Single-dish antenna at (sub)mm wavelengths

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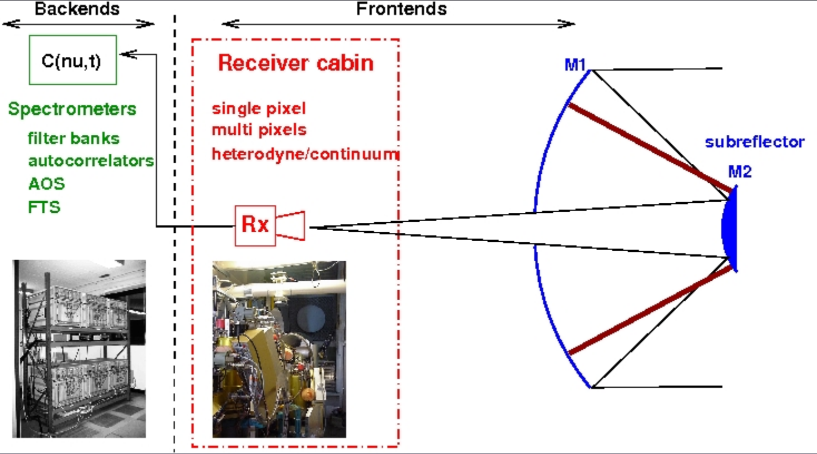
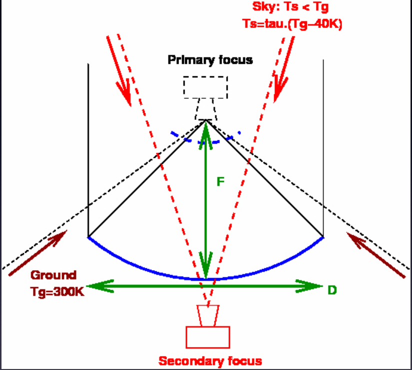
Outline

A single-dish antenna



Why Cassegrain configuration ?

- $F/D \longrightarrow F_e/D = m \times F/D$
  IRAM-30m, $m = 27.8$
  $F/D = 0.35$, $F_e/D \approx 10$
- Rx alignement easier ; focal plane arrays
- increase effective area (or on-axis gain)
- decrease spillover
- *but* increase mechanical load
- obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam



Main single-dish antenna at λmm

Large aperture: f/D$\lesssim$ 1

Obs.	D (m)	ν (GHz)	λ (mm)	HPBW ($''$)	Latitude
IRAM	30	70 – 345	0.7 – 4	7 – 35	$+37^\circ$
JCMT	15	210 – 710	0.2 – 2	8 – 20	$+20^\circ$
APEX	12	230 – 1200	0.3 – 1.3	6 – 30	-22°
CSO	10.4	230 – 810	0.4 – 1.3	10 – 30	$+20^\circ$

Some terminology (1): receivers

- central frequency

$$\nu_0 = 80 - 2000\,\mathrm{GHz}$$

- heterodyne receivers $\approx$ monochromatic

$$\Delta\nu \sim 0.5 - 4\mathrm{GHz} \ll \nu_0$$

- bolometers: not monochromatic

$$\Delta\nu \approx 50\,\mathrm{GHz}$$

- one polarization (linear, circular)
- taper (apodization at the rim)

Some terminology (2): backends

- spectrometers:
  - filter banks (FB)
  - acousto-optic (AOS)
  - autocorrelators (AC)
  - Fast Fourier Transform Spectrometer (FFTS)
- spectral resolution $\delta\nu \approx 3 - 2000\,\text{kHz}$

$\Rightarrow$ largest resolution power
$R = \nu_0/\delta\nu \approx 10^5 - 10^8$

Outline

Collected power from a source

What is the power received from an unpolarized (point) source of **flux density** S_ν $(\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}})$?

- S_ν measured in Jy: 1 Jy = $10^{-26}\ \mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$
- Monochromatic (and monomode) power collected by an area A_e:
$$p_\nu = \frac{1}{2} A_e \cdot S_\nu \qquad [\mathrm{W\,Hz^{-1}}]$$
- Power in the bandwidth $\Delta\nu$: $p = \frac{1}{2} A_e \cdot S_\nu \cdot \Delta\nu \qquad [\mathrm{W}]$
- Effective area of the antenna: $A_e \leq A_{\mathrm{geom}}$
- Question: $A_e = ?$
- Answer: $A_e = \eta_A A_{\mathrm{geom}} = \eta_i \eta_s ... A_{\mathrm{geom}}$
- Determine η_i, η_s...

