Imaging, Deconvolution & Image Analysis
I. Theory

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Scientific Analysis of a mm Interferometer Output

mm interferometer output:
Calibrated visibilities in the uv plane ($\simeq$ the Fourier plane).

2 possibilities:

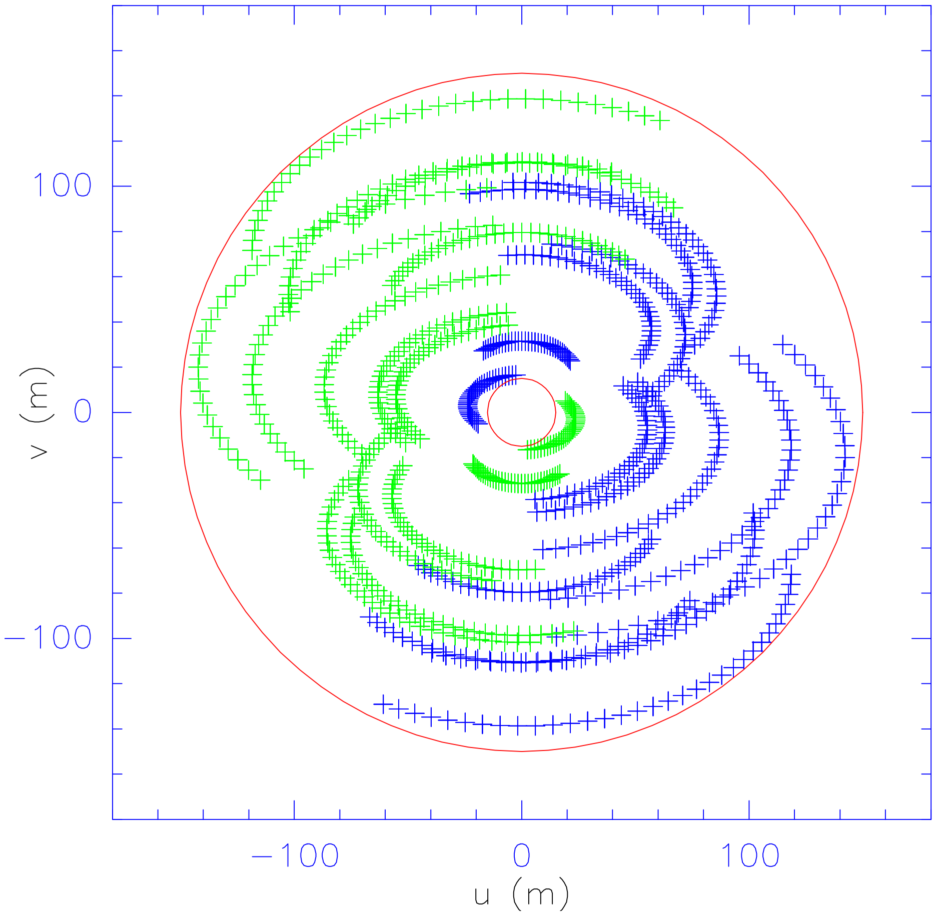
- uv plane analysis (cf. Lecture by A. Castro-Carrizo):
  Always better . . . when possible!
  (in practice for "simple" sources as point sources or disks)
- Image plane analysis:
  $\Rightarrow$ Mathematical transforms to go from uv to image plane!

Goal: Understand effects of the imaging process on

- The resolution;
- The field of view (single pointing or mosaicing, cf. Lecture by F. Gueth);
- The reliability of the image;
- The noise level and repartition (cf. lecture by S.Guilloteau).

From Calibrated Visibilities to Images:
I. Comparison Visibilities/Source Fourier Transform

$$V_{ij}(b_{ij}) = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}(b_{ij}) + N$$



- Primary Beam
  ⇒ Distorted source information.
- Noise ⇒ Sensitivity problems.
- Irregular, limited sampling
  ⇒ incomplete source information:
  - Support limited at:
    * High spatial frequency
      ⇒ limited resolution;
    * Low spatial frequency ⇒ problem of wide field imaging;
  - Inside the support, incomplete (*i.e.* Nyquist's criterion not respected) sampling ⇒ lost of information.

From Calibrated Visibilities to Images:
II. Effect of Irregular, Limited Sampling

Definitions:

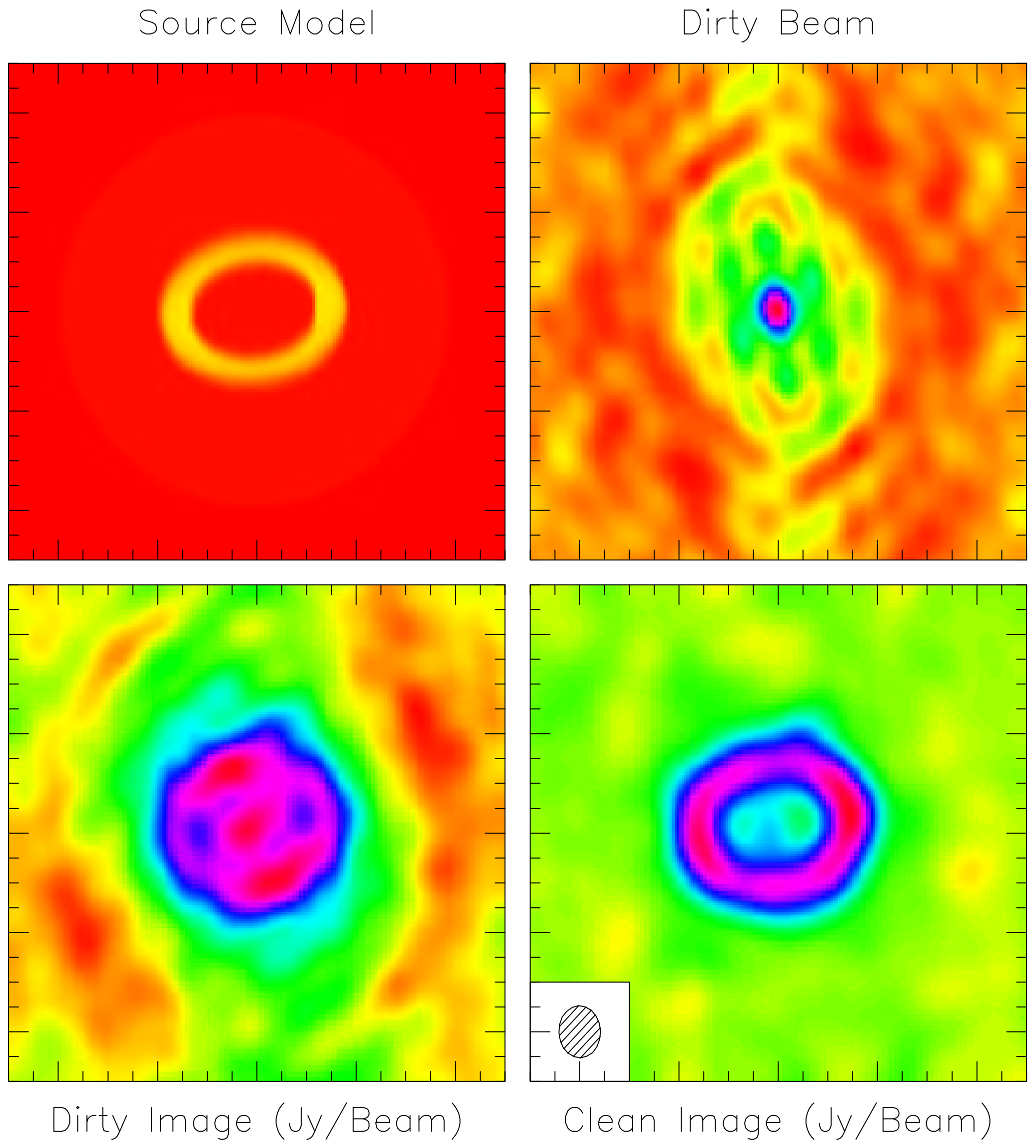
- $V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$;
- Irregular, limited sampling function:
  - $S(u,v) = 1$ at (u,v) points where visibilities are measured;
  - $S(u,v) = 0$ elsewhere;
- $B_{\text{dirty}} = \text{2D FT}^{-1}\{S\}$;
- $I_{\text{meas}} = \text{2D FT}^{-1}\{S.V\}$.

Fourier Transform Property #1:

$I_{\text{meas}} = B_{\text{dirty}} * \left\{B_{\text{primary}}.I_{\text{source}}\right\}$.

B_{dirty}: Point Spread Function (PSF) of the interferometer
(*i.e.* if the source is a point, then $I_{\text{meas}} = I_{\text{tot}}.B_{\text{dirty}}$).

From Calibrated Visibilities to Images:
III. Why Deconvolving?



- Difficult to do science on dirty image.
- Deconvolution $\Rightarrow$ a clean image compatible with the sky intensity distribution.

From Calibrated Visibilities to Images: Summary

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
$\Downarrow$ Fourier Transform	`GO UVSTAT, GO UVMAP`
Dirty beam & image	
$\Downarrow$ Deconvolution	`GO CLEAN`
Clean beam & image	
$\Downarrow$ Visualization	`GO BIT, GO VIEW`
$\Downarrow$ Image analysis	`GO NOISE, GO FLUX, GO MOMENTS`
Physical information on your source	

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Direct vs. Fast Fourier Transform

Direct FT:

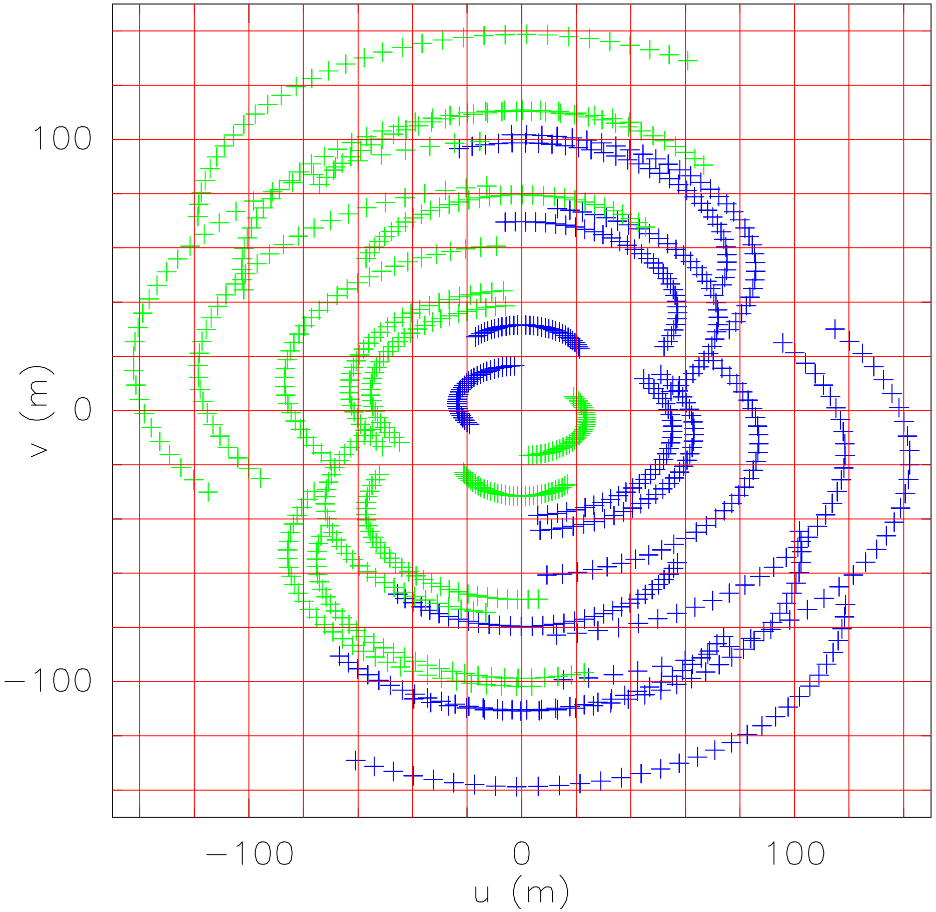
- Advantage: Direct use of the irregular sampling;
- Inconvenient: Slow.

Fast FT:

- Inconvenient: Needs a regular sampling $\Rightarrow$ Gridding;
- Advantage: Quick for images of size $2^M \times 2^N$.

$\Rightarrow$ In practice, everybody use FFT.

Gridding: I. Interpolation Scheme



Convolution because:

- Visibilities = noisy samples of a smooth function.
  $\Rightarrow$ Some smoothing is desirable.
- Nearby visibilities are not independent.
  - $V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$
    $= \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}}$;
  - FWHM(convolution kernel) $<$ FWHM($\tilde{B}_{\text{primary}}$)
    $\Rightarrow$ No real information lost.

Gridding: II. Convolution Equation is Kept Through Gridding

Demonstration:

- $I_{\text{meas}}^{\text{grid}} \overset{\text{2D FT}}{\rightleftharpoons} G * (S.V) \quad \Leftrightarrow \quad I_{\text{meas}}^{\text{grid}} = \tilde{G}.(\widetilde{S.V}) = \tilde{G}.(\tilde{S} * \tilde{V});$
- $B_{\text{dirty}}^{\text{grid}} \overset{\text{2D FT}}{\rightleftharpoons} G * S \quad \Leftrightarrow \quad B_{\text{dirty}}^{\text{grid}} = \tilde{G}.\tilde{S};$

$\Rightarrow I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}}.I_{\text{source}} \right\}$

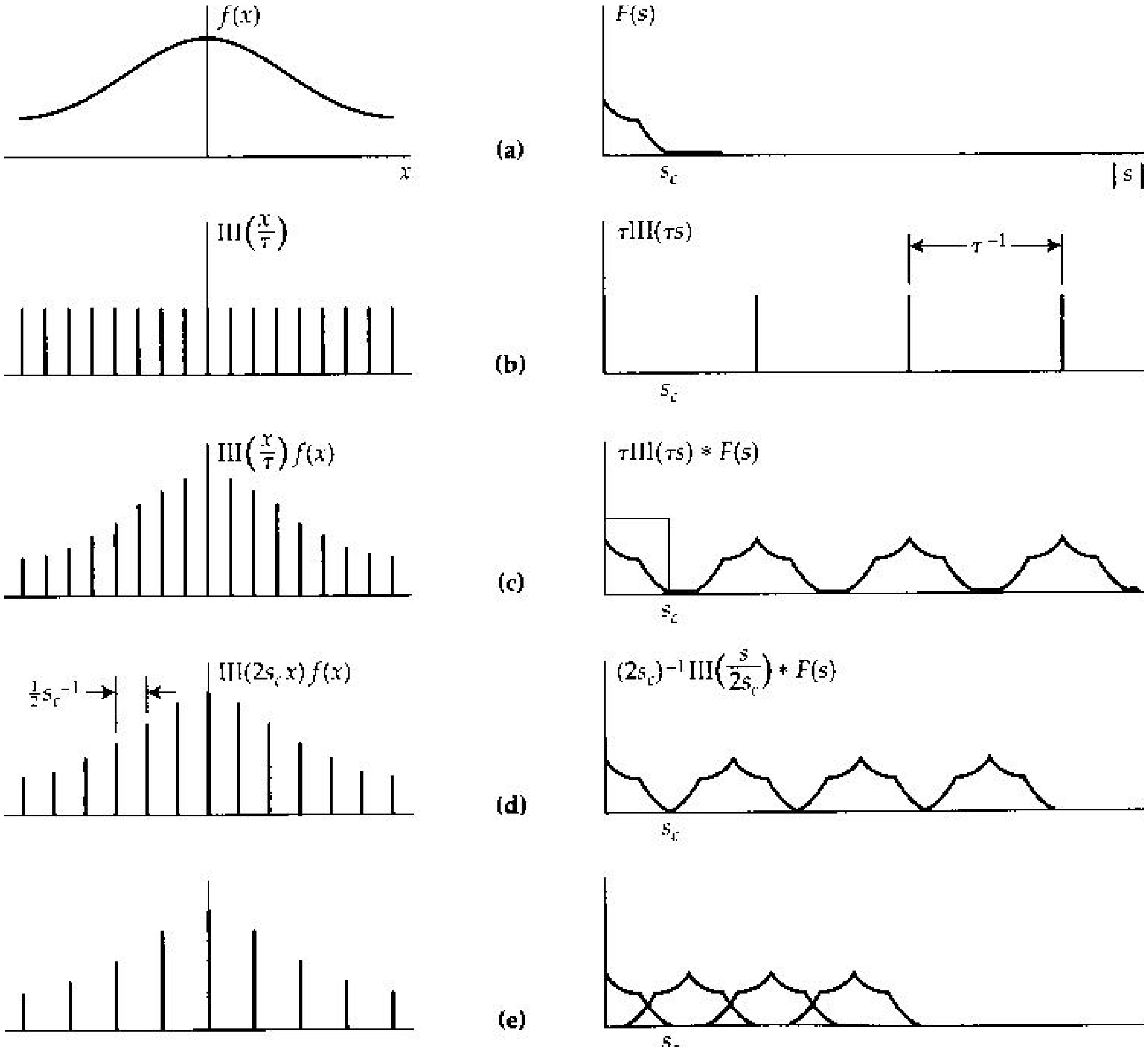
with $I_{\text{meas}} = I_{\text{meas}}^{\text{grid}} / \tilde{G}$

and $B_{\text{dirty}} = B_{\text{dirty}}^{\text{grid}} / \tilde{G}$.

