Single-dish antenna at radio wavelength

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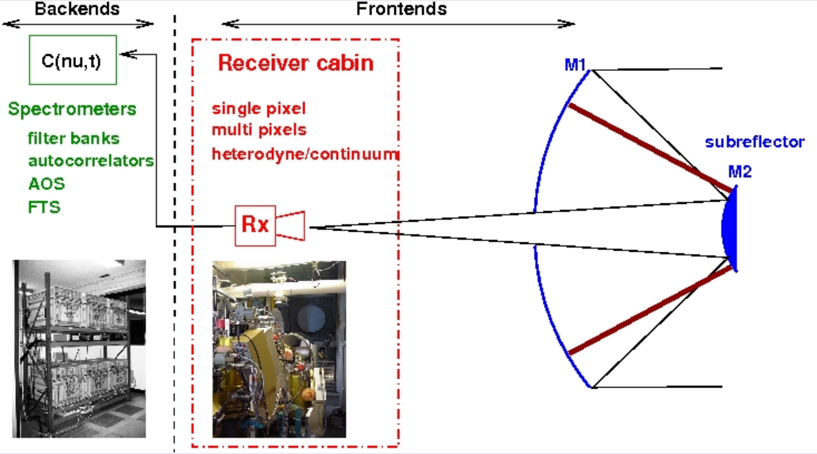
Millimétrique

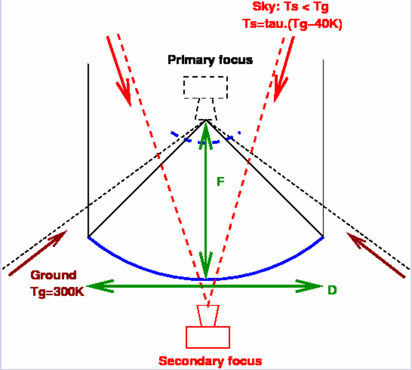
Outline

A typical single-dish antenna



Why Cassegrain configuration ?

- $F/D \longrightarrow F_e/D = m \times F/D$
  IRAM-30m, $m = 27.8$
  $F/D = 0.35$, $F_e/D \approx 10$
- Rx alignement easier ; focal plane arrays
- increase effective area (or on-axis gain)
- decrease spillover
- *but* increase mechanical load
- obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam



Main single-dish antenna at mm wavelength

Large aperture: f/D$\lesssim$ 1

Institute	Diameter (m)	Frequency (GHz)	Wavelength (mm)	HPBW ($''$)	Latitude
IRAM	30	70 – 280	1 – 4	9 – 35	+37°
IRAM		70 – 345	0.8 – 4	7 – 35	
JCMT	15	210 – 710	0.2 – 2	8 – 20	+20°
APEX	12	230 – 1200	0.3 – 1.3	6 – 30	−22°
CSO	10.4	230 – 810	0.4 – 1.3	10 – 30	+20°

Some terminology

Receivers (Rx)

- bandwidth $\Delta\nu = 0.5\text{-}4$ GHz
- central frequency $\nu_0 = 100 - 1200$ GHz
- heterodyne receivers: $\Delta\nu \ll \nu_0$ $\Rightarrow\approx$ monochromatic
- bolometers: $\Delta\nu \approx 50$ GHz not monochromatic
- one polarization (linear, circular)
- taper (apodization at the rim)

Backends

- spectrometers:
  - filter banks (FB)
  - acousto-optic (AOS)
  - autocorrelators (AC)
  - Fast Fourier Transform Spectrometer (FFTS)
- spectral resolution $\delta\nu \approx 3 - 2000\,\mathrm{kHz}$

$\Rightarrow$ largest resolution power $R = \nu_0/\delta\nu \approx 10^5 - 10^8$

Outline

Power received

What is the power received from an unpolarized (point) source of **flux density** S_ν $(\mathrm{W\,m^{-2}\,Hz^{-1}})$?

