Millimeter-wave Interferometry

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2006 October

A Millimeter-wave Interferometer

consists of

- Antennas
- Heterodyne Receivers
- Correlators
- a lot of Cables and Oscillators
- Computer(s)
- on a fine site

plus quite a few people to operate it...

Contents

Heterodyne Detection (1)

- The antenna produces an output voltage proportional to the incident electric field pattern. For a simple monochromatic case:

$$U(t) = E\cos\left(2\pi\nu t + \varphi\right) \tag{1}$$

- In a mixer, in the receiver, the antenna output is combined with the output of a local oscillator:

$$U_{\mathrm{LO}}(t) = Q\cos\left(2\pi\nu_{\mathrm{LO}} t + \varphi_{\mathrm{LO}}\right) \tag{2}$$

- The mixer is a non-linear element (diode) whose output is

$$I(t) = a_0 + a_1(U(t) + U_{\mathrm{LO}}(t)) + a_2(U(t) + U_{\mathrm{LO}}(t))^2 + a_3(U(t) + U_{\mathrm{LO}}(t))^3 + \ldots \tag{3}$$

The second order (quadratic) term of Eq.3 can be expressed as

$$\begin{aligned} I(t) &= a_2E^2\cos^2(2\pi\nu t + \varphi) + 2a_2EQ\cos(2\pi\nu t + \varphi)\cos(2\pi\nu_{\mathrm{LO}} t + \varphi_{\mathrm{LO}}) \\ &+ a_2Q^2\cos^2(2\pi\nu_{\mathrm{LO}} t + \varphi_{\mathrm{LO}}) \end{aligned} \tag{4}$$

Heterodyne Detection (2)

- Developing the product of the two cosine functions, we obtain

$$I(t) = ... + a_2 EQ\cos(2\pi(\nu - \nu_{\rm LO})t + \varphi - \varphi_{\rm LO}) + ... \tag{5}$$

There are obviously other terms in $\nu + \nu_{\rm LO}$, $2\nu_{\rm LO}$, 2ν, $3\nu_{\rm LO} \pm \nu$, etc... in the above equation, as well as terms at very different frequencies like ν, 3ν, etc...

- By inserting a filter at the output of the mixer, we can select only the term such that

$$\nu_{\rm IF} - \Delta\nu/2 \leq |\nu - \nu_{\rm LO}| \leq \nu_{\rm IF} + \Delta\nu/2 \tag{6}$$

where $\nu_{\rm IF}$, the Intermediate Frequency, is a frequency which is significantly different from the original signal frequency ν (which is often called the Radio Frequency $\nu_{\rm RF}$).

- Hence, after mixing and filtering, the output of the receiver is

$$I(t) \propto EQ\cos\left(\pm(2\pi(\nu - \nu_{\rm LO})t + \varphi - \varphi_{\rm LO}\right) \tag{7}$$

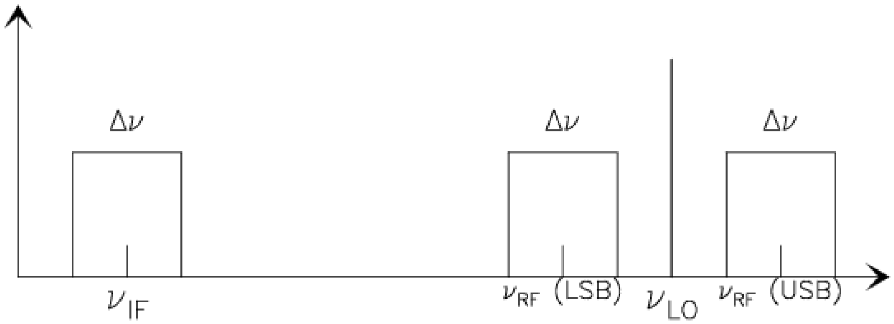
Heterodyne Detection (3)

- The receiver output is

$$I(t) \propto EQ \cos\left(\pm(2\pi(\nu - \nu_{\rm LO})t + \varphi - \varphi_{\rm LO})\right) \qquad (8)$$

  - changed in frequency: $\nu \longrightarrow \nu - \nu_{\rm LO}$ or $\nu \longrightarrow \nu_{\rm LO} - \nu$
  - proportional to the original electric field of the incident wave: $\propto E$
  - with a phase relation with this electric field: $\varphi \longrightarrow \varphi - \varphi_{\rm LO}$ or $\varphi \longrightarrow \varphi_{\rm LO} - \varphi$
  - proportional to the local oscillator voltage: $\propto Q$

- The frequency change, usually toward a lower frequency, allows to select $\nu_{\rm IF}$ such that amplifiers and transport elements are easily available for further processing.