Remark: Gridding may be hidden in equations but it is still there.
$\Rightarrow$ Artifacts due to gridding! (cf. next transparencies)

Gridding:
III. Effect of a Regular Sampling (Periodic Replication)

uv Plane | Image Plane



$B_{\text{primary}}.I_{\text{source}}$

Regular Sampling function

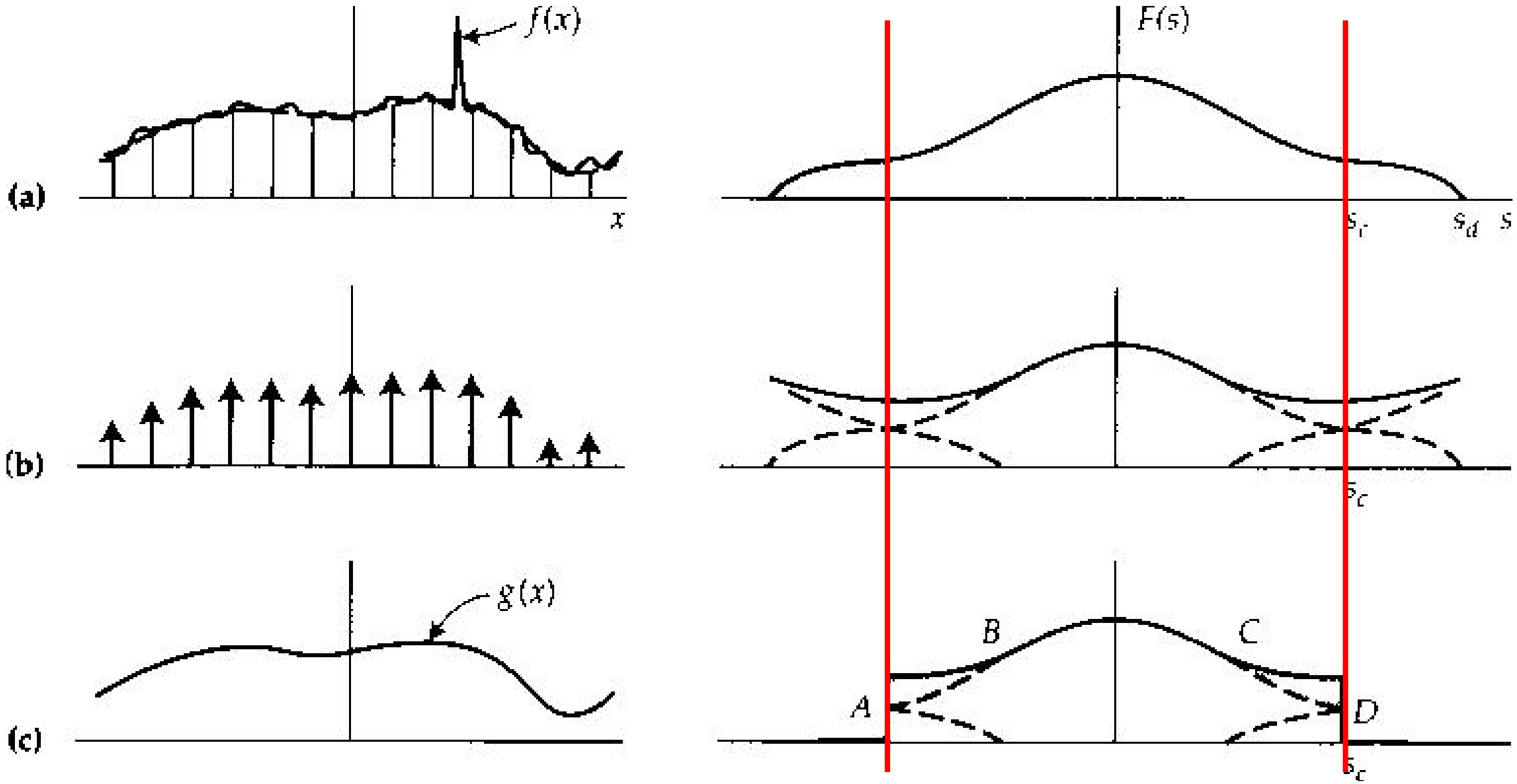
Result for a fine sampling

Result for critical sampling
(Nyquist's criterion)

Result for a coarse sampling

Gridding: III. Effect of a Regular Sampling (Aliasing)

uv Plane | Image Plane



Aliasing = Folding of intensity outside the image size into the image.

⇒ Image size must be large enough.

Gridding: IV. Pixel and Image Sizes

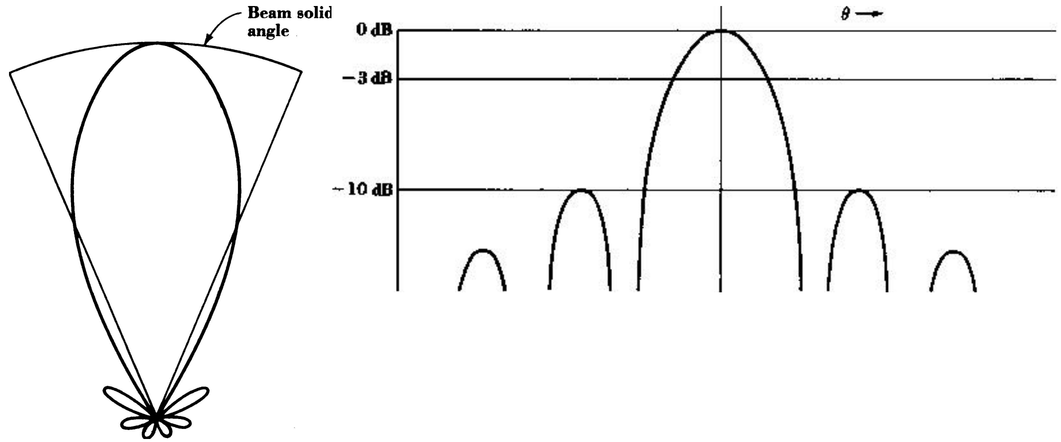
Pixel size: Between 1/4 and 1/5 of the synthesized beam size (*i.e.* more than the Nyquist's criterion in image plane to ease deconvolution).

Image size:

- $=$ uv plane sampling rate (FT property # 2);
- Natural resolution in the uv plane: $\tilde{B}_{\text{primary}}$ size;

$\Rightarrow$ At least twice the B_{primary} size (*i.e.* Nyquist's criterion in uv plane).

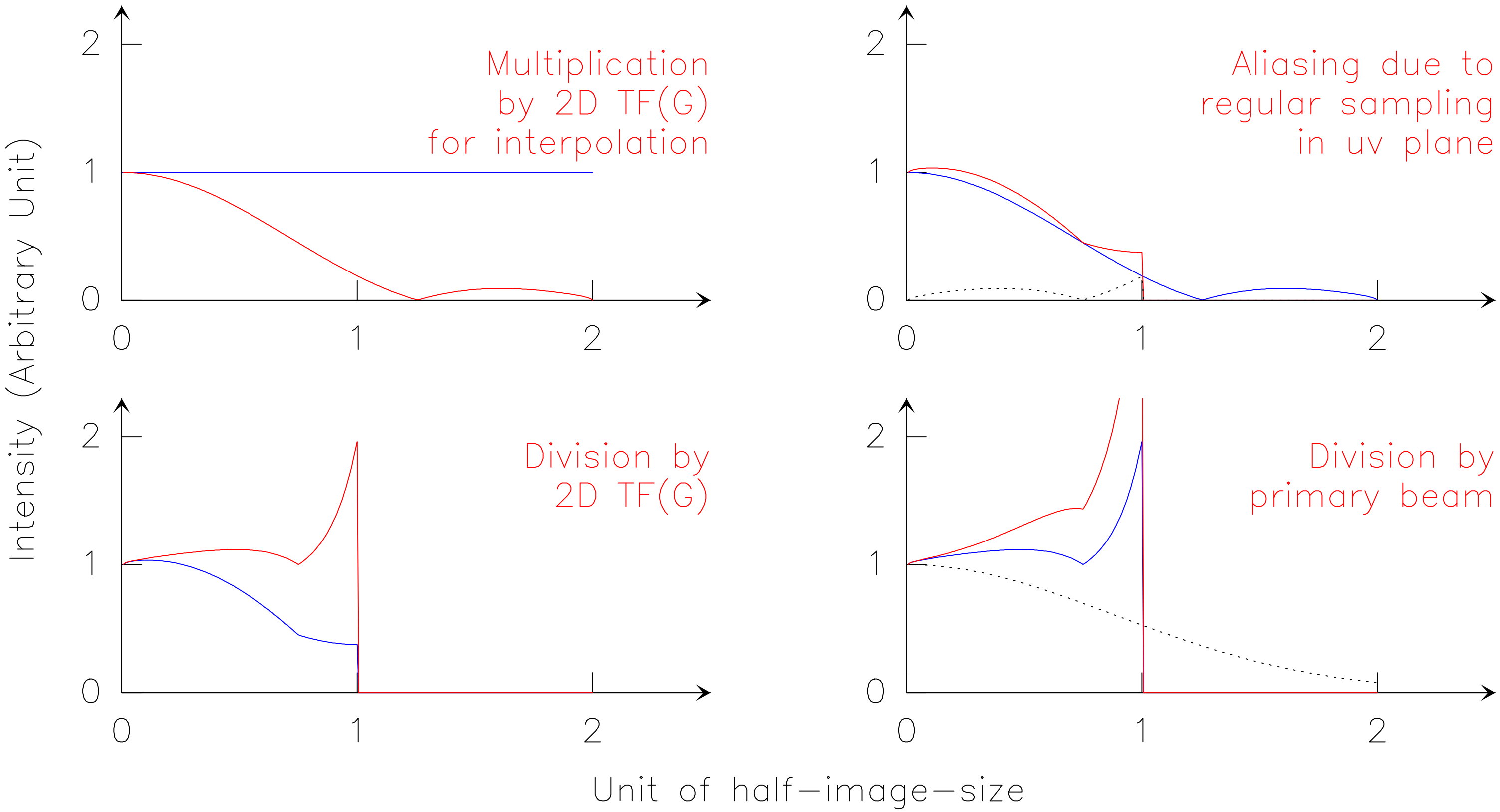
Gridding: V. Bright Sources in B_{primary} Sidelobes



Bright Sources in B_{primary} sidelobes
outside image size will be aliased into image.
⇒ Spurious source in your image!

Solution: Increase the image size.
(Be careful: only when needed for efficiency reasons!)

Gridding: VI. Noise Distribution



Gridding: VII. Choice of Gridding function

Gridding function must:

- Fall off quickly in image plane (to avoid noise aliasing);
- Fall off quickly in uv plane (to avoid too much smoothing).

$\Rightarrow$ Define a mathematical class of functions: Spheroidal functions.

GILDAS implementation: In `GO UVMAP`

- Spheroidal functions = Default gridding function;
- Tabulated values are used for speed reasons.

Dirty Beam Shape and Image Quality

$$B_{\text{dirty}} = \text{2D FT}^{-1}\{S\}.$$

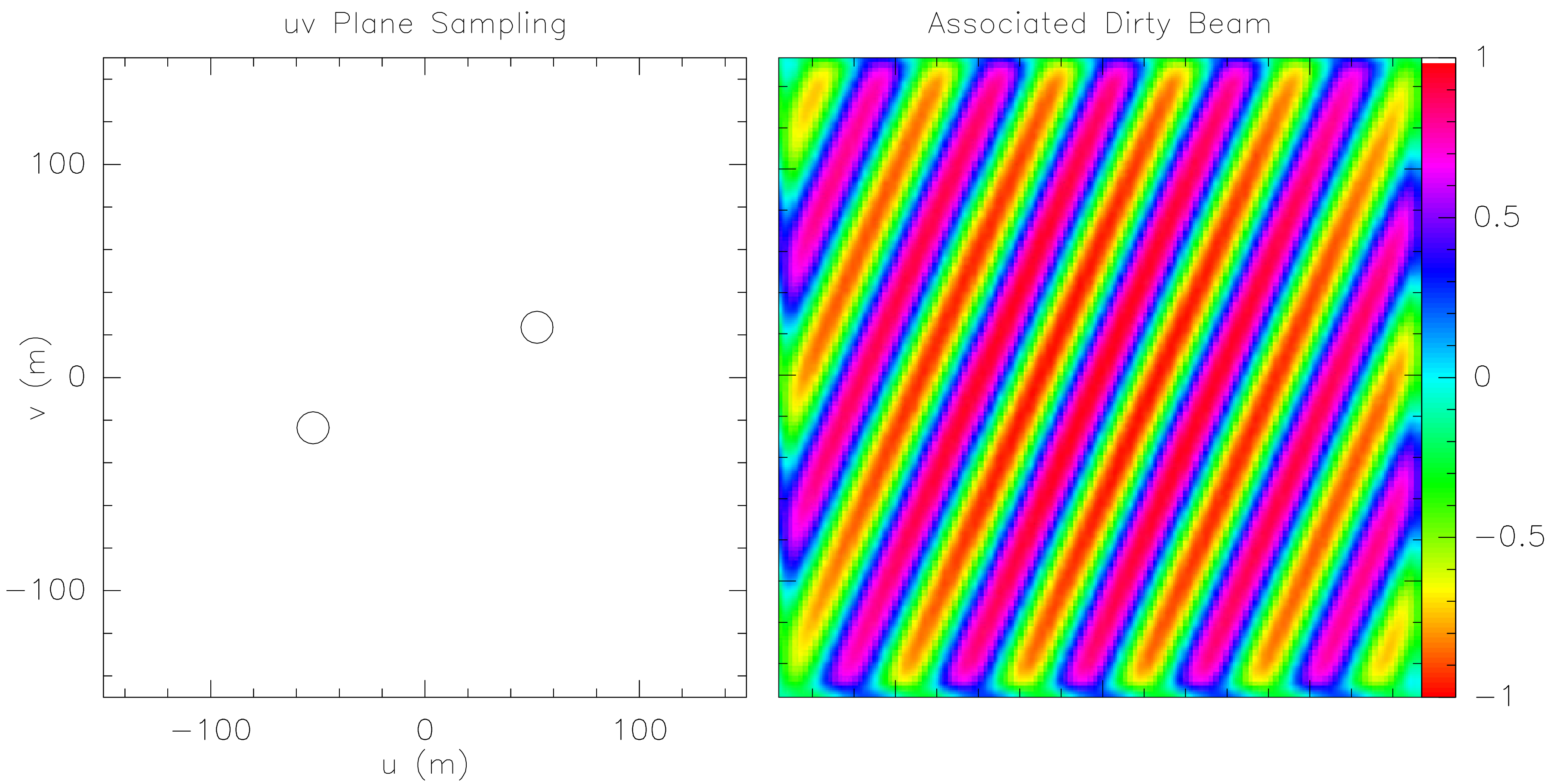
Importance of the Dirty Beam Shape:

- Deconvolving a dirty image is a delicate stage;
- The closest to a Gaussian B_{dirty} is, the easier the deconvolution;
- Extreme case:
  B_{dirty} = Gaussian $\Rightarrow$ No deconvolution needed at all!

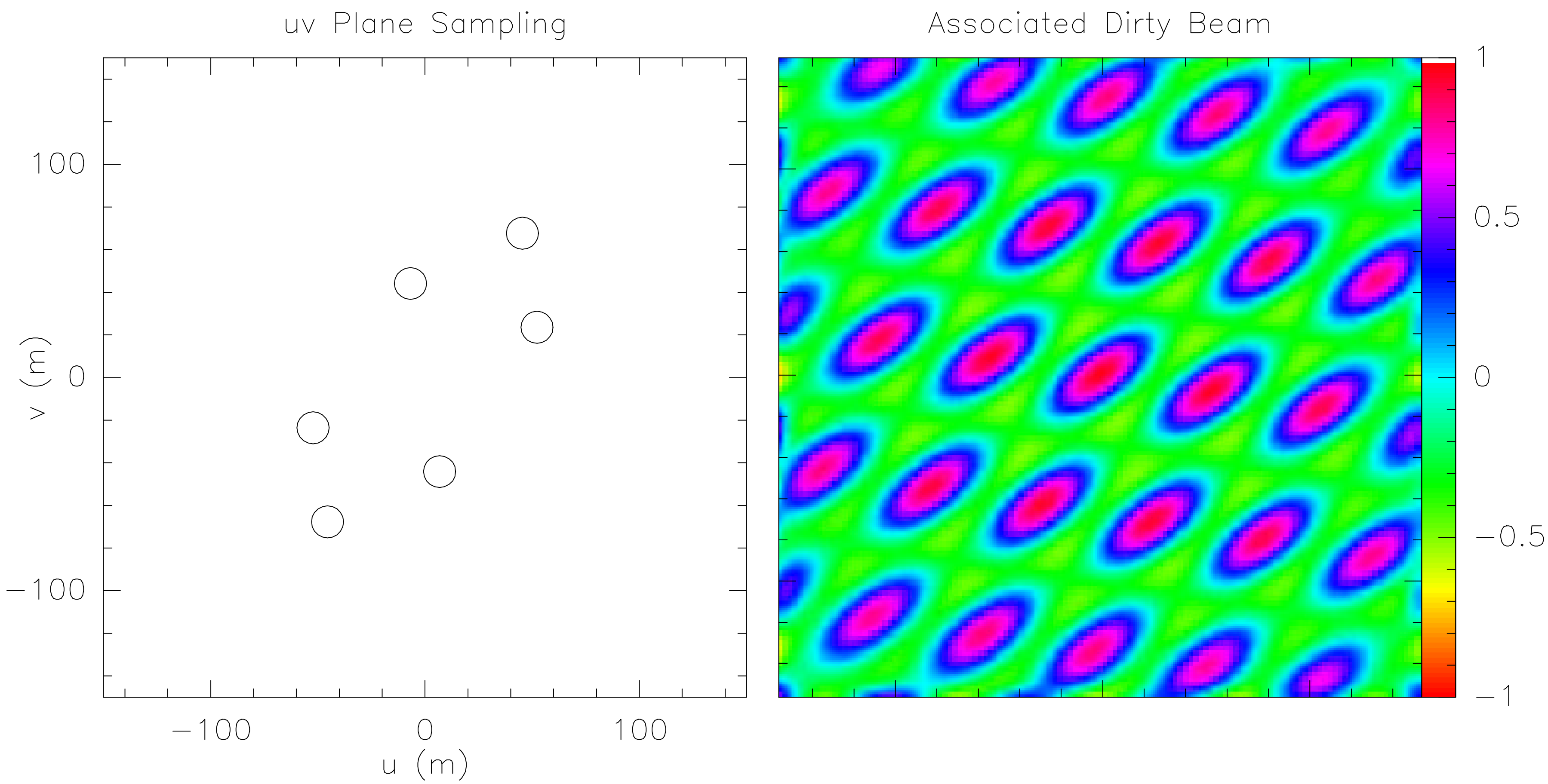
Ways to improve (at least change) B_{dirty} shape:

- Increase the number of antenna (costly).
- Change the antenna layout (technically difficult).
- Weight the irregular, limited sampling function S (the only thing you can do in practice).