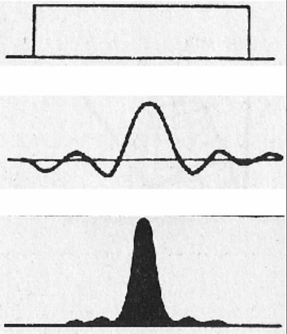
Emitted power by an antenna

- Diffraction theory (Huygens-Fresnel, Fraunhoffer approx.):

$$E_{\mathrm{f-f}}(l, m) \propto \mathcal{F}[E_{\mathrm{ant}}(x, y)]$$

- $E_{\mathrm{ant}}(x, y)$ (grading): bounded on a finite domain Δr
  $\Rightarrow$ $E_{\mathrm{f-f}}(l, m)$ concentrated on a finite domain $\Delta\Omega$
  ($\Delta r \cdot \Delta\Omega \sim 1$)
- sharp cut of the antenna domain $\Rightarrow$ oscillations (side-lobes)
- apodization or taper: decrease the level of the sidelobes, to the cost of increasing $\Delta\Omega$

Antenna power pattern

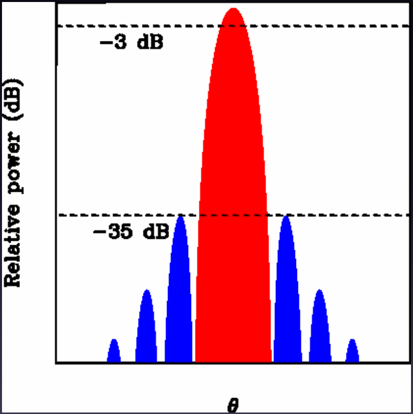


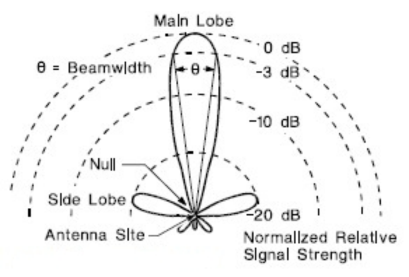
- Reciprocity: antenna in emission
- Distribution of electric field on the dish: $E_{\mathrm{ant}}(x,y)$
- Far-field radiated by the dish: $E_{\mathrm{f-f}}(l,m) \propto \mathcal{F}[E_{\mathrm{ant}}(x,y)]$
- Power emitted is a function of direction: $\propto |E_{\mathrm{f-f}}(l,m)|^2$

- Power pattern: $\mathcal{P}(l,m) \propto |E_{\mathrm{f-f}}(l,m)|^2$
- Beam solid angle: $\Omega_A = \int_{4\pi} \mathcal{P}(\Omega)\,\mathrm{d}\Omega \leq 4\pi$
- Effective area: $A(l,m) = A_e \cdot \mathcal{P}(l,m) \leq A_e$
- Fundamental relation:

$$\boxed{A_e \Omega_A = \lambda^2}$$

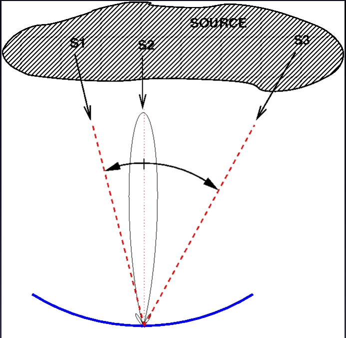
Power pattern $\mathcal{P}(l, m)$





Power collected by an antenna (2)

Given a source of brightness $I_\nu(l, m) = I_\nu(\Omega)$



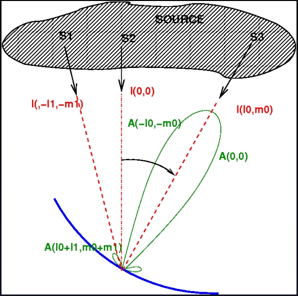
- Source flux density:
  $S_\nu = \int_{\Omega_s} I_\nu(\Omega)\ \mathrm{d}\Omega$
- Observed flux density:

$$S_{\rm obs} = \int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega < S_\nu$$

- Power received from $\mathrm{d}\Omega_i$:
  $\mathrm{d}p_\nu = \frac{1}{2} A(\Omega_i)\ I_\nu(\Omega_i)\ \mathrm{d}\Omega_i$
- Incoherent emission: add intensities

Pointing towards a fixed position of the source at a fixed position

$$p_\nu(l = 0, m = 0) = \frac{A_e}{2} \int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega = \frac{1}{2} A_e S_{\rm obs}$$



- antenna tilted towards $\Omega_0 = (l_0, m_0)$
- Power received from the direction Ω_i

$$\mathrm{d}p_\nu(\Omega_i) = \frac{1}{2} A(\Omega_0 - \Omega_i)\, I_\nu(\Omega_i)\, \mathrm{d}\Omega_i$$