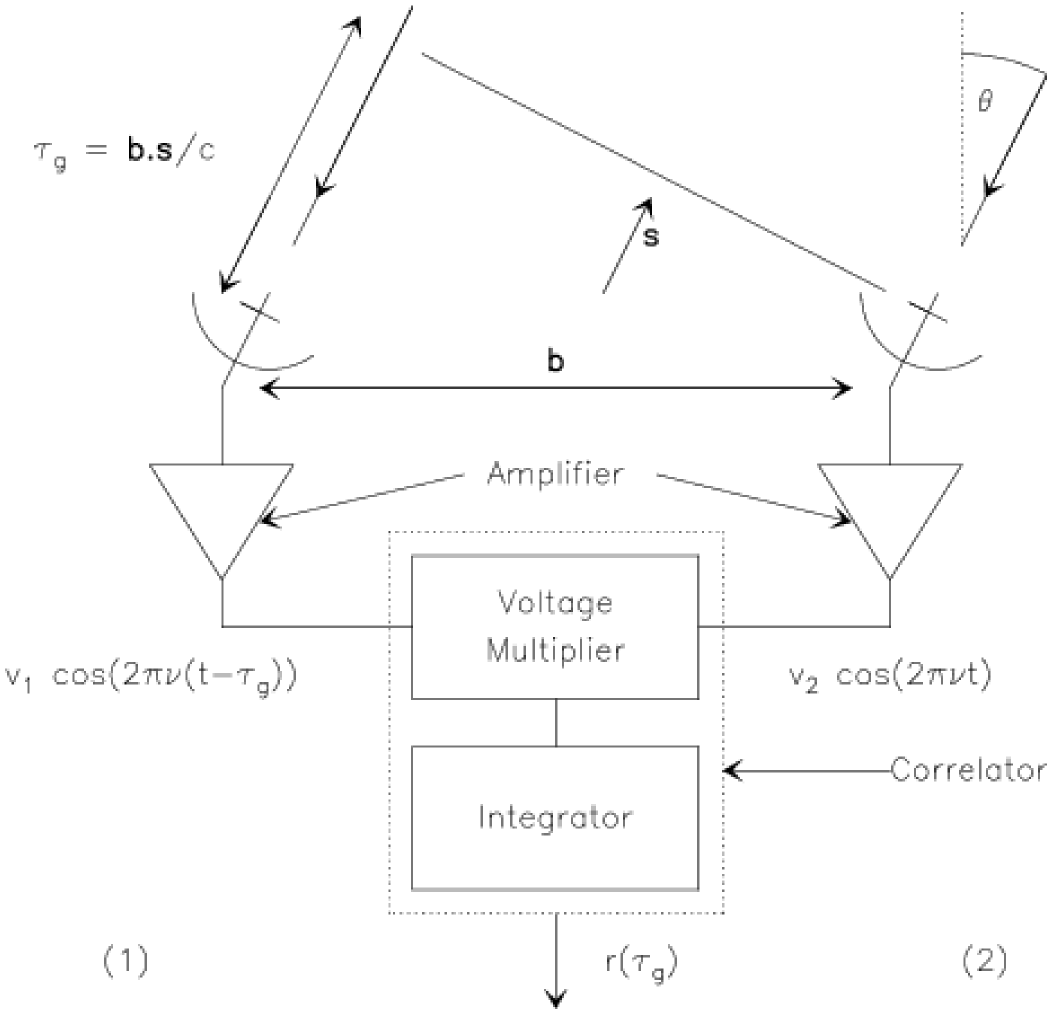
Relation between the IF, RF and local oscillator frequencies in an heterodyne system

Heterodyne Detection (4)

- The mixer described before accepts simultaneously frequencies which are
  - higher than the local oscillator frequency: this is called Upper Side Band (USB) reception
  - lower than the local oscillator frequency: this is called Lower Side Band (LSB) reception

  and cannot *a priori* distinguish between them. This is called Double Side Band (DSB) reception.
- Some receivers are actually insensitive to one of the frequency range, either because a filter has been placed at the receiver input, or because their response is very strongly frequency dependent. Such receivers are called Single Side Band (SSB) receivers. There are even *dual side band (2SB)* receivers.
- An important property of the receiving system expressed by Eq.8 is that the sign of the phase is changed for LSB conversion.
- This property can be easily retrieved recognizing that the angular frequency $2\pi\nu$ is the time derivative of the phase φ.

