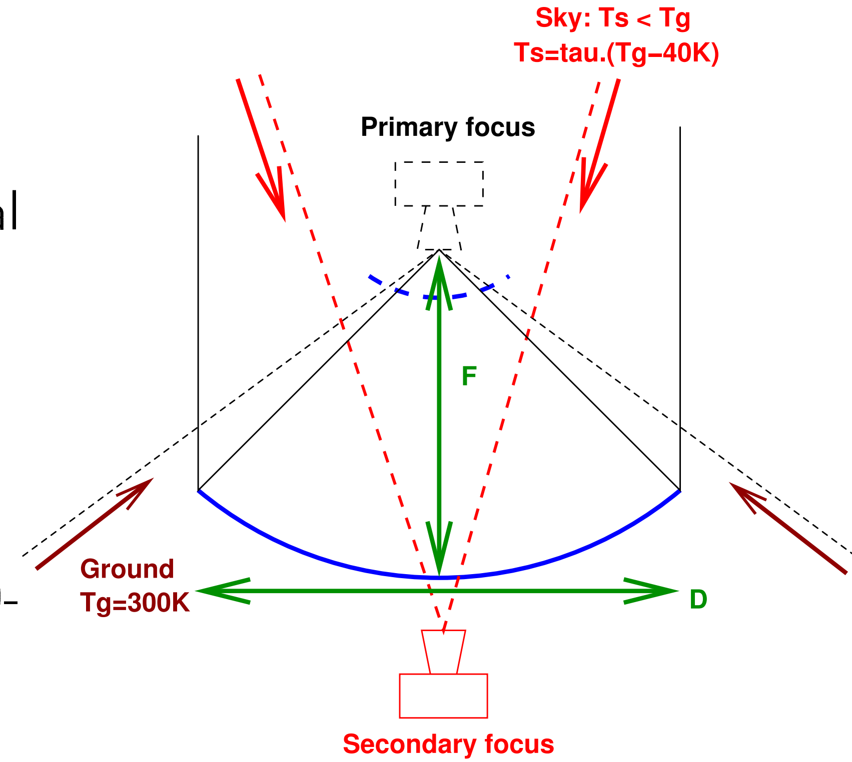
- increase effective area (or on-axis gain)
- use secondary focus: decrease spillover
- Rx alignement more easily acheived; focal plane arrays
- *but increase mechanical load*

$\Rightarrow$ **Cassegrain configuration**

$+$ Effective ratio $F_e/D = m(F/D)$

$-$ obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam

- 30m antenna: $F/D = 0.35$, $m = 27.8$, $F_e/D \approx 10$



Main single-dish antenna

Large aperture: f/D$\lesssim 1$

Institute	Diameter (m)	Frequency (GHz)	Wavelength (mm)	HPBW ($''$)	Latitude
Max-Planck	100	$0.09 - 1.15$	$3 - 300$	$11 - 680$	$+47°$
IRAM	30	$80 - 280$	$1 - 3$	$9 - 30$	$+37°$
JCMT	15	$210 - 710$	$0.2 - 2$	$8 - 20$	$+20°$
APEX	12	$230 - 1200$	$0.3 - 1.3$	$6 - 30$	$-22°$
CSO	10.4	$230 - 810$	$0.4 - 1.3$	$10 - 30$	$+20°$

Terminology

Receivers (Rx)

- bandwidth
  $\Delta\nu$ = 0.5-2 GHz to 50 GHz
- central frequency
  $\nu_0 = 100 - 1200$ GHz
- $\Delta\nu \ll \nu_0 \Rightarrow \approx$ monochromatic
- one polarization (linear, circular)
- taper (apodization at the rim)

Backends:

- spectrometers:
  - filter banks (FB),
  - autocorrelators (AC)
  - acousto-optic (AOS), fourier transform (FTS)
- spectral resolution $\delta\nu \approx 0.01 -$ 1 MHz

$\Rightarrow$ resolution power
$R = \nu_0/\delta\nu \approx 10^5 - 10^8$

Power received

What is the power received from a (point) source of **flux density** S_ν $(\mathrm{W\,m^{-2}\,Hz^{-1}})$?

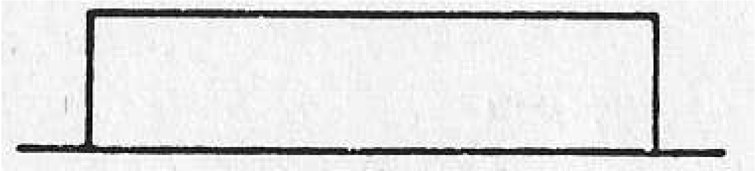
- S_ν measured in Jy: 1 Jy $= 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$
- Monochromatic power: $p_\nu = \frac{1}{2} A_e \cdot S_\nu \qquad [\mathrm{W\,Hz^{-1}}]$
- Power in the bandwidth $\Delta\nu$: $p = \frac{1}{2} A_e \cdot S_\nu \cdot \Delta\nu \qquad [\mathrm{W}]$
- effective area of the antenna: $A_e \leq A_{\mathrm{geom}}$

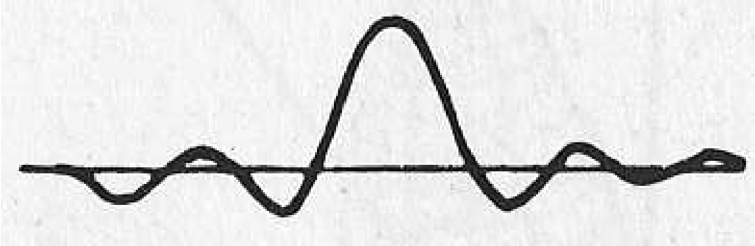
Question: $A_e = ?$

Ideal beam pattern

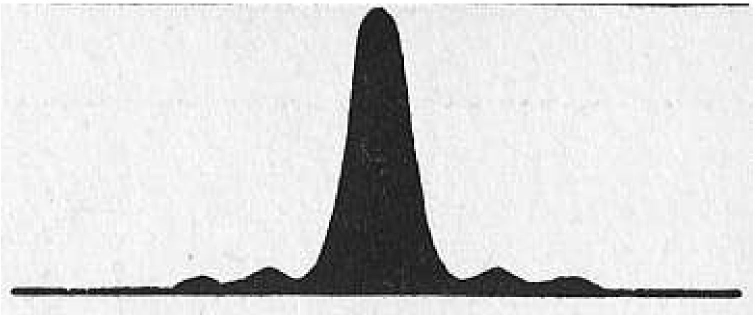
- Diffraction theory
  (Huygens-Fresnel, Fraunhoffer approx.)