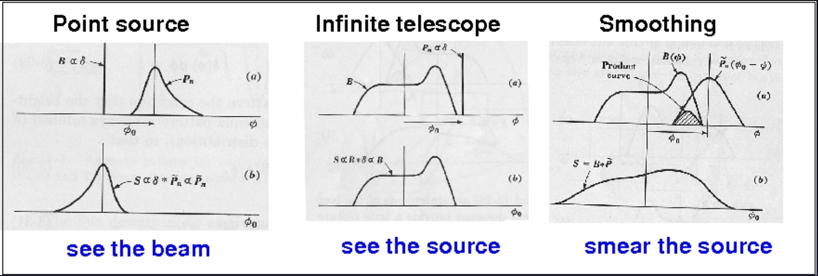
- Incoherent emission: add intensities

Scanning a source leads to a convolution

$$S_{\mathrm{obs}}(\Omega_0) = \int_{\Omega_S} \mathcal{P}(\Omega - \Omega_0) I_\nu(\Omega)\, \mathrm{d}\Omega$$

$$p_\nu(\Omega_0) = \frac{A_e}{2} \int_{\Omega_S} \mathcal{P}(\Omega_0 - \Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega = \frac{1}{2} A_e S_{\mathrm{obs}}(\Omega_0)$$

Convolution: consequences



$$\theta_{\rm obs} = \sqrt{\theta_{\rm mb}^2 + \theta_{\rm sou}^2}$$

When quoting sizes from observations, must quote the deconvolved size (when needed)

Outline

1. Introduction
2. Perfect antenna
3. Real antenna
4. Temperature scales
5. Calibration

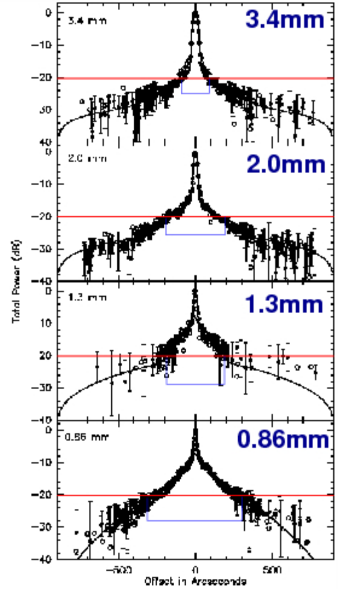
Beam pattern

- secondary lobes (finite surface antenna)
- error lobes (surface irregularities)
- main-beam collects less power
- if correlation length ℓ
  $\Rightarrow$ Gaussian error-beam
  $\Theta_{\rm EB} \approx \lambda/\ell$

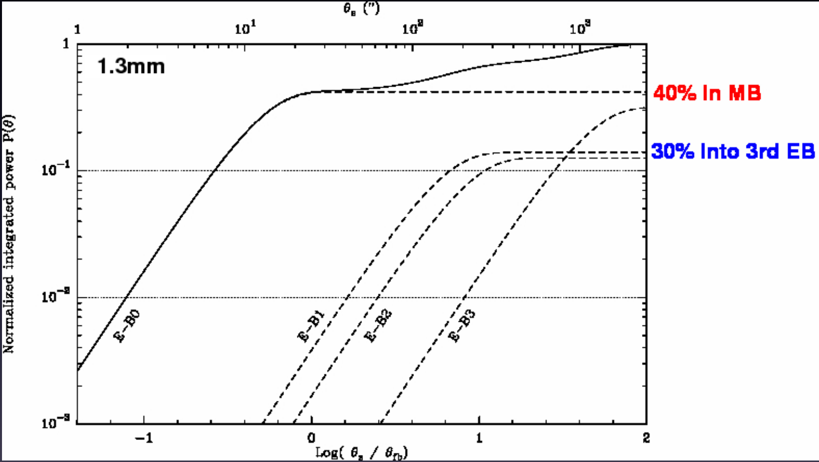
  real beam = main-beam + error-beam(s)
- Questions:

  Power collected in each e-beam ?

  FWHMs of the e-beams ?



Error-Beams at IRAM-30m



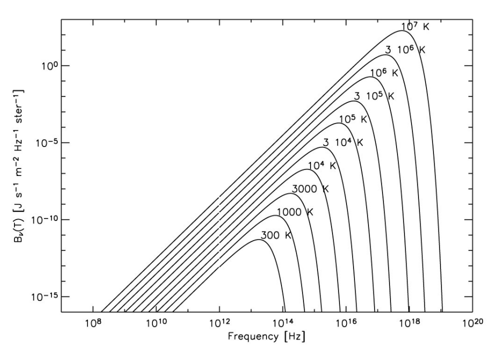
Greve et al A&A 1998

Outline

Speaking in terms of temperatures

- Black-body radiation at temperature T

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$



Speaking in terms of temperatures

- Black-body radiation (*i.e.* photons in thermal equilibrium with matter at temperature T)

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1} \qquad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$