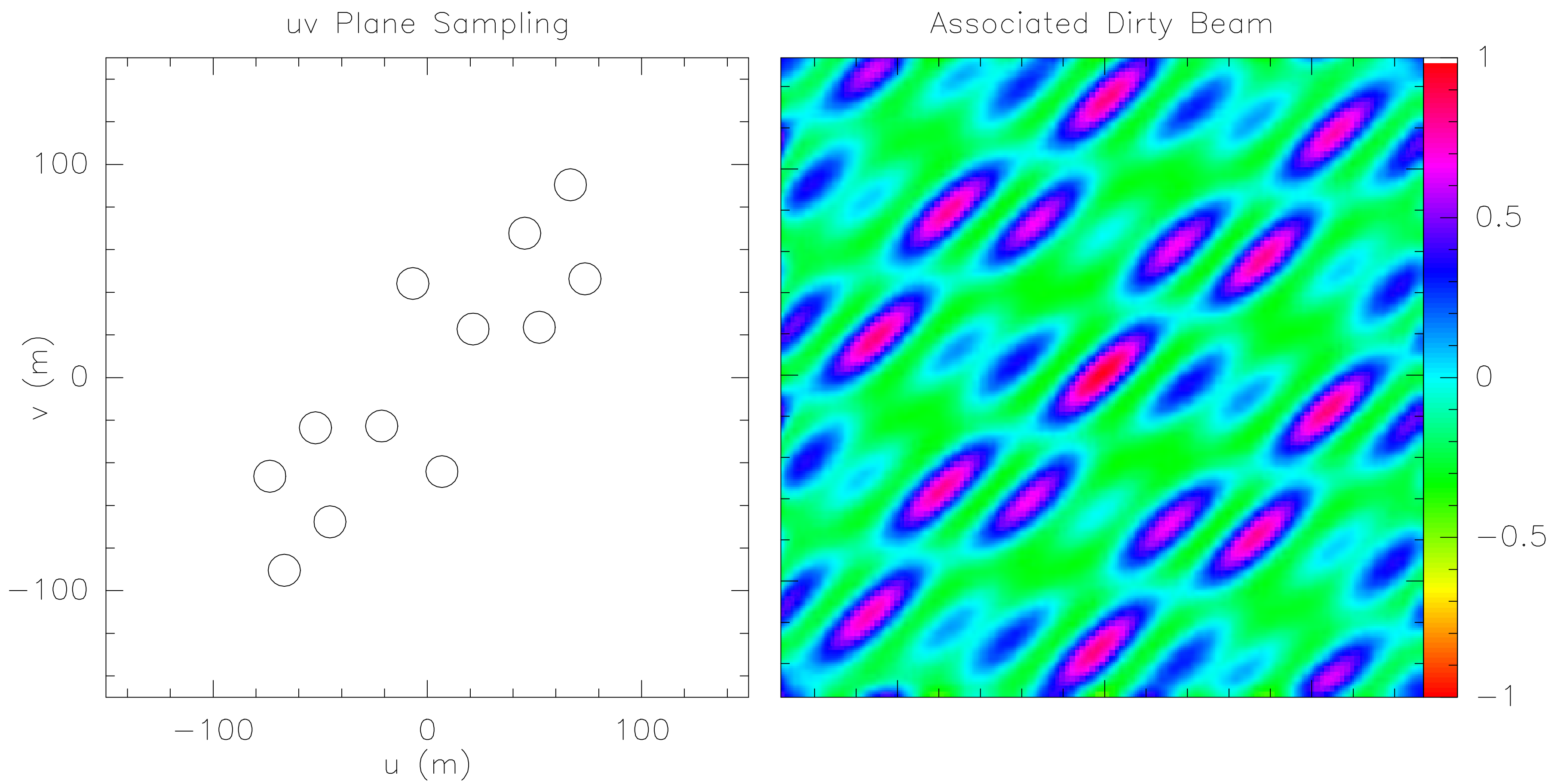
Dirty Beam Shape and Number of Antenna:
2 Antenna



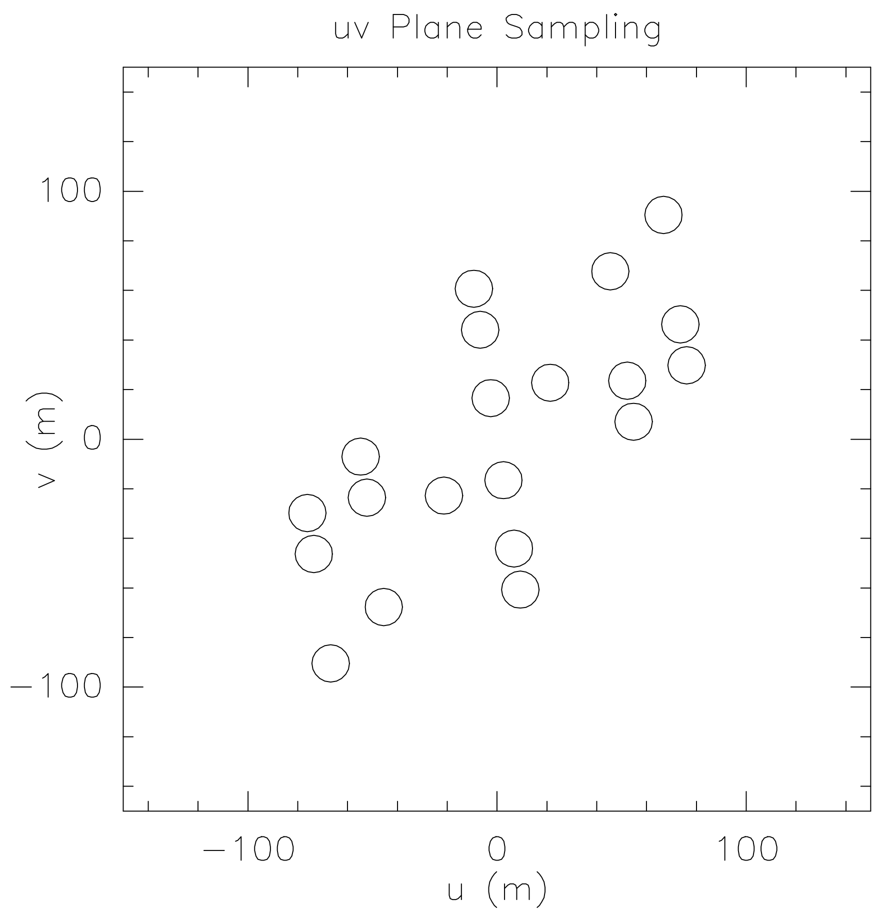
Dirty Beam Shape and Number of Antenna:
3 Antenna

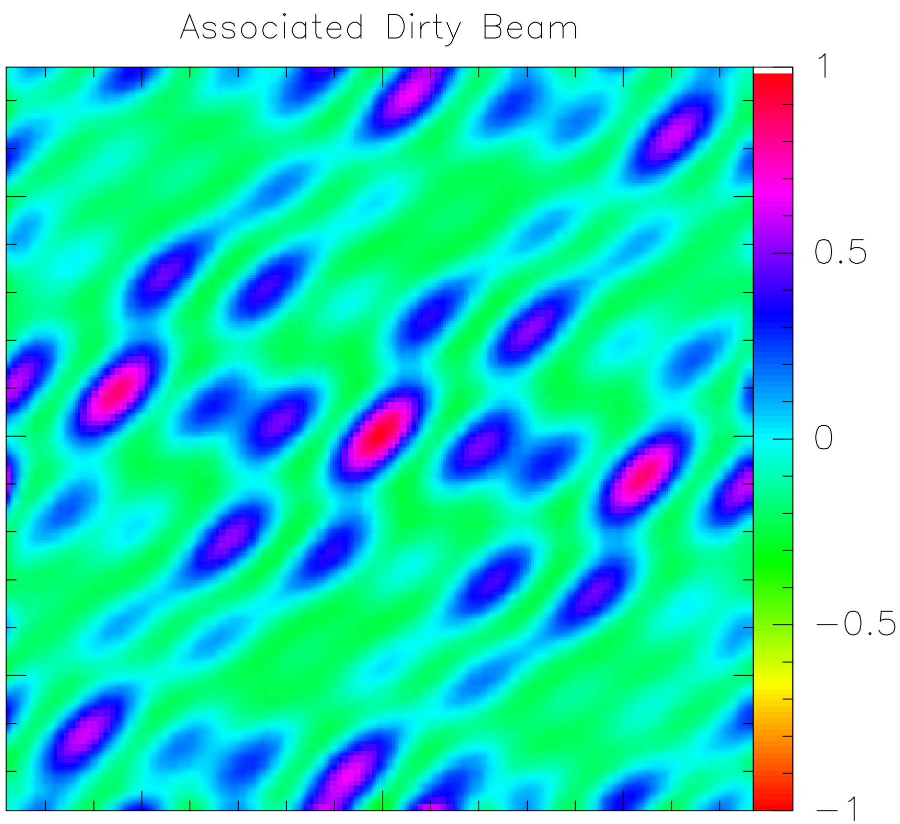


Dirty Beam Shape and Number of Antenna:
4 Antenna

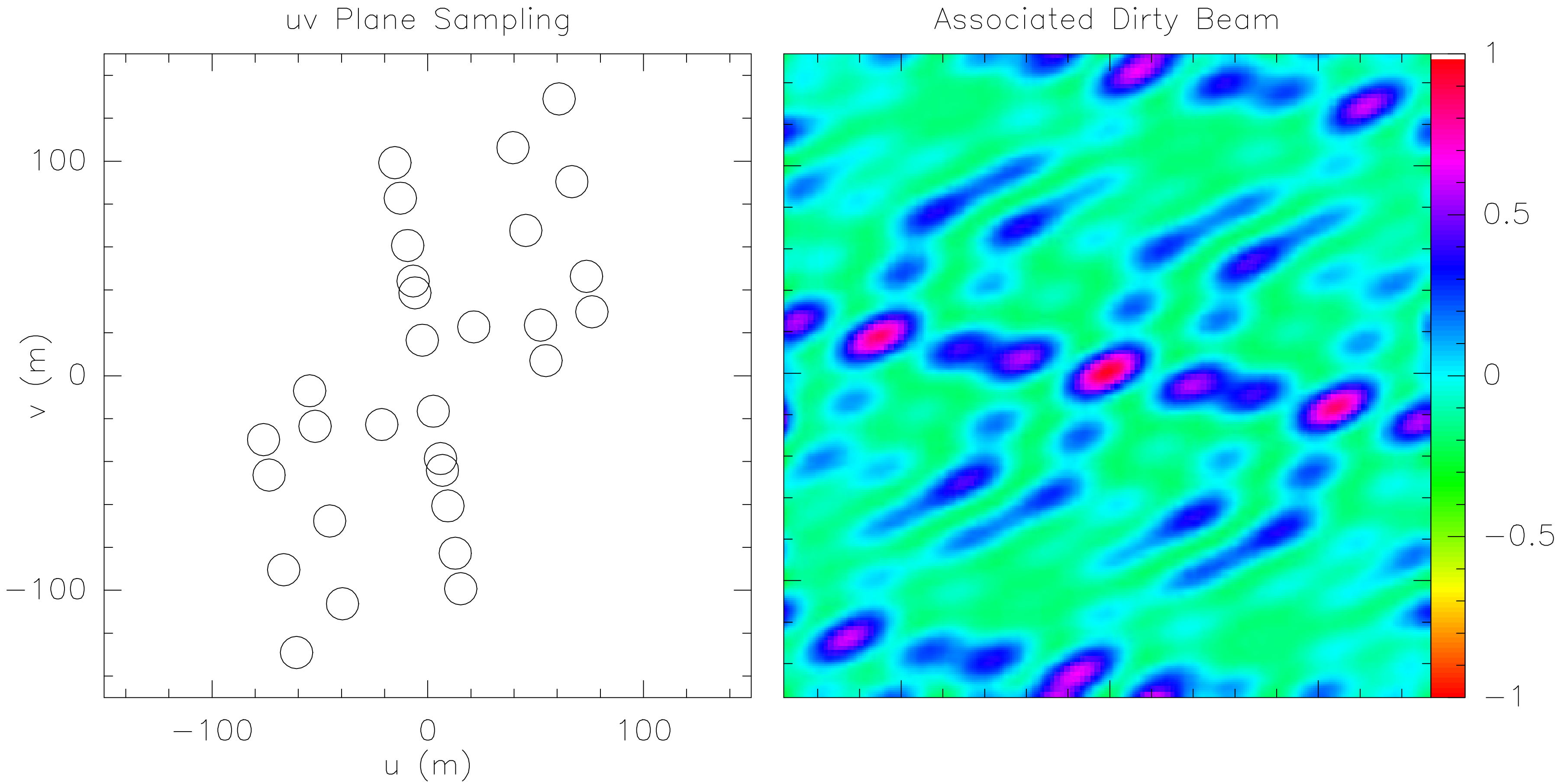


Dirty Beam Shape and Number of Antenna:
5 Antenna

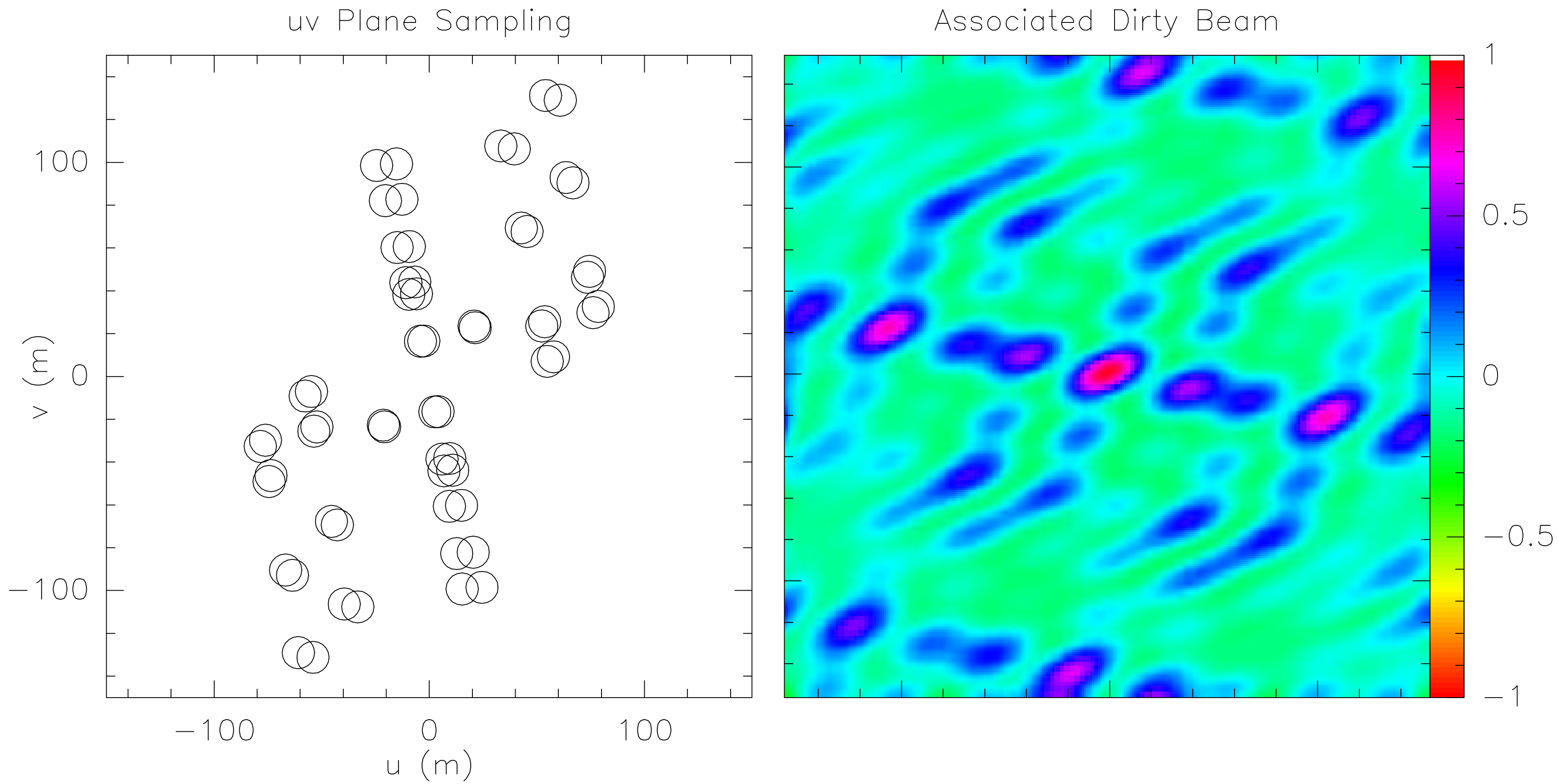




Dirty Beam Shape and Number of Antenna:
6 Antenna



Dirty Beam Shape and Super Synthesis



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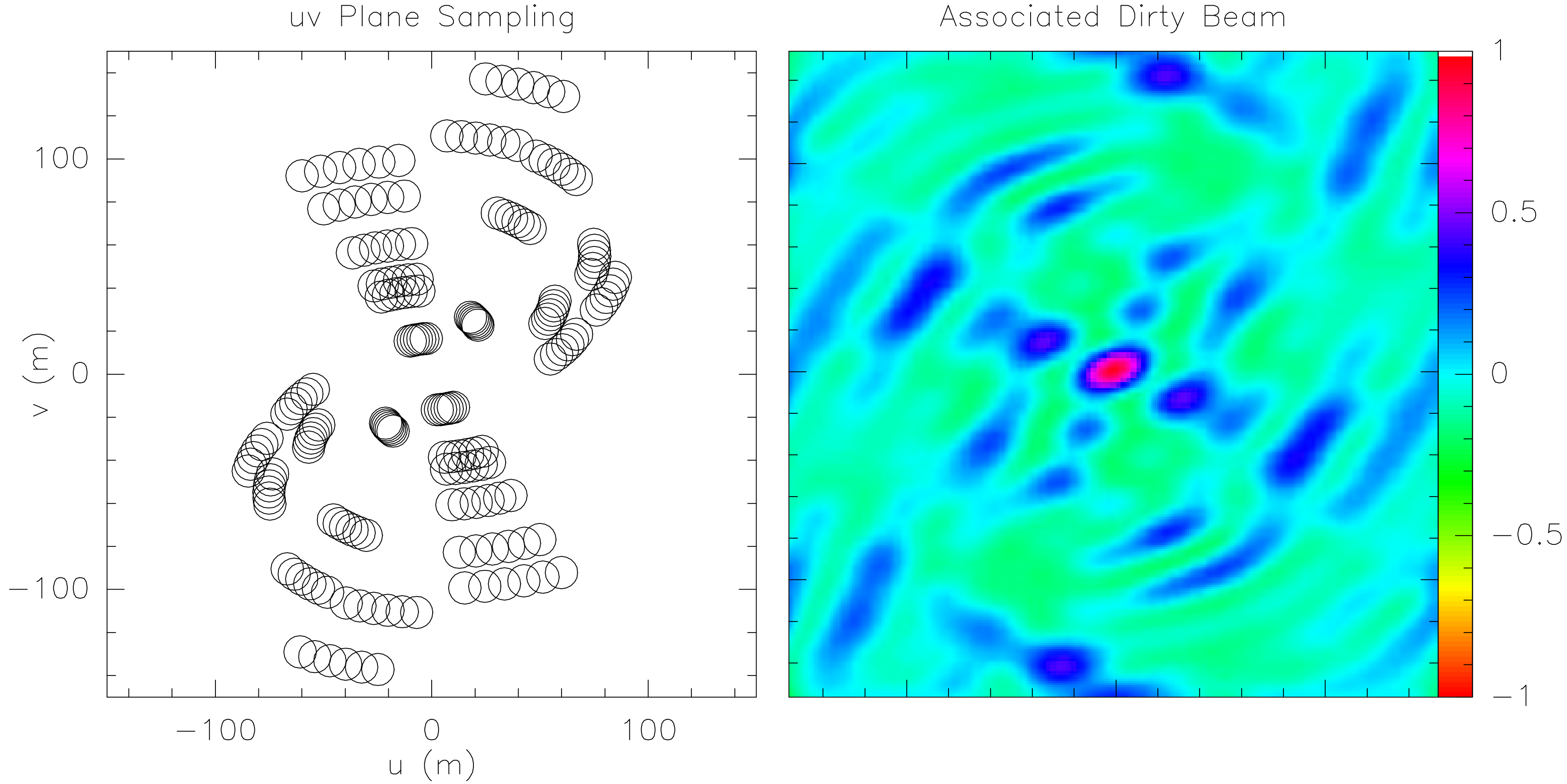