The Heterodyne Interferometer (1)



Schematic diagram of the two-antenna radio interferometer. $\tau_g = \mathbf{b}.\mathbf{s}/c$ is called the geometrical delay

Let us forget the frequency conversion for some time, i.e. assume $\nu_{\rm IF} = \nu_{\rm RF}$...

The Heterodyne Interferometer (2)

- The input (amplified) signals from 2 elements of the interferometer are processed by a correlator, which is just a voltage multiplier followed by a time integrator.
- With one incident plane wave, the output $r(t)$ is

$$r(t) = < v_1 \cos(2\pi\nu(t - \tau_g(t)))v_2 \cos(2\pi\nu t) > = v_1 v_2 \cos(2\pi\nu\tau_g(t)) \quad (9)$$

  where τ_g is the geometrical delay $\tau_g(t) = (\mathbf{b}.\mathbf{s})/c$

- As τ_g varies slowly because of Earth rotation, $r(t)$ oscillates as a cosine function, and is thus called the fringe pattern. As we had shown before that v_1 and v_2 were proportional to the electric field of the wave, the correlator output (fringe pattern) is thus proportional to the power of the wave.

Source Size Effects (1)

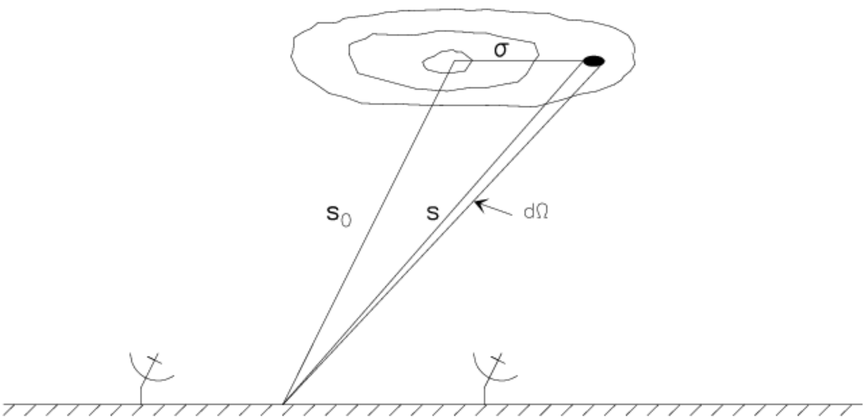
- The signal power received from a sky area $d\Omega$ in direction $\mathbf{s}$ is (see Fig. for notations)

$$A(\mathbf{s})I(\mathbf{s})d\Omega\delta\nu \tag{10}$$

over bandwidth $d\nu$, where $A(\mathbf{s})$ is the antenna power pattern (assumed identical for both elements, more precisely $A(\mathbf{s}) = \mathcal{A}_i(\mathbf{s})\mathcal{A}_j(\mathbf{s})$ with $\mathcal{A}_i$ the voltage pattern of antenna i), and $I(\mathbf{s})$ is the sky brightness distribution

$$dr = A(\mathbf{s})I(\mathbf{s})d\Omega\delta\nu\cos(2\pi\nu\tau_g) \tag{11}$$

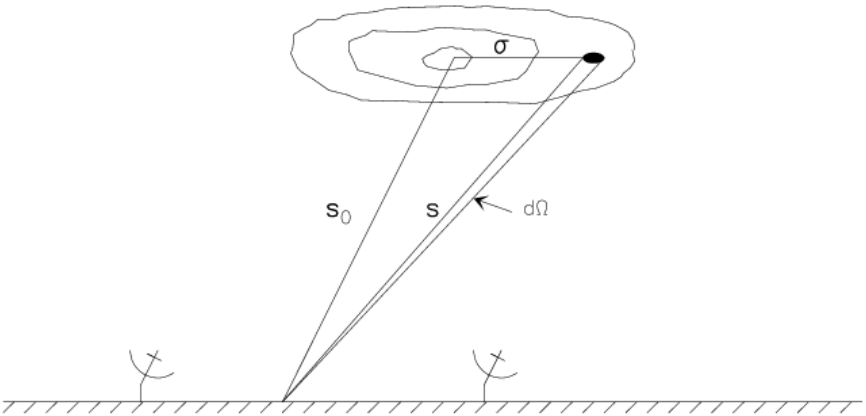
$$r = \delta\nu\int_{Sky} A(\mathbf{s})I(\mathbf{s})\cos(2\pi\nu\mathbf{b.s}/c)d\Omega \tag{12}$$



Position vectors used for the expression of the interferometer response to an extended source, schematically represented by the iso-contours of the sky brightness distribution.