- S_ν measured in Jy: 1 Jy $= 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$
- Monochromatic power:
  $p_\nu = \frac{1}{2} A_e \cdot S_\nu \qquad [\mathrm{W\,Hz^{-1}}]$
- Power in the bandwidth $\Delta\nu$:
  $p = \frac{1}{2} A_e \cdot S_\nu \cdot \Delta\nu \qquad [\mathrm{W}]$
- effective area of the antenna: $A_e \leq A_{\mathrm{geom}}$
- Question: $A_e =?$
- Answer: $A_e = \eta_A A_{\mathrm{geom}} = \eta_i \eta_s \ldots A_{\mathrm{geom}}$
- Determine η_i, $\eta_s \ldots$

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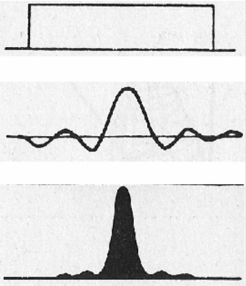
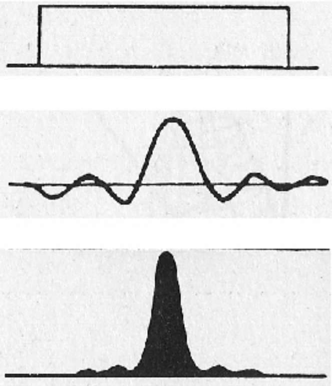
Ideal beam pattern: illumination and taper

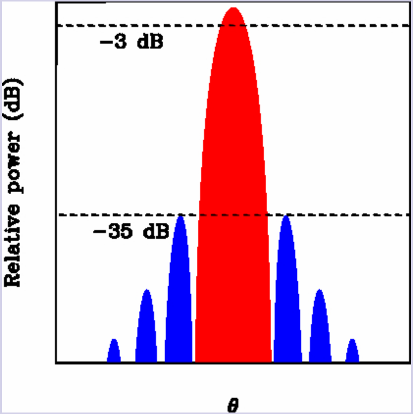
- Diffraction theory (Huygens-Fresnel, Fraunhoffer approx.):
  $E_{\mathrm{f-f}}(l, m) \propto \mathcal{F}[E_{\mathrm{ant}}(x, y)]$
- $E_{\mathrm{ant}}(x, y)$ (grading)
  bounded on a finite domain $\Delta 1 \Rightarrow E_{\mathrm{f-f}}(l, m)$ concentrated on a finite domain $\Delta 2$ ($\Delta 1 \cdot \Delta 2 \sim 1$)
  sharp cut of the antenna domain $\Rightarrow$ oscillations (side-lobes) $\Rightarrow$ taper

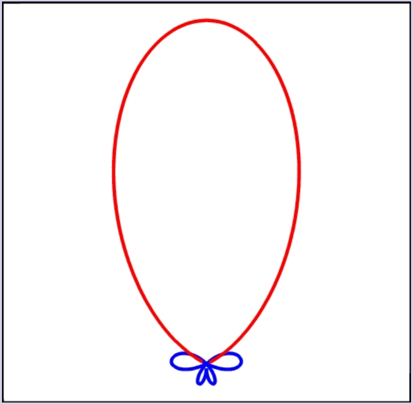


- Reciprocity: antenna in emission
- pattern of the transmitted emission depends on the direction (l, m):
  Power pattern
  $\mathcal{P}(l, m) \propto |E_{\mathrm{f-f}}(l, m)|^2$
  Effective area
  $A(l, m) = A_{\mathrm{max}} \cdot \mathcal{P}(l, m)$
- example: circular aperture
  $\mathcal{P}(l, m) \propto$ Airy disk



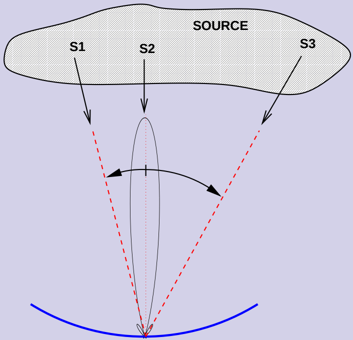
Power pattern





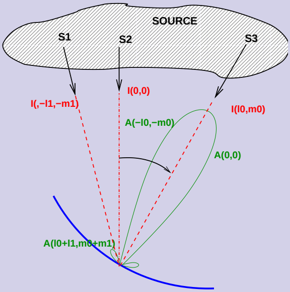
Brightness distribution with total-power telescope

- point source: flux density
  $S_\nu \quad [\mathrm{W\,m^{-2}\,Hz^{-1}}]$
- extended source: brightness
  $I_\nu(l,m) \quad [\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$
  $I_\nu = \mathrm{d}S_\nu / \mathrm{d}\Omega$
- from the direction (l_i, m_i):
  $\mathrm{d}p_\nu = A(l_i,m_i)\, I_\nu(l_i,m_i)\, \mathrm{d}\Omega_i$
- incoherent emission: add intensities