$$E_{\mathrm{f-f}}(l,m) \propto \mathcal{F}[E_{\mathrm{ant}}(x,y)]$$





- $E_{\mathrm{ant}}(x,y)$ (grading)
  - bounded on a finite domain $\Delta 1$
    $\Rightarrow$ $E_{\mathrm{f-f}}(l,m)$ concentrated on a finite domain $\Delta 2$ $(\Delta 1 \cdot \Delta 2 \sim 1)$
  - sharp cut of the antenna domain
    $\Rightarrow$ oscillations (side-lobes) $\Rightarrow$ taper



Ideal beam pattern

- Reciprocity: antenna in emission:
  - pattern of the transmitted emission depends on the direction (l, m):

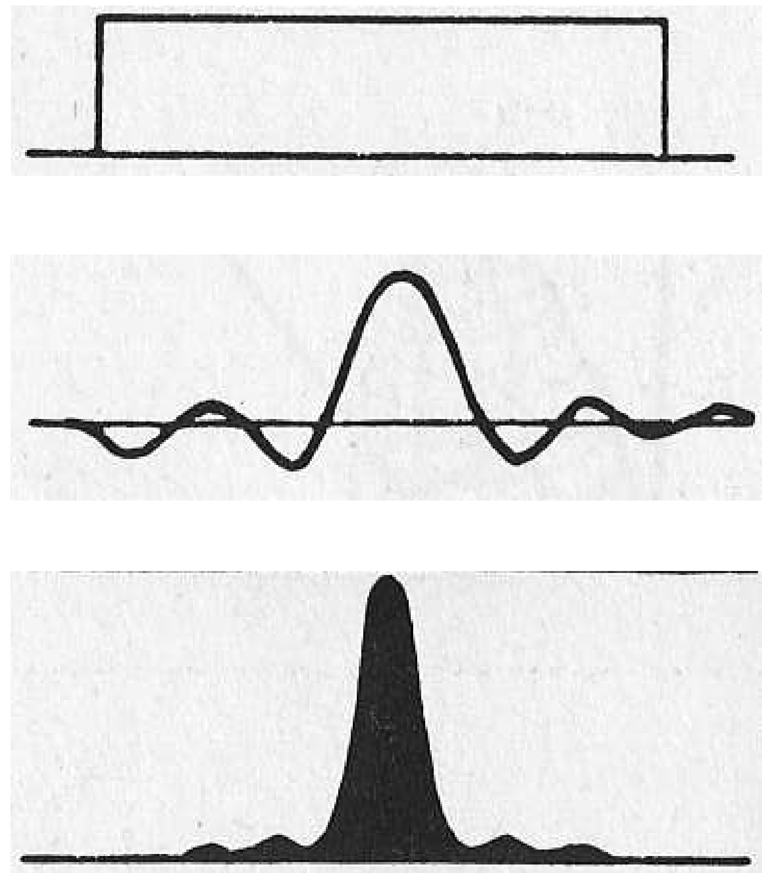
    Power pattern

    $\mathcal{P}(l, m) \propto |E_{\mathrm{f-f}}(l, m)|^2$

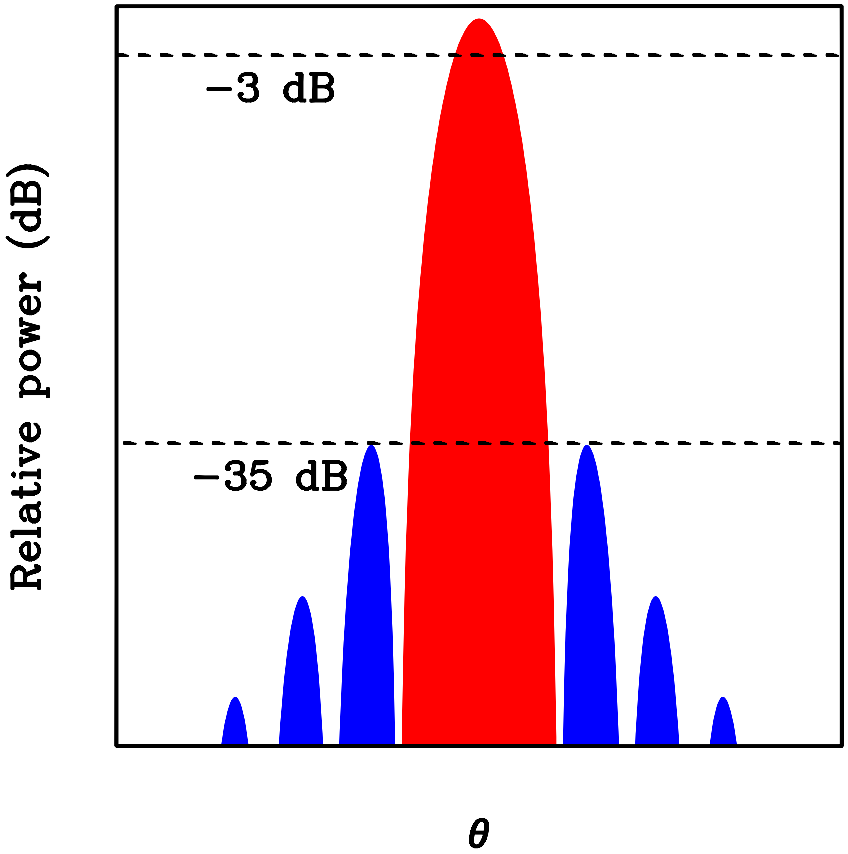
    Effective area

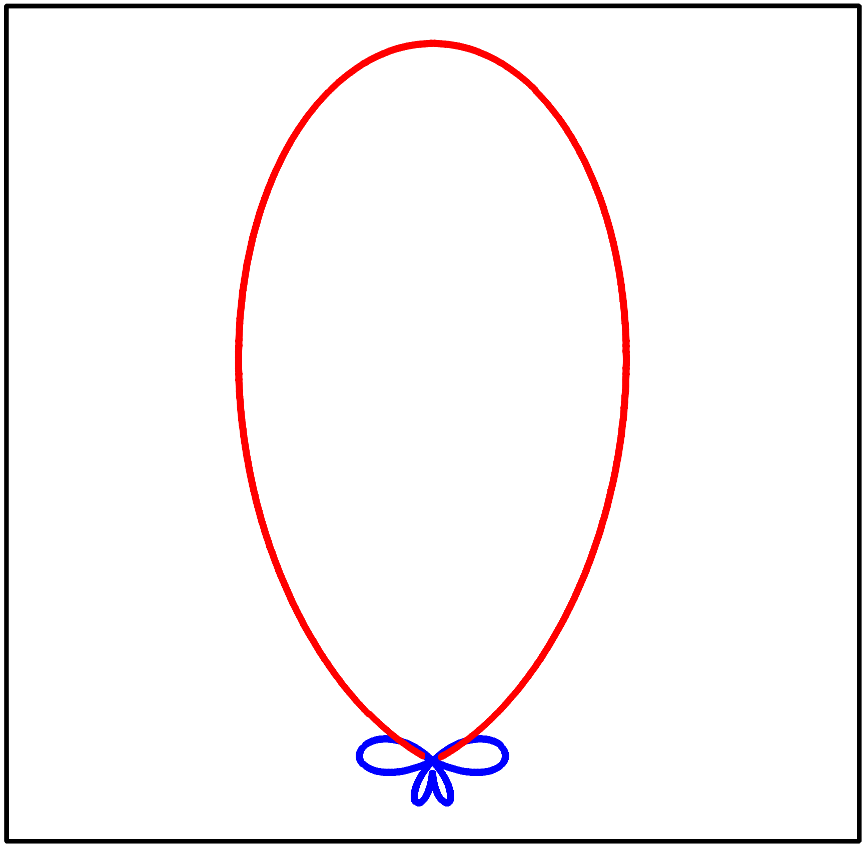
    $A(l, m) = A_{\mathrm{max}} \cdot \mathcal{P}(l, m)$
  - example: circular aperture

    $\mathcal{P}(l, m) \propto$ Airy disk



