The Development of High-Resolution Imaging in Radio Astronomy

Jim Moran

Harvard-Smithsonian Center for Astrophysics

7th IRAM Interferometry School, Grenoble, October 4–7, 2010

It is an honor to give this lecture in the city where Joseph Fourier did the work that is so fundamental to our craft.

Outline of Talk

I. Origins of Interferometry

II. Fundamental Theorem of Interferometry
(Van Cittert-Zernike Theorem)

III. Limits to Resolution (*uv* plane coverage)

IV. Quest for High Resolution in the 1950s

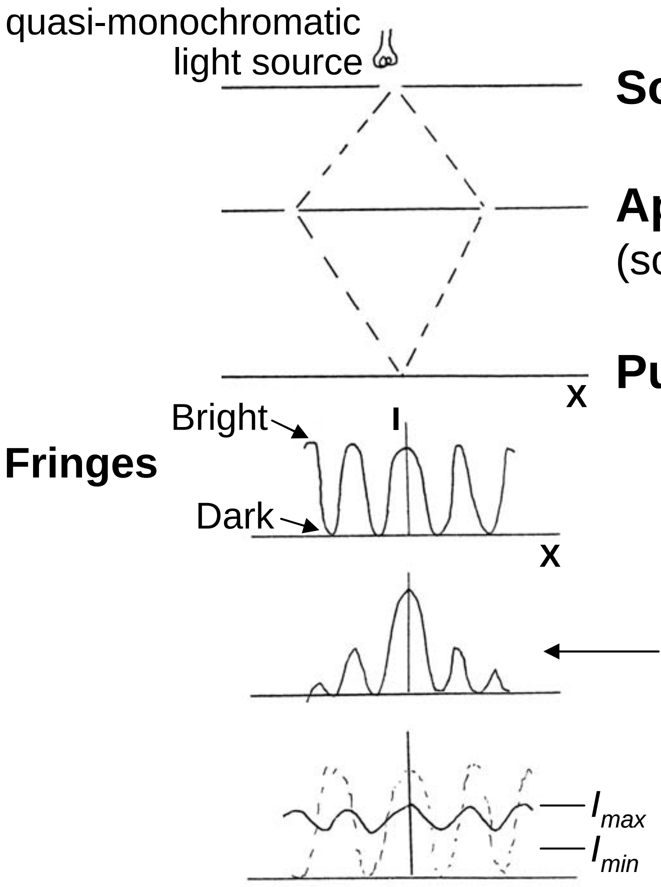
V. Key Ideas in Image Calibration and Restoration

VI. Back to Basics – Imaging Sgr A* in 2010 and beyond

I. Origins of Interferometry

A. Young’s Two-Slit Experiment	Thomas Young (1773–1829)
B. Michelson’s Stellar Interferometer	Albert Michelson (1852–1931)
C. Basic Radio Implementation	
D. Ryle’s Correlator	Martin Ryle (1918–1984)
E. Sea Cliff Interferometer	John Bolton (1922–1993)
F. Earth Rotation Synthesis	Martin Ryle (1918–1984)

Young’s Two-Slit Experiment (1805)



Source plane

Aperture plane
(screen with 2 small holes)

Pupil or Image plane

$$I = \langle (E_1 + E_2)^2 \rangle$$
$$= \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2\langle (E_1 E_2) \rangle$$

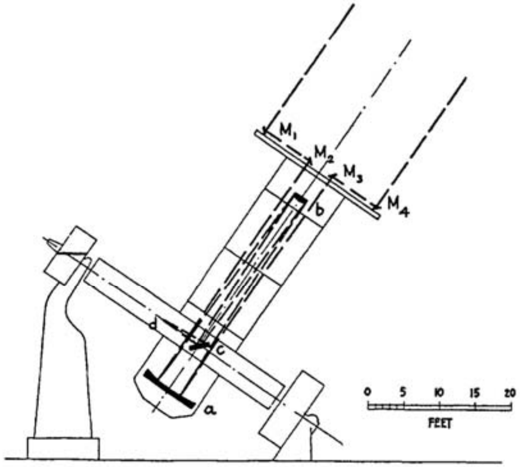
interference term

← bandwidth effect $\Delta\omega \Delta t \sim 1$

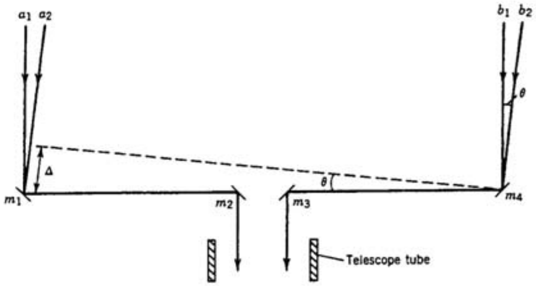
$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

1. Move source ⇒ shift pattern (phase)
2. Change aperture hole spacing ⇒ change period of fringes
3. Enlarge source plane hole ⇒ reduce visibility

Michelson-Pease Stellar Interferometer (1890-1920)

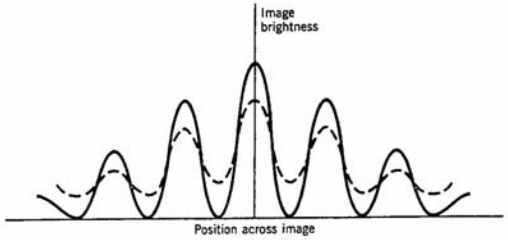


Two outrigger mirrors
on the Mount Wilson
100 inch telescope



(a)

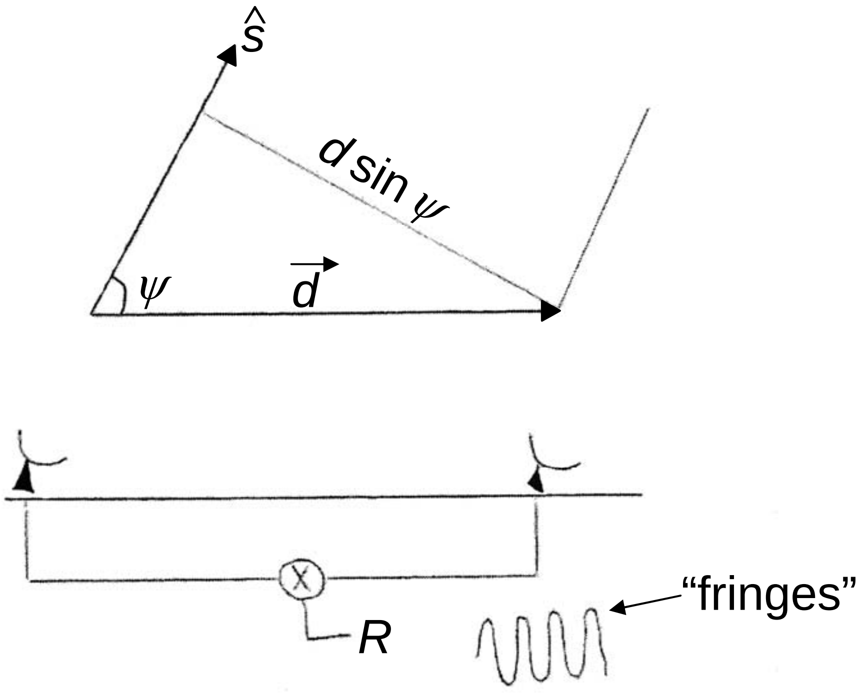
Paths for on axis ray and
slightly offset ray



(b)

Image plane fringe pattern.
Solid line: unresolved star
Dotted line: resolved star

Simple Radio Interferometer



$$R = I \cos \phi$$

$$\phi = \frac{2\pi}{\lambda} \vec{d} \cdot \hat{s} = \frac{2\pi d}{\lambda} \cos \psi$$

$$\frac{d\phi}{dt} = \frac{2\pi d}{\lambda} \omega_e \sin \psi$$

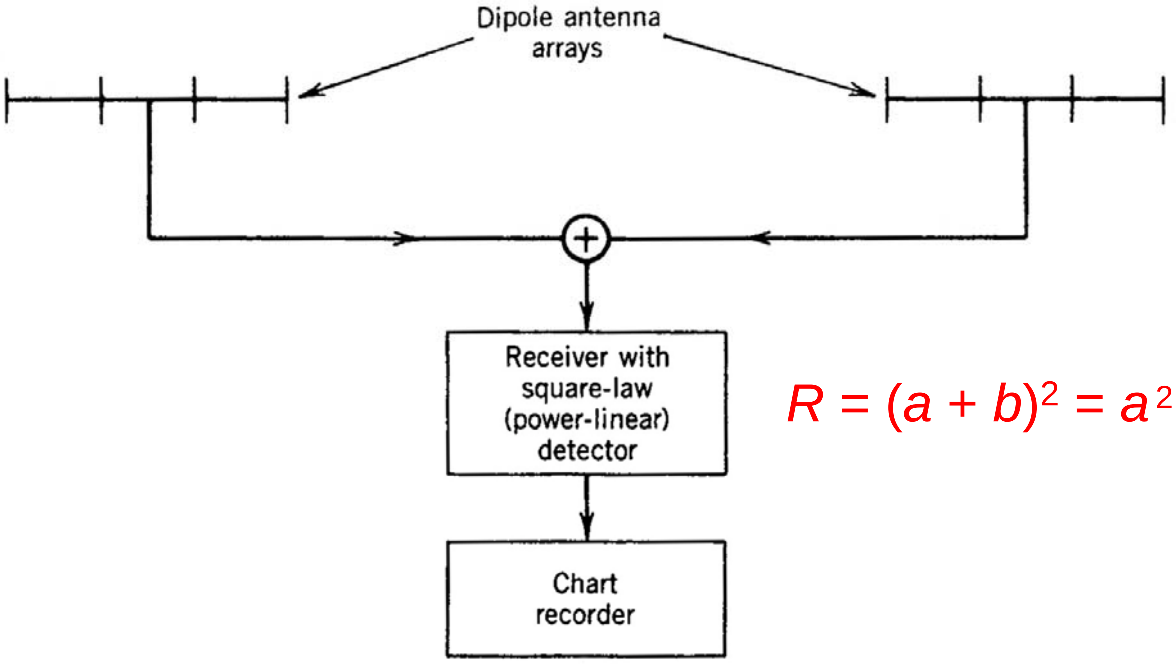
$$\frac{d\phi}{d\psi} = \frac{2\pi d}{\lambda} \sin \psi$$