Source Size Effects (2)

- Two implicit assumptions have been made in deriving Eq.12.
  - We assumed incident plane waves, which implies that the source must be in the far field of the interferometer.
  - We used a linear superposition of the fringes from the incident waves, which implies that the source must be spatially incoherent.
- These assumptions are quite valid for most astronomical sources, but may be violated under special circumstances (e.g. VLBI observations of solar system objects would violate the first assumption, or masers for the second one.)



Position vectors used for the expression of the interferometer response to an extended source, schematically represented by the iso-contours of the sky brightness distribution.

Source Size Effects (3)

- When the interferometer is tracking a source in direction $\mathbf{s}_o$, with $\mathbf{s} = \mathbf{s}_o + \sigma$

$$\begin{aligned} r &= \delta\nu \cos\left(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c}\right) \int_{Sky} A(\sigma) I(\sigma) \cos(2\pi\nu\mathbf{b}.\sigma/c) d\Omega \\ &- \delta\nu \sin\left(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c}\right) \int_{Sky} A(\sigma) I(\sigma) \sin(2\pi\nu\mathbf{b}.\sigma/c) d\Omega \end{aligned} \tag{13}$$

- We define the *Complex Visibility*

$$V = |V| e^{i\varphi_\mathrm{V}} = \int_{Sky} A(\sigma) I(\sigma) e^{-2i\pi\nu\mathbf{b}.\sigma/c} d\Omega \tag{14}$$

which resembles a Fourier Transform...

- We thus have

$$\begin{aligned} r &= \delta\nu \ \cos(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c})|V| \cos(\varphi_\mathrm{V}) - d\nu \ \sin(2\pi\nu\frac{\mathbf{b}.\mathbf{s}_o}{c})|V| \sin(\varphi_\mathrm{V}) \\ &= \delta\nu \ |V| \cos(2\pi\nu\tau_g - \varphi_\mathrm{V}) \end{aligned} \tag{15}$$

i.e. the correlator output is proportional to the amplitude of the visibility, and contains a phase relation with the visibility.

Finite Bandwidth

- Integrating over a finite bandwidth $\Delta\nu$,

$$R = \frac{1}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} |V| \cos(2\pi\nu\tau_g - \varphi_\mathrm{V}) d\nu \tag{16}$$

- Using $\nu = \nu_0 + n$

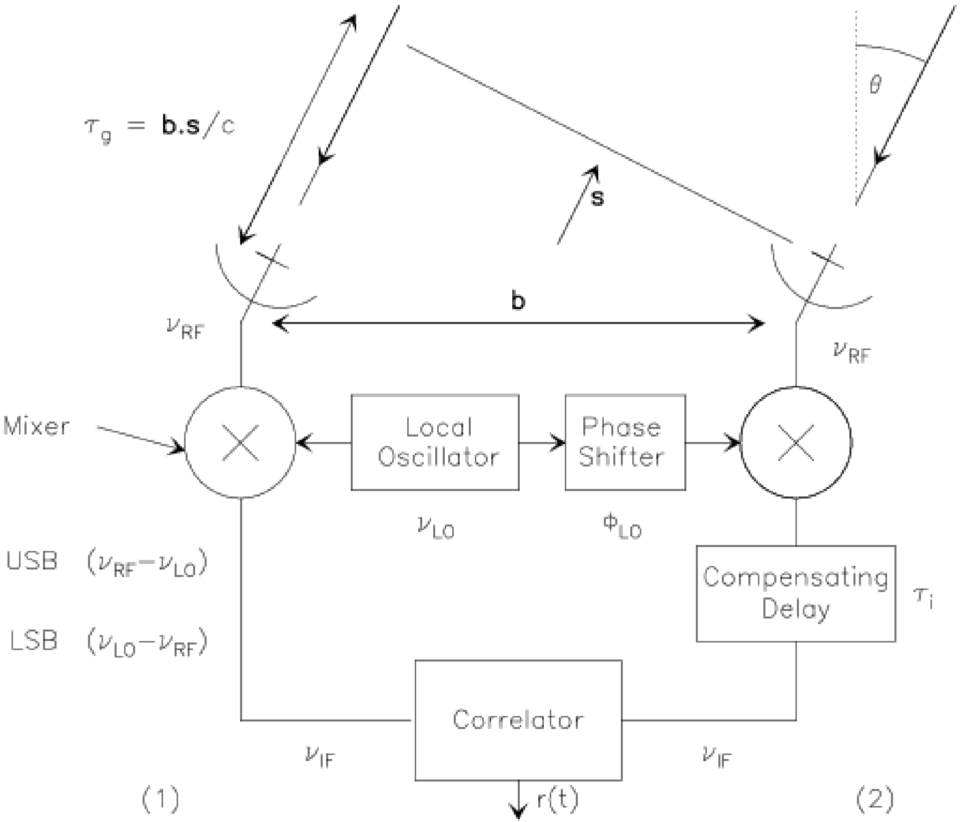
$$R = \frac{1}{\Delta\nu} \int_{-\Delta\nu/2}^{\Delta\nu/2} |V| \cos(2\pi\nu_0\tau_g - \varphi_\mathrm{V} + 2\pi n\tau_g) dn \tag{17}$$