Power pattern





Brightness distribution with total-power telescope

- point source: flux density $S_\nu (\mathrm{W\,m^{-2}\,Hz^{-1}})$
- extended source: brightness $I_\nu(l, m)$

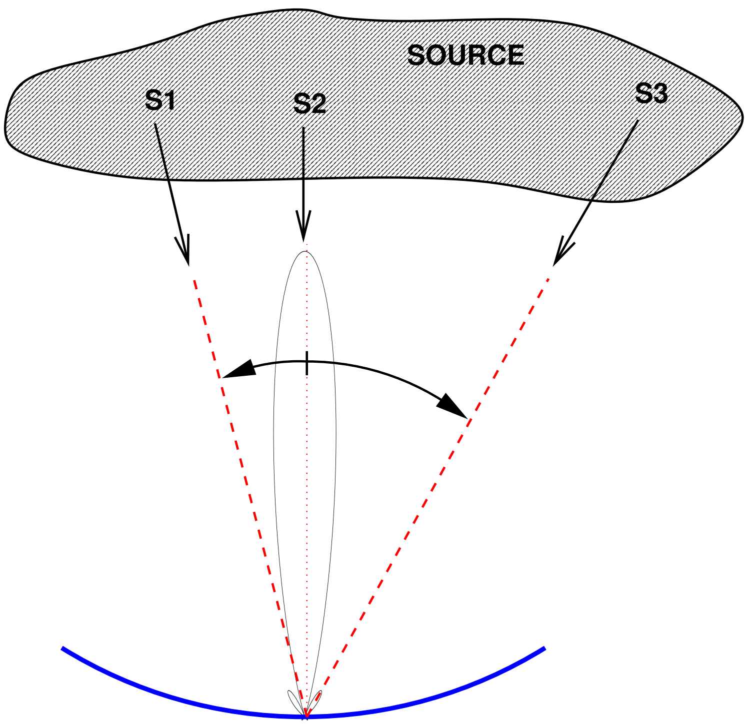
  $$I_\nu = \mathrm{d}S_\nu / \mathrm{d}\Omega \; (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$$

- from the direction (l_i, m_i):

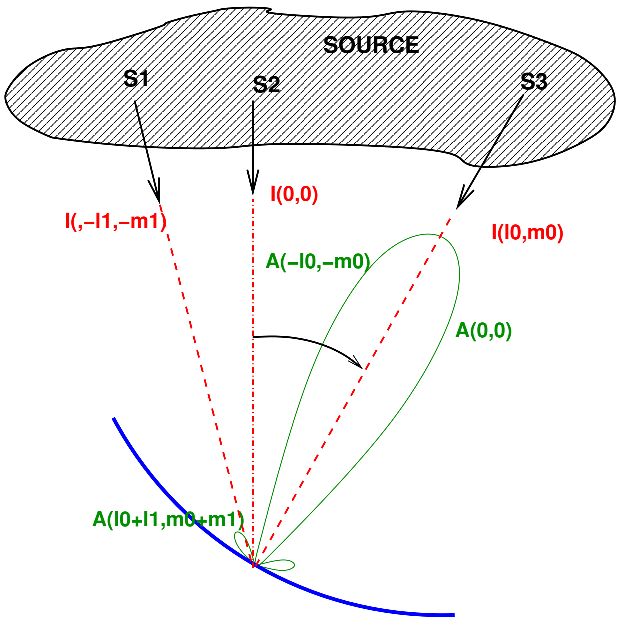
  $$\mathrm{d}p_\nu = A(l_i, m_i)\, I_\nu(l_i, m_i)\, \mathrm{d}\Omega_i$$

- incoherent emission: add intensities

  $$p_\nu(0,0) = \iint A(l, m)\, I_\nu(l, m)\, \mathrm{d}\Omega$$



Brightness distribution with total-power telescope



- point source: flux density $S_\nu (\mathrm{W\,m^{-2}\,Hz^{-1}})$
- extended source: brightness $I_\nu(l,m)$

$$I_\nu = \mathrm{d}S_\nu / \mathrm{d}\Omega \; (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$$