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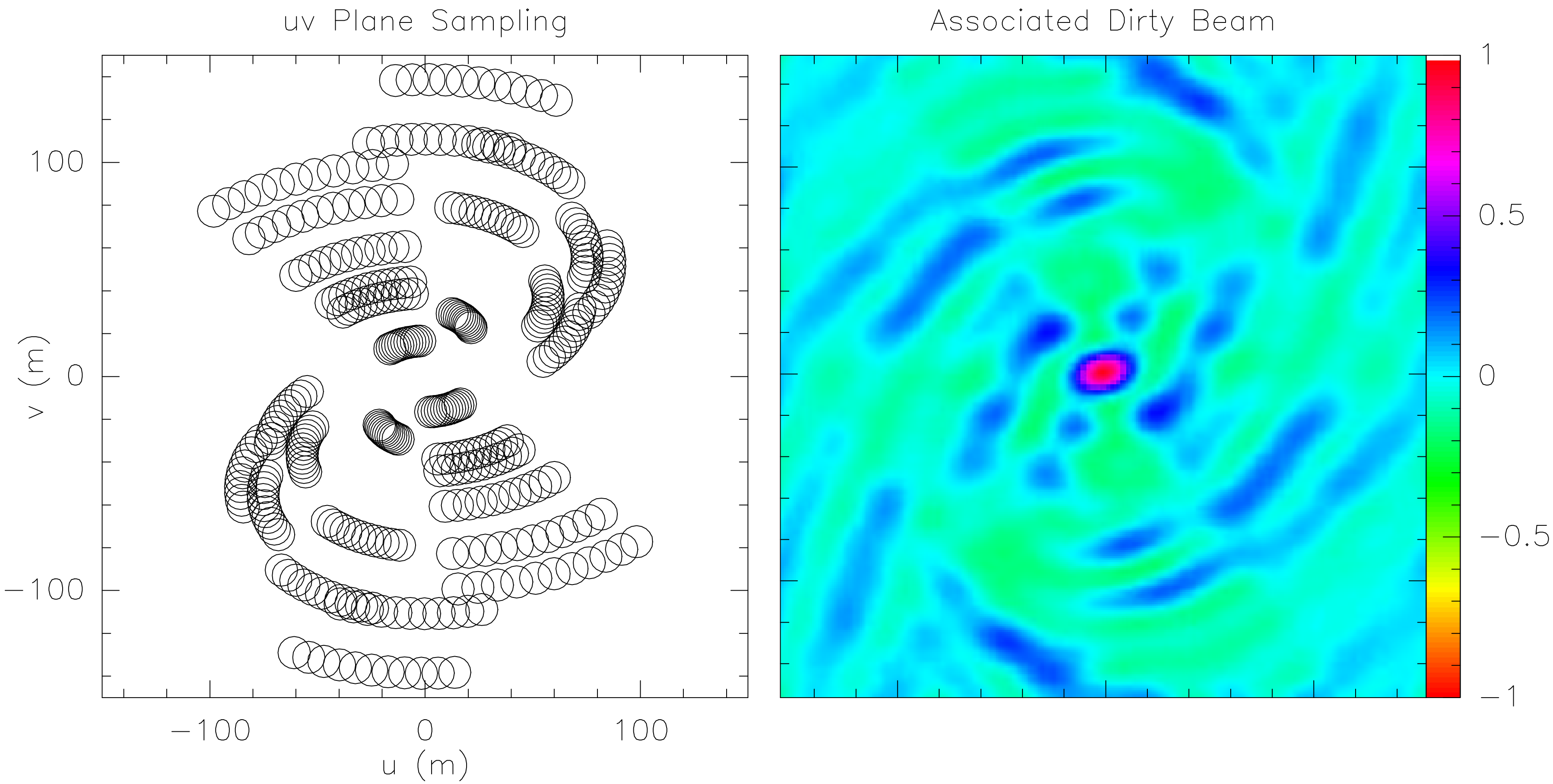

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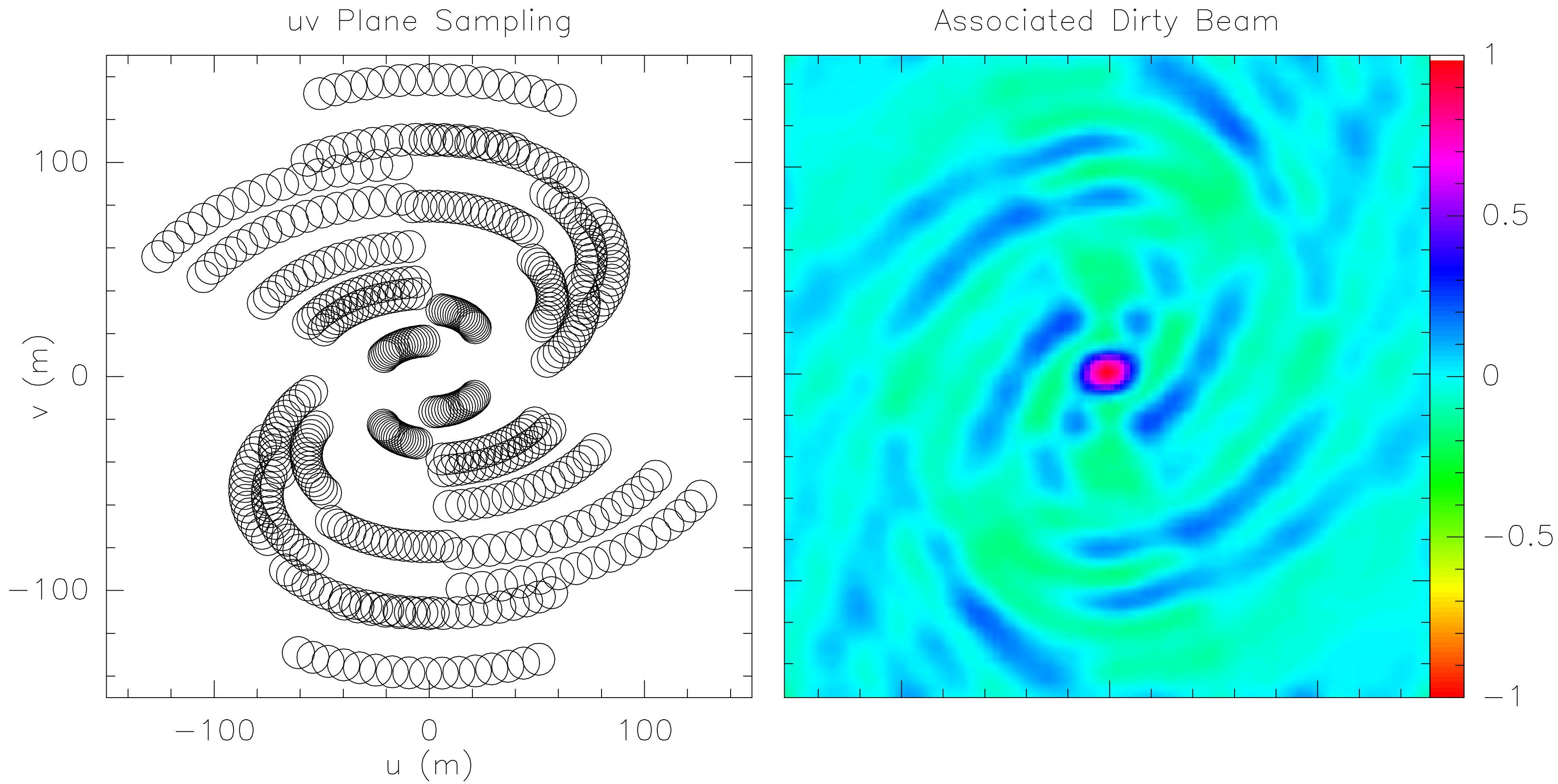

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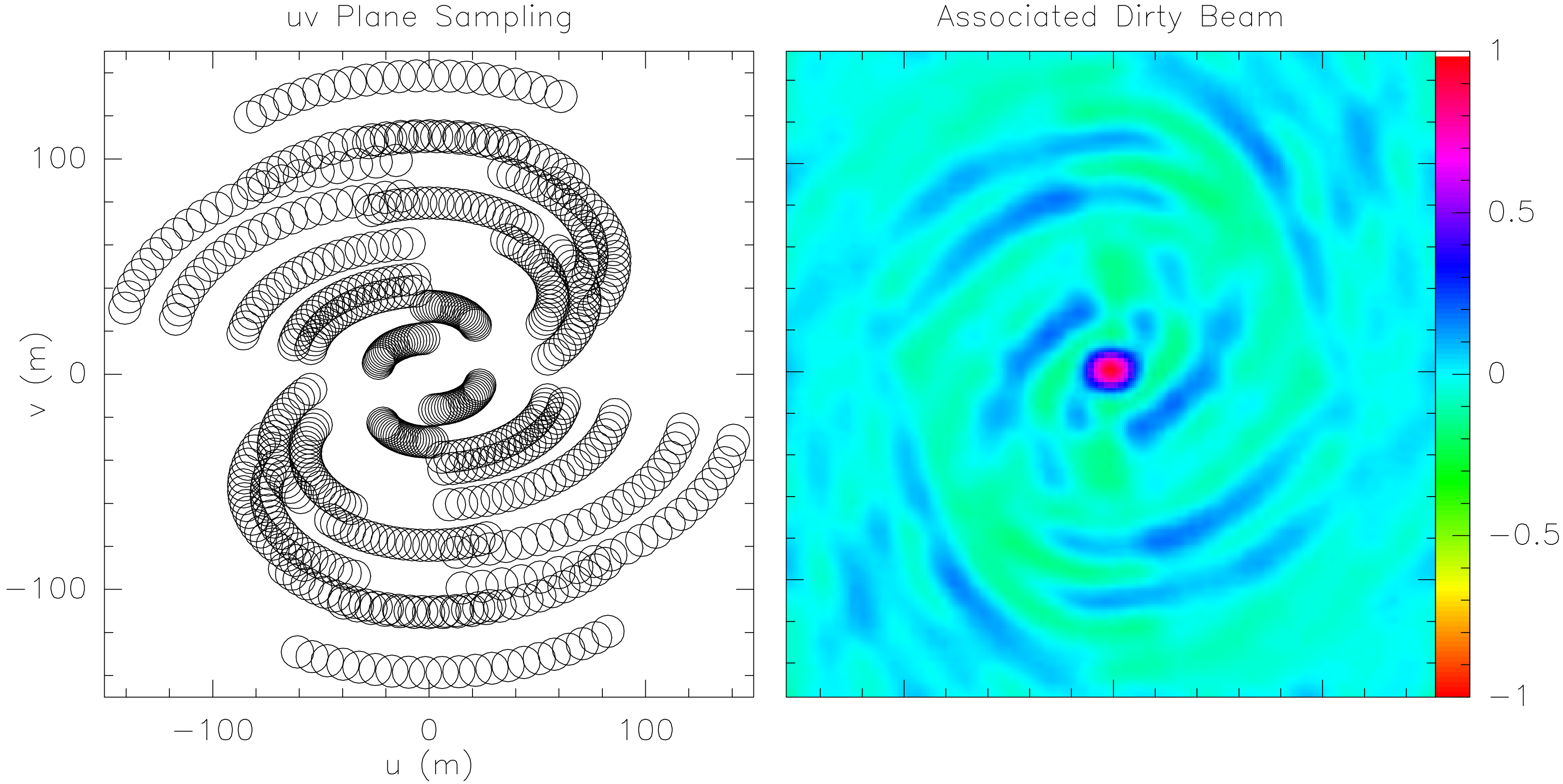

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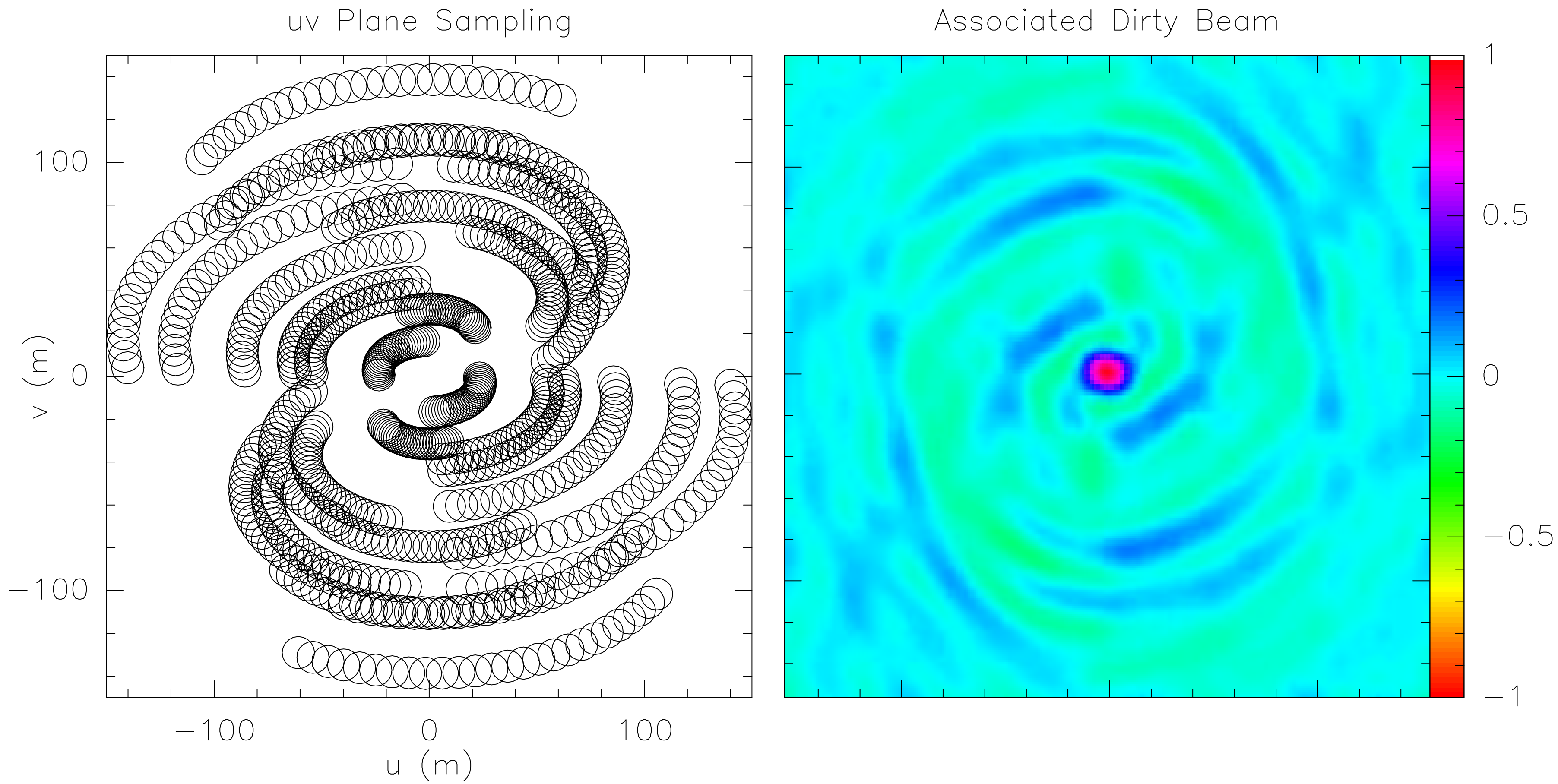

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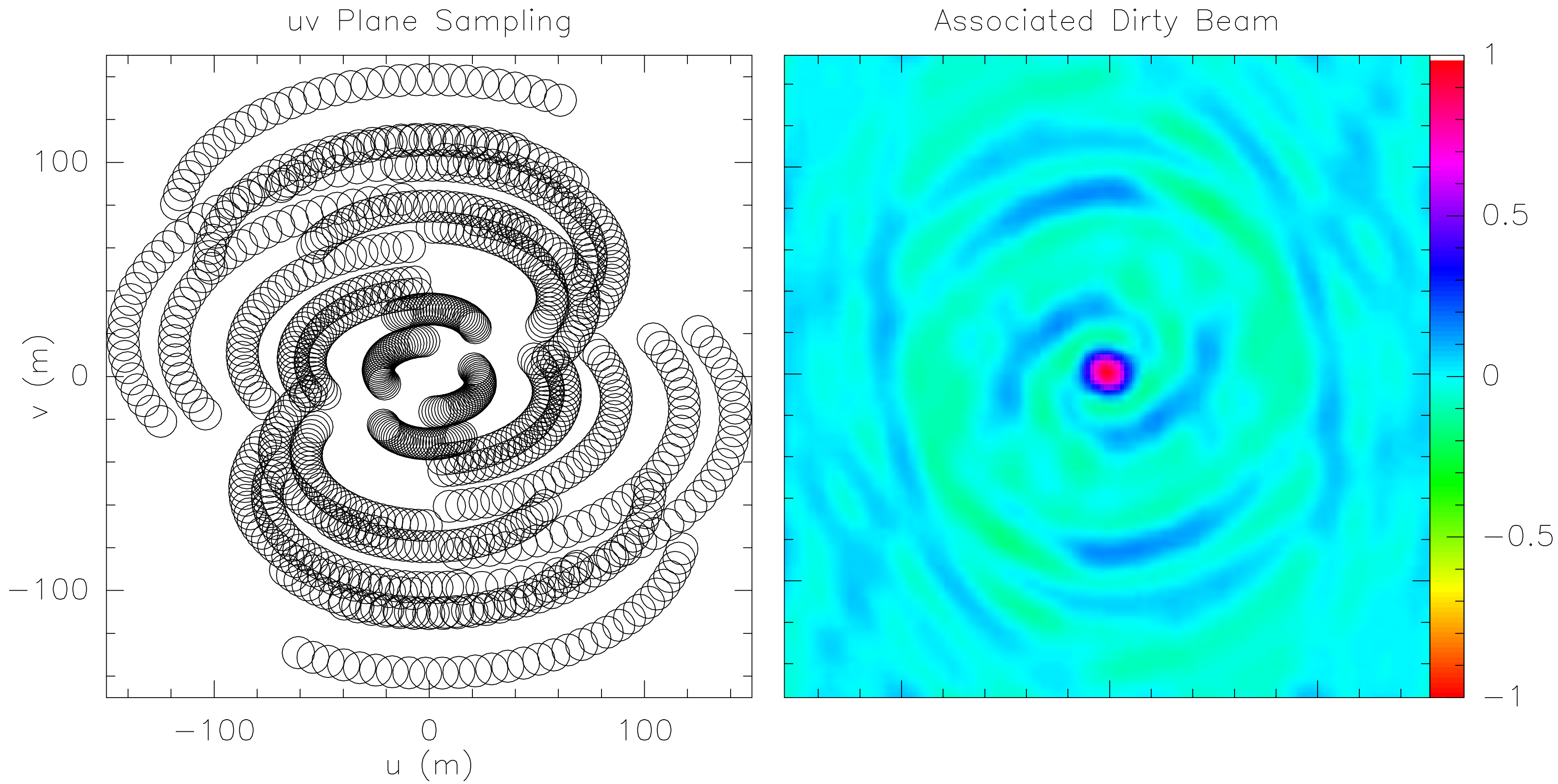