- Dimensions of a brightness
- Rayleigh-Jeans approximation: $h\nu \ll kT$

$$B_\nu(T) \approx \frac{2k}{\lambda^2}\,T \qquad [\mathrm{W\,m^{-2}\,Hz^{-1}}]$$

- Flux density of a black-body:

$$S_\nu = \int_S B_\nu(T,\Omega)\,\mathrm{d}\Omega \qquad = 4\pi B_\nu(T)$$

Definitions

- *Brightness temperature* $T_{\rm B}$ of source brightness I_ν:

$$\boxed{I_\nu(\Omega) = B_\nu(T_{\rm B})}$$

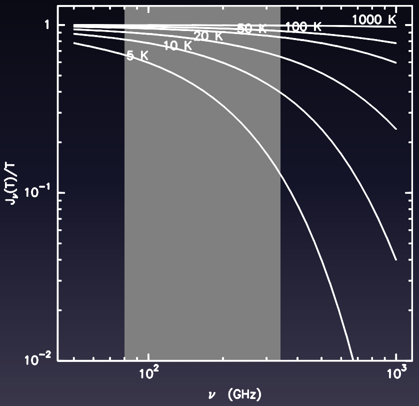
- *Radiation temperature*, $T_{\rm R}$, in the Rayleigh-Jeans regime approximation

$$I_\nu(\Omega) = \frac{2k\nu^2}{c^2} T_{\rm R}(\Omega) = \frac{2k}{\lambda^2} T_{\rm R} \qquad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$

- Relationship between $T_{\rm B}$ and $T_{\rm R}$:

$$T_{\rm R} = J_\nu(T_{\rm B}) = \frac{h\nu}{k} \frac{1}{\exp(h\nu/kT_{\rm B}) - 1} = \frac{T_0}{\exp(T_0/T_{\rm B}) - 1}$$

R-J approximation in the (sub)mm



Definitions

- *Brightness temperature* T_{B} of source brightness I_ν:

$$\boxed{I_\nu(\Omega) = B_\nu(T_{\mathrm{B}}, \Omega)}$$

- *Radiation temperature*, T_{R}, in the Rayleigh-Jeans regime approximation

$$I_\nu(\Omega) = \frac{2k\nu^2}{c^2} T_{\mathrm{R}}(\Omega) = \frac{2k}{\lambda^2} T_{\mathrm{R}}(\Omega) \qquad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$

- Relationship between T_{B} and T_{R}:

$$T_{\mathrm{R}} = J_\nu(T_{\mathrm{B}}) = \frac{h\nu}{k} \frac{1}{\exp(h\nu/kT_{\mathrm{B}}) - 1} = \frac{T_0}{\exp(T_0/T_{\mathrm{B}}) - 1}$$

- In the following: $I_\nu(\Omega) \rightarrow T_{\mathrm{R}}(\Omega)$

Consequences

- Monochromatic power received by the antenna:

$$p_\nu(\Omega_0) = \frac{kA_e}{\lambda^2}\int_S \mathcal{P}(\Omega)\, T_\mathrm{R}(\Omega_0 - \Omega)\,\mathrm{d}\Omega$$

- Observed flux density

$$S_\mathrm{obs}(\Omega_0) = \frac{2k}{\lambda^2}\int_S \mathcal{P}(\Omega)\, T_\mathrm{R}(\Omega_0 - \Omega)\,\mathrm{d}\Omega$$

- Consider a source of finite angular extent (Ω_S) and flux density $S_\nu \quad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}]$:

$$S_\nu = \frac{2k}{\lambda^2}\int_S T_\mathrm{B}(\Omega)\,\mathrm{d}\Omega$$

Antenna temperature

- Johnson noise in terms of an equivalent temperature: the average power transferred from a conductor (in thermal equilibrium) to a line within $\delta\nu$: $\delta p = k\,T\,\delta\nu$
- Antenna temperature defined by

$$p_\nu = kT_A \qquad [\mathrm{W}\cdot\mathrm{Hz}^{-1}] = [\mathrm{J}] = [\mathrm{J}\cdot\mathrm{K}^{-1}][\mathrm{K}]$$

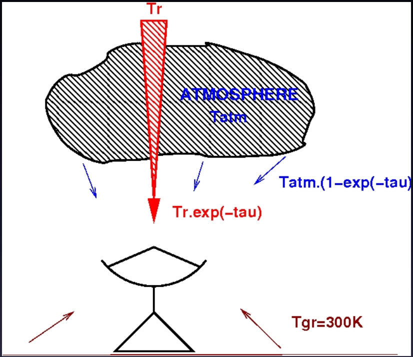
- On the other hand: $p_\nu = A_e/2\,(\mathcal{P} * I_\nu) = kA_e/\lambda^2(\mathcal{P} * T_\mathrm{R})$
- Hence, the antenna temperature reads

$$\boxed{T_A(\Omega_0) = \frac{A_e}{\lambda^2}\int_S \mathcal{P}(\Omega)\,T_\mathrm{R}(\Omega - \Omega_0)\,\mathrm{d}\Omega}$$