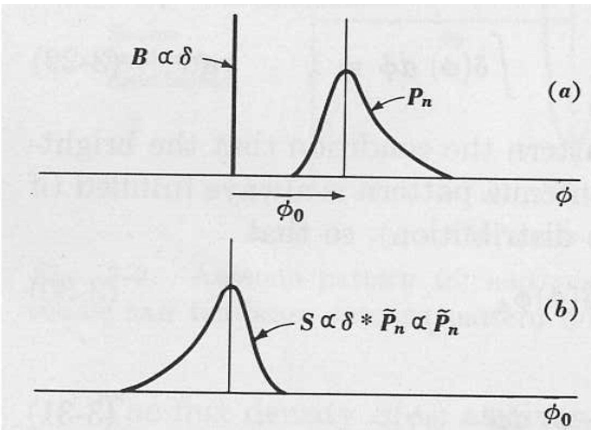
- antenna tilted towards (l_0, m_0)
- from the direction (l_i, m_i)
  $\mathrm{d}p_\nu = A(l_0 - l_i, m_0 - m_i)\, I_\nu(l_i, m_i)\, \mathrm{d}l_i \mathrm{d}m_i$
- incoherent emission: add intensities
- convolution

$$p_\nu(l_0, m_0) = \iint A(l_0 - l, m_0 - m)\, I_\nu(l,m)\, \mathrm{d}l\mathrm{d}m$$

$$I'_\nu = \mathcal{P} * I_\nu$$

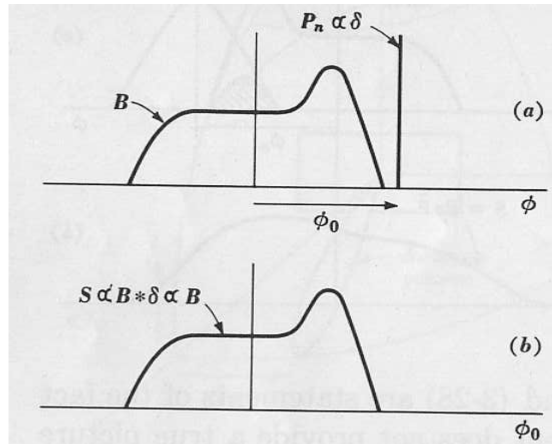
Convolution: consequences

Point source



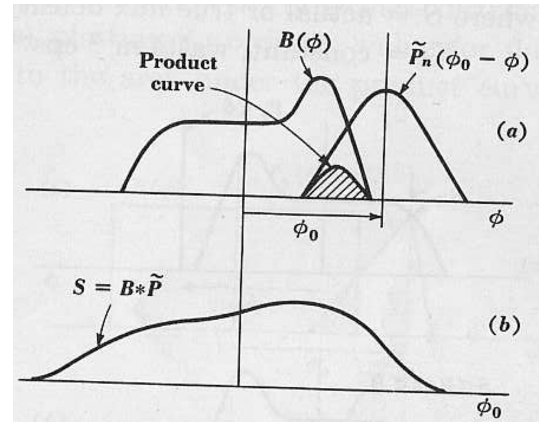
→ *see the beam*

Infinite telescope



→ *see the source*

Smoothing



→ *smear the source*

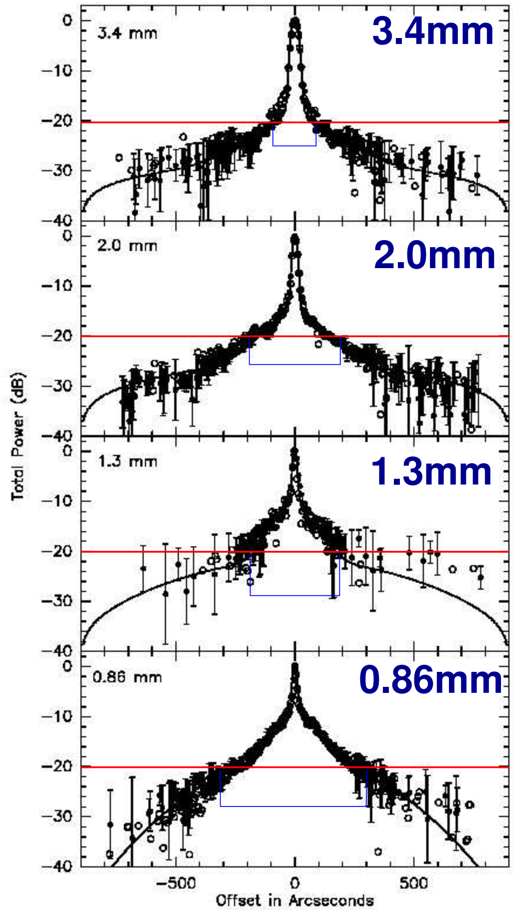
$$\theta_{\rm obs} = \sqrt{\theta_{\rm mb}^2 + \theta_{\rm sou}^2}$$