Pointing a source at a fixed position:

$$p_\nu(0,0) = \iint_{4\pi} A(l,m)\, I_\nu(l,m)\, \mathrm{d}\Omega$$



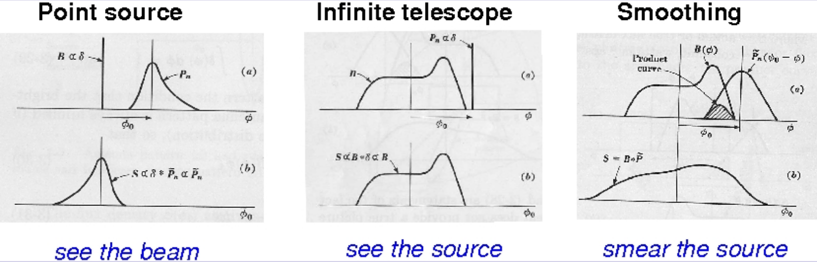
- point source: flux density $S_\nu (\mathrm{W\,m^{-2}\,Hz^{-1}})$
- extended source: brightness $I_\nu(l, m)$ $I_\nu = \mathrm{d}S_\nu / \mathrm{d}\Omega \; (\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}})$
- antenna tilted towards (l_0, m_0)
- from the direction (l_i, m_i) $\mathrm{d}p_\nu = A(l_0 - l_i, m_0 - m_i)\, I_\nu(l_i, m_i)\, \mathrm{d}l_i\, \mathrm{d}m_i$
- incoherent emission: add intensities
- convolution

Scanning a source leads to a convolution:

$$p_\nu(l_0, m_0) = \iint A(l_0 - l, m_0 - m)\, I_\nu(l, m)\, \mathrm{d}l\, \mathrm{d}m$$

$$I'_\nu = \mathcal{P} * I_\nu$$

Convolution: consequences



$$\theta_{\rm obs} = \sqrt{\theta_{\rm mb}^2 + \theta_{\rm sou}^2}$$

Outline

Beam pattern

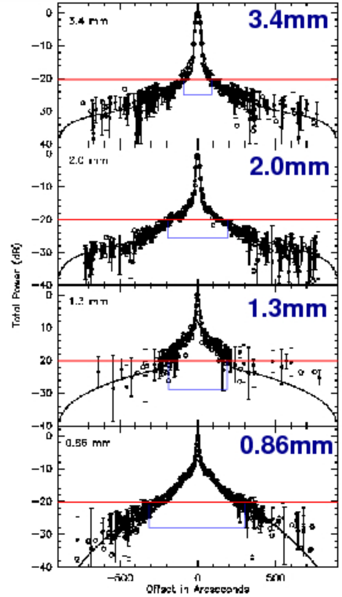
- secondary lobes (finite surface antenna)
- error lobes (surface irregularities)
  - main-beam collects less power
  - if correlation length l $\Rightarrow$ Gaussian error-beam $\Theta_{\mathrm{EB}} \approx \lambda/\ell$

real beam = main-beam + error-beam(s)

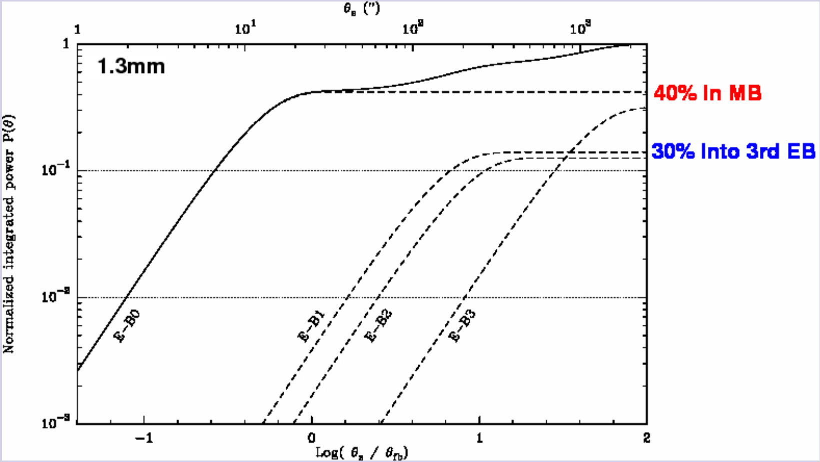
- Questions:

  What power is collected in each beam ?