$$= |V| \cos(2\pi\nu_0\tau_g - \varphi_\mathrm{V}) \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g} \tag{18}$$

- The fringe visibility is attenuated by a $\sin(x)/x$ envelope, called the bandwidth pattern, which falls off rapidly.
  1% loss in visibility $\rightarrow |\Delta\nu\tau_g| \simeq 0.078$, or, with $\Delta\nu = 1$GHz: a path length difference of 2cm.
- The ability to track a source for a significant hour angle coverage requires proper compensation of the geometrical delay when a finite bandwidth is desired.

Delay Tracking and Frequency Conversion (1)

- Delay lines with mirrors (as in optics...) are impractical: avoiding diffraction losses requires $D_m^2 \gg B\lambda$, i.e. $D_m \gg 9$ m for ALMA ($\lambda = 7$ mm, $B = 12$ km).
- Millimeter waveguides are lossy
- The compensating delay must be introduced after one (or several) frequency conversion(s)



2-element heterodyne interferometer with delay tracking after frequency conversion

Delay Tracking and Frequency Conversion (2)

- For USB conversion, the phase changes of the input signals before reaching the correlator are

$$\varphi_1 = 2\pi\nu\tau_g = 2\pi(\nu_{\rm LO} + \nu_{\rm IF})\tau_g \quad (19)$$
$$\varphi_2 = 2\pi\nu_{\rm IF}\tau_i + \varphi_{\rm LO} \quad (20)$$

since the delay is inserted at frequency $\nu_{\rm IF}$

- $\Delta\tau = \tau_g - \tau_i$ being the delay tracking error, the correlator output is

$$r = |V|\cos(\varphi_1 - \varphi_2 - \varphi_{\rm V}) \quad (21)$$
$$r_{\rm USB} = |V|\cos(2\pi(\nu_{\rm LO}\tau_g + \nu_{\rm IF}\Delta\tau) - \varphi_{\rm V} - \varphi_{\rm LO}) \quad (22)$$
$$r_{\rm LSB} = |V|\cos(2\pi(\nu_{\rm LO}\tau_g - \nu_{\rm IF}\Delta\tau) - \varphi_{\rm V} - \varphi_{\rm LO}) \quad (23)$$

- When the two sidebands are superposed,

$$r_{\rm DSB} = 2|V|\cos(2\pi\nu_{\rm LO}\tau_g - \varphi_{\rm V} - \varphi_{\rm LO})\cos(2\pi\nu_{\rm IF}\Delta\tau) \quad (24)$$

i.e. the amplitude is modulated by the delay tracking error. Also the visibility phase $\varphi_{\rm V}$ cannot be measured by varying $\Delta\tau$

- We use sideband separation to avoid this problem. The delay tracking error should then be kept small compared to the bandwidth $\Delta\tau \ll 1/\Delta\nu$.

Fringe Stopping

- With the Earth rotation, the cosine term of Eq.18 modulates the correlator output with a *natural fringe rate* of

$$\nu_{\rm LO}\frac{d\tau_g}{dt} \simeq \Omega_{earth}\frac{\mathrm{b}\nu_{\rm LO}}{c} \tag{25}$$

  which is of order of 50 Hz for $b = 800$ m baselines and $\nu_{\rm LO} = 250$ GHz, or $2''$ angular resolution (Earth rotates at $15''$/s; the fringe rate only depends on the effective angular resolution).