$$\Delta\phi = 2\pi = \frac{2\pi d \sin \psi}{\lambda} \Delta\psi$$

$$\Delta\psi = \frac{\lambda}{d \sin \psi}$$

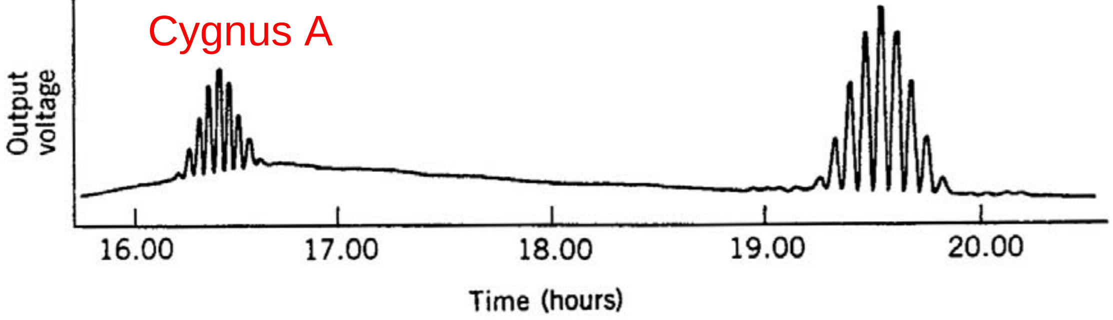
projected
baseline

Simple Adding Interferometer (Ryle, 1952)



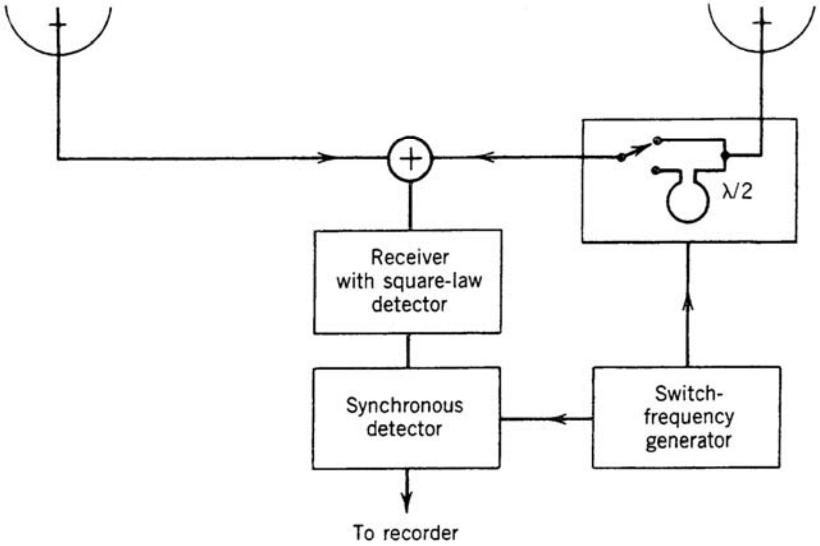
(a)

$R = (a + b)^2 = a^2 + b^2 + 2ab$

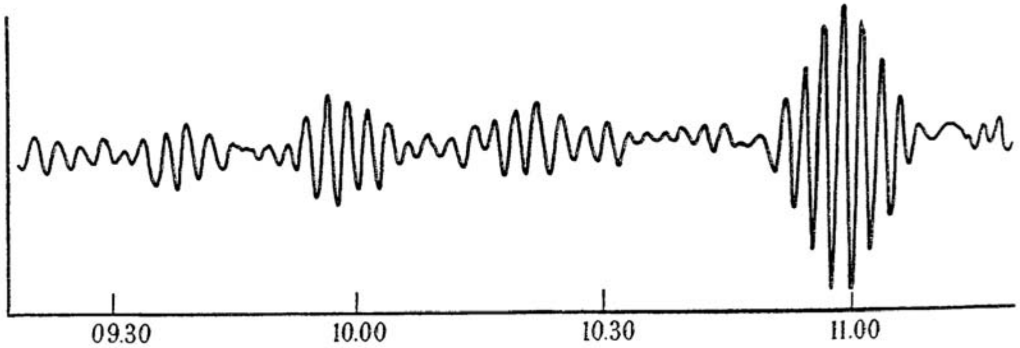


(b)

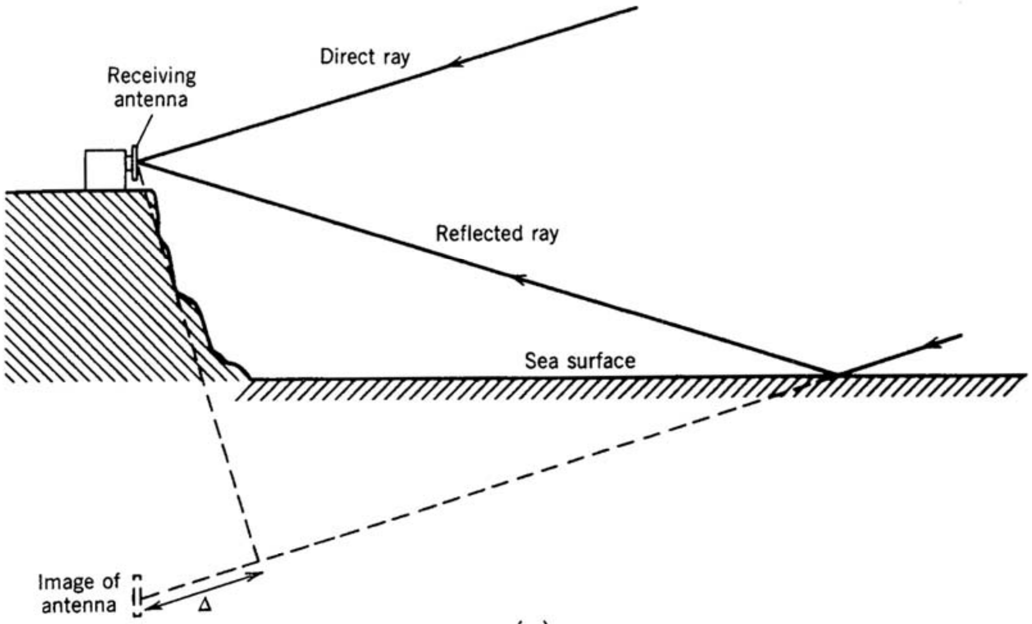
Phase Switching Interferometer (Ryle, 1952)



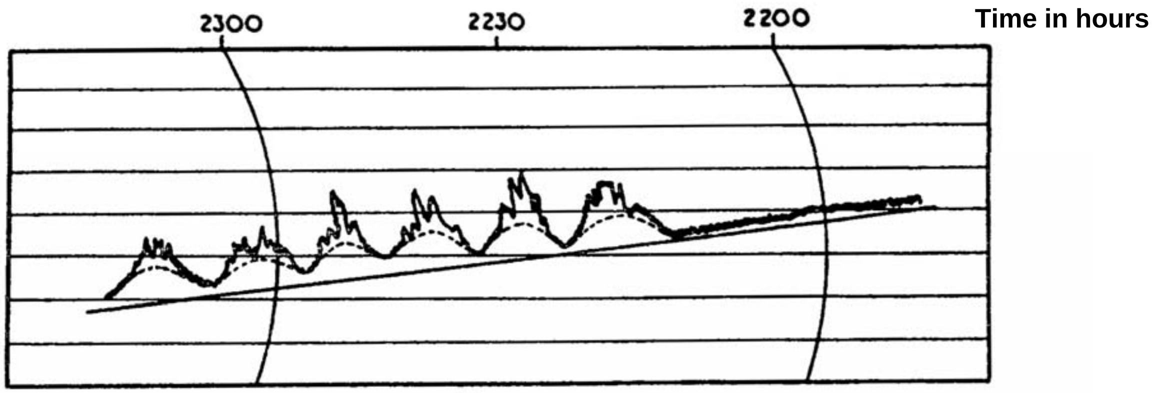
$$R = (a + b)^2 – (a – b)^2 = 4ab$$



Sea Cliff Interferometer (Bolton and Stanley, 1948)



(a)



(b)

Response to Cygnus A at 100 MHz (Nature, 161, 313, 1948)

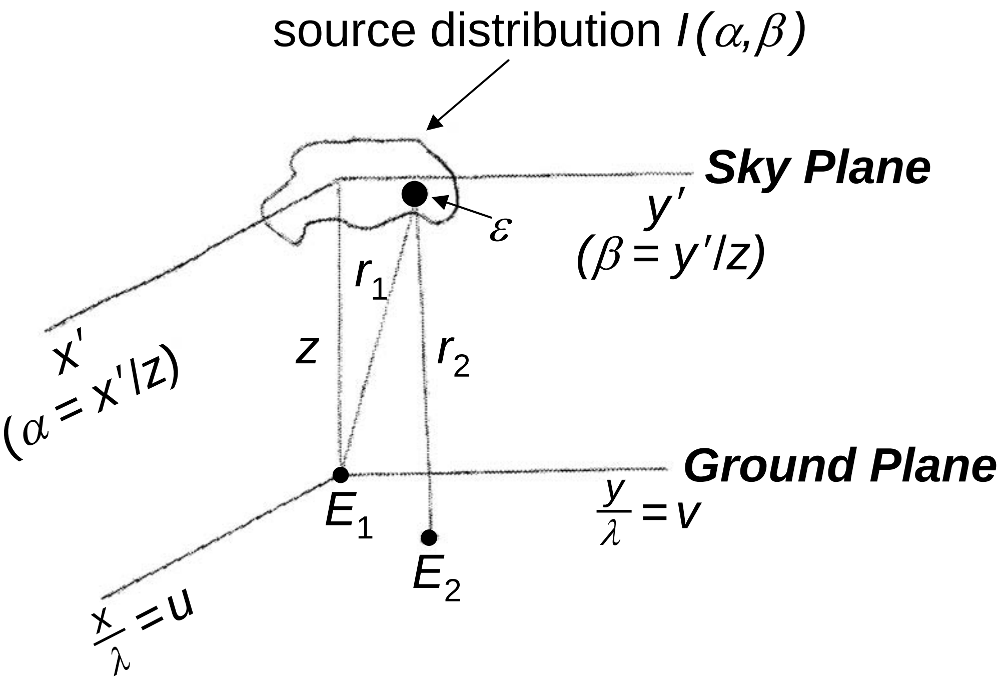
II. Fundamental Theorem

A. Van Cittert-Zernike Theorem

B. Projection Slice Theorem

C. Some Fourier Transforms

Van Cittert–Zernike Theorem (1934)



$$E_1 = \frac{\varepsilon}{r_1} e^{i\frac{2\pi}{\lambda}r_1}$$

$$E_2 = \frac{\varepsilon}{r_2} e^{i\frac{2\pi}{\lambda}r_2}$$

Huygen’s principle

$$\langle E_1 E_2^* \rangle = \frac{\varepsilon^2}{r_1 r_2} e^{i\frac{2\pi}{\lambda}(r_1 - r_2)}$$

$$\varepsilon^2 = I(\alpha,\beta) \qquad r_1 r_2 \sim z^2$$

Integrate over source

$$V(u,v) = \int I(\alpha,\beta)\, e^{i2\pi(\alpha u + \beta v)}\, d\alpha\, d\beta$$