- Using $A_e\Omega_A = \lambda^2$, we may write:

$$\boxed{T_A(\Omega_0) = \frac{1}{\Omega_A}\int_S \mathcal{P}(\Omega)\,T_\mathrm{R}(\Omega - \Omega_0)\,\mathrm{d}\Omega}$$

Atmospheric emission/absorption



At frequency ν, Antenna temperature given by:

$$T_A = \eta_s \left\{ T_{\rm R} e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) T_{\rm atm} \right\} + (1 - \eta_s) T_{\rm gr}$$

Correct for **atmospheric attenuation**:

$$T_A \rightarrow T'_{\rm A} = T_A e^{\tau_\nu}$$

From T'_{A} to T^*_{A}

- Correct for rear-sidelobes: measure the monochromatic power recieved from the forward 2π sr. Hence, $\Omega_A \rightarrow \mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega)\,\mathrm{d}\Omega$:

$$T^*_{\mathrm{A}}(\Omega_0) = \frac{T'_{\mathrm{A}}}{F_{\mathrm{eff}}} = \frac{\int_S \mathcal{P}(\Omega)\, T_{\mathrm{R}}(\Omega_0 - \Omega)\,\mathrm{d}\Omega}{\mathcal{P}_{2\pi}}$$

- Forward efficiency: $F_{\mathrm{eff}} = \mathcal{P}_{2\pi}/\mathcal{P}_{4\pi}$
- $T_{\mathrm{A}} = F_{\mathrm{eff}}\, e^{-\tau_\nu}\, T^*_{\mathrm{A}}$

From T_A' to T_mb

- Take into account main-beam and error-lobes
- Same as T_A^* but in Ω_mb instead of 2π. Hence, $\Omega_A \rightarrow \mathcal{P}_\mathrm{mb} = \int_\mathrm{mb} \mathcal{P}(\Omega)\,\mathrm{d}\Omega$:

$$T_\mathrm{mb}(\Omega_0) = \frac{T_\mathrm{A}'}{B_\mathrm{eff}} = \frac{\int_S \mathcal{P}(\Omega)\; T_\mathrm{R}(\Omega_0 - \Omega)\,\mathrm{d}\Omega}{\mathcal{P}_\mathrm{mb}},$$

- Beam efficiency: $B_\mathrm{eff} = \mathcal{P}_\mathrm{mb}/\mathcal{P}_{4\pi}$
- Useful relation for a Gaussian beam:

$$\Omega_\mathrm{mb} = \int_\mathrm{mb} \mathcal{P}(\Omega)\,\mathrm{d}\Omega = 1.133\theta_\mathrm{mb}^2$$

Temperature scales

Definitions

$$\text{Forward efficiency:} \quad F_{\text{eff}} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{4\pi}}$$

$$\text{Beam efficiency:} \quad B_{\text{eff}} = \frac{\mathcal{P}_{\text{mb}}}{\mathcal{P}_{4\pi}}$$

Consequences

$$T_{\text{mb}} = \frac{F_{\text{eff}}}{B_{\text{eff}}} \; T_{\text{A}}^{*} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{\text{mb}}} \; T_{\text{A}}^{*}$$

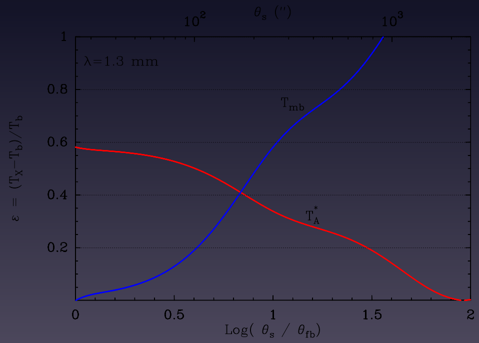
What you measure is T_{A}^{*} or T_{mb} (usually $\neq T_{\text{R}}$)