  What are the FWHMs of the beams ?



IRAM 30-m antenna: Error-Beams power



Greve et al A&A 1998

Outline

Radioastronomy: speaking in terms of temperatures

- Black-body radiation:
  $B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$ $[\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$
- express energy of a transition into temperature: $T_0 = \frac{h\nu}{k}$
- Rayleigh-Jeans approximation: $h\nu \ll kT$
  $B_\nu(T) \approx \frac{2kT}{\lambda^2}$ $[\mathrm{W\,m^{-2}\,Hz^{-1}}]$
- radiation temperature, T_{RJ}, Rayleigh-Jeans approximation

$$I_\nu = \frac{2k\nu^2}{c^2} T_{\mathrm{RJ}} = \frac{2k}{\lambda^2} T_{\mathrm{RJ}} \qquad [\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$

- relation $T_{\mathrm{B}} - T_{\mathrm{RJ}}$:
  $T_{\mathrm{RJ}} = J_\nu(T_{\mathrm{B}}) = \frac{h\nu}{k}\frac{1}{\exp(h\nu/kT_{\mathrm{B}})-1} = \frac{T_0}{\exp(T_0/T_{\mathrm{B}})-1}$
- in the following: $I_\nu(l,m) \rightarrow T_{\mathrm{RJ}}(l,m)$
- consequence: power $\propto T_{\mathrm{RJ}}$

$$p_\nu(l_0,m_0) = \frac{k}{\lambda^2}\iint_{4\pi} A(l,m)\, T_{\mathrm{RJ}}(l_0-l, m_0-m)\, \mathrm{d}\Omega$$

Brightness temperature

- T_B defined by
  $I_\nu = B_\nu(T_\mathrm{B}) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_\mathrm{B}}-1} \qquad [\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$
- Consider a source of a finite angular extent (Ω_S) and flux density S_ν $\quad [\mathrm{W\,m^{-2}\,Hz^{-1}}]$:
  $S_\nu = \frac{2k}{\lambda^2}\int_{\Omega_S} T_\mathrm{B}(\Omega)\,\mathrm{d}\Omega$

Antenna temperature

- Johnson noise in terms of an equivalent temperature

average power transferred from a conductor to a line within $\delta\nu$: $= k\, T\, \delta\nu$

- Antenna temperature: antenna as a conductor

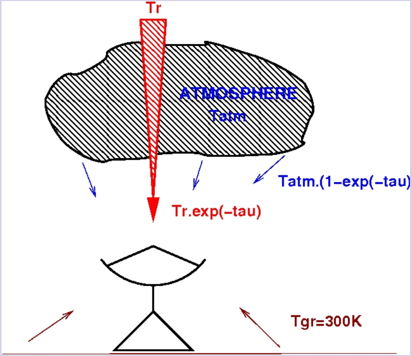
$$p_\nu = k\, T_A \qquad [\mathrm{W} \cdot \mathrm{Hz}^{-1}] = [\mathrm{J}] = [\mathrm{J} \cdot \mathrm{K}^{-1}][\mathrm{K}]$$

- $A_{\max} = \lambda^2 / \iint_{4\pi} P(l,m)\, \mathrm{d}\Omega$

- Therefore: $T_A(l,m) = \frac{1}{\lambda^2} \iint_{4\pi} A(l,m)\, T_{\mathrm{RJ}}(l_0 - l, m_0 - m)\, \mathrm{d}\Omega$

$$T_A(l,m) = \iint_{4\pi} P(l,m)\, T_{\mathrm{RJ}}(l_0 - l, m_0 - m)\, \mathrm{d}\Omega / \iint_{4\pi} P(l,m)\, \mathrm{d}\Omega$$

Correct for atmospheric emission/absorption



Antenna temperature given by:

$$T_A = \eta_s \left\{ T_{\mathrm{RJ}} e^{-\tau_\nu} + (1 - e^{-\tau_\nu})\, T_{\mathrm{atm}} \right\} + (1 - \eta_s)\, T_{\mathrm{gr}}$$

see lecture on Atmospheric transmission

Correcting for spillover and beam pattern: $T_{\rm A}^*$ and $T_{\rm mb}$

- corrects for **atmospheric attenuation**: $\times\,\exp(\tau_\nu)$
  $T'_A = T_A e^{\tau_\nu}$
- takes into account rear side-lobes:
  forward signal only (2π sr)
- Antenna temperature: $T_{\rm A}^* = \frac{T'_A}{F_{\rm eff}}$
- Consequence:

$$T_{\rm A}^*(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\rm RJ}(\Omega_0 - \Omega)\,{\rm d}\Omega}{\mathcal{P}_{2\pi}}, \qquad \mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega)\,{\rm d}\Omega$$