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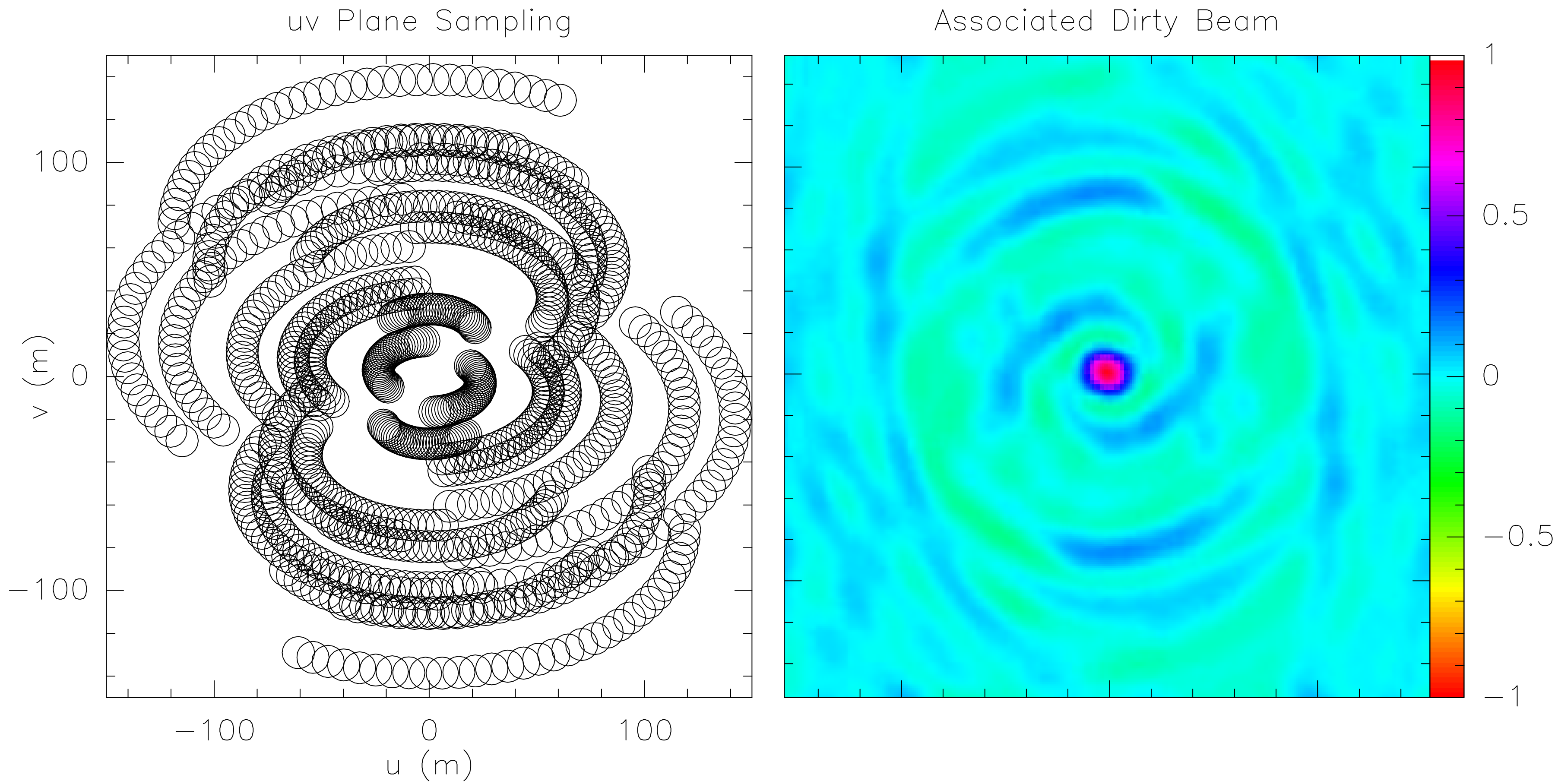


Dirty Beam Shape and Weighting

Natural Weighting: Default definition of the irregular sampling function at uv table creation.

- $S(u,v) = 1/\sigma^2$ at (u,v) points where visibilities are measured;
- $S(u,v) = 0$ elsewhere;

with $\sigma^2(u,v)$ the noise variance of the visibility.

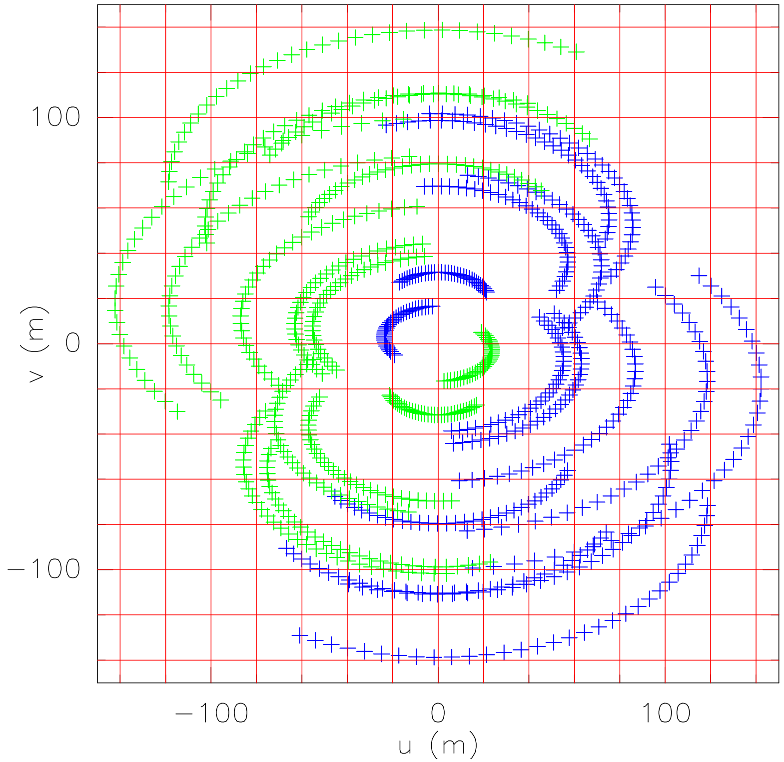
Introduction of a weighting function $W(u,v)$:

- $B_{\text{dirty}} = \text{2D FT}^{-1}\{W.S\}$;
- Robust weighting: W enhance the large baseline contribution;
- Tapering: W enhance the small baseline contribution.

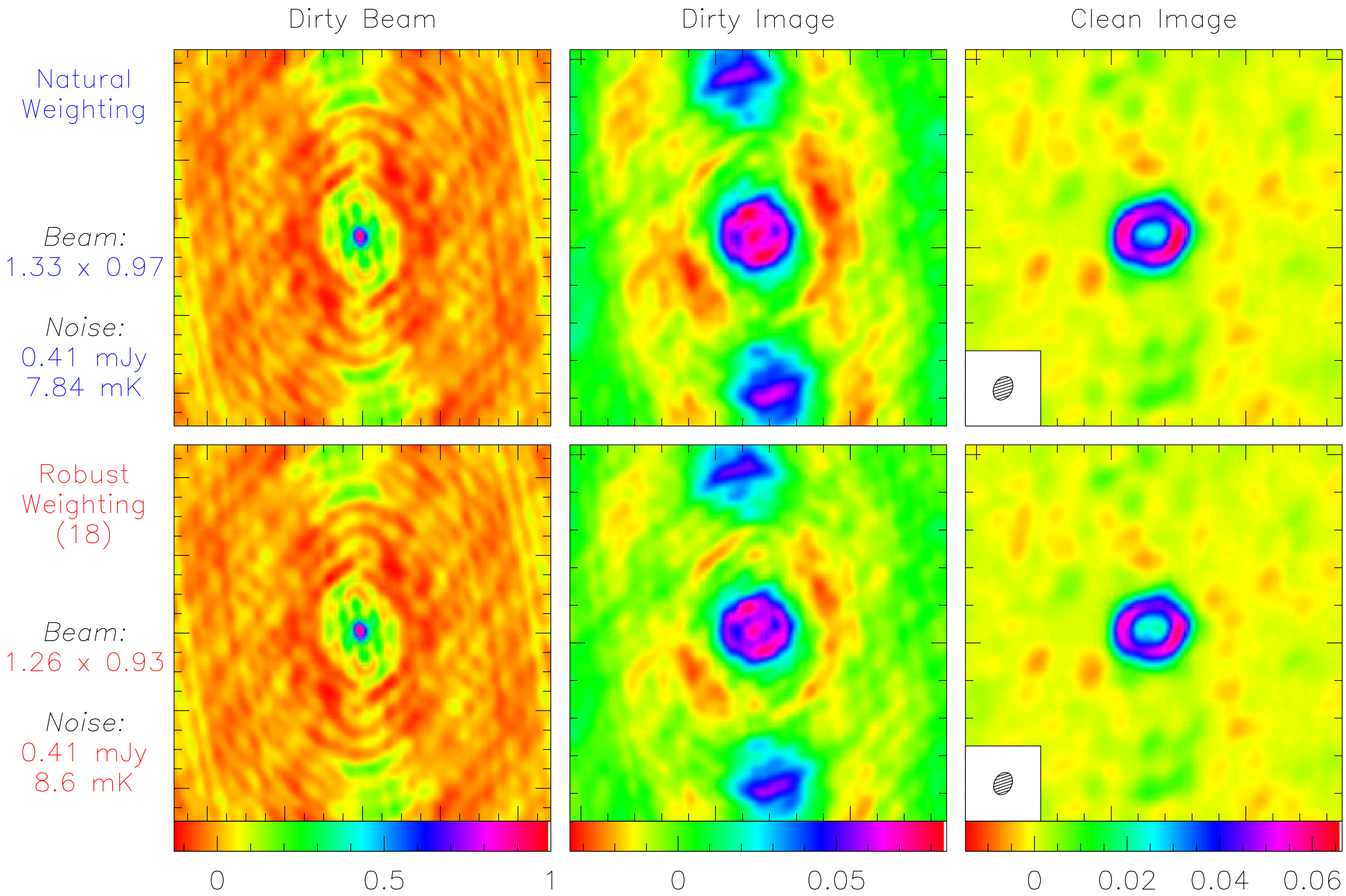
Robust Weighting: I. Definition

Definitions:

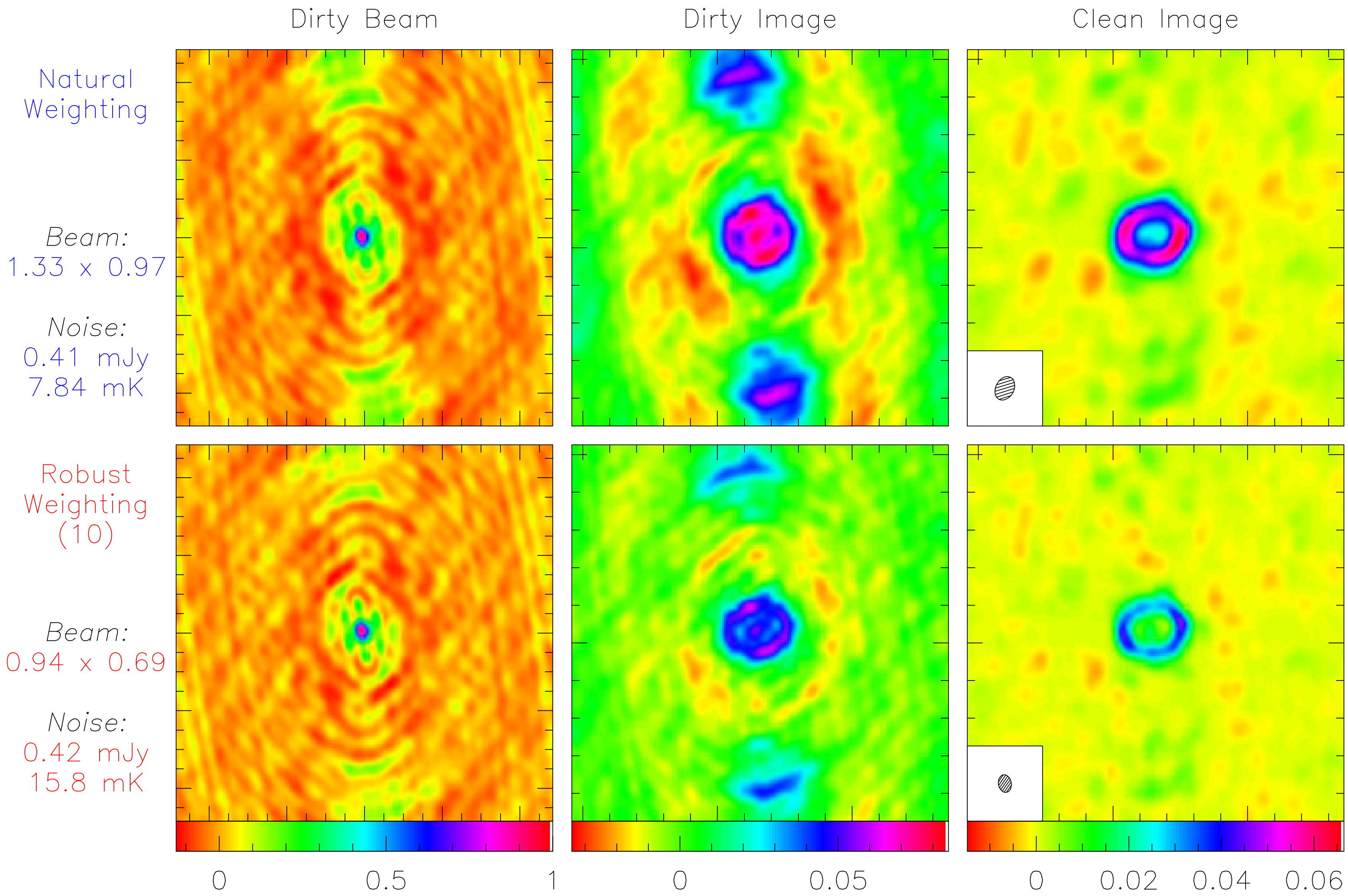
- $\text{Natural} = \sum_{(u,v)\in\text{Cell}} S;$
- $$\sum_{(u,v)\in\text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \geq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$$
- In practice, the cell size is $0.5D$ where D is the single-dish antenna diameter (*i.e.* 15m for PdBI).