Limiting cases

- Small sources: $\Omega_S \ll \Omega_{\rm mb}$
  - $T_{\rm mb} \approx \frac{P(0)\Omega_S T_{\rm R}}{\mathcal{P}_{\rm mb}} = T_{\rm R}\frac{\Omega_S}{\mathcal{P}_{\rm mb}}$
  - Gaussian sources & beam:
    $\int_S \mathcal{P}(\Omega) T_R(\Omega_0 - \Omega)\, {\rm d}\Omega = 1.133(\theta^2_{\rm sou} + \theta^2_{\rm mb})$ and
    $\mathcal{P}_{\rm mb} = 1.133\theta^2_{\rm mb}$ hence $T_{\rm R} = T_{\rm mb}\frac{\theta^2_{\rm mb}}{\theta^2_{\rm sou}+\theta^2_{\rm mb}}$: beam dilution
- Large sources: $\Omega_S \gg \mathcal{P}_{\rm mb}$
  - $T^*_{\rm A} \approx T_{\rm R}\frac{\int_{2\pi} P(\Omega)\, {\rm d}\Omega}{\mathcal{P}_{2\pi}} \approx T_{\rm R}$
- Special case: $\Omega_S = \mathcal{P}_{\rm mb}$
  - $T_{\rm mb} = T_{\rm R}\frac{\int_S P(\Omega)\, {\rm d}\Omega}{\mathcal{P}_{\rm mb}} = T_{\rm R}$
  - Main-beam temperature gives the source brightness
- General (worse) case: $\Omega_S \sim \mathcal{P}_{\rm mb}$
  - $T^*_{\rm A} = \frac{T_{\rm R}}{\mathcal{P}_{2\pi}} \int_S P(\Omega)\, {\rm d}\Omega$
  - Main-beam temperature usually quoted
  - If source of uniform brightness and beam pattern known, feasible, but in real life... Which scale to use: $T^*_{\rm A}$, $T_{\rm mb}$?

Which temperature scale ?

Source size	Temperature scales
$\Omega_S = 2\pi$	$T_\mathrm{R} = T_\mathrm{A}^*$
$\Omega_S = \Omega_\mathrm{mb}$	$T_\mathrm{R} = T_\mathrm{mb}$
$2\pi < \Omega_S$	$T_\mathrm{R} < T_\mathrm{A}^*$
$\Omega_\mathrm{mb} < \Omega_S < 2\pi$	$T_\mathrm{A}^* < T_\mathrm{R} < T_\mathrm{mb}$
$\Omega_\mathrm{mb} > \Omega_S$	$T_\mathrm{mb} < T_\mathrm{R}$



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Goals of the calibration

Overview

- Atmospheric calibration:
  - atmospheric emission/abs at frequency ν: radiative transfer
  - turbulence affects the intensity through phase shifts
- Full detection chain calibration (antenna, receiver,)
  - Antenna-sky coupling: F_{eff}
  - Receiver: gain, noise, stability
  - Cables, backends (*e.g.* dark currents)

Goals

- Input is an e-m field and output at backends are counts
- Question 1: how to convert from counts to power in physical units ?
- Question 2: how to correct for the atmospheric contribution ?

Notations

Telescope pointing at a source receives

$$\begin{aligned} C_{\rm sou} &= \chi \left\{ T_{\rm rec} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} + T_{\rm sky} \right\} \\ T_{\rm sky} &= F_{\rm eff}(1 - e^{-\tau_\nu})\, T_{\rm atm} + (1 - F_{\rm eff})\, T_{\rm gr} \end{aligned}$$

- $T_{\rm rec}$: noise contribution from the receiver
- $F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou}$: signal from the scientific target after propagation through the atmosphere
- $T_{\rm sky}$: signal emitted by the atmosphere ($T_{\rm atm}$) and the ground ($T_{\rm gr}$)
- Orders of magnitude (IRAM-30m):
  - $T_{\rm atm} \approx T_{\rm gr} \approx 290$ K
  - $T_{\rm rec} \approx 50 - 70$ K at $100 - 350$ GHz at the IRAM-30m
  - $F_{\rm eff}(\nu) \approx 90 - 80\%$, $B_{\rm eff}(\nu) \approx 80 - 35\%$, $\nu_{\rm GHz} = 100 - 350$
  - $T_{\rm sky} \approx 30 - 100$ K at $100 - 230$ GHz
  - Note: if $\tau_\nu \ll 1$ (good weather), $T_{\rm sky} \approx F_{\rm eff}\, \tau_\nu\, T_{\rm atm}$

The "Chopper Wheel" method

Perform $3+1$ measurements: hot, cold, empty sky, source

$$
\begin{aligned}
C_{\rm sou} &= \chi\left\{T_{\rm rec} + T_{\rm sky} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou}\right\} \\
C_{\rm atm} &= \chi\left\{T_{\rm rec} + T_{\rm sky}\right\} \\
C_{\rm hot} &= \chi\left\{T_{\rm rec} + T_{\rm hot}\right\} \\
C_{\rm col} &= \chi\left\{T_{\rm rec} + T_{\rm col}\right\}
\end{aligned}
$$

Making differences:

$$
\begin{aligned}
\Delta C_{\rm sig} &= C_{\rm sou} - C_{\rm atm} = \chi F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} \\
\Delta C_{\rm cal} &= C_{\rm hot} - C_{\rm atm} = \chi(T_{\rm hot} - T_{\rm sky})
\end{aligned}
$$

$$
\boxed{T_{\rm sou} = T_{\rm cal}\,\frac{\Delta C_{\rm sig}}{\Delta C_{\rm cal}}}
$$