Correcting for spillover and beam pattern: T_{A}^* and T_{mb}

- corrects for **atmospheric attenuation**: $\times \exp(\tau_\nu)$
  $T_A' = T_A e^{\tau_\nu}$
- takes into account rear side-lobes:
  forward signal only (2π sr)
- Antenna temperature: $T_{\mathrm{A}}^* = \frac{T_A'}{F_{\mathrm{eff}}}$
- Consequence:

$$T_{\mathrm{A}}^*(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{RJ}}(\Omega_0 - \Omega)\, \mathrm{d}\Omega}{\mathcal{P}_{2\pi}}, \qquad \mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega)\, \mathrm{d}\Omega$$

- take into account main-beam and error-lobes
- same as T_{A}^* but in Ω_{mb} instead of 2π: $T_{\mathrm{mb}} = \frac{T_A'}{B_{\mathrm{eff}}}$

$$T_{\mathrm{mb}}(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{RJ}}(\Omega_0 - \Omega)\, \mathrm{d}\Omega}{\mathcal{P}_{\mathrm{mb}}}, \qquad \mathcal{P}_{\mathrm{mb}} = \int_{\Omega_{\mathrm{mb}}} \mathcal{P}(\Omega)\, \mathrm{d}\Omega$$

Temperature scales

Definitions

$$\text{Forward efficiency:} \quad F_{\rm eff} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{4\pi}}$$

$$\text{Beam efficiency:} \quad B_{\rm eff} = \frac{\mathcal{P}_{\rm mb}}{\mathcal{P}_{4\pi}}$$

Consequences

$$T_{\rm mb} = \frac{F_{\rm eff}}{B_{\rm eff}}\ T_{\rm A}^* = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{\rm mb}}\ T_{\rm A}^*$$

What you measure is $T_{\rm A}^*$ or $T_{\rm mb}$ (usually $\neq T_{\rm RJ}$)

Consequences

Consider limiting cases:

- Small source: $\Omega_S \ll \mathcal{P}_{\mathrm{mb}}$
  $T_{\mathrm{A}}^* = \frac{P(0)\Omega_S T_{\mathrm{RJ}}}{\mathcal{P}_{2\pi}} = T_{\mathrm{RJ}} \frac{\Omega_S}{\mathcal{P}_{2\pi}}$
  $\longrightarrow$ beam dilution

Consequences

Consider limiting cases:

- Small source: $\Omega_S \ll \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{P(0)\Omega_S T_{\rm RJ}}{\mathcal{P}_{2\pi}} = T_{\rm RJ}\frac{\Omega_S}{\mathcal{P}_{2\pi}}$
  $\rightarrow$ beam dilution
- Large source: $\Omega_S \gg \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{\mathcal{P}_{2\pi} T_{\rm RJ}}{\mathcal{P}_{2\pi}} \approx T_{\rm RJ}$
  $\rightarrow$ Antenna temperature gives the source brightness

Consequences

Consider limiting cases:

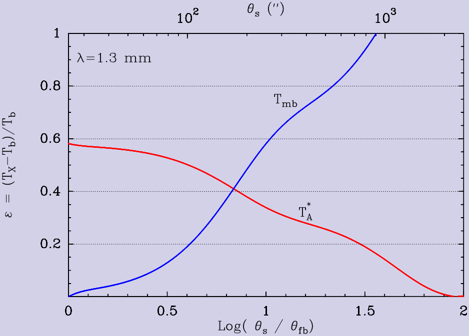
- Small source: $\Omega_S \ll \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{P(0)\Omega_S T_{\rm RJ}}{\mathcal{P}_{2\pi}} = T_{\rm RJ}\frac{\Omega_S}{\mathcal{P}_{2\pi}}$
  $\longrightarrow$ beam dilution
- Large source: $\Omega_S \gg \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{\mathcal{P}_{2\pi} T_{\rm RJ}}{\mathcal{P}_{2\pi}} \approx T_{\rm RJ}$
  $\longrightarrow$ Antenna temperature gives the source brightness
- Special case: $\Omega_S = \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{T_{\rm RJ}}{\mathcal{P}_{2\pi}} \int_{\Omega_S} P(\Omega)\,{\rm d}Omega \Rightarrow T_{\rm mb} = T_{\rm RJ}$
  Main-beam temperature gives the source brightness