- The modulated visibility could be sampled digitally. This is for example done in VLBI.
- But the usual technique is to rotate the phase of the local oscillator $\varphi_{\rm LO}$ such that

$$\varphi_{\rm LO}(t) = 2\pi\nu_{\rm LO}\tau_g(t) \tag{26}$$

  at any given time. Then the output phase is slowly varying, actually constant for a point source at the reference position (also called the delay tracking center). This process is called *Fringe Stopping*.

Complex Correlator

- After fringe stopping

$$r_r = A_o|V|\cos(\pm 2\pi\nu_{\mathrm{IF}}\Delta\tau - \varphi_{\mathrm{V}}) \tag{27}$$

we can no longer measure the amplitude $|V|$ and the phase φ_{V} separately.

- A second correlator, with one signal phase shifted by $\pi/2$ becomes necessary. Its output is

$$r_i = A_o|V|\sin(\pm 2\pi\nu_{\mathrm{IF}}\Delta\tau - \varphi_{\mathrm{V}}) \tag{28}$$

- With both correlators, we measure directly the real r_r and imaginary r_i parts of the complex visibility r. The device is thus called a *complex correlator*.
- **Note:** A delay tracking error $\Delta\tau$ appears as a phase slope as a function of frequency, with

$$\varphi(\nu_{\mathrm{IF}}) = \pm 2\pi\nu_{\mathrm{IF}}\Delta\tau \tag{29}$$

with the $+$ sign for USB conversion, and the $-$ sign for LSB conversion.

Spectroscopy

- Observing spectral lines requires to make synthesis images at a large number of closely spaced frequencies
- This can be done by implementing a large number of multipliers to calculate the correlation function as a function of delay τ. Noting that each delay step results in a phase shift $2\pi\nu\tau$ at frequency ν. $V(u, v, \tau)$ is given by:

$$V_\nu(u, v, \tau) = \int V(u, v, \nu) e^{2i\pi\tau\nu} d\nu \tag{30}$$

  This Fourier transform can be inverted to retrieve the complex visibility $V(u, v, \nu)$ (as we measure both positive and negative values of τ).

- This is feasible because the complex visibility $V(u, v, \nu)$ of the source varies on a timescale (dominated by Earth rotation) which is significantly larger than the delay step $\tau = 1/\Delta\nu$.
- An additional delay machinery allows coarse compensation of the geometric delay. Compensation of the residual fine delay $\Delta\tau = \mathrm{mod}(\tau_g, 1/\Delta\nu)$ can be done *a posteriori* by applying a simple correcting phase slope as function of frequency.
- For N delay steps, the spectral channel width is $\Delta\nu/N$ and the low loss delay tracking condition $\Delta\nu\Delta\tau \ll 1$ is satisfied if $N \gg 1$.

Sensitivity

- Measurement of visibilities is limited by noise emitted by atmosphere, antenna, ground, receivers.
- The r.m.s. noise in each component of the visibility for the baseline ij is given by:

$$\delta S_{ij} = \frac{2k}{A\eta_{\mathrm{A}}\eta_{\mathrm{Q}}\eta_{\mathrm{P}}} \cdot \frac{\sqrt{T_{\mathrm{SYS}i}T_{\mathrm{SYS}j}}}{\sqrt{2B\tau}} \tag{31}$$

  - A antenna physical aperture
  - η_{A} antenna aperture efficiency
  - η_{Q} efficiency for the correlator
  - η_{P} phase decorrelation factor (LO jitter)
  - $T_{\mathrm{SYS}i}$ system noise temperature (single dish)
  - B bandwidth
  - τ integration time

For the whole array of N identical antenna/receivers, the point-source sensitivity is:

$$\delta S = \frac{2k}{A\eta_{\mathrm{A}}\eta_{\mathrm{Q}}\eta_{\mathrm{P}}} \cdot \frac{T_{\mathrm{SYS}}}{\sqrt{N(N-1)B\tau}} \tag{32}$$

Field of View: Fourier Transform

- The *Complex Visibility* is

$$V = |V|e^{i\varphi_{\rm V}} = \int_{Sky} A(\sigma) I(\sigma) e^{-2i\pi\nu\mathbf{b}.\sigma/c} d\Omega \tag{33}$$