Assumptions

1. Incoherent source
2. Far field $z > d^2_{max}/\lambda$; $d = 10^4$ km, $\lambda = 1$ mm, $z > 3$ pc !
3. Small field of view
4. Narrow bandwidth $\Delta\nu \Rightarrow$ field = $\left(\frac{\lambda}{d_{max}}\right)\frac{\nu}{\Delta\nu}$

Projection-Slice Theorem (Bracewell, 1956)

$$F\,(u,v) = \iint f(x,y)\, e^{-i2\pi(ux+vy)}\, dx\, dy$$

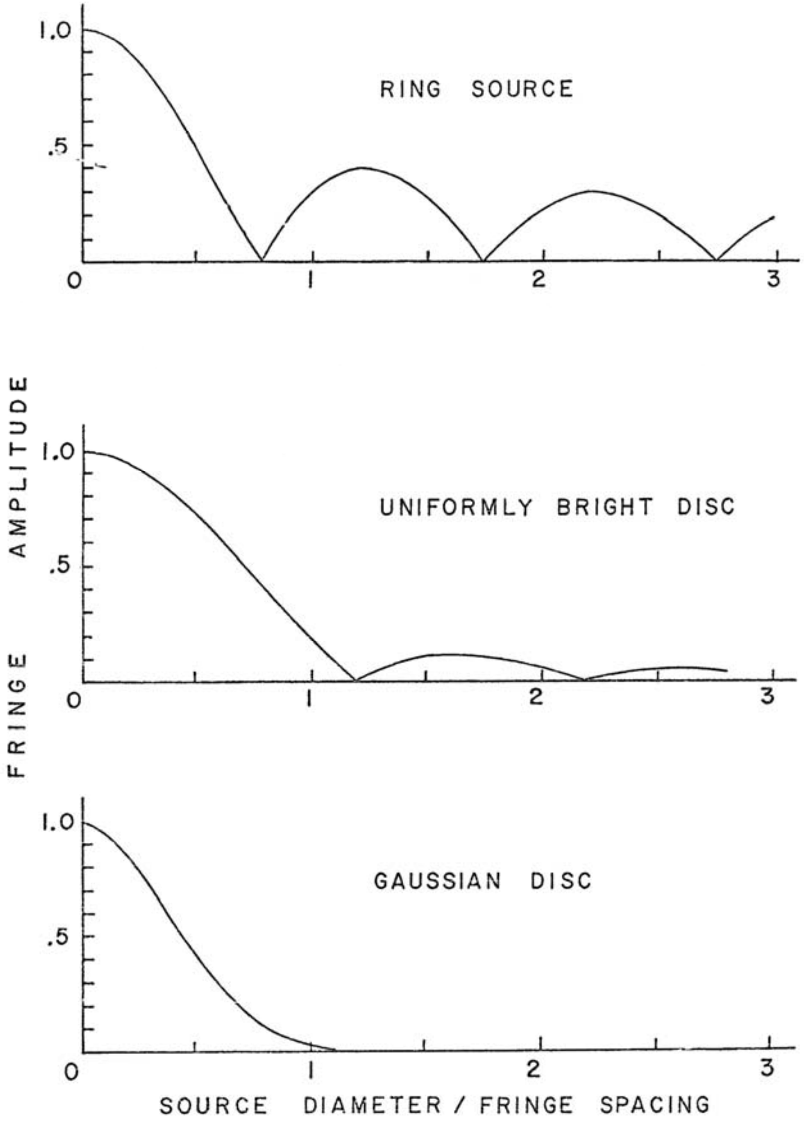
$$F(u,0) = \int \Big[\int f(x,y)\, dy \Big]\, e^{-i2\pi ux}\, dx$$

$$F(u,0) \leftrightarrow \underset{\substack{\uparrow \\ \text{“strip” integral}}}{f_s(x)}$$

Works for any arbitrary angle

Strip integrals, also called back projections,
are the common link between radio interferometry
and medical tomography.

Visibility (Fringe) Amplitude Functions for Various Source Models



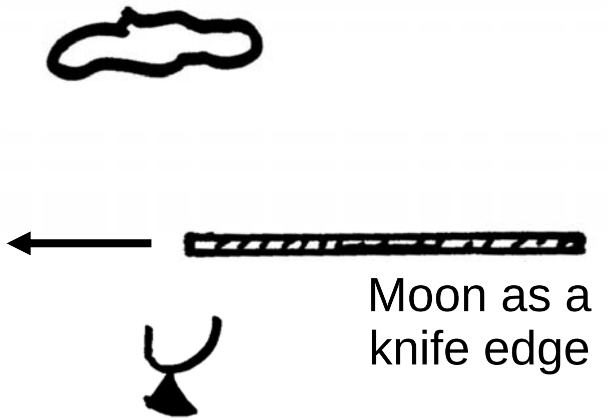
Moran, PhD thesis, 1968

III. Limits to Resolution (uv plane coverage)

A. Lunar Occultation

B. *uv* Plane Coverage of a Single Aperture

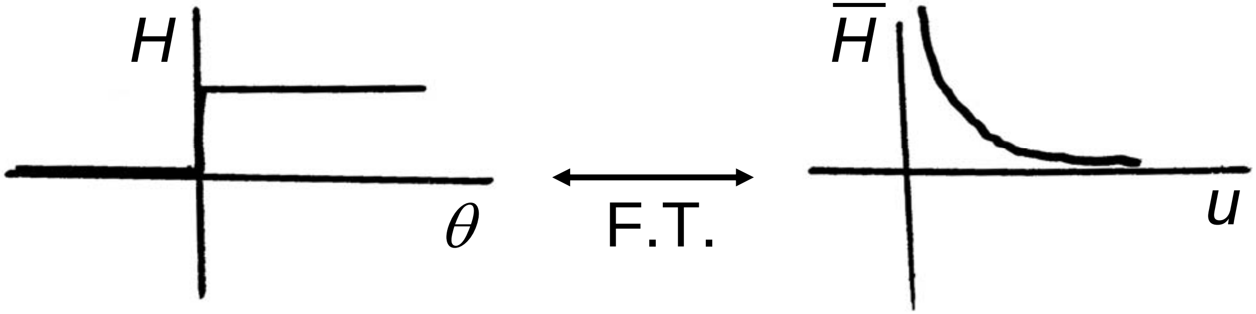
Lunar Occultation



Geometric optics ⇒ one-dimension integration of source intensity

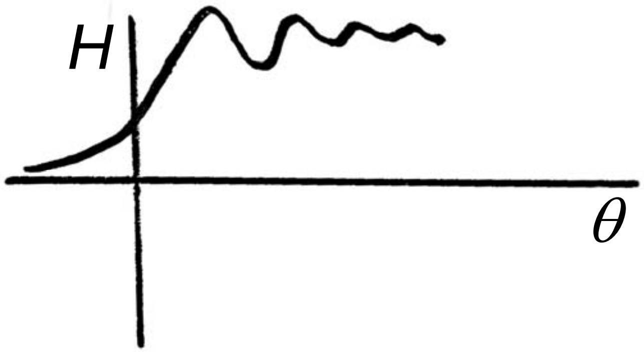
$$I_m(\theta) = I(\theta) \otimes H(\theta)$$

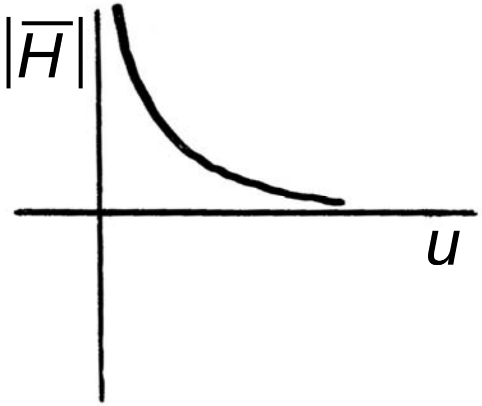
$$\overline{I}_m(u) = \overline{I}(u)\ \overline{H}(u) \quad \text{where } H(u) = 1/u$$



MacMahon (1909)

Criticized by Eddington (1909)





$\theta_F = \sqrt{\dfrac{\lambda}{2R}} \sim \Delta\theta$ (wiggles) (R = earth-moon distance)