Consequences

Consider limiting cases:

- Small source: $\Omega_S \ll \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{P(0)\Omega_S T_{\rm RJ}}{\mathcal{P}_{2\pi}} = T_{\rm RJ}\frac{\Omega_S}{\mathcal{P}_{2\pi}}$
  $\rightarrow$ beam dilution
- Large source: $\Omega_S \gg \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{\mathcal{P}_{2\pi} T_{\rm RJ}}{\mathcal{P}_{2\pi}} \approx T_{\rm RJ}$
  $\rightarrow$ Antenna temperature gives the source brightness
- Special case: $\Omega_S = \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{T_{\rm RJ}}{\mathcal{P}_{2\pi}} \int_{\Omega_S} P(\Omega)\,{\rm d}Omega \Rightarrow T_{\rm mb} = T_{\rm RJ}$
  Main-beam temperature gives the source brightness
- General (worse) case: $\Omega_S \sim \mathcal{P}_{\rm mb}$
  $T_{\rm A}^* = \frac{T_{\rm RJ}}{\mathcal{P}_{2\pi}} \int_{\Omega_S} P(\Omega)\,{\rm d}Omega$
  If source of uniform brightness and beam pattern known, feasible, but in real life... Which scale to use: $T_{\rm A}^*$, $T_{\rm mb}$?

Which temperature scale ?



Source size	Temperature scales
$\Omega_S = 2\pi$	$T_{\rm RJ} = T_{\rm A}^*$
$\Omega_S = \Omega_{\rm mb}$	$T_{\rm RJ} = T_{\rm mb}$
$2\pi < \Omega_S$	$T_{\rm RJ} < T_{\rm A}^*$
$\Omega_{\rm mb} < \Omega_S < 2\pi$	$T_{\rm A}^* < T_{\rm RJ} < T_{\rm mb}$
$\Omega_{\rm mb} > \Omega_S$	$T_{\rm mb} < T_{\rm RJ}$

Outline

Goal of the calibration

- Atmosphere: opacity τ_ν
- Antenna-sky coupling: $F_{\rm eff}$
- Output at backends: "counts"
- Question: **counts $\longrightarrow$ Temperature ?**
  $C = \chi T \Longrightarrow \chi = ?$
- Telescope pointing at a source: how many counts ?

$$\begin{aligned} C_{\rm sou} &= \chi \left\{ T_{\rm rec} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} + T_{\rm emi} \right\} \\ T_{\rm emi} &= F_{\rm eff}(1 - e^{-\tau_\nu})\, T_{\rm atm} + (1 - F_{\rm eff})\, T_{\rm gr} \end{aligned}$$

- source signal very weak: $T_{\rm atm} = 290$ K, $T_{\rm rec} \approx 50 - 150$ K, $T_{\rm emi} \approx 70$ K

Chopper Wheel method

$$
\begin{aligned}
C_{\rm sou} &= \chi \left\{ T_{\rm rec} + T_{\rm emi} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} \right\} \\
C_{\rm atm} &= \chi \left\{ T_{\rm rec} + T_{\rm emi} \right\} \\
C_{\rm hot} &= \chi \left\{ T_{\rm rec} + T_{\rm hot} \right\} \\
C_{\rm col} &= \chi \left\{ T_{\rm rec} + T_{\rm col} \right\}
\end{aligned}
$$

Making differences

$$
\begin{aligned}
C_{\rm sou} - C_{\rm atm} &= \chi F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} \\
C_{\rm hot} - C_{\rm atm} &= \chi (T_{\rm hot} - T_{\rm emi})
\end{aligned}
$$

Definition of $T_{\rm cal}$:

$$
T_{\rm sou} = \frac{C_{\rm sou} - C_{\rm atm}}{C_{\rm hot} - C_{\rm atm}}\; T_{\rm cal}
$$

$$
\Rightarrow T_{\rm cal} = (T_{\rm hot} - T_{\rm emi})\, \frac{e^{\tau_\nu}}{F_{\rm eff}}
$$