Beam pattern

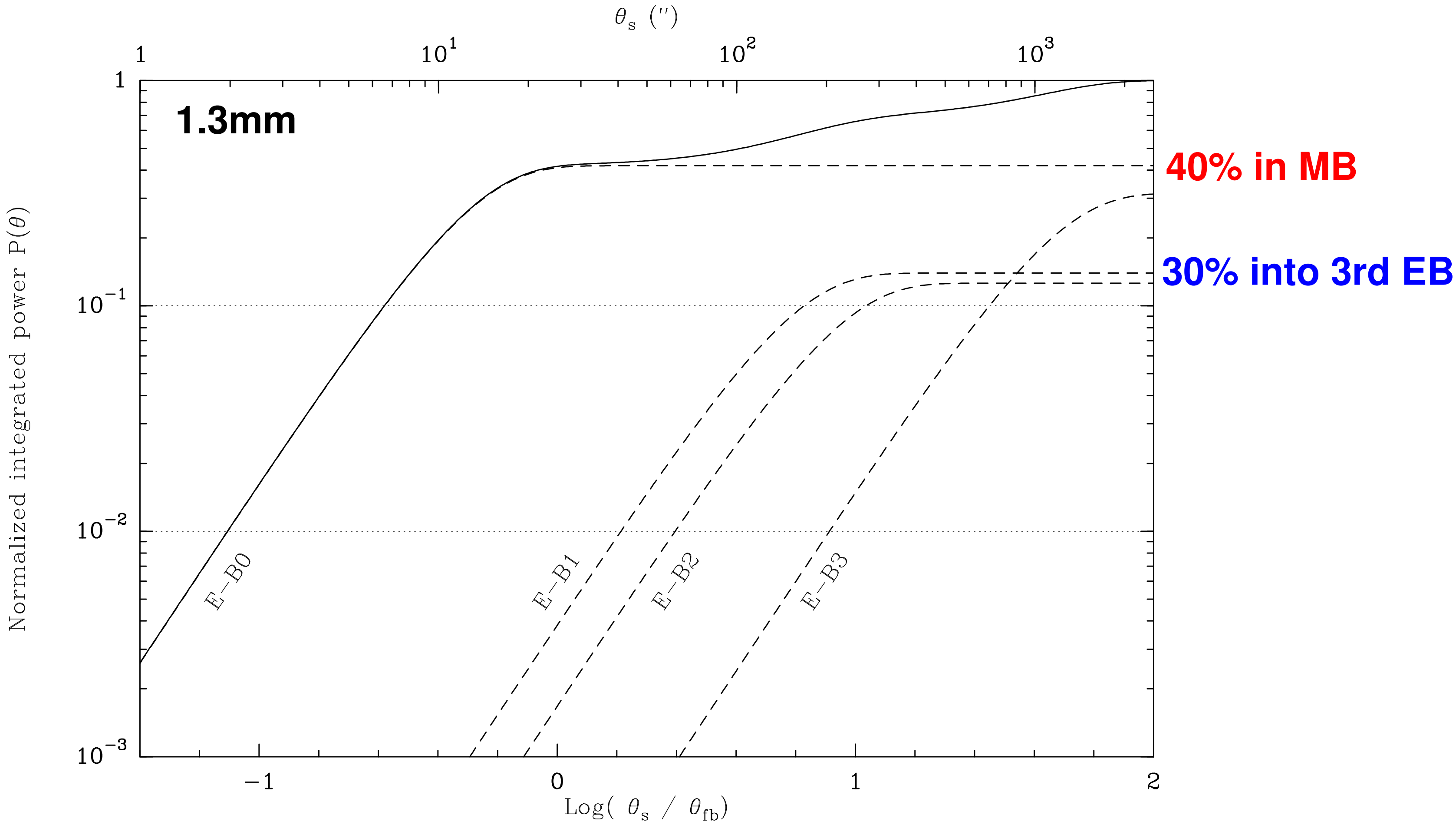
- secondary lobes (finite surface antenna)
- error lobes (surface irregularities)
  - main-beam collects less power
  - if correlation length l
    $\Rightarrow$ one Gaussian error-beam $\Theta_{\rm EB} \approx \frac{1}{2} \cdot \lambda/\ell$

real beam = main-beam + error-beam(s)

- Questions:
  - What power is collected in each beam ?
  - What are the FWHMs of the beams ?



IRAM 30-m antenna: Error-Beams power



Brightness temperature

- T_{B} defined by $I_\nu = B_\nu(T_{\mathrm{B}}) \quad [\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$
- radiation temperature, T_R, Rayleigh-Jeans approximation

$$I_\nu = \frac{2k\nu^2}{c^2} T_R \qquad (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$$

- relation $T_{\mathrm{B}} - T_R$: $T_R = J_\nu(T_{\mathrm{B}}) = \frac{h\nu}{k}\frac{1}{\exp(h\nu/kT_{\mathrm{B}})-1}$
- in the following: $I_\nu(l,m) \rightarrow T_R(l,m)$
- consequence: power $\propto T_R$

$$p_\nu(l_0,m_0) = \frac{k}{\lambda^2}\iint_{4\pi} A(l,m)\,T_R(l_0-l,m_0-m)\,\mathrm{d}\Omega$$

Antenna temperature

- Johnson noise in terms of an equivalent temperature

  average power transferred from a conductor to a line within $\delta\nu$: $= k\,T\,\delta\nu$

- Antenna temperature: antenna as a conductor

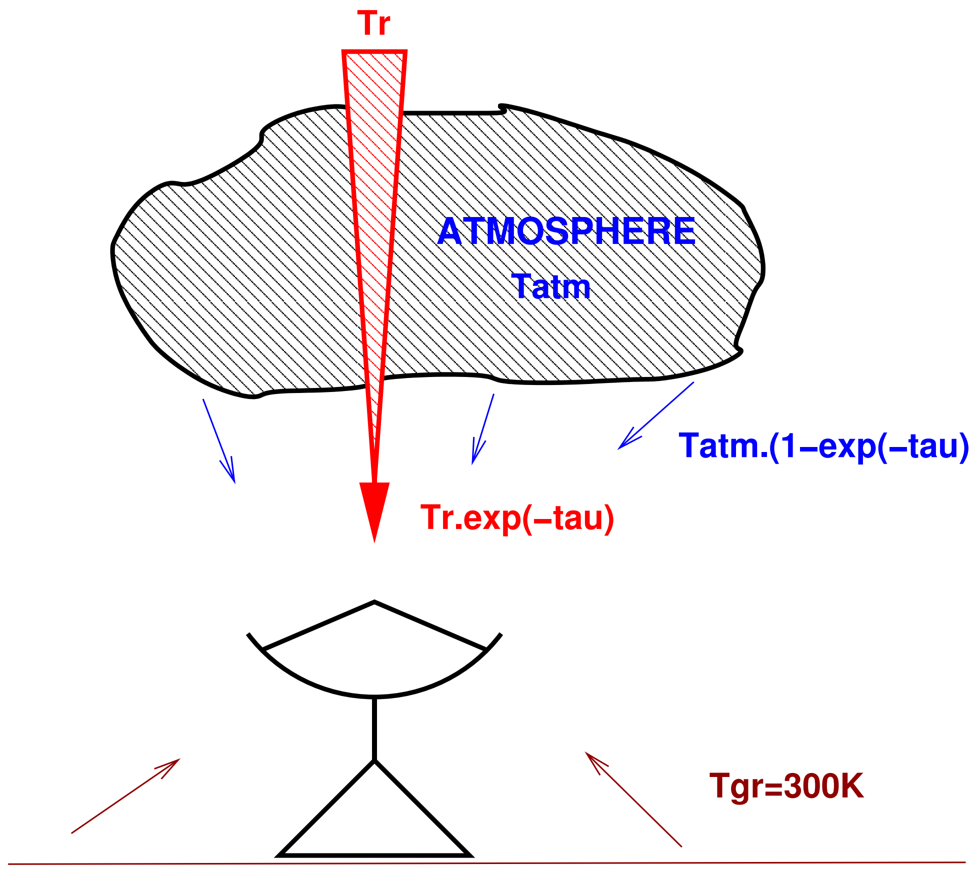
$$p_\nu = k\,T_A \qquad [\mathrm{W} \cdot \mathrm{Hz}^{-1}] = [\mathrm{J}] = [\mathrm{J} \cdot \mathrm{K}^{-1}][\mathrm{K}]$$