- Let (u,v,w) be the coordinate of the baseline vector, in units of the observing wavelength ν, in a frame of the phase tracking vector s_o, with w along s_o. (x,y,z) are the coordinates of the source vector s in this frame. Then

$$\begin{aligned} \nu\mathbf{b}.\mathbf{s}/c = ux + vy + wz \quad & \nu\mathbf{b}.\mathbf{s}_o/c = w \\ z = \sqrt{1-x^2-y^2} \quad & d\Omega = \frac{dxdy}{z} = \frac{dxdy}{\sqrt{1-x^2-y^2}} \end{aligned} \tag{34}$$

$$V(u,v,w) = \iint A(x,y) I(x,y) \frac{e^{-2i\pi[ux+vy+w(\sqrt{1-x^2-y^2}-1)]}}{\sqrt{1-x^2-y^2}} dxdy \tag{35}$$

Field of View: 2-D Fourier Transform (1)

- If (x, y) are sufficiently small,

$$(\sqrt{1-x^2-y^2}-1)w \simeq (x^2+y^2)w/2 \simeq 0 \tag{36}$$

and Eq.35 becomes

$$\begin{aligned} V(u,v) &= \iint A'(x,y)I(x,y)e^{-2i\pi(ux+vy)}e^{-i\pi(x^2+y^2)w}dxdy \\ \text{with} \quad & A'(x,y) = \frac{A(x,y)}{\sqrt{1-x^2-y^2}} \end{aligned} \tag{37}$$

i.e. basically a 2-D Fourier Transform of AI, but with a phase error term $\pi(x^2+y^2)w$.

Field of View: 2-D Fourier Transform (2)

- We want $|\pi(x^2 + y^2)w| \ll 1$ in all circumstances. Note that

$$w < w_{\rm max} \simeq \frac{b_{\rm max}}{\lambda} \simeq \frac{1}{\theta_{\rm s}} \quad (38)$$

- If $\theta_{\rm f}$ is the field of view to be synthesized, the maximum phase error is

$$\Delta\varphi = \frac{\pi\theta_{\rm f}^2}{4\theta_{\rm s}} \quad (39)$$

- Using $\Delta\varphi < 0.1$ radian results in the condition

$$\theta_{\rm f} < 0.35\sqrt{\theta_{\rm s}} \quad (40)$$

Field of View: Bandwidth Smearing

- Assume u, v are computed for the center frequency ν_0. At frequency ν_0, we have

$$V(u,v) \rightleftharpoons AI(x,y) \tag{41}$$

The similarity theorem on Fourier pairs give

$$V(\frac{\nu_0}{\nu}u, \frac{\nu_0}{\nu}v) = (\frac{\nu}{\nu_0})^2 I(\frac{\nu}{\nu_0}x, \frac{\nu}{\nu_0}y) \tag{42}$$

- Averaging of bandwidth $\Delta\nu$, there is a *radial smearing* equal to

$$\sim \frac{\Delta\nu}{\nu_0}\sqrt{x^2+y^2} \tag{43}$$

and hence the constraint

$$\sqrt{x^2+y^2} \leq 0.1\frac{\theta_s \nu_0}{\Delta\nu} \tag{44}$$

Field of View: Time Averaging

- Assume observations of the Celestial Pole. The baselines cover a sector of angular width $\Omega_{earth}\Delta t$, where Ω_{earth} is the Earth rotation speed, and Δt the integration time.
- The smearing is *circumferential* and of magnitude $\Omega_{earth}\Delta t\sqrt{x^2+y^2}$, hence the constraint

$$\sqrt{x^2+y^2} \leq 0.1\frac{\theta_{\mathrm{s}}}{\Omega_{earth}\Delta t} \tag{45}$$

- For other declinations, the smearing is no longer rotational, but of similar magnitude.
- Values for Plateau de Bure

θ_{s}	ν (GHz)	2-D Field	0.5 GHz Bandwidth	1 Min Averaging	Primary Beam
$5''$	80	$5'$	$80''$	$2'$	$60''$
$2''$	80	$3.5'$	$30''$	$45''$	$60''$
$2''$	220	$3.5'$	$1.5'$	$45''$	$24''$
$0.5''$	230	$1.7'$	$22''$	$12''$	$24''$

Phase stability requirements

- Short term phase errors in the local oscillators (jitter) will cause a decorrelation of the signal and reduce the visibility amplitude by a factor

$$\eta_{12} = e^{-({\sigma_1}^2 + {\sigma_2}^2)/2} = \eta_1 \eta_2 \quad (46)$$

where σ_1 is the rms phase fluctuation of the LO in one of the antennas (σ_2 in the other). $\eta_1 = e^{-{\sigma_1}^2/2}$ is the decorrelation factor for one antenna; typical requirements on σ_1 are:

η_1	0.99	0.98	0.95	0.90
σ_1 (degrees)	8.1	11.5	18.3	26.4

- The phase stability required on the *LO reference frequency* (at 1.8 GHz) is $\sigma_1/(230/1.8)) \sim 0.15^\circ$ for a 0.95 efficiency at 1.3mm
- Very stable oscillators are needed.