Outputs of calibration procedure: $T_{\rm cal}$

Rewrite $T_{\rm emi}$

$$T_{\rm emi} = T_{\rm gr} + F_{\rm eff}(T_{\rm atm} - T_{\rm gr}) - F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm atm}$$

$$\begin{aligned} C_{\rm hot} - C_{\rm atm} &= \chi\{(T_{\rm hot} - T_{\rm gr}) + F_{\rm eff}(T_{\rm gr} - T_{\rm atm}) \\ &+ F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm atm}\} \end{aligned}$$

- Assume $T_{\rm hot} = T_{\rm atm} = T_{\rm gr} \Rightarrow \{\chi, \tau_\nu\} \rightarrow \{\chi e^{-\tau_\nu}\}$
  $\Rightarrow$ 3 unknowns $\Rightarrow$ *e.g.* don't need to solve for τ_ν
  (Penzias & Burrus ARAA 1973)

$$\boxed{T_{\rm cal} = T_{\rm atm}}$$

- General case: different $T_{\rm atm}$, $T_{\rm hot}$ and $T_{\rm gr}$
  $\Rightarrow$ solve for the 4 unknowns

Outputs of calibration procedure: T_{rec}

Hot & cold loads $\longrightarrow$ T_{rec}:

$$
\begin{aligned}
Y &= \frac{C_{\mathrm{hot}}}{C_{\mathrm{col}}} = \frac{T_{\mathrm{rec}} + T_{\mathrm{hot}}}{T_{\mathrm{rec}} + T_{\mathrm{col}}} \\
T_{\mathrm{rec}} &= \frac{T_{\mathrm{hot}} - Y T_{\mathrm{col}}}{Y - 1}
\end{aligned}
$$

Outputs of calibration procedure: T_{sys}

System temperature: describes the noise including all sources from the sky down to the backends

- used to determine the total statistical noise. For heterodyne receivers, noise is given by the "radiometer formula":

$$\sigma_T = \frac{\kappa \cdot T_{\mathrm{sys}}}{\sqrt{\delta\nu\,\Delta t}}$$

- δ_ν: spectral resolution
- Δt: total integration time
- κ depends on the observing mode:
  example: position switching
  $\mathtt{ON\text{-}OFF} \Rightarrow \sqrt{2}$
  $t_{\mathtt{ON}} = t_{\mathtt{OFF}} \Rightarrow \Delta t = 2t_{\mathtt{ON}} \Rightarrow \sqrt{2}$
  $\Rightarrow \kappa = 2$

From T_{mb} to I_ν, from Kelvin to Jansky

- flux density: $S_\nu = \int_{\Omega_r} I_\nu(\Omega)\,\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_{\Omega_r} T_{\mathrm{mb}}\,\mathrm{d}\Omega$
- power received: $kT'_A = k\frac{T_{\mathrm{A}}^*}{F_{\mathrm{eff}}} = \frac{1}{2}S_\nu A_e$
  $\Rightarrow \frac{S_\nu}{T_{\mathrm{A}}^*} = \frac{2k}{A}\frac{F_{\mathrm{eff}}}{\eta_A}$
- values of S_ν / T_{A}^* are tabulated *e.g.* on IRAM-30m web page ($\approx$ 6 @ 100 GHz, $\approx$ 9 @ 230 GHz)

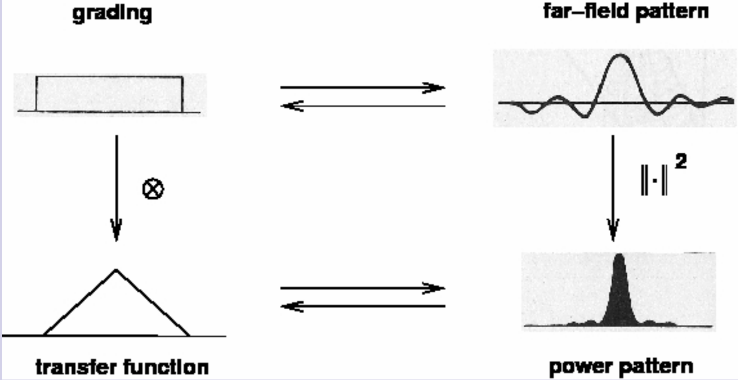
From $T_{\rm mb}$ to I_ν, from Kelvin to Jansky