- $A_{\max} = \lambda^2 / \iint_{4\pi} P(l,m)\mathrm{d}\Omega$

- Therefore: $T_A(l,m) = \frac{1}{\lambda^2} \iint_{4\pi} A(l,m)\,T_R(l_0 - l, m_0 - m)\,\mathrm{d}\Omega$

$$T_A(l,m) = \iint_{4\pi} T_R(l_0 - l, m_0 - m)\,\mathrm{d}\Omega / \iint_{4\pi} P(l,m)\mathrm{d}\Omega$$

Atmosphere



$$T = \alpha \left\{ T_R e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) T_{\mathrm{atm}} \right\} + (1-\alpha) T_{\mathrm{gr}}$$

see J-M Winters lecture

$T_{\rm A}^*$ and $T_{\rm mb}$

- Antenna temperature: $T_{\rm A}^*$
  - takes into account **rear side-lobes**:**FORWARD SIGNAL ONLY (2π sr)**
  - corrects for **atmospheric attenuation**: $\times \exp(\tau_\nu)$

$$T_{\rm A}^*(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_R(\Omega_0 - \Omega) {\rm d}\Omega}{\mathcal{P}_{2\pi}} \qquad \mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega) {\rm d}\Omega$$

- $T_{\rm mb}$: Equivalent in **main-beam** instead of 2π

$$T_{\rm mb}(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_R(\Omega_0 - \Omega) {\rm d}\Omega}{\mathcal{P}_{\rm mb}} \qquad \mathcal{P}_{\rm mb} = \int_{\Omega_{\rm mb}} \mathcal{P}(\Omega) {\rm d}\Omega$$

Temperature scales

Definitions

$$F_{\text{eff}} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{4\pi}}$$

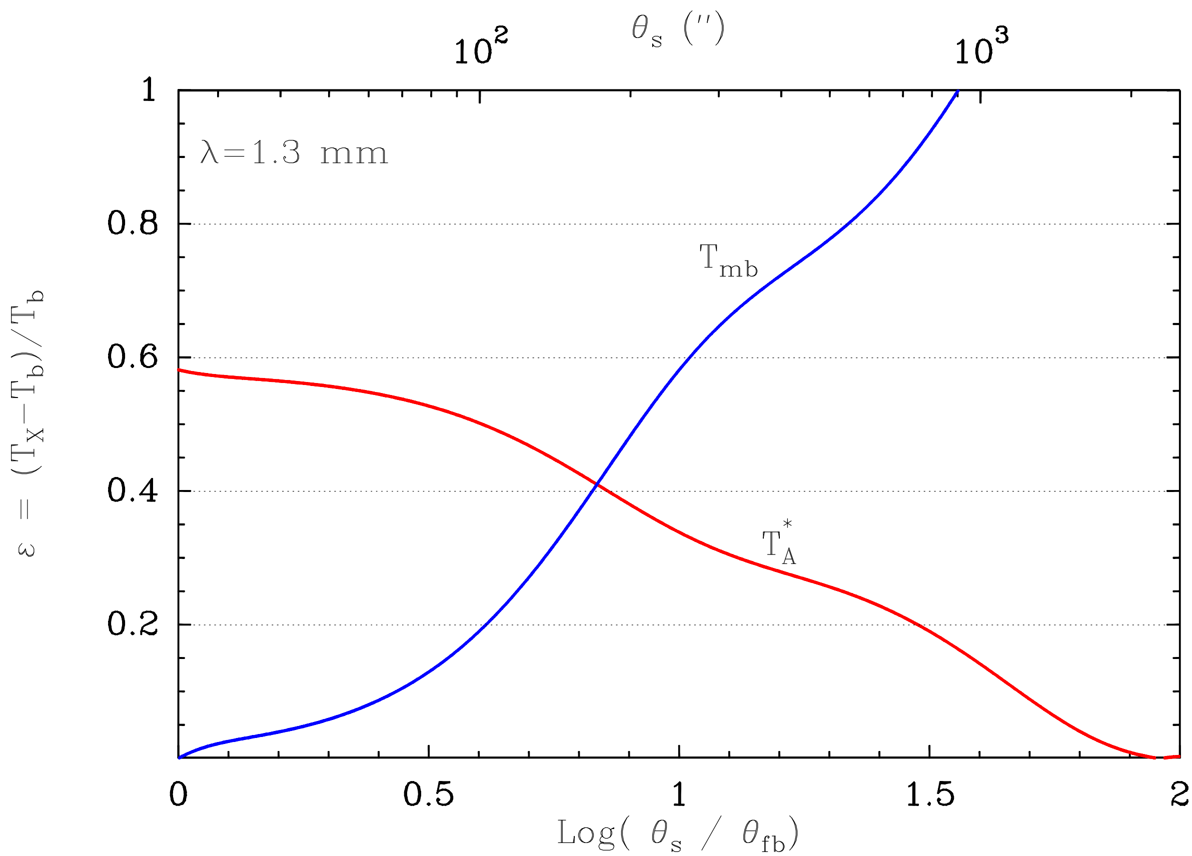
$$B_{\text{eff}} = \frac{\mathcal{P}_{\text{mb}}}{\mathcal{P}_{4\pi}}$$

Consequences

$$T_{\text{mb}} = \frac{F_{\text{eff}}}{B_{\text{eff}}}\, T_{\text{A}}^{*} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{\text{mb}}}\, T_{\text{A}}^{*}$$

What you measure is T_{A}^{*} or T_{mb} (usually $\neq T_R$)

Which temperature scale ?