Robust Weighting: II. Examples



Robust Weighting: II. Examples



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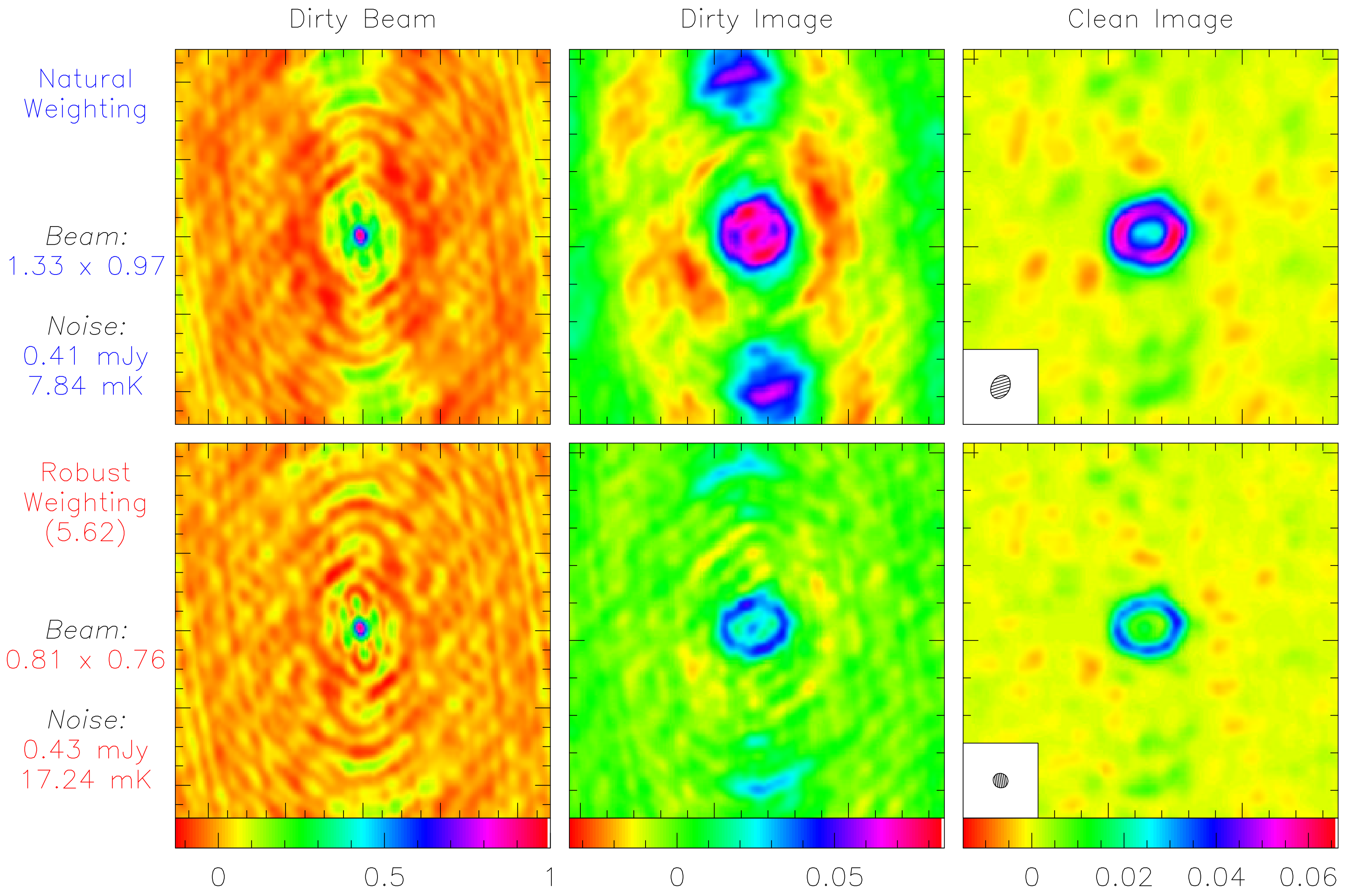

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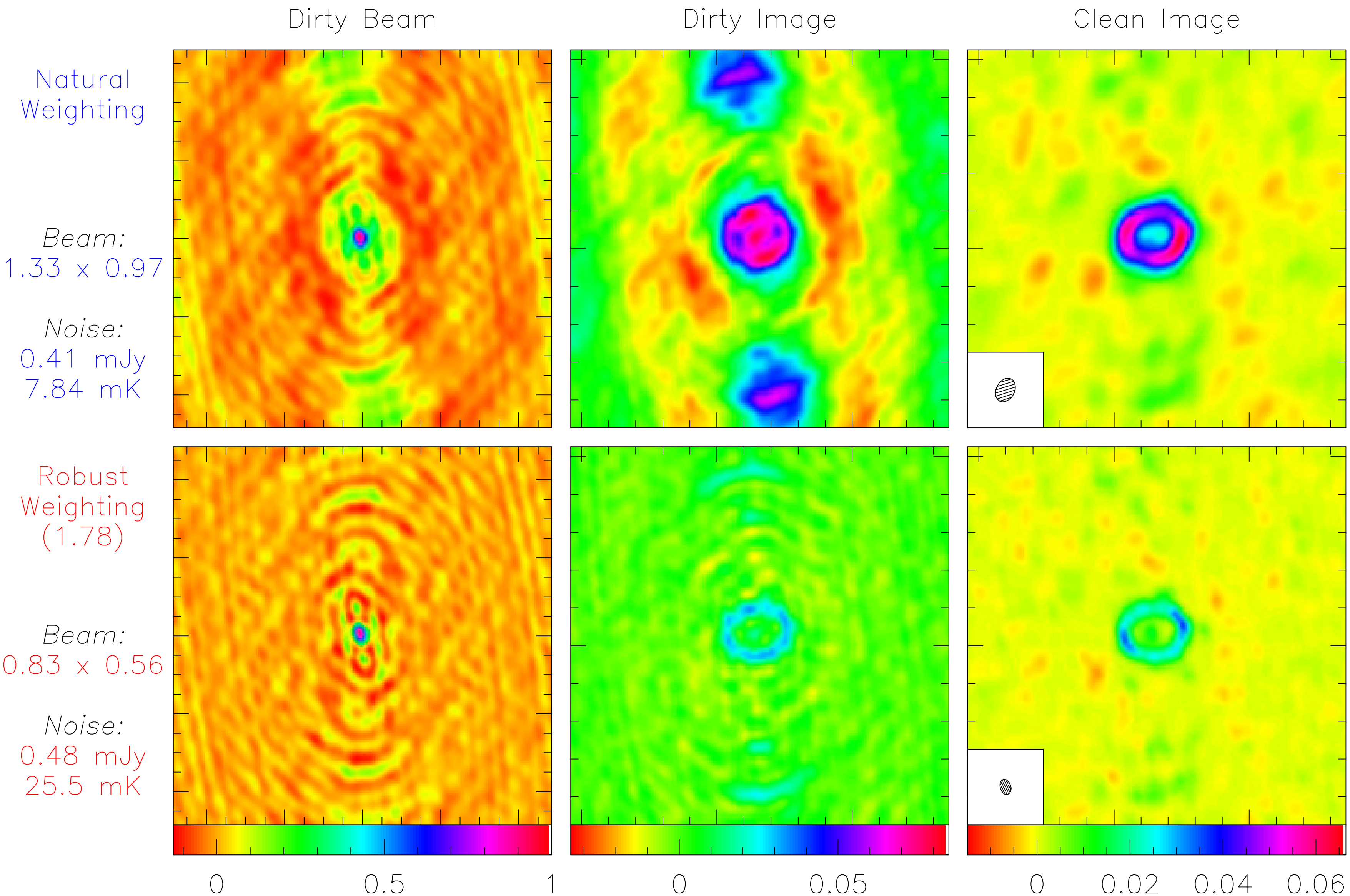


Robust Weighting: III. Definition and Properties

Definitions:

- Natural $= \sum_{(u,v)\in\text{Cell}} S$;
- $\sum_{(u,v)\in\text{Cell}} W.S = \begin{cases} \text{Constant} & \text{if (Natural} \leq \text{Threshold);} \\ \text{Natural} & \text{else;} \end{cases}$
- In practice, the cell size is $0.5D$.

Properties:

- Increase the resolution;
- Lower the sidelobes;
- Degrade point source sensitivity.

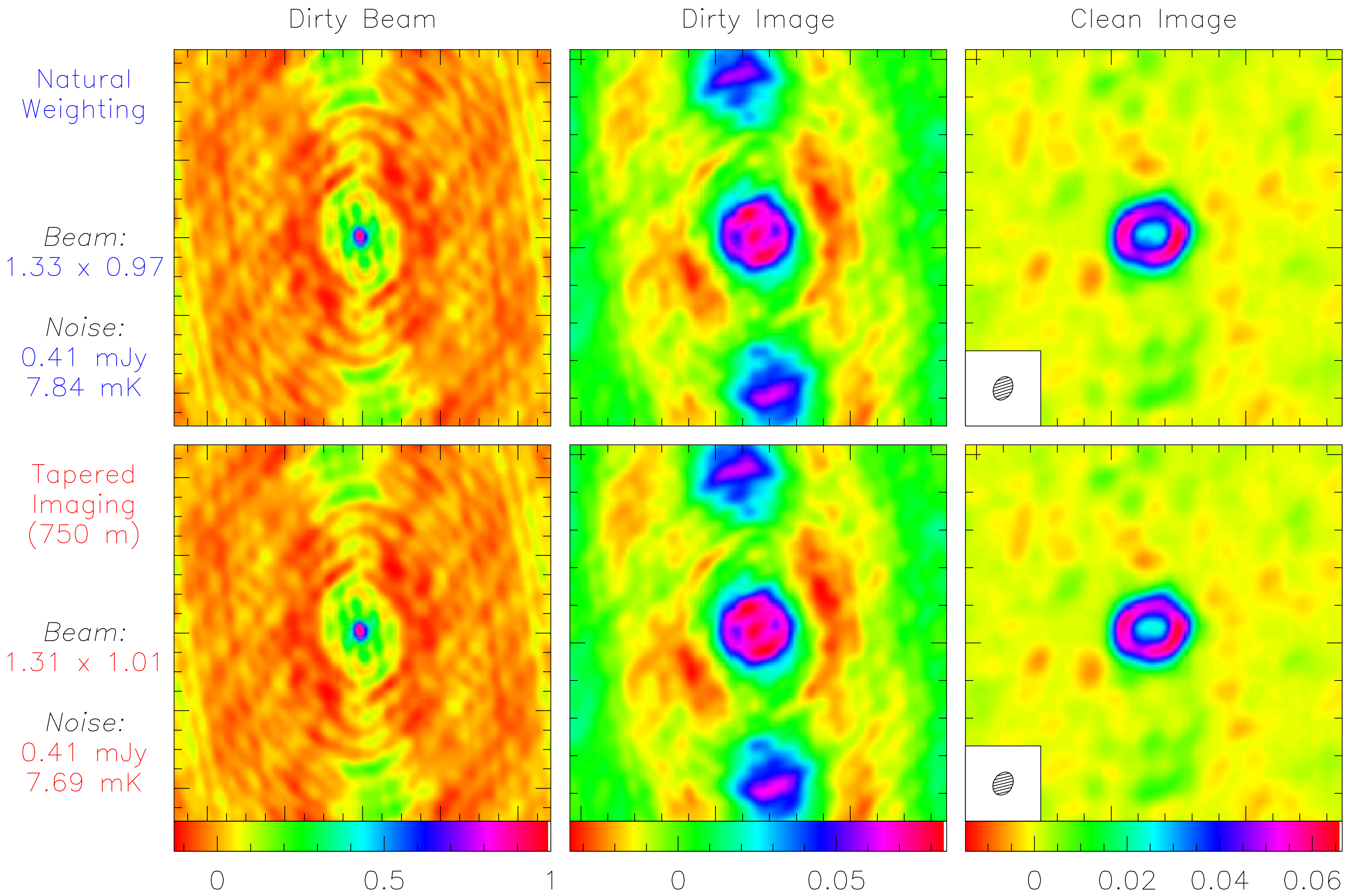
Tapering: I Definition

Definition:

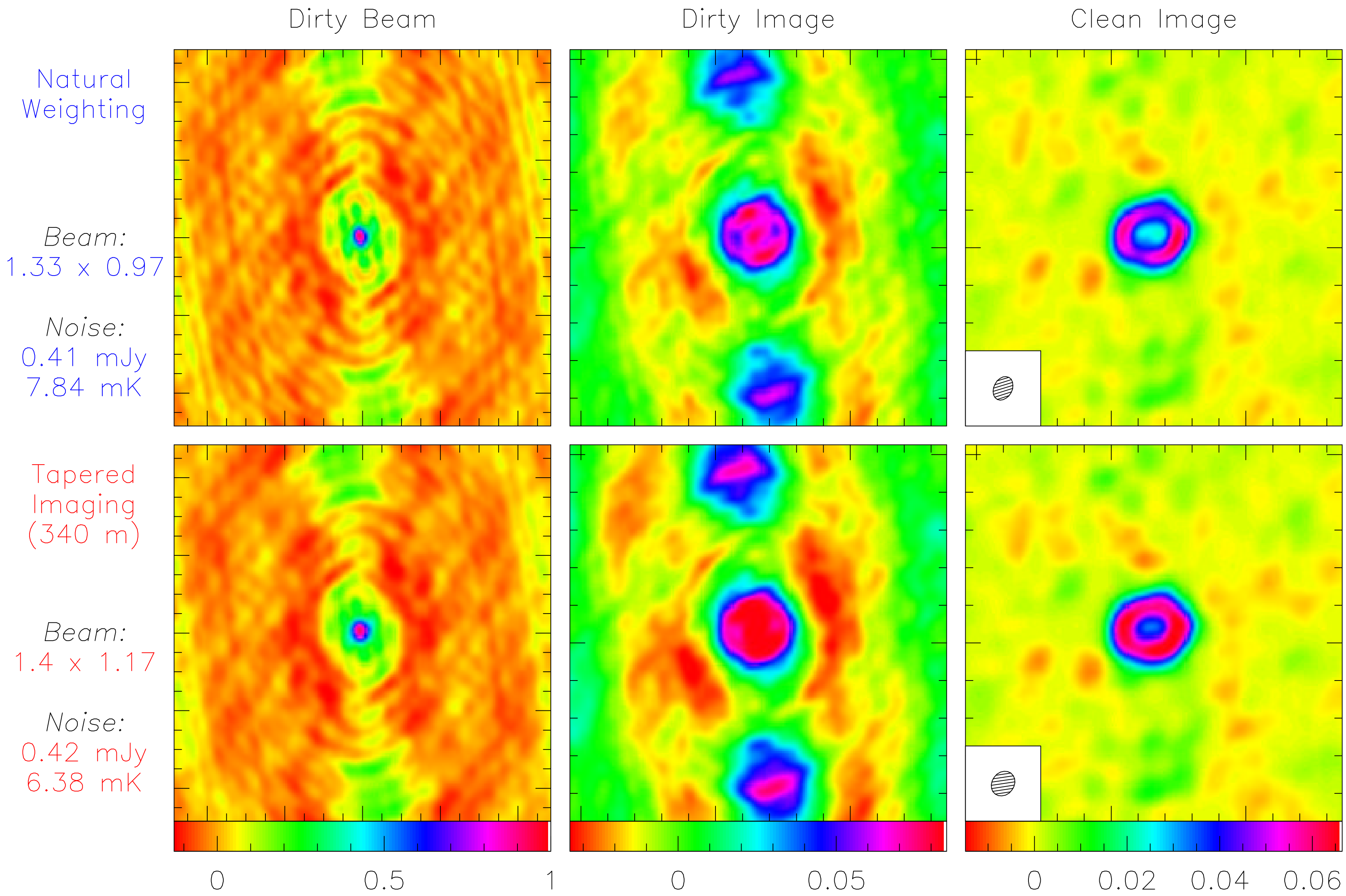
- Apodization of the uv coverage in general by a Gaussian;
- $W = \exp\left\{-\frac{\left(u^2+v^2\right)}{t^2}\right\}$ where t = tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

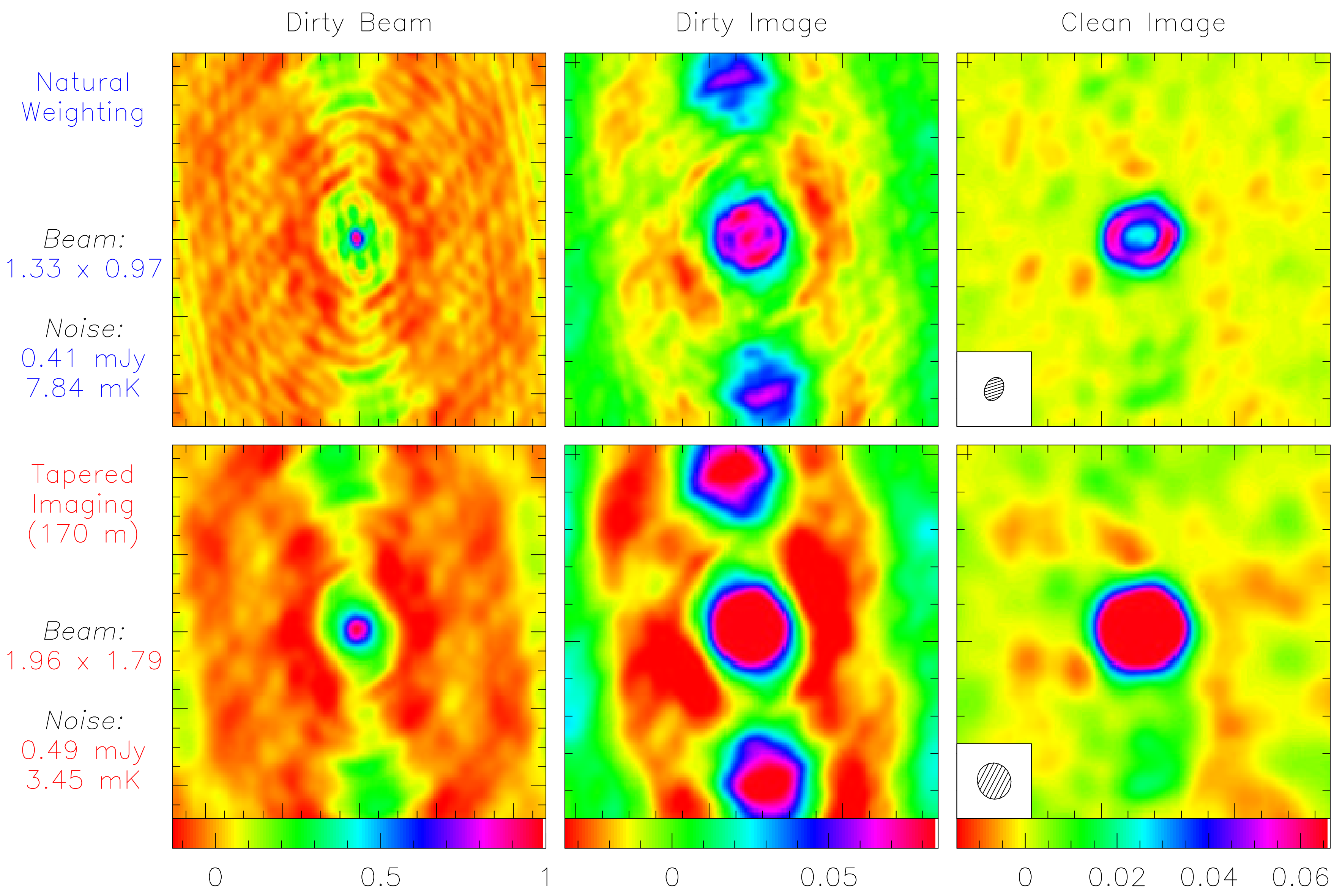
Tapering: II. Examples



Tapering: II. Examples



Tapering: II. Examples



Tapering: III. Definition and Properties

Definition:

- Apodization of the uv coverage in general by a Gaussian;
- $W = \exp\left\{-\frac{\left(u^2+v^2\right)}{t^2}\right\}$ where t = tapering distance.

$\Rightarrow$ Convolution (*i.e.* smoothing) of the image by a Gaussian.

Properties:

- Decrease the resolution;
- Degrade point source sensitivity;
- Increase sensitivity to "medium size" structures.

Inconvenient: Throw out some information.

$\Rightarrow$ To increase sensitivity to extended sources, use compact arrays not tapering.

Weighting and Tapering: Summary

	Robust	Natural	Tapering
Resolution	High	Medium	Low
Side Lobes	$\searrow$	Medium	?
Point Source Sensitivity	$\searrow$	Maximum	$\searrow$
Extended Source Sensitivity	$\searrow$	Medium	$\nearrow$

Non-circular tapering:

Sometimes $\Rightarrow$ Better (*i.e.* more circular) beams.

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The two key issues in imaging.

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Dirty beam & image	
$\Downarrow$ Deconvolution	`GO CLEAN`
Clean beam & image	
$\Downarrow$ Visualization	`GO BIT, GO VIEW`
$\Downarrow$ Image analysis	`GO NOISE, GO FLUX, GO MOMENTS`
Physical information on your source	

Deconvolution: I. Philosophy

$$I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}} . I_{\text{source}} \right\} + N.$$

Information lost:

- Irregular, incomplete sampling $\Rightarrow$ convolution by B_{dirty};
- Noise $\Rightarrow$ Low signal structures undetected.