Definition of $T_{\rm cal}$ (correcting for atm contrib. and spillover)

$$
\boxed{T_{\rm cal} = (T_{\rm hot} - T_{\rm sky})\,\frac{e^{\tau_\nu}}{F_{\rm eff}}}
$$

Calibration outputs (1): $T_{\rm cal}$

Rewrite $T_{\rm sky}$ and *DeltaC*$_{\rm cal}$:

$$
\begin{aligned}
T_{\rm sky} &= T_{\rm gr} + F_{\rm eff}(T_{\rm atm} - T_{\rm gr}) - F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm atm} \\
\Delta C_{\rm cal} &= \chi\{\,(T_{\rm hot} - T_{\rm gr}) + F_{\rm eff}(T_{\rm gr} - T_{\rm atm}) + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm atm}\}
\end{aligned}
$$

- Assume $T_{\rm hot} = T_{\rm atm} = T_{\rm gr} = T$. Then $T_{\rm cal} = T$, no need to know $e^{-\tau_\nu}$ (Penzias & Burrus ARAA 1973):

$$T_{\rm sou} = T\frac{\Delta C_{\rm sig}}{\Delta C_{\rm cal}}, \quad T_{\rm cal} = T_{\rm atm} = T_{\rm gr} = T_{\rm hot}$$

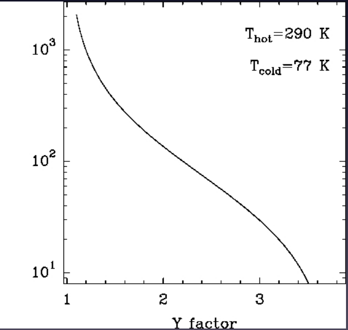
- General case: different $T_{\rm atm}$, $T_{\rm hot}$ and $T_{\rm gr}$ $\Rightarrow$ must solve for $e^{-\tau_\nu}$. This is done using a model of the atmosphere trying to reproduce $T_{\rm sky}$ varying the amount of dominant species in the model (hence providing us with pwv).

Calibration outputs (2): $T_{\rm rec}$

Using hot & cold loads measurements lead to $T_{\rm rec}$:

$$T_{\rm rec} = \frac{T_{\rm hot} - Y T_{\rm col}}{Y - 1}$$

$$Y = \frac{C_{\rm hot}}{C_{\rm col}} = \frac{T_{\rm rec} + T_{\rm hot}}{T_{\rm rec} + T_{\rm col}}$$



Calibration outputs (3): T_{sys}

System temperature: describes the noise including all sources from the sky down to the backends

- used to determine the total statistical noise. For heterodyne receivers, noise is given by the “radiometer formula”:

$$\boxed{\sigma_T = \frac{\kappa \cdot T_{\mathrm{sys}}}{\sqrt{\delta\nu\,\Delta t}}}$$

- δ_ν: spectral resolution
- Δt: total integration time
- κ depends on the observing mode:
- example: position switching
  $\mathrm{ON\text{-}OFF} \Rightarrow \sqrt{2}$
  $t_{\mathrm{ON}} = t_{\mathrm{OFF}} \Rightarrow \Delta t = 2t_{\mathrm{ON}} \Rightarrow \sqrt{2}$
  $\Rightarrow \kappa = 2$

From T_{mb} to S_ν, from Kelvin to Jansky

- flux density: $S_\nu = \int_S I_\nu(\Omega)\,\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_S T_{\mathrm{mb}}\,\mathrm{d}\Omega$
- power received by the antenna: $kT'_A = k\frac{T_{\mathrm{A}}^*}{F_{\mathrm{eff}}} = \frac{1}{2}S_\nu A_e$

$$\frac{S_\nu}{T_{\mathrm{A}}^*} = \frac{2k}{A}\frac{F_{\mathrm{eff}}}{\eta_A} \qquad \mathrm{Jy/K}$$

- 1 Jy = $10^{-26}\ \mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$
- values of S_ν/T_{A}^* are tabulated *e.g.* on IRAM-30m web page ($\approx$ 6 @ 100 GHz, $\approx$ 11 @ 340 GHz)