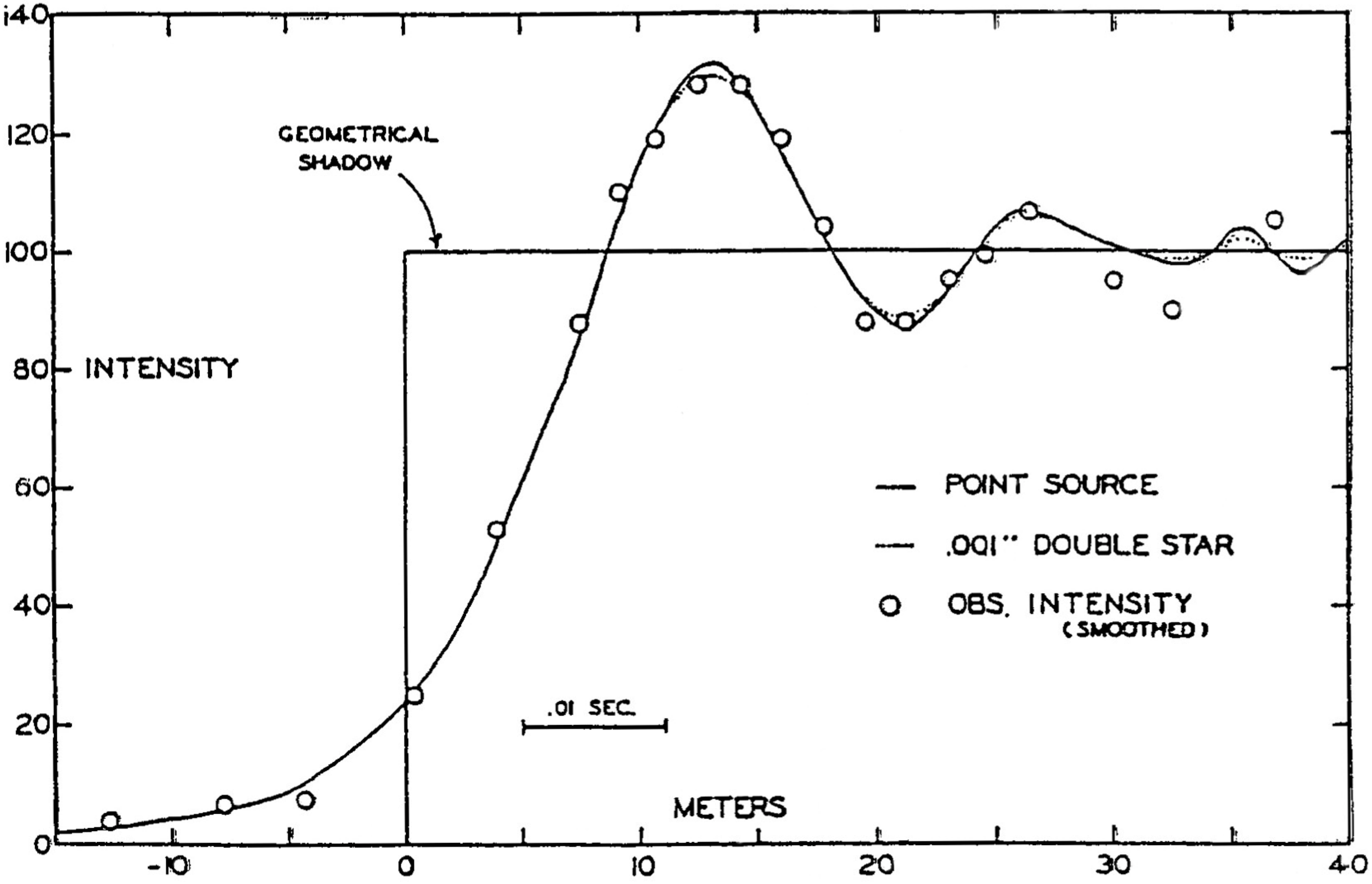
$$H = \frac{1}{\mu}\, e^{-i\theta_F u^2 \,\mathrm{sign}(u)}$$

Same amplitude as response in geometric optics, but scrambled phase

θ_F = 5 mas @ 0.5μ wavelength

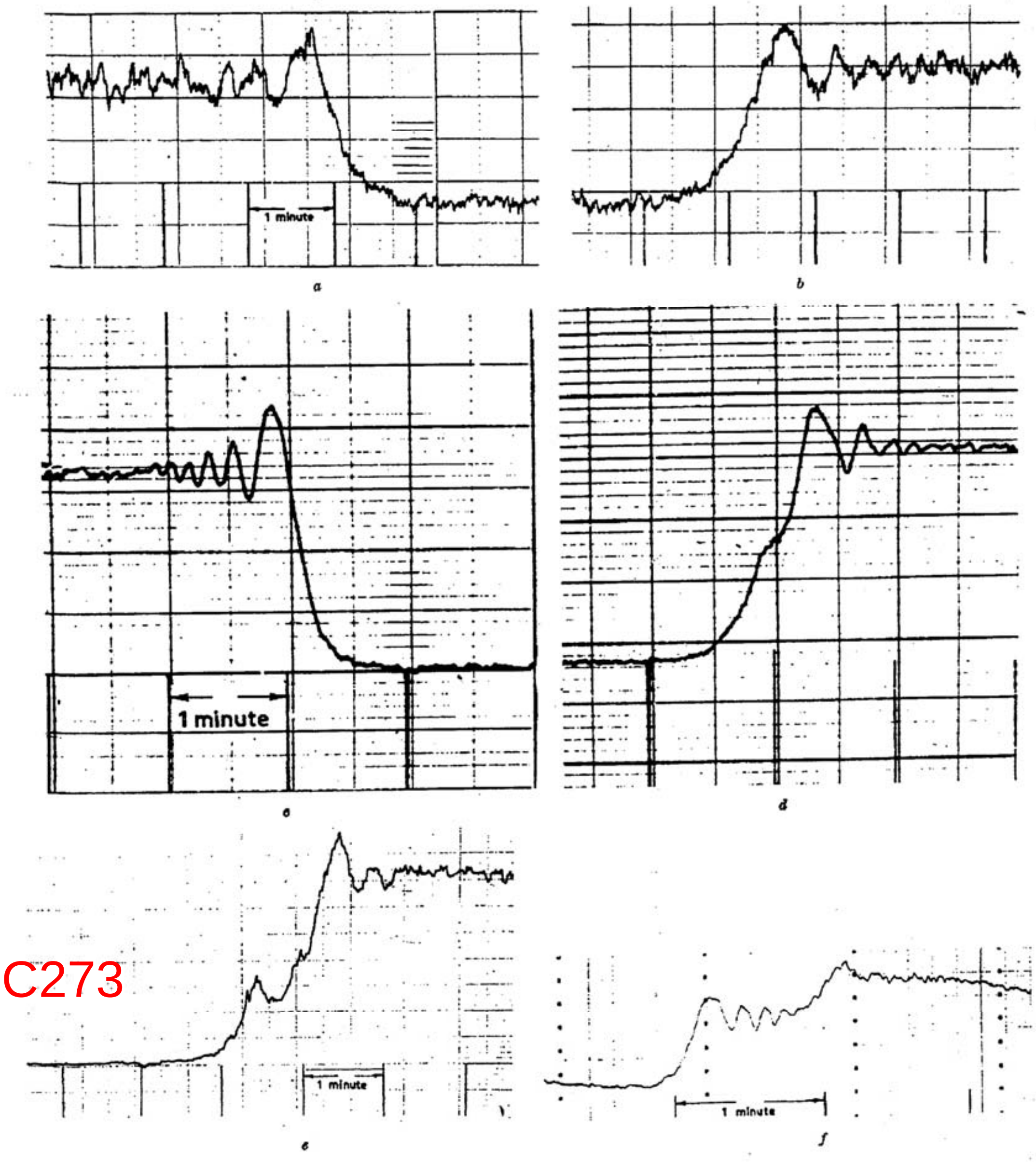
θ_F = 2″ @ 10 m wavelength

Occultation of Beta Capricorni with Mt. Wilson 100 Inch Telescope and Fast Photoelectric Detector



Whitford, Ap.J., 89, 472, 1939

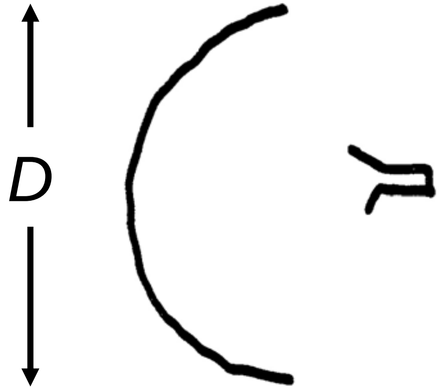
Radio Occultation Curves (Hazard et al., 1963)