$\Rightarrow$ 1. Impossible to recover the intrinsic source structure!

$\Rightarrow$ 2. Infinite number of solutions!

$$\left\{ \begin{array}{c} S \text{ solution } (\textit{i.e. } I_{\text{meas}} = B_{\text{dirty}} * S + N) \\ B_{\text{dirty}} * R = 0 \end{array} \right\} \Rightarrow (S+R) \text{ solution.}$$

Deconvolution: I. Philosophy (continued)

$$I_{\text{meas}} = B_{\text{dirty}} * \left\{ B_{\text{primary}} . I_{\text{source}} \right\} + N.$$

Information lost:

$\Rightarrow$ 1. Impossible to recover the intrinsic source structure!

$\Rightarrow$ 2. Infinite number of solutions!

Deconvolution goal: Finding a sensible intensity distribution compatible with the intrinsic source one.

Deconvolution needs:

- Some *a priori* assumptions about the source intensity distribution;
- As much as possible knowledge of
  - B_{dirty} (OK in radioastronomy);
  - Noise properties.

The best solution: A Gaussian $B_{\text{dirty}} \Rightarrow$ No deconvolution needed!

Deconvolution: II. MEM principle

a priori assumptions: Smoothed and positive intensity.

Idea:

"Select from the images that agree with the measured visibilities to within the noise level the one that maximizes entropy."

Algorithm:

- Entropy:
  $\mathcal{S} = -\sum_{ij} I_{ij}\ \log(I_{ij}/M_{ij})$ with $M =$ first guess image.
- Constraint:
  $\sum_k \frac{|V(u_k,v_k)-\tilde{I}(u_k,v_k)|^2}{\sigma_k^2} =$ number of visibilities
  with $\tilde{I} =$ 2D FT(I).

Deconvolution: II. MEM properties

Advantages:

- Fast:
  Computational load $\propto N \ln(N)$ with $N =$ number of pixels.
- Easy to generalize (Arrays with different antenna diameters).
- Flatten low-level extended emission.
- Resolve peaks.

Inconvenients:

- Angular resolution increases with peak height.
- Unable to clean ripples (*e.g.* point source sidelobes) in extended emission.
- Biased residuals:
  $\Rightarrow$ Noise increase and spurious emission at low signal.
- Impossibility to deal with absorption features.
- Poor performance with limited uv coverage
  $\Rightarrow$ Not used at PdBI.

Deconvolution: III. The Basic CLEAN Algorithm

a priori assumption: Source = Collection of point sources.

Idea: “Matching pursuit”.

Algorithm:

1 Initialize

- the residual map to the dirty map;
- the Clean component list to an empty (NULL) value;

2 Identify pixel of $|I_{\max}|$ in residual map as a point source;

3 Add $\gamma.I_{\max}$ to clean component list;

4 Subtract $\gamma.I_{\max}$ from residual map;

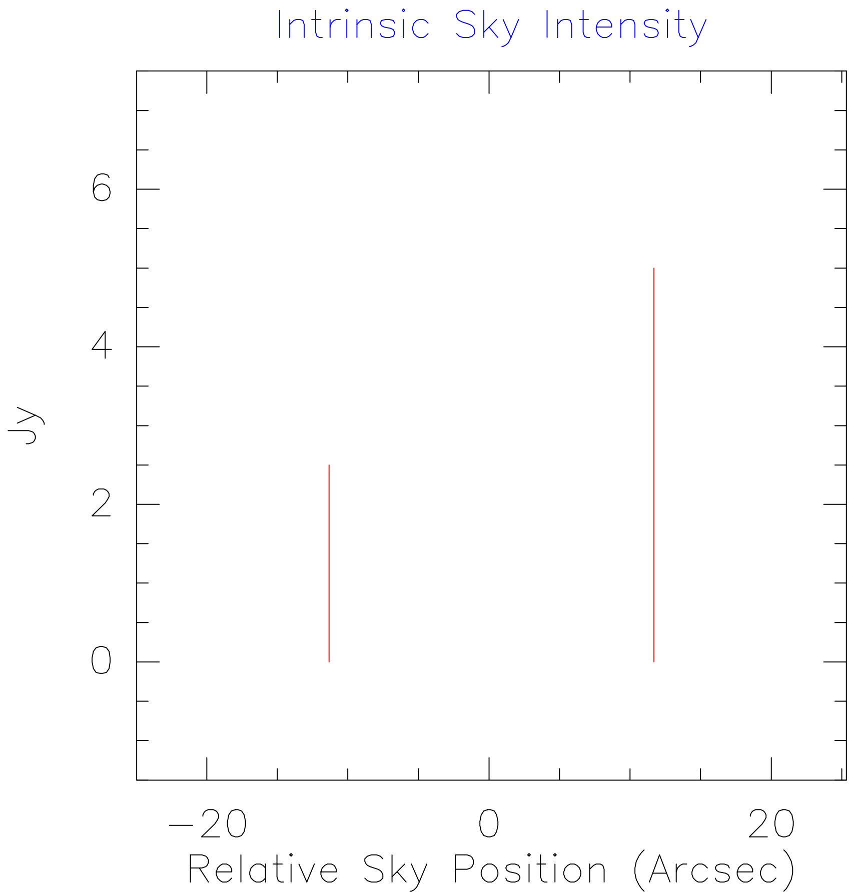
5 Go back to point 2 while stopping criterion is not matched;

6 Convolution by Clean beam (*a posteriori* regularization);

5 Addition of residual map to enable:

- Correction when cleaning is too superficial;
- Noise estimation.

Deconvolution: III. The Basic Clean Algorithm
1. First Illustration



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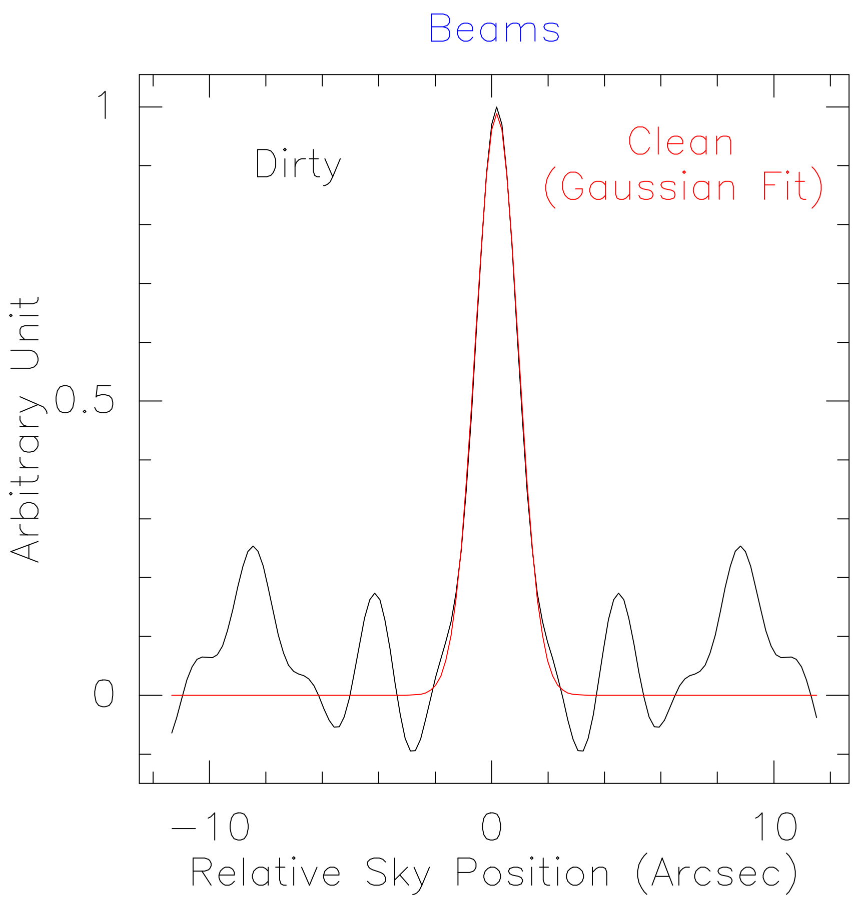

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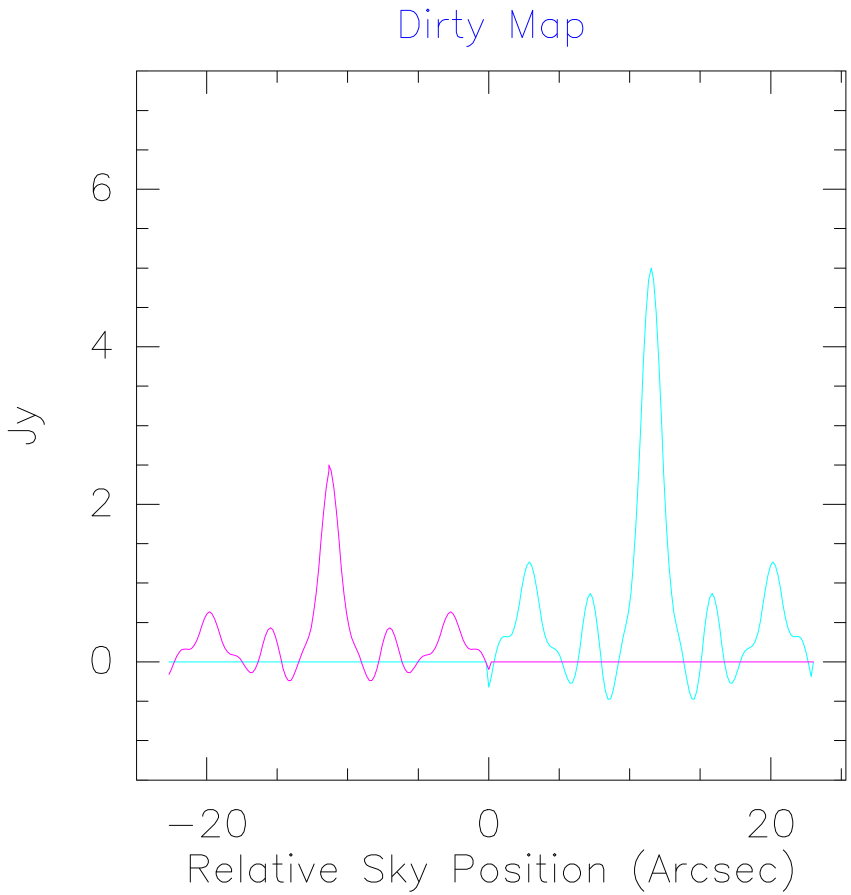

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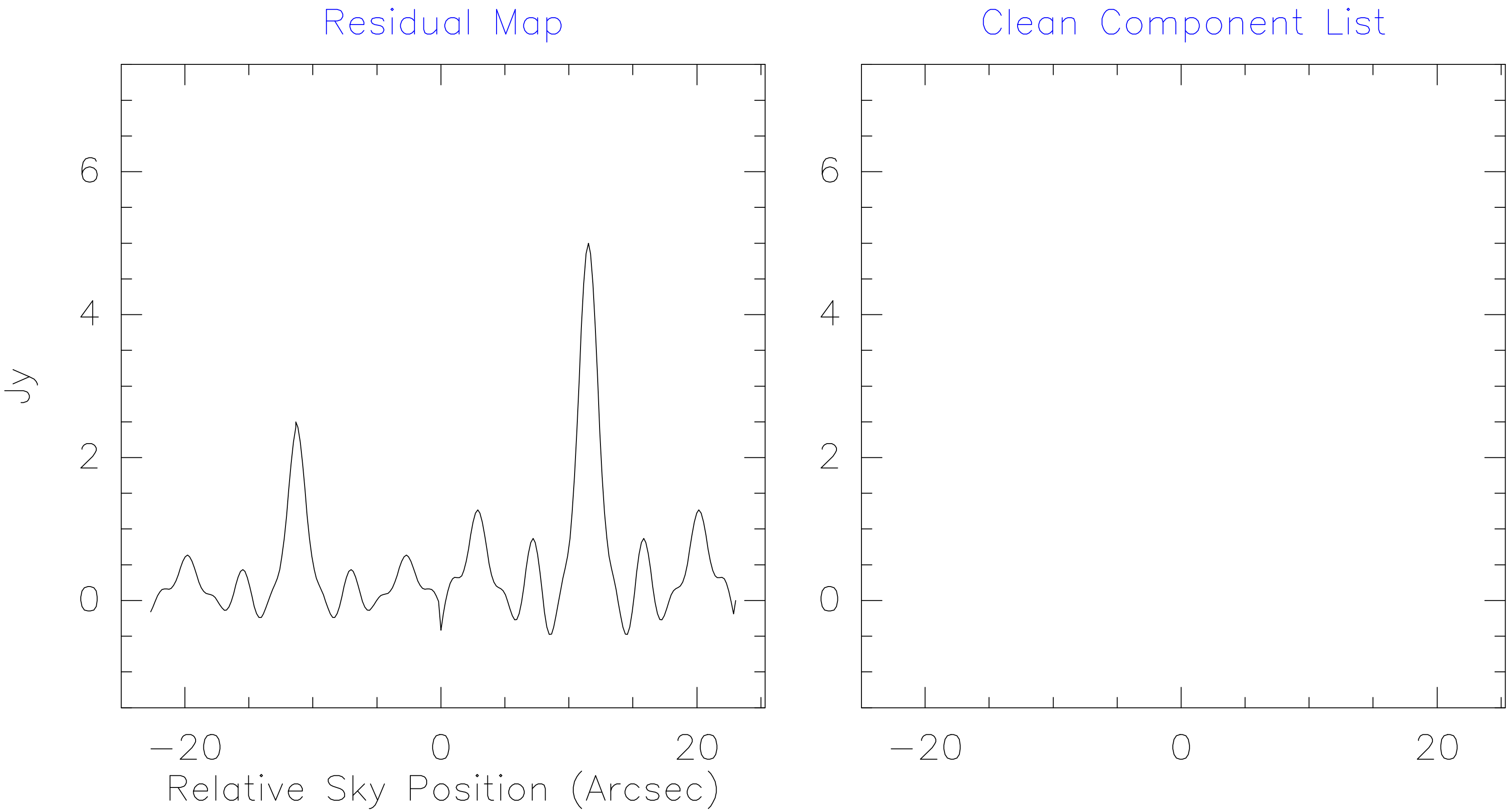

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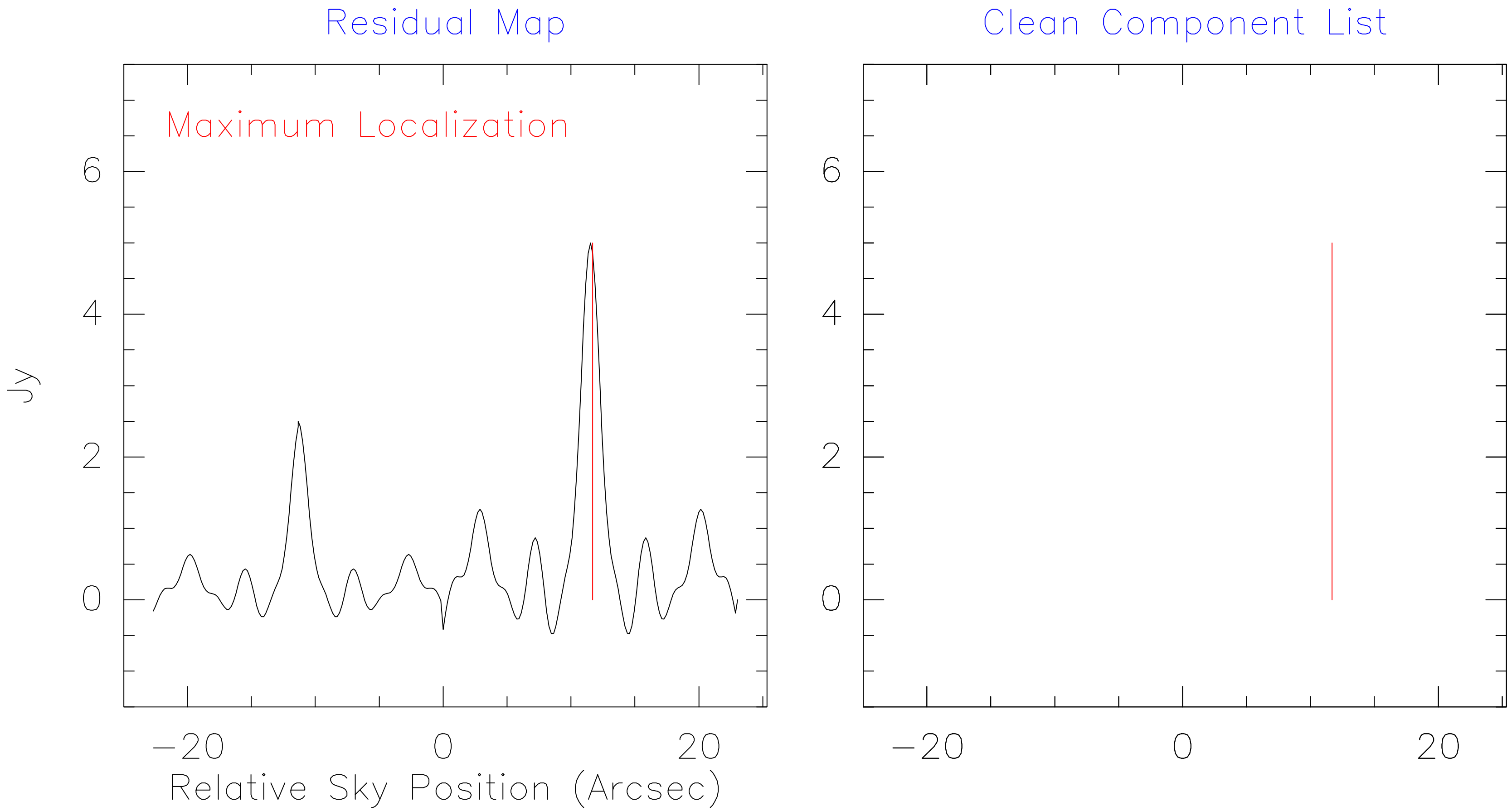