- flux density: $S_\nu = \int_{\Omega_r} I_\nu(\Omega)\,\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_{\Omega_r} T_{\rm mb}\,\mathrm{d}\Omega$
- power received: $kT'_A = k\frac{T^*_{\rm A}}{F_{\rm eff}} = \frac{1}{2}S_\nu A_e$
  $\Rightarrow \frac{S_\nu}{T^*_{\rm A}} = \frac{2k}{A}\frac{F_{\rm eff}}{\eta_A}$
- values of $S_\nu/T^*_{\rm A}$ are tabulated *e.g.* on IRAM-30m web page (≈ 6 @ 100 GHz, ≈ 9 @ 230 GHz)
- How to convert the temperatures into $\mathrm{W\,m^{-2}\,Hz^{-1}}$?
- Gaussian source of uniform radiation temperature T_R:
  $S_\nu = 8.2\times10^{-3}\left(\frac{\nu}{100\ GHz}\right)^2\left(\frac{\theta_r}{1''}\right)^2\left(\frac{T_R}{K}\right) \quad \mathrm{Jy}$

$$1\ \mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$$

Outline

Image formation: total power telescope



- antenna scans the source
- image: convolution of I_0 by beam pattern $I'_\nu = \mathcal{P} * I_{0,\nu}$
- measure directly the brightness distribution I_0

Interferometer field of view

$$F = D * (\mathcal{P} \times I) + N$$

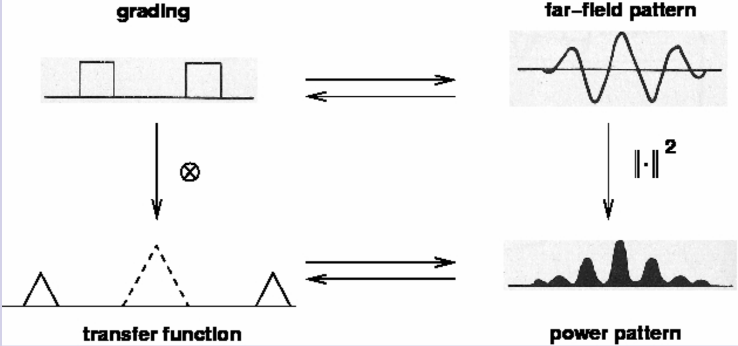
$$\begin{aligned} F &= \text{dirty map} = \text{FT of observed visibilities} \\ D &= \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ \mathcal{P} &= \text{power pattern of single-dish } \textit{(primary beam B in the following)} \\ I &= \text{sky brightness distribution} \\ N &= \text{noise distribution} \end{aligned}$$

- An interferometer measures the product $\mathcal{P} \times I$
- $\mathcal{P}$ has a finite support $\longrightarrow$ limits the size of the field of view

Summary

- full-aperture antenna: $\mathcal{P} * \mathbf{I}$
- interferometry sensitive to $\mathcal{P} \times \mathbf{I}$
- **amplitude calibration:**
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- **lobe = main-lobe + error-lobes** (*e.g.* as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the **temperature scale** to use ($T_{\rm A}^*$, $T_{\rm mb}$,...)

Image formation: correlation telescope



- antennas fixed w.r.t. the source
- correlation temperature: $\mathcal{T}(0,0)$ Fourier transform of $I_0 \times \mathcal{P}$
- measure the Fourier transform of the brightness distribution I_0
- image built afterwards

Interferometer field of view

Measurement equation of an interferometric observation:

$$F = D * (B \times I) + N$$

$$\begin{aligned} F &= \text{dirty map} = \text{FT of observed visibilities} \\ D &= \text{dirty beam } (\longrightarrow \text{deconvolution}) \\ B &= \text{primary beam} \\ I &= \text{sky brightness distribution} \\ N &= \text{noise distribution} \end{aligned}$$

- An interferometer measures the product $B \times I$
- B has a finite support $\longrightarrow$ limits the size of the field of view
- B is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

$$\begin{array}{ccc} \text{Aperture function} & \rightleftharpoons & \text{Voltage pattern} \\ \star\downarrow & & \downarrow |\cdot|^2 \\ \text{Transfert function } T(u,v) & \rightleftharpoons & \text{Power pattern } B(\ell,m) \\ & & = \text{Primary beam} \end{array}$$

Gaussian illumination $\Longrightarrow$ to a good approximation, B is a Gaussian of $1.2\,\lambda/D$ FWHM

Plateau de Bure
$D = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	58″
100 GHz	3.0 mm	50″
115 GHz	2.6 mm	43″
215 GHz	1.4 mm	23″
230 GHz	1.3 mm	22″
245 GHz	1.2 mm	20″