Atmospheric Phase

- Cannot (usually) be calibrated because
  - Time scale is short (typically 100 sec)
  - Large antennas cannot usually re-point so quickly
- However, atmospheric phases are usually small enough, i.e. much less than 1 radian.
- More precisely
  - the phase error is $\propto \nu$, because the atmosphere actually introduces a varying path length δl
  - these differential path length fluctuations go as $D^{0.8-0.3}$
  - up to an outer scale of $D \approx 1000 - 2000$ m
  - the timescale is simply $D/v_{\rm wind}$ (frozen atmosphere model), typically 30 seconds for 300 m.
  - $\delta l(300 \text{ m}) \lessapprox 200 - 300\mu$m, $\ll \lambda$.
  - this result in an the atmospheric seeing of about 0.3 – 1″, nearly frequency independent.
- Can be predicted by Water Vapor Monitoring, because water is (by large) the dominant refractive component of the atmosphere.

Other Instrumental Issues

- Delay tracking and fringe stopping require good knowledge of the geometrical delay, i.e. of the baseline coordinates.
- Accurate phase lock systems are required to control φ_{LO}; this usually implies correcting in real time for LO propagation effects (in cables or fibers).
- Elaborate phase switching is used additionally to cancel offsets, separate/reject side bands, ...
- Antenna deformations (e.g. thermal expansion) may change the geometrical delay. Proper control of the antenna focus is thus required.
- The primary beam is not so large, and uncorrected pointing errors will affect the spatial intensity distribution. Accurate pointing is mandatory for precision mapping.
- and of course, there is noise... One needs sensitive receivers.

Pre-Calibrations

- Antenna positions must be accurately measured:
  - in order to track geometric phases and delays
  - on an ensemble of point sources of known direction (since $\varphi = \mathbf{b}.\mathbf{s}/c$, you can determine $\mathbf{b}$ if you have an ensemble of $\mathbf{s}$)
  - phases may be corrected *a posteriori*
- Delay offsets must be accurately known (may be corrected *a posteriori* if not too big)
  - Delay Calibration: measure phase slope vs frequency on a strong point source
- Antenna pointing:
  - a good pointing model is needed
  - reference pointing every ~ 1 hour is usually required
  - short term pointing errors (refraction fluctuations, wind) are much more difficult to deal with...
- Antenna focus:
  - means also frequent recalibration (typically 1-2 h)

Post-Calibrations

- Unpredictable instrumental phase and amplitude drifts:
  - φ_{LO} may vary with time
  - The local oscillator power may vary with time
  - The atmospheric transparency may vary with time.
  - The atmosphere introduces a non-geometric delay, because of varying water vapor
  - Observe sources for which you know the answer (i.e. the visibility: point sources). Phase and amplitude calibration every 30 min. or so.
  - This does not usually correct for atmospheric delay fluctuations
- Atmospheric transparency: changes as a function of elevation are better split from instrumental amplitude drifts if a good atmospheric model is applied first; based on regular observation of known temperature load(s)
- The amplitude and phase response as $f(\nu)$ must be known.
  - Bandpass Calibration on a strong continuum point source; much less often (typ. once a session) as it is costly in time.

In summary...

- mm-wave interferometry is not so difficult to understand
- A few simple points
  - The sign of phase depends on sideband conversion
  - The angular frequency is the time derivative of the interferometer phase
  - A finite bandwidth implies delay tracking
  - A delay error produces a linear phase slope as function of frequency
  - Fringe stopping makes life easier, but implies a complex correlator
  - High resolution spectroscopy is natural
  - Delays can be implemented digitally
  - A complex correlator directly measures the visibility
  - The visibility is *almost* the Fourier transform of the sky brightness multiplied by the primary beam...
  - ... provided the data is properly handled! Field of view can be limited otherwise.
- Proper calibration is essential.
- but atmospheric phases cannot (yet) be calibrated out.