3C273

Single Aperture

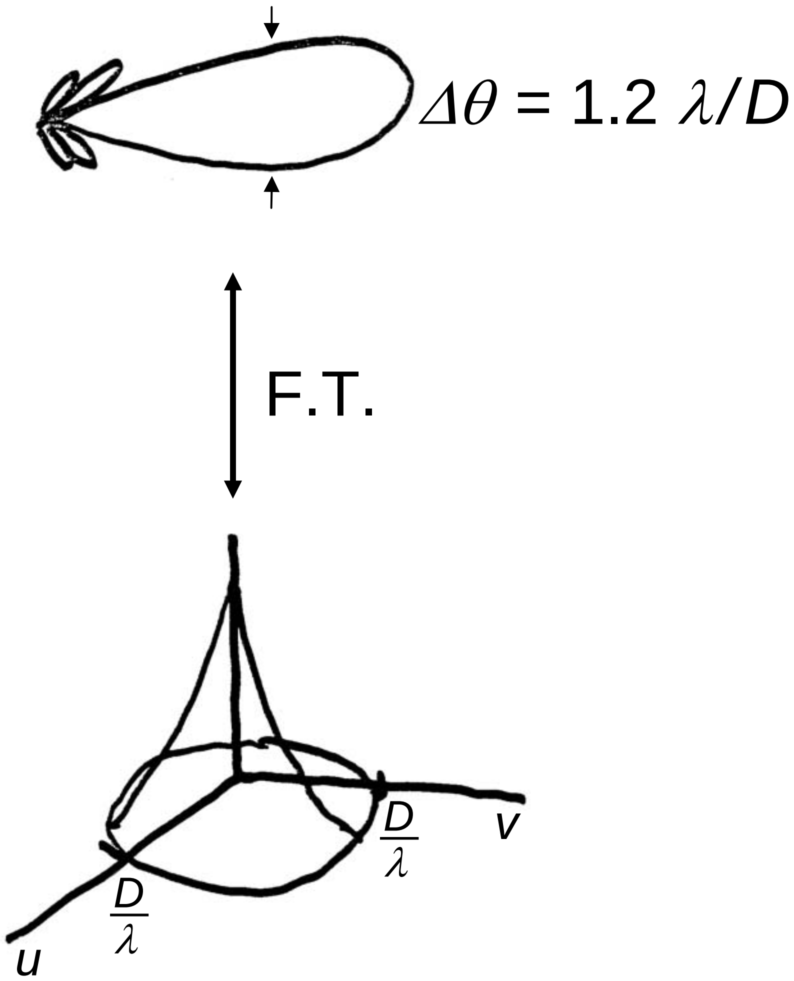
Single Pixel



Restore high
spatial frequencies
up to $u = D/\lambda$

$\Rightarrow$ no super resolution

Airy Pattern



Chinese Hat Function

IV. Quest for High Resolution in the 1950s

A. Hanbury Brown’s Three Ideas

B. The Cygnus A Story

Hanbury Brown’s Three Ideas for High Angular Resolution

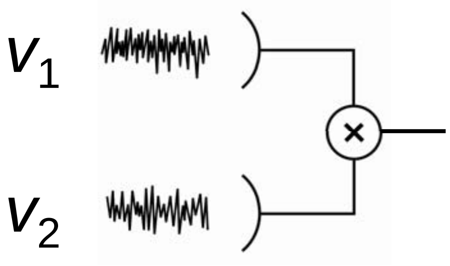
In about 1950, when sources were called “radio stars,” Hanbury Brown had several ideas of how to dramatically increase angular resolution to resolve them.

1. Let the Earth Move (250 km/s, but beware the radiometer formula!)

2. Reflection off Moon (resolution too high)

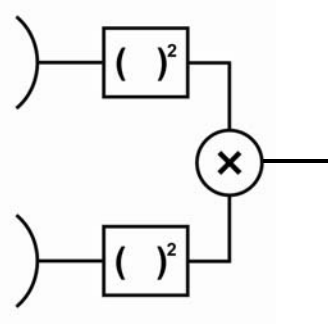
3. Intensity Interferometer (inspired the field of quantum optics)

Intensity Interferometry



Normal Interferometer

$$\rho = \langle v_1 v_2 \rangle \equiv V$$



Intensity Interferometer

$$\rho = \langle v_1^2 v_2^2 \rangle$$

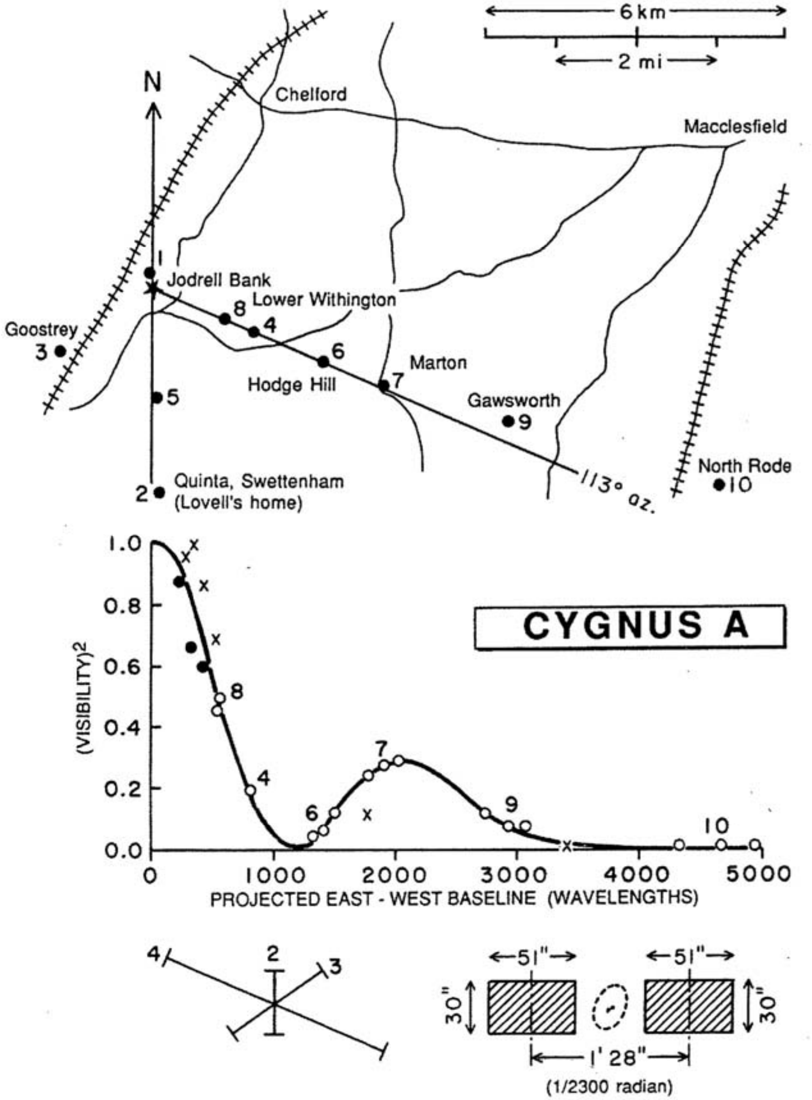
Fourth-order moment theorem for Gaussian processes

$$\langle v_1 v_2 v_3 v_4 \rangle = \langle v_1 v_2 \rangle \, \langle v_3 v_4 \rangle + \langle v_1 v_3 \rangle \, \langle v_2 v_4 \rangle + \langle v_1 v_4 \rangle \, \langle v_2 v_3 \rangle$$

$$\rho = \langle v_1^2 v_2^2 \rangle = \langle v_1^2 \rangle \, \langle v_2^2 \rangle + \langle v_1 v_2 \rangle^2$$

$$\rho = \underset{\substack{\uparrow \\ \text{constant}}}{P_1} P_2 + \underset{\substack{\uparrow \\ \text{square of visibility}}}{V^2}$$