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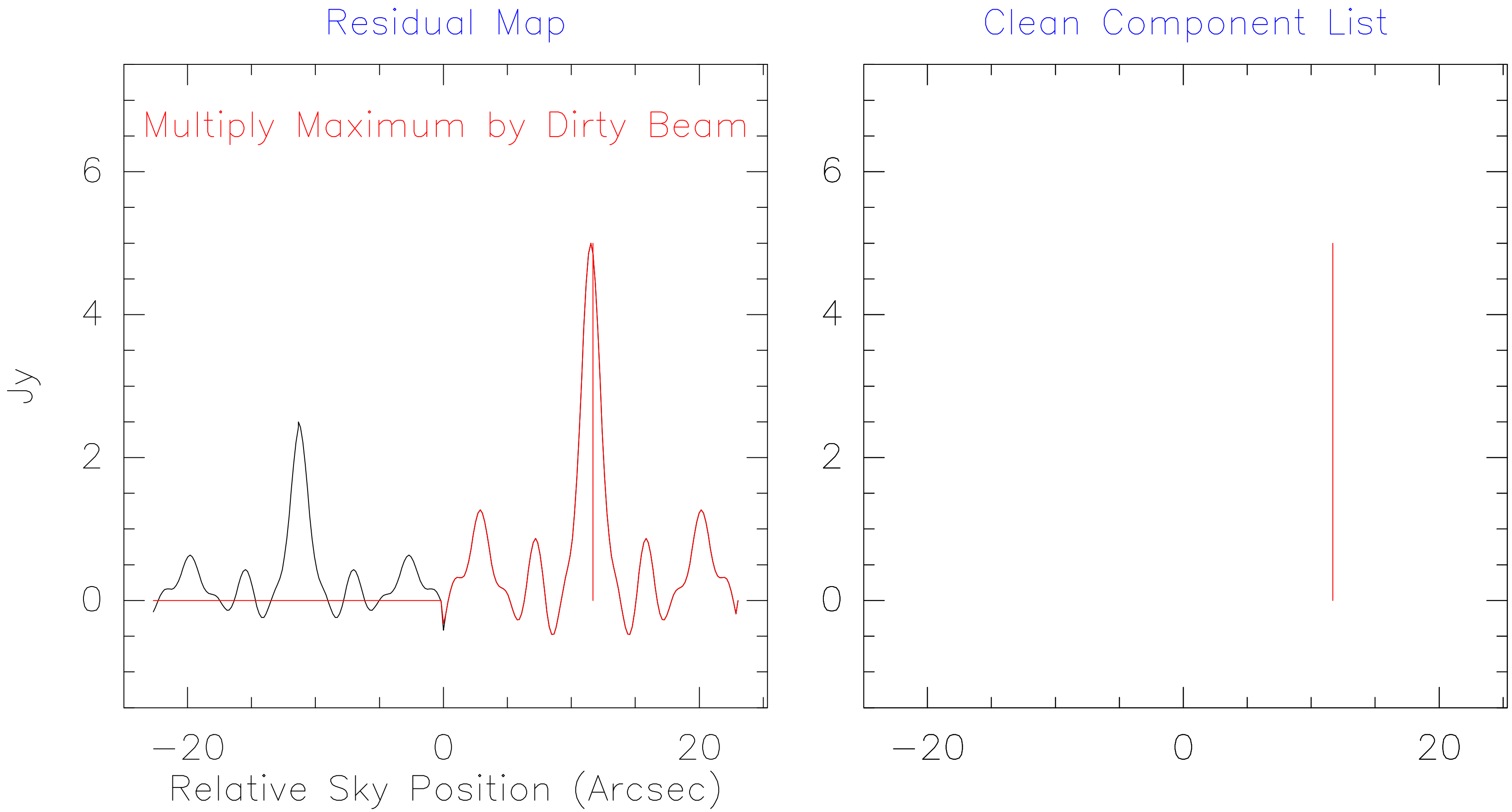

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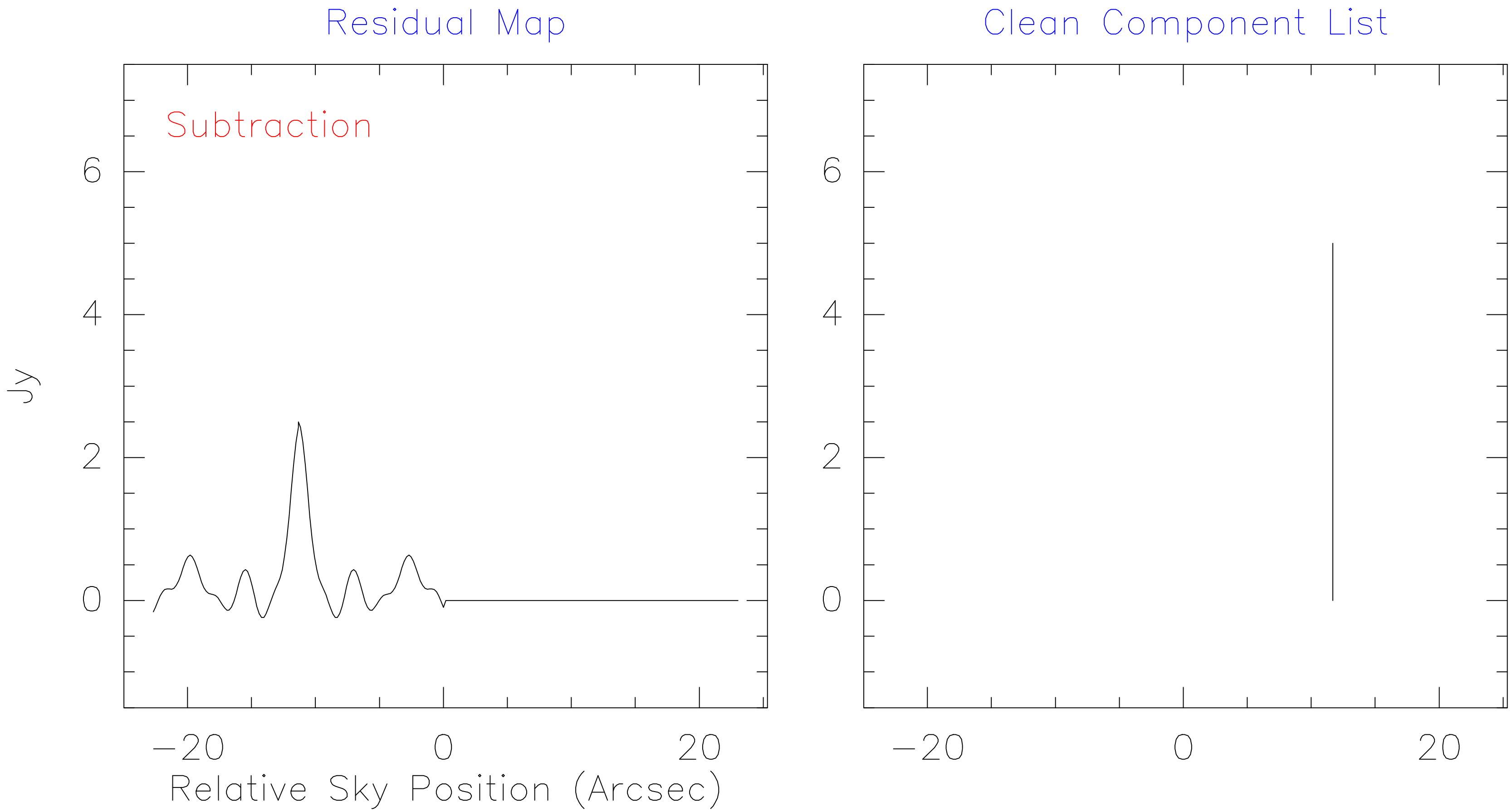
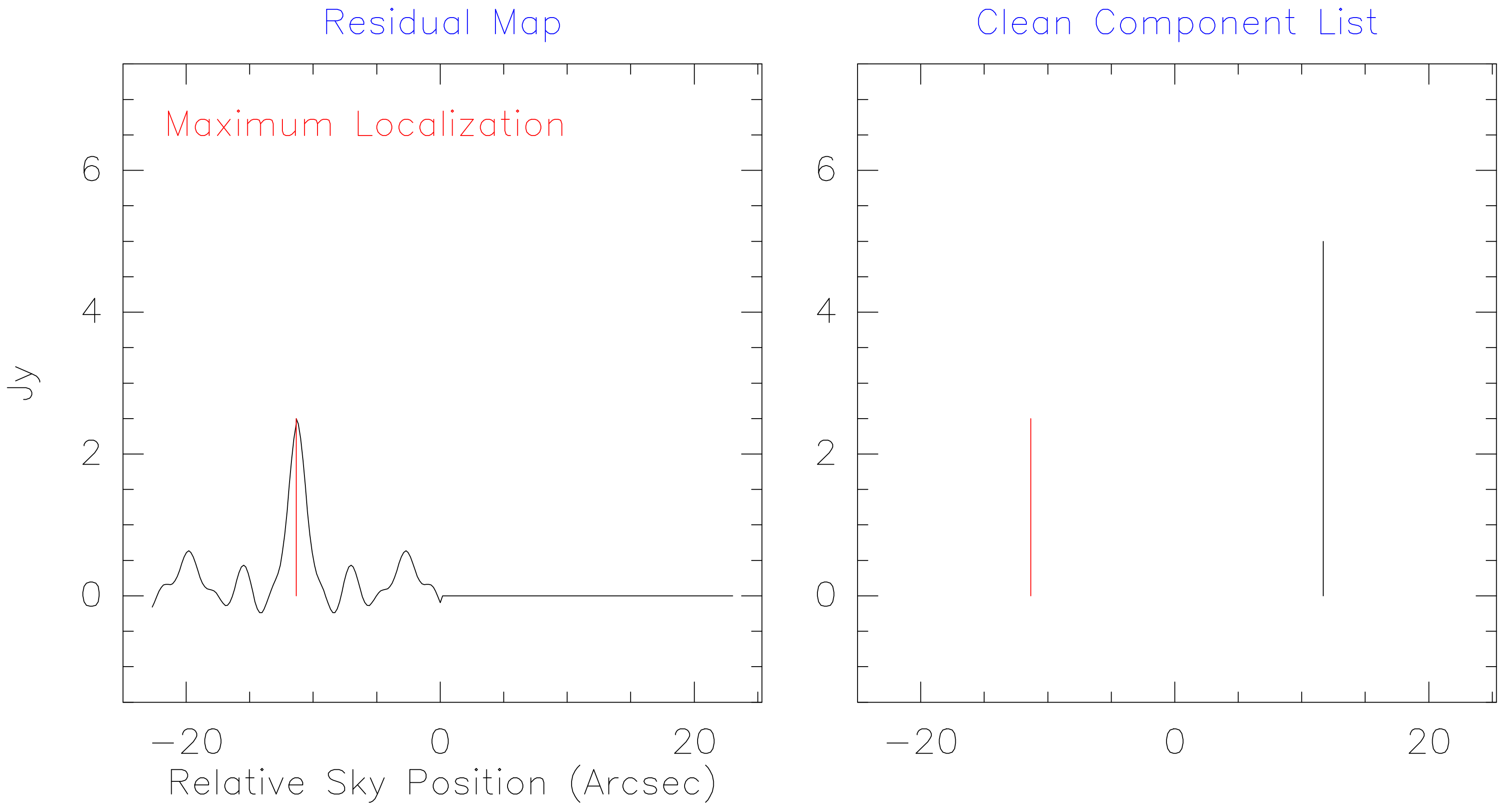


Deconvolution: III. The Basic Clean Algorithm
1. First Illustration



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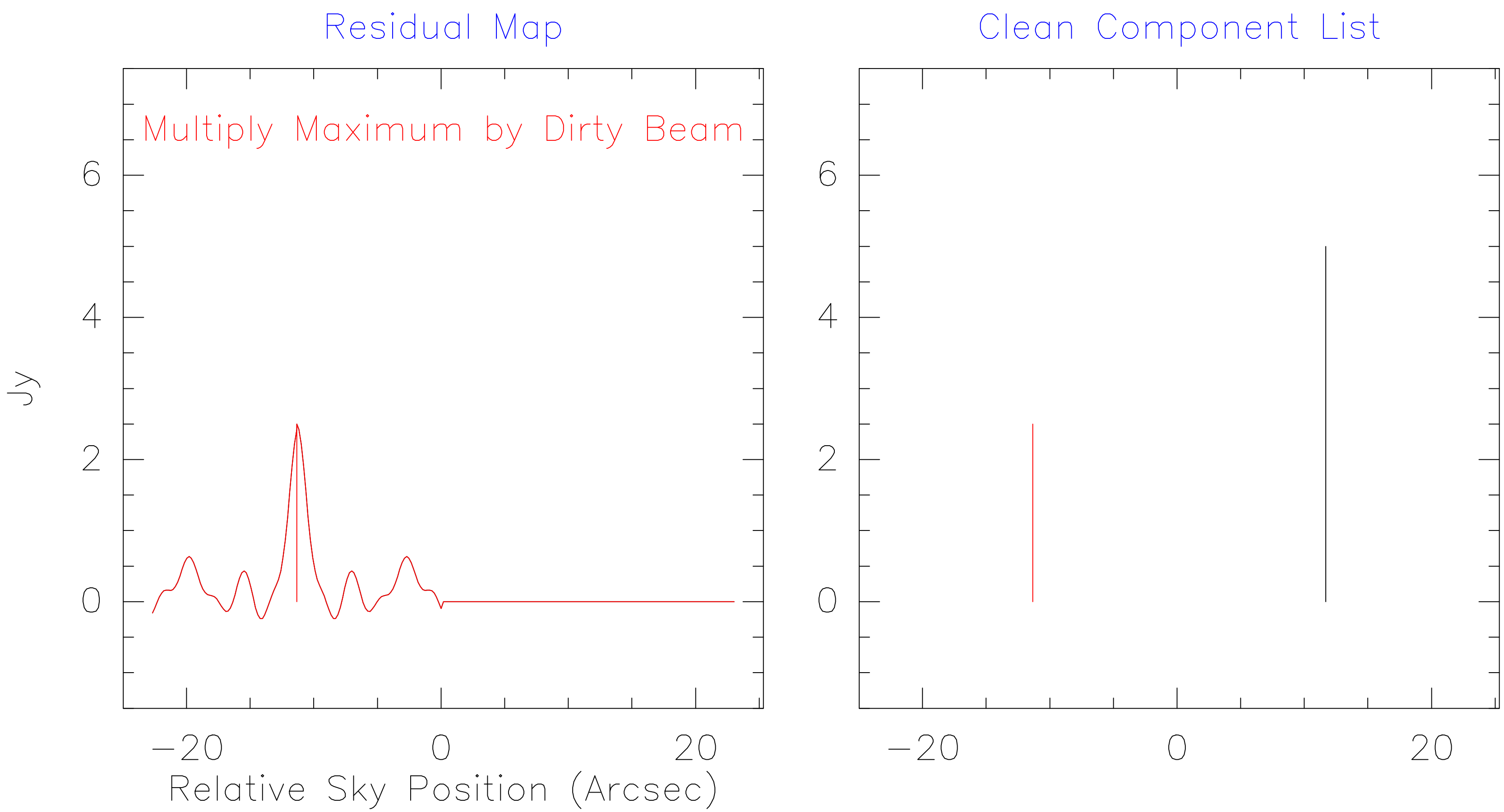

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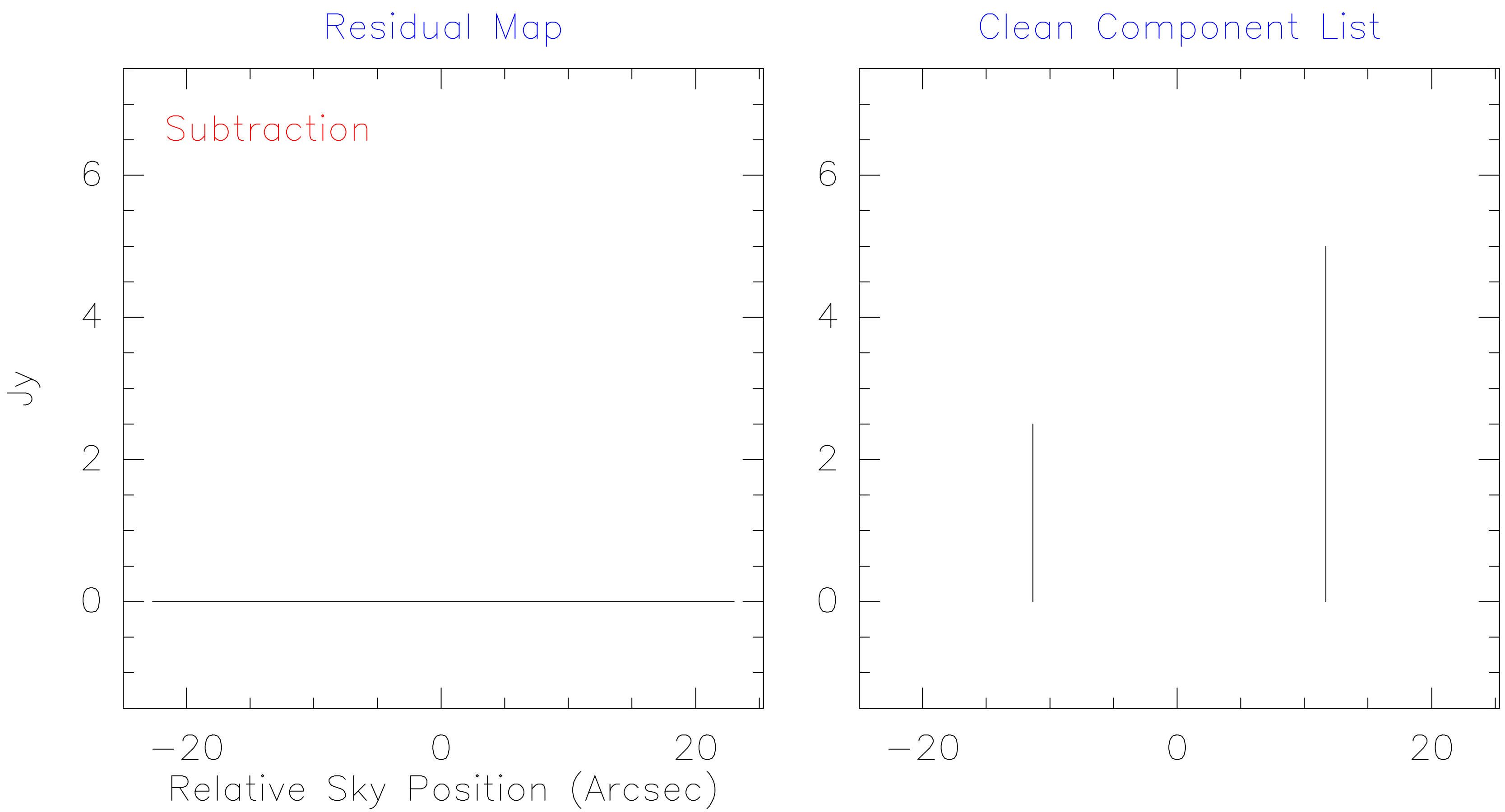

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