Source size	Temperature scales
$\Omega_S = 2\pi$	$T_R = T_A^*$
$\Omega_S = \Omega_{mb}$	$T_R = T_{mb}$
$2\pi < \Omega_S$	$T_R < T_A^*$
$\Omega_{mb} < \Omega_S < 2\pi$	$T_A^* < T_R < T_{mb}$
$\Omega_{mb} > \Omega_S$	$T_{mb} < T_R$

Goal of the calibration

- Atmosphere: opacity τ_ν
- Antenna-sky coupling: $F_{\rm eff}$
- Output at backends: "counts"

Question: **counts $\longrightarrow$ Temperature ?**

$$C = \chi T \Longrightarrow \chi = ?$$

$$
\begin{aligned}
C_{\rm sou} &= \chi \left\{ T_{\rm rec} + F_{\rm eff} e^{-\tau_\nu} T_{\rm sou} + T_{\rm emi} \right\} \\
T_{\rm emi} &= F_{\rm eff}(1 - e^{-\tau_\nu}) T_{\rm atm} + (1 - F_{\rm eff}) T_{\rm gr}
\end{aligned}
$$

$\Rightarrow$ How many unknowns ? **4 unknowns**

$$\boxed{\{\chi,\ \tau_\nu,\ \mathbf{T}_{\rm sou},\ \mathbf{T}_{\rm rec}\}}$$

4 unknowns $\Rightarrow$ 4 equations $\Rightarrow$ 4 measurements:

$$\boxed{T_{\rm sou},\ T_{\rm atm},\ T_{\rm hot}\ \text{ and } T_{\rm col}}$$

"Chopper Wheel"

$$
\begin{aligned}
C_{\rm sou} &= \chi \left\{T_{\rm rec} + T_{\rm emi} + F_{\rm eff} e^{-\tau_\nu} T_{\rm sou}\right\} \\
C_{\rm atm} &= \chi \left\{T_{\rm rec} + T_{\rm emi}\right\} \\
C_{\rm hot} &= \chi \left\{T_{\rm rec} + T_{\rm hot}\right\} \\
C_{\rm col} &= \chi \left\{T_{\rm rec} + T_{\rm col}\right\}
\end{aligned}
$$

Making differences

$$
\begin{aligned}
C_{\rm sou} - C_{\rm atm} &= \chi F_{\rm eff} e^{-\tau_\nu} T_{\rm sou} \\
C_{\rm hot} - C_{\rm atm} &= \chi (T_{\rm hot} - T_{\rm emi})
\end{aligned}
$$

Definition of $T_{\rm cal}$:

$$
T_{\rm sou} = \frac{C_{\rm sou} - C_{\rm atm}}{C_{\rm hot} - C_{\rm atm}} \, T_{\rm cal}
$$

$$
\Rightarrow T_{\rm cal} = (T_{\rm hot} - T_{\rm emi}) \, \frac{e^{\tau_\nu}}{F_{\rm eff}}
$$

Outputs of calibration procedure: $T_{\rm rec}$

Hot & cold loads $\longrightarrow T_{\rm rec}$:

$$
\begin{aligned}
Y &= \frac{C_{\rm hot}}{C_{\rm col}} \\
T_{\rm rec} &= \frac{T_{\rm hot} - Y T_{\rm col}}{Y - 1}
\end{aligned}
$$

Outputs of calibration procedure: $T_{\rm cal}$

Rewrite $T_{\rm emi}$

$$T_{\rm emi} = T_{\rm gr} + F_{\rm eff}(T_{\rm atm} - T_{\rm gr}) - F_{\rm eff} e^{-\tau_\nu} T_{\rm atm}$$

$$\begin{aligned} C_{\rm hot} - C_{\rm atm} &= \chi\{(T_{\rm hot} - T_{\rm gr}) + F_{\rm eff}(T_{\rm gr} - T_{\rm atm}) \\ &+ F_{\rm eff} e^{-\tau_\nu} T_{\rm atm}\} \end{aligned}$$

- Assume $T_{\rm hot} = T_{\rm atm} = T_{\rm gr} \Rightarrow \{\chi, \tau_\nu\} \rightarrow \{\chi e^{-\tau_\nu}\}$
  $\Rightarrow$ 3 unknowns $\Rightarrow$ *e.g.* don't need to solve for τ_ν
  (Penzias & Burrus 1973)

$$\boxed{T_{\rm cal} = T_{\rm atm}}$$

- General case: different $T_{\rm atm}$, $T_{\rm hot}$ and $T_{\rm gr}$
  $\Rightarrow$ solve for the 4 unknowns

Outputs of calibration procedure: T_{sys}

System temperature: describes the noise including all sources from the sky down to the backends

$$\sigma_T = \frac{\kappa \cdot T_{\mathrm{sys}}}{\sqrt{\delta\nu\,\Delta t}}$$

- κ depends on the observing mode: ON-OFF $t_{\mathtt{ON}} = t_{\mathtt{OFF}} \Rightarrow \kappa = \sqrt{2}$
- δ_ν: spectral resolution
- Δt: integration time ($t_{\mathtt{ON}}$=$t_{\mathtt{OFF}}$)

From $T_{\rm mb}$ to I_ν

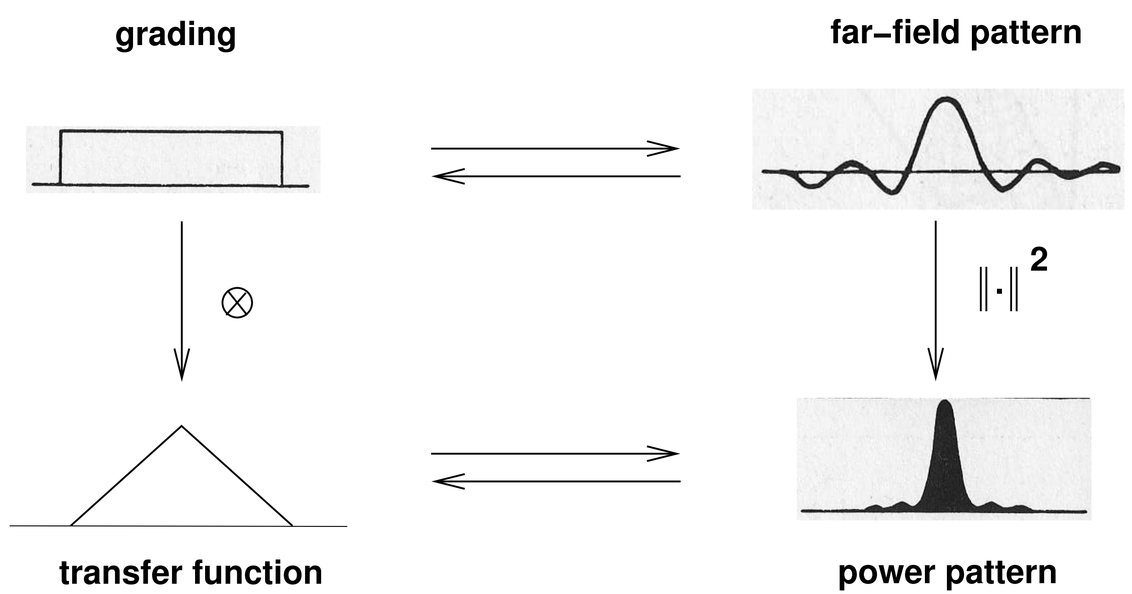
How to convert the temperatures into $\rm W\,m^{-2}\,Hz^{-1}$?