Observations of Cygnus A with Jodrell Bank Intensity Interferometer



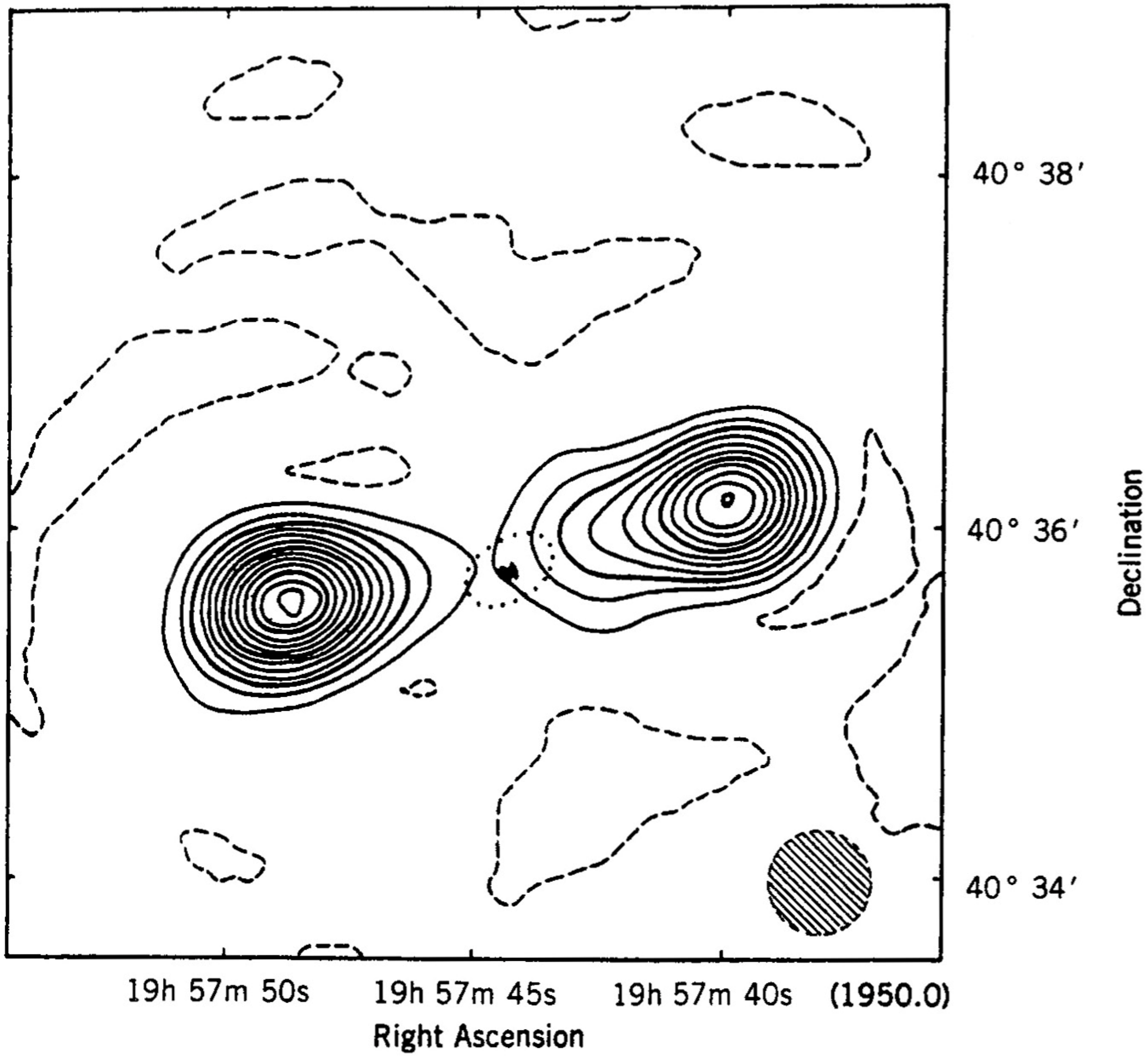
Square of Visibility
at 125 MHz

Jennison and Das Gupta, 1952, see Sullivan 2010

Cygnus A with Cambridge 1-mile Telescope at 1.4 GHz

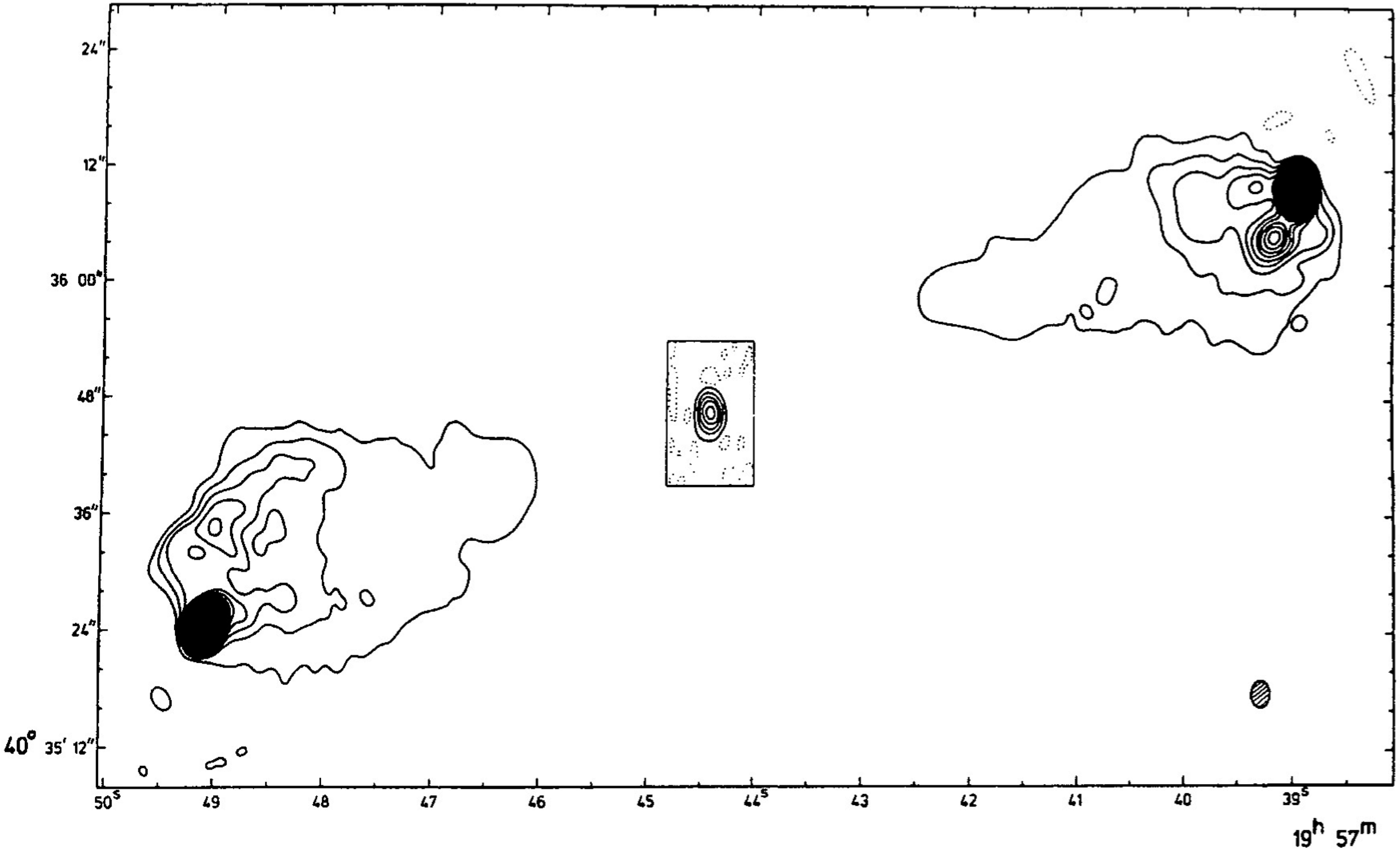
3 telescopes

20 arcsec resolution



Ryle, Elsmore, and Neville, Nature, 205, 1259, 1965

Cygnus A with Cambridge 5 km Interferometer at 5 GHz



16 element E-W Array, 3 arcsec resolution

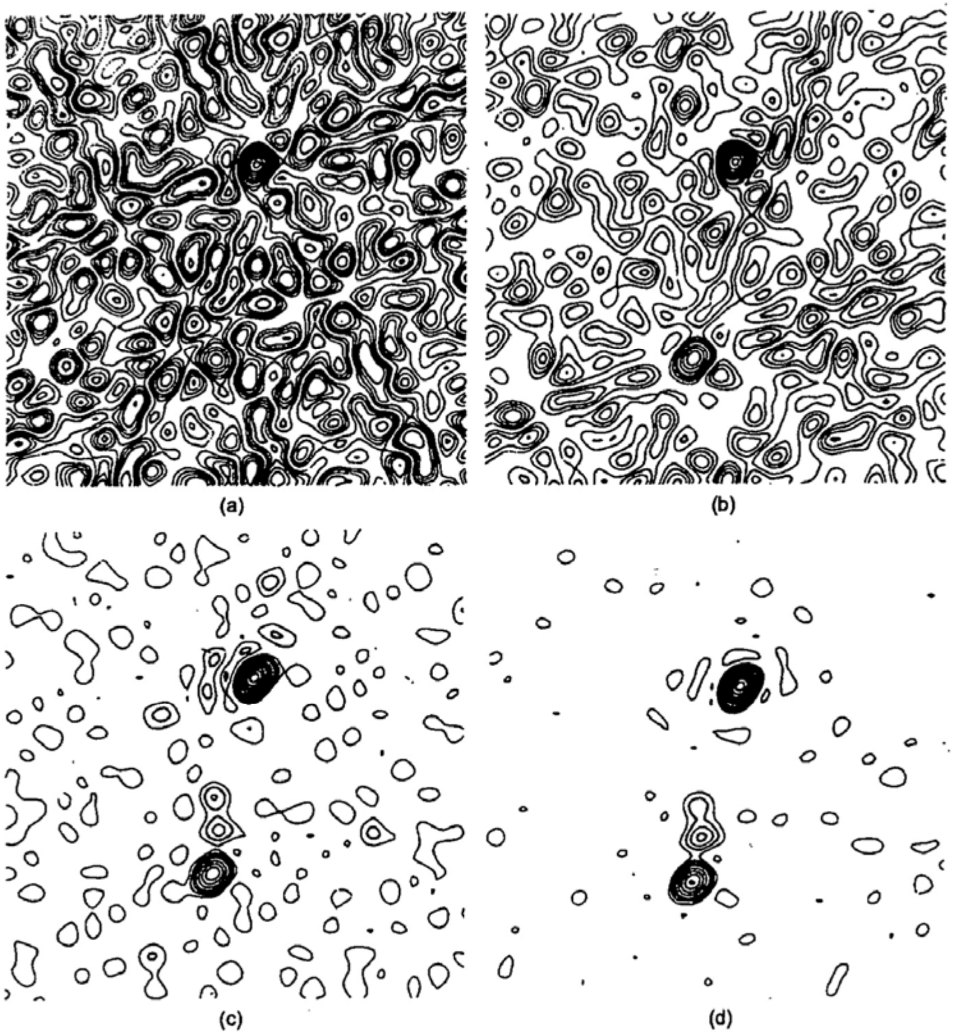
Hargrave and Ryle, MNRAS, 166, 305, 1974

V. Key Ideas in Image Calibration and Restoration

A. CLEAN	Jan Högbom (1930–)
B. Phase and Amplitude Closure	Roger Jennison (1922–2006) Alan Rogers (1942–)
C. Self Calibration	several
D. Mosaicking	Ron Eker (~1944–) Arnold Rots (1946–)
E. The Cygnus A Story Continued	

First Illustration of Clean Algorithm on 3C224.1 at 2.7 GHz with Green Bank Interferometer

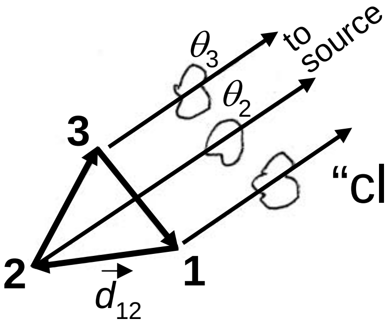
Zero, 1, 2 and 6 iterations



J. Högbom, Astron. Astrophys. Suppl., 15, 417, 1974

Closure Phase

“Necessity is the Mother of Invention”



“cloud” with phase shift $\theta_1 = 2\pi\nu\,\Delta t$

$$\vec{d}_{12} + \vec{d}_{23} + \vec{d}_{31} = 0$$

Observe a Point Source

$$\phi_{12} = \frac{2\pi}{\lambda}\vec{d}_{12}\cdot\hat{s} + \theta_1 - \theta_2$$

$$\phi_C = \phi_{12} + \phi_{23} + \phi_{31} = \frac{2\pi}{\lambda}\overset{0}{\overbrace{[\vec{d}_{12} + \vec{d}_{23} + \vec{d}_{31}]}}\cdot\hat{s}$$

Arbitrary Source Distribution

$$\phi_{m_{ij}} = \phi_{v_{ij}} + (\theta_i - \theta_j) + \overset{\text{noise}}{\varepsilon_{ij}}$$

$$\phi_C = \phi_{m_{12}} + \phi_{m_{23}} + \phi_{m_{31}} = \phi_{v_{12}} + \phi_{v_{23}} + \phi_{v_{31}} + \text{noise}$$