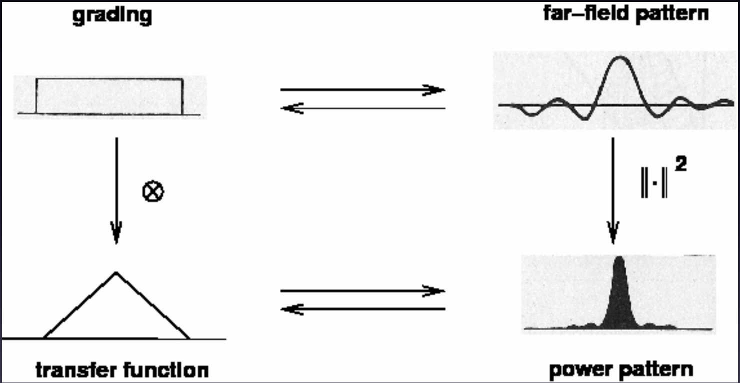
From T_{mb} to S_ν, from Kelvin to Jansky

- flux density: $S_\nu = \int_S I_\nu(\Omega)\,\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_S T_{\mathrm{mb}}\,\mathrm{d}\Omega$
- power received by the antenna: $kT'_A = k\frac{T_{\mathrm{A}}^*}{F_{\mathrm{eff}}} = \frac{1}{2}S_\nu A_e$

$$\frac{S_\nu}{T_{\mathrm{A}}^*} = \frac{2k}{A}\frac{F_{\mathrm{eff}}}{\eta_A} \qquad \mathrm{Jy/K}$$

- 1 Jy = $10^{-26}\ \mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$
- values of S_ν/T_{A}^* are tabulated *e.g.* on IRAM-30m web page ($\approx$ 6 @ 100 GHz, $\approx$ 11 @ 340 GHz)
- How to convert the temperatures into $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}$?
  - $I_\nu(\Omega) = \frac{2k}{\lambda^2}\,T_R(\Omega)$

Image formation: total power telescope



- antenna scans the source
- image: convolution of I_0 by beam pattern $I'_\nu = \mathcal{P} * I_{0,\nu}$
- measure directly the brightness distribution I_0

Interferometer field of view

$$F = D * (\mathcal{P} \times I) + N$$

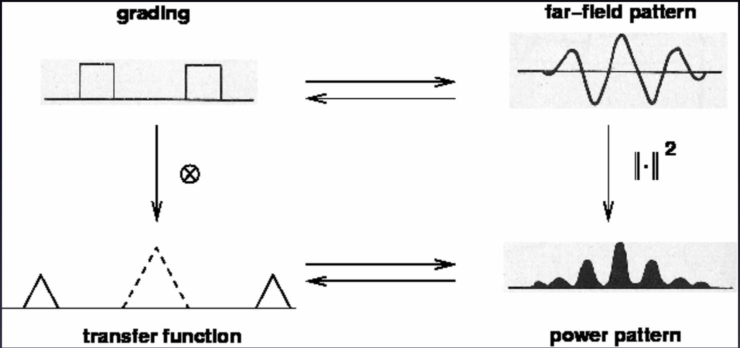
F	=	dirty map = FT of observed visibilities
D	=	dirty beam ($\longrightarrow$ deconvolution)
$\mathcal{P}$	=	power pattern of single-dish *(primary beam B in the followin*
I	=	sky brightness distribution
N	=	noise distribution

- **An interferometer measures the product $\mathcal{P} \times I$**
- $\mathcal{P}$ has a finite support $\longrightarrow$ limits the size of the field of view

Summary

- full-aperture antenna: $\mathcal{P} * \mathbf{I}$
- interferometry sensitive to $\mathcal{P} \times \mathbf{I}$
- **amplitude calibration:**
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- **lobe = main-lobe + error-lobes** (*e.g.* as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the **temperature scale** to use (T_A^*, T_{mb},...)

Image formation: correlation telescope



- antennas fixed w.r.t. the source
- correlation temperature: $\mathcal{T}(0,0)$ Fourier transform of $I_0 \times \mathcal{P}$
- measure the Fourier transform of the brightness distribution I_0
- image built afterwards

Interferometer field of view

Measurement equation of an interferometric observation:

$$F = D * (B \times I) + N$$

F	=	dirty map = FT of observed visibilities
D	=	dirty beam ($\longrightarrow$ deconvolution)
B	=	primary beam
I	=	sky brightness distribution
N	=	noise distribution

- **An interferometer measures the product** $B \times I$
- B has a finite support $\longrightarrow$ limits the size of the field of view
- B is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

Aperture function	$\rightleftharpoons$	Voltage pattern
$\star\downarrow$		$\downarrow \lvert\cdot\rvert^2$
Transfert function $T(u,v)$	$\rightleftharpoons$	Power pattern $B(\ell,m)$ **= Primary beam**

Gaussian illumination $\Longrightarrow$ to a good approximation, B is a Gaussian of $1.2\,\lambda/D$ FWHM

Plateau de Bure

$D = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	$58''$
100 GHz	3.0 mm	$50''$
115 GHz	2.6 mm	$43''$
215 GHz	1.4 mm	$23''$
230 GHz	1.3 mm	$22''$
245 GHz	1.2 mm	$20''$