$$S_\nu = \int_{\Omega_r} I_\nu(\Omega)\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_{\Omega_r} T_{\rm mb}\,\mathrm{d}\Omega$$

Gaussian source of uniform radiation temperature T_R:

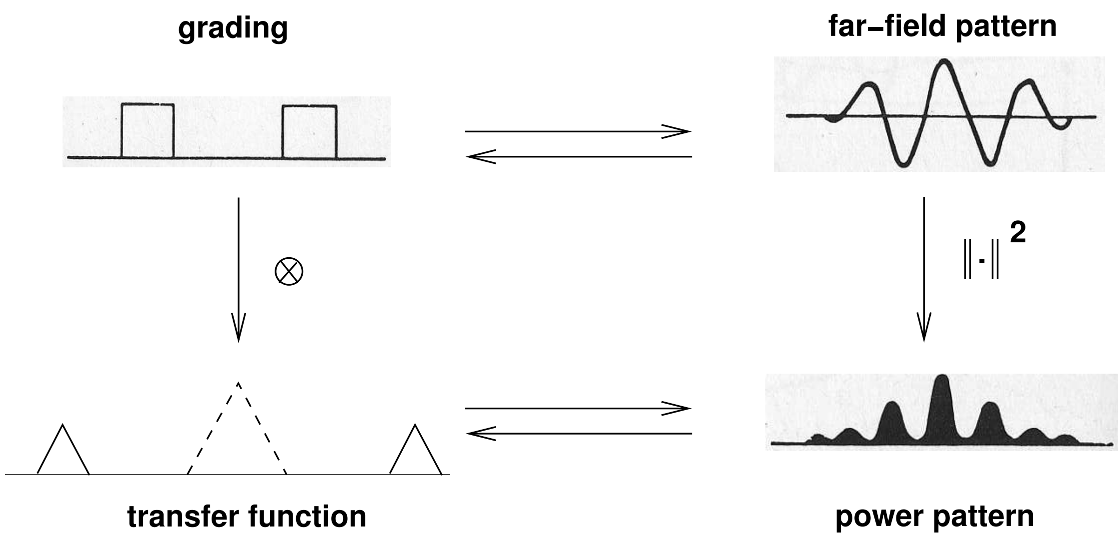
$$S_\nu = 8.2\times 10^{-3}\left(\frac{\nu}{100\ GHz}\right)^2\left(\frac{\theta_r}{1''}\right)^2\left(\frac{T_R}{K}\right)$$

Image formation: total power telescope



- antenna scans the source
- image: convolution of I_0 by beam pattern $I'_\nu = \mathcal{P} * I_{0,\nu}$
- measure directly the brightness distribution I_0

Image formation: correlation telescope



- antennas fixed w.r.t. the source
- correlation temperature: $\mathcal{T}(0,0)$ Fourier transform of $I_0 \times \mathcal{P}$
- measure the Fourier transform of the brightness distribution I_0
- image built afterwards

Interferometer field of view

$$F = D * (\mathcal{P} \times I) + N$$

$$\begin{aligned} F &= \text{dirty map = FT of observed visibilities} \\ D &= \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ \mathcal{P} &= \text{power pattern of single-dish } \textit{(primary beam } B \textit{ in the following)} \\ I &= \text{sky brightness distribution} \\ N &= \text{noise distribution} \end{aligned}$$

- An interferometer measures the product $\mathcal{P} \times I$
- $\mathcal{P}$ has a finite support $\longrightarrow$ limits the size of the field of view
- $\mathcal{P}$ is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

$$
\begin{array}{ccc}
E_{\text{ant}}(x,y) & \rightleftharpoons & \text{Voltage pattern } F(l,m) \\
\star\big\downarrow & & \big\downarrow |\cdot|^2 \\
\text{Transfert function } T(u,v) & \rightleftharpoons & \text{Power pattern } \mathcal{P}(\ell,m) \\
 & & = \textbf{Primary beam}
\end{array}
$$

Summary

- full-aperture antenna: $\mathcal{P} * \mathbf{I}$
- interferometry sensitive to $\mathcal{P} \times \mathbf{I}$
- **amplitude calibration:**
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- **lobe** $=$ **main-lobe** $+$ **error-lobes** (*e.g.* as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the **temperature scale** to use (T_{A}^{*}, T_{mb},...)

Interferometer field of view

Measurement equation of an interferometric observation:

$$F = D * (B \times I) + N$$

$$\begin{array}{lcl} F & = & \text{dirty map = FT of observed visibilities} \\ D & = & \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ B & = & \text{primary beam} \\ I & = & \text{sky brightness distribution} \\ N & = & \text{noise distribution} \end{array}$$

- An interferometer measures the product $B \times I$
- B has a finite support $\longrightarrow$ limits the size of the field of view
- B is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

$$\begin{array}{ccc} \text{Aperture function} & \rightleftharpoons & \text{Voltage pattern} \\ \star\downarrow & & \downarrow |\cdot|^2 \\ \text{Transfert function } T(u,v) & \rightleftharpoons & \text{Power pattern } B(\ell,m) \\ & & = \text{Primary beam} \end{array}$$

Gaussian illumination $\Longrightarrow$ to a good approximation, B is a Gaussian of $1.2\,\lambda/D$ FWHM

Plateau de Bure
$D = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	$58''$
100 GHz	3.0 mm	$50''$
115 GHz	2.6 mm	$43''$
215 GHz	1.4 mm	$23''$
230 GHz	1.3 mm	$22''$
245 GHz	1.2 mm	$20''$