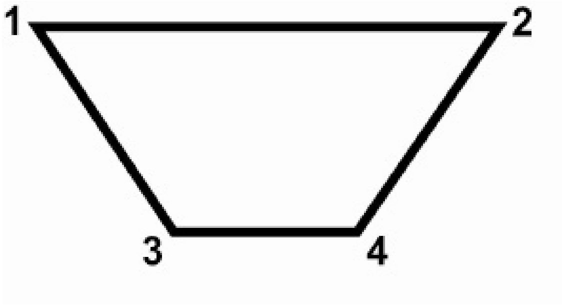
N stations $\Rightarrow \frac{N(N-1)}{2}$ baselines, $\frac{1}{2}(N-1)(N-2)$ closure conditions

fraction of phases $\boxed{f = 1 - \frac{2}{N}}$ $N = 27,\ f \sim 0.9$

R. Jennison, 1952 (thesis): MNRAS, 118, 276, 1956; A. Rogers et al., Ap.J., 193, 293, 1974

Closure Amplitude

$N \geq 4$



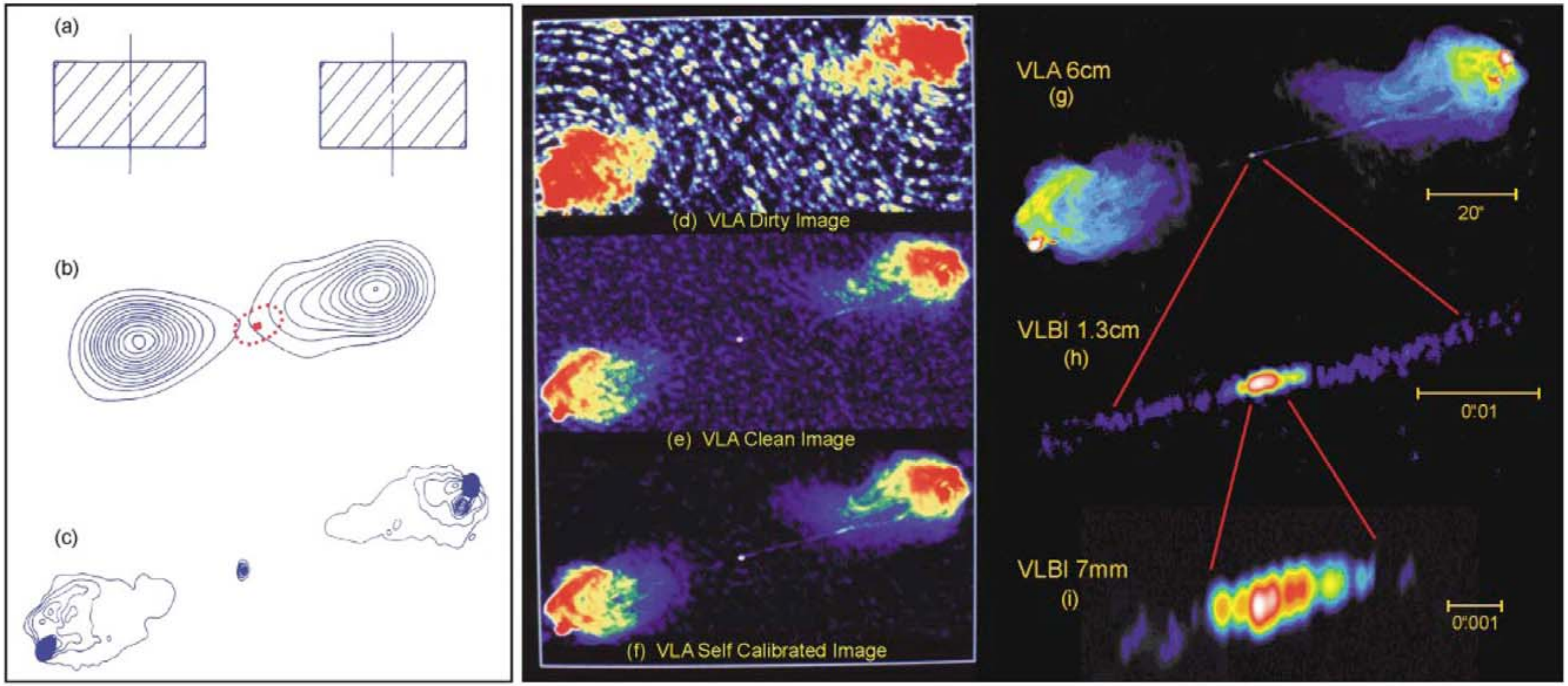
Unknown voltage gain factors for each antenna g_i $(i = 1–4)$

$$V_C = \frac{(g_1 g_2 V_{12})\,(g_3 g_4 V_{34})}{(g_1 g_3 V_{13})\,(g_2 g_4 V_{24})}$$

$$V_C = \frac{V_{12}\ V_{34}}{V_{13}\ V_{24}}$$

$$f = \frac{N-3}{N-1}$$

A Half Century of Improvements in Imaging of Cygnus A

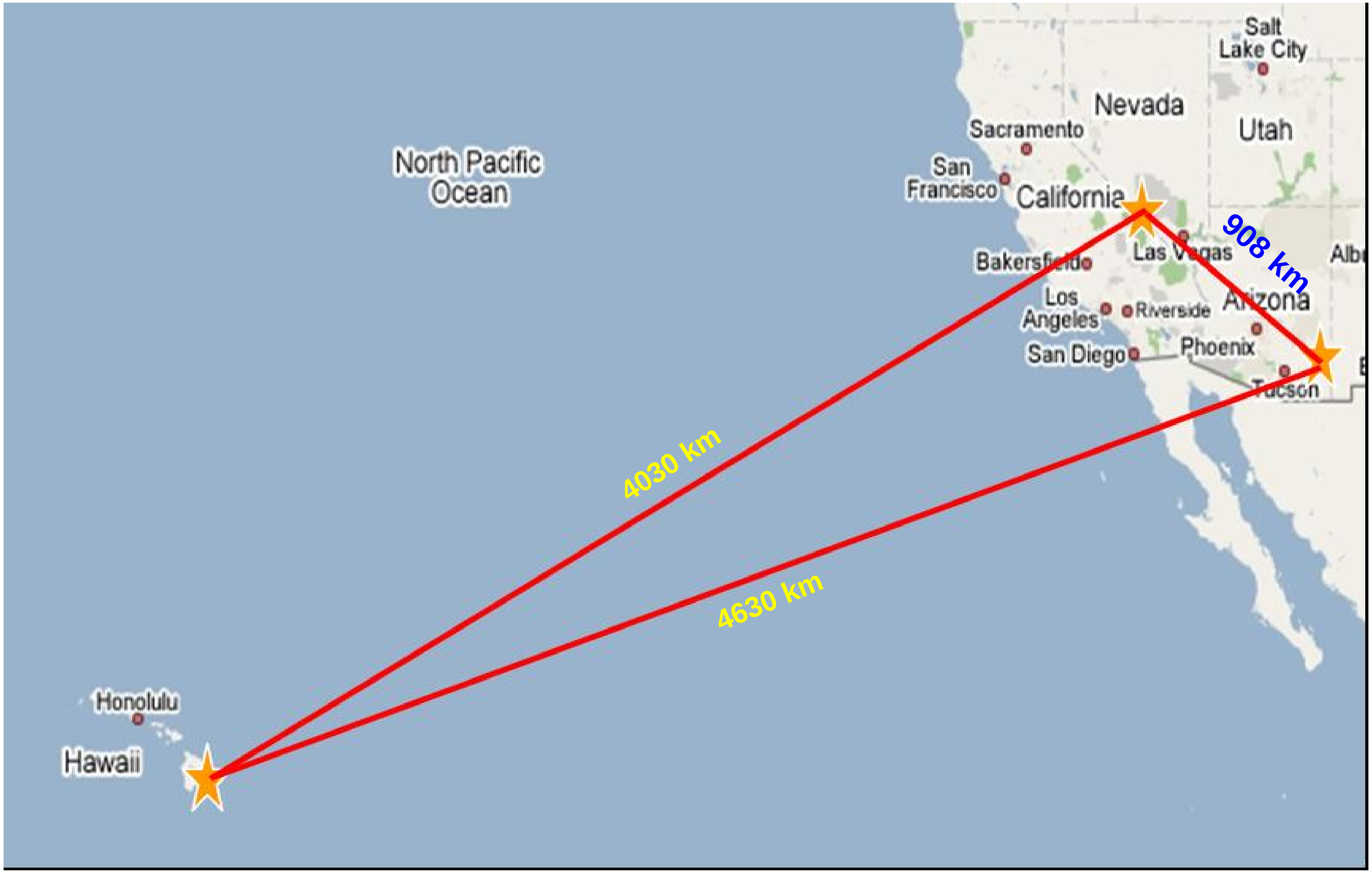


Kellermann and Moran, Ann. Rev. Astron., 39, 457, 2001

VI. Back to Basics

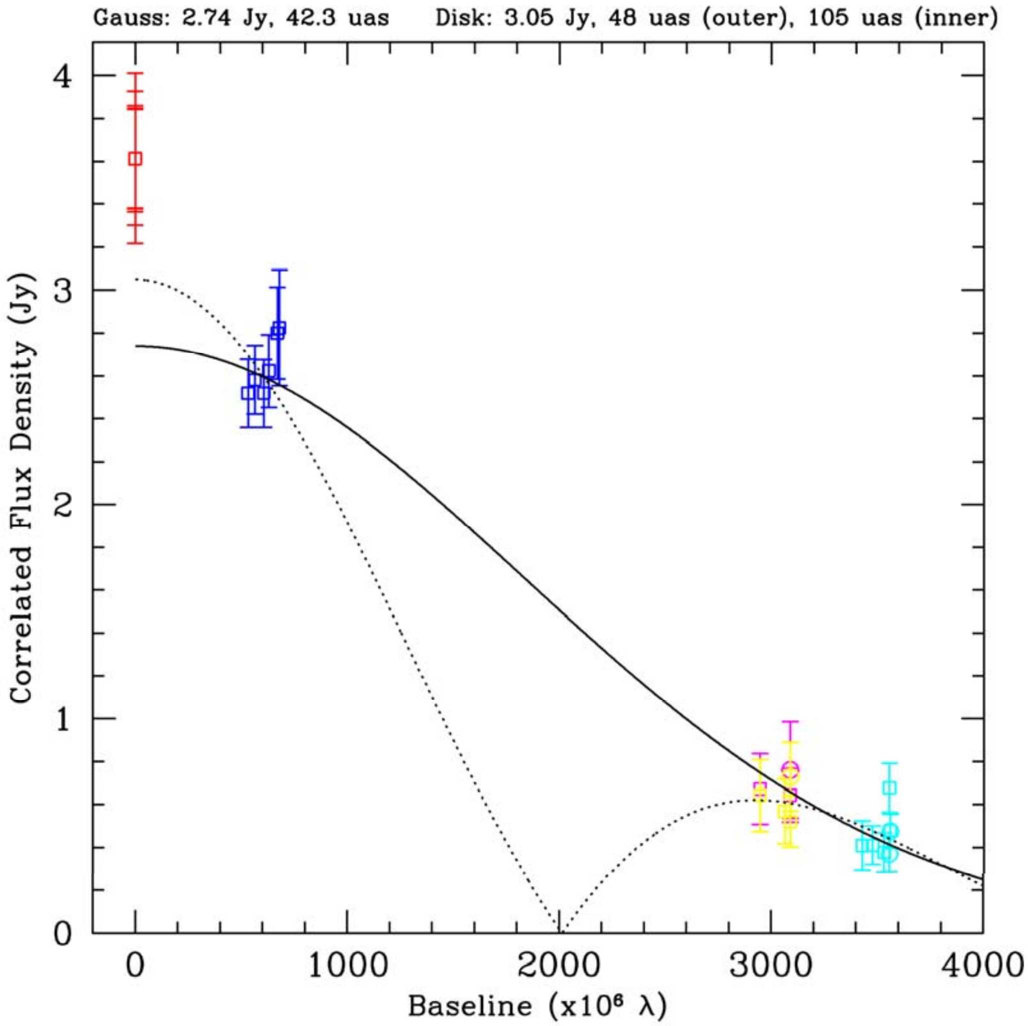
Imaging Sgr A* in 2010 and beyond

230 GHz Observations of SgrA*



VLBI program led by large consortium led by Shep Doeleman, MIT/Haystack

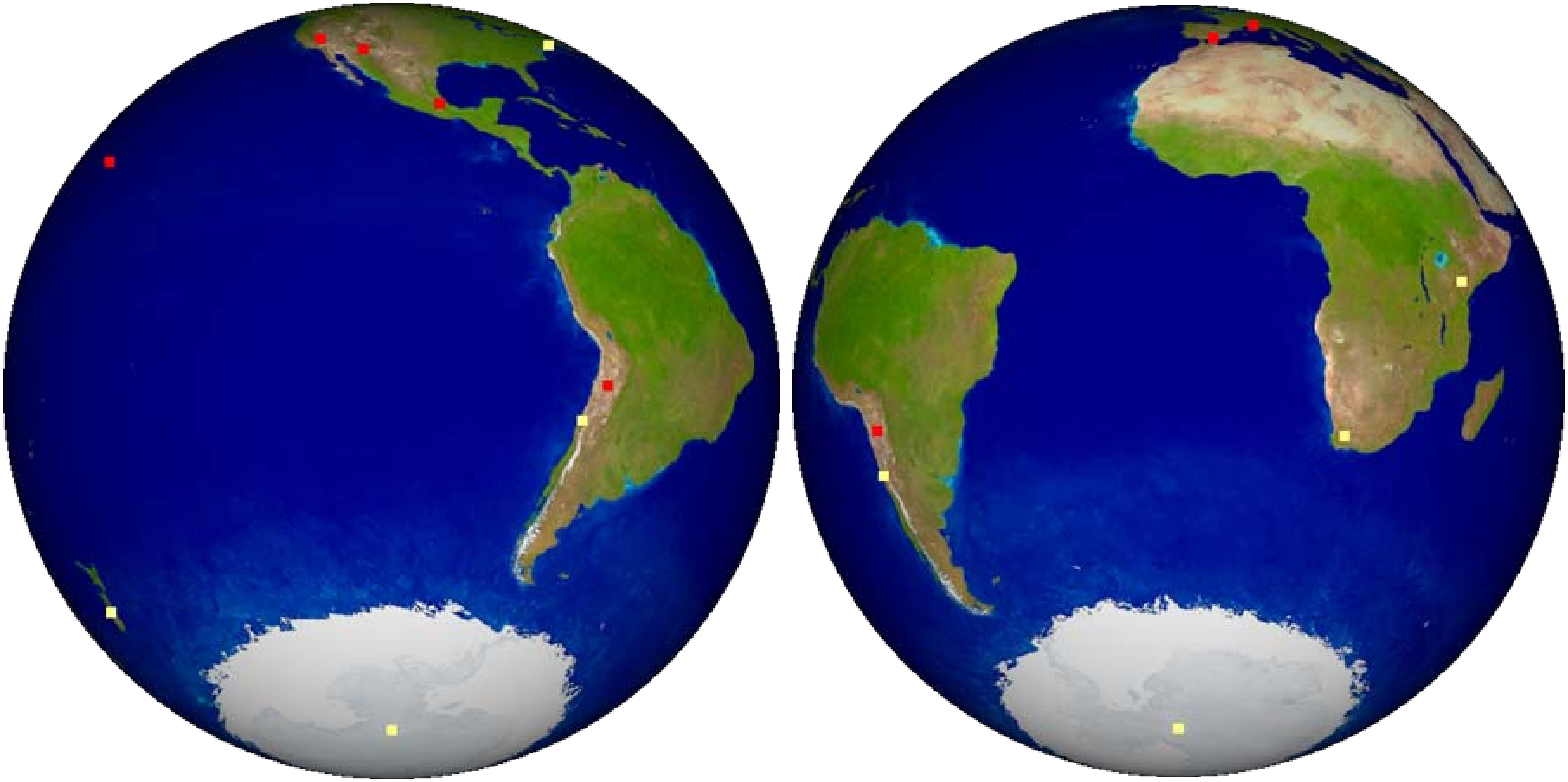
Visibility Amplitude on SgrA* at 230 GHz, March 2010



Model fits: (solid) Gaussian, 37 uas FWHM; (dotted) Annular ring, 105/48 μas diameter – both with 25 μas of interstellar scattering

Doeleman et al., private communication

New (sub)mm VLBI Sites

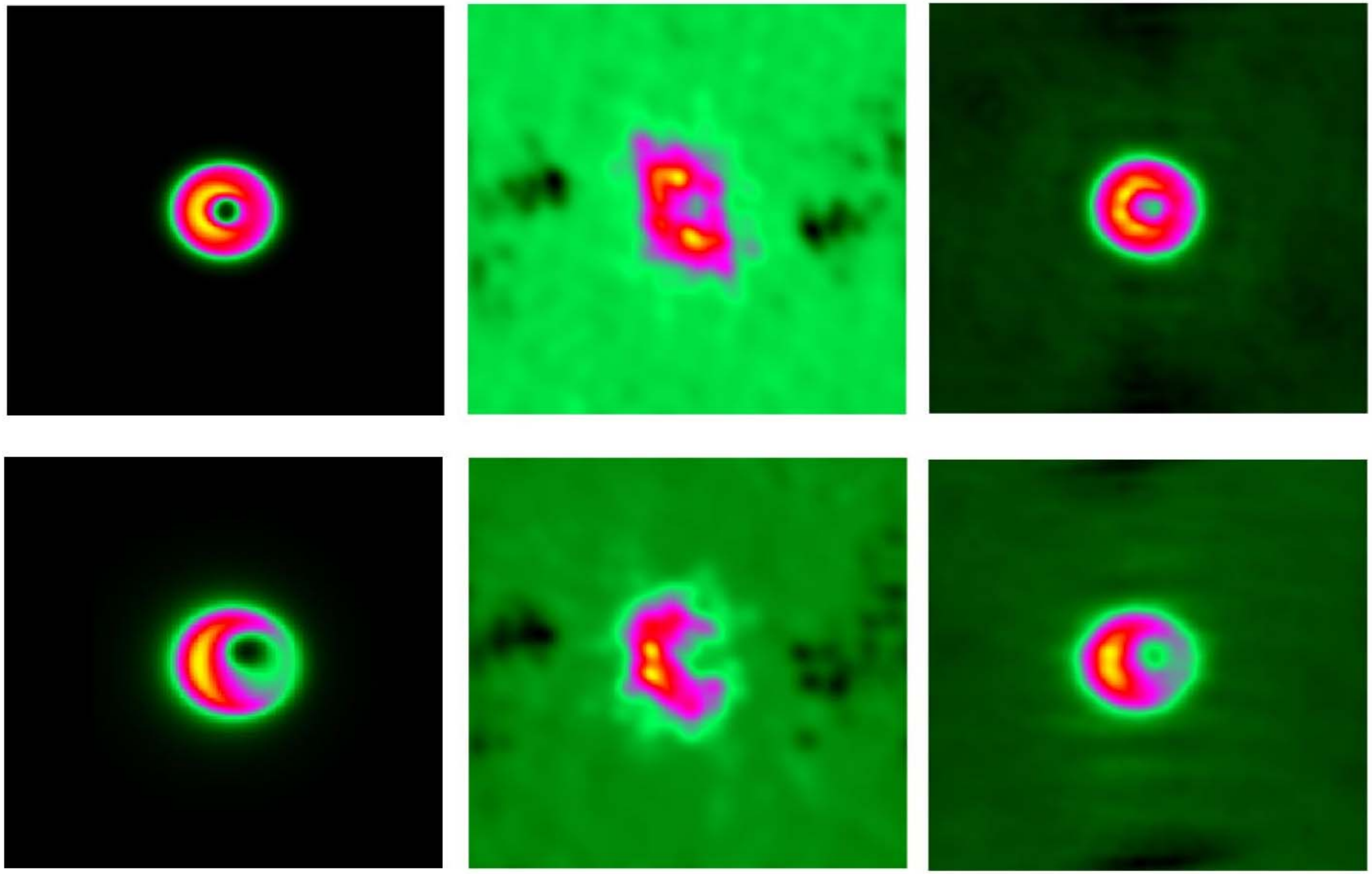


Phase 1: 7 Telescopes (+ IRAM, PdB, LMT, Chile)

Phase 2: 10 Telescopes (+ Spole, SEST, Haystack)

Phase 3: 13 Telescopes (+ NZ, Africa)

Progression to an Image



GR Model **7 Stations** **13 Stations**

Doeleman et al., “The Event Horizon Telescope,” Astro2010: The Astronomy and Astrophysics Decadal Survey, Science White Papers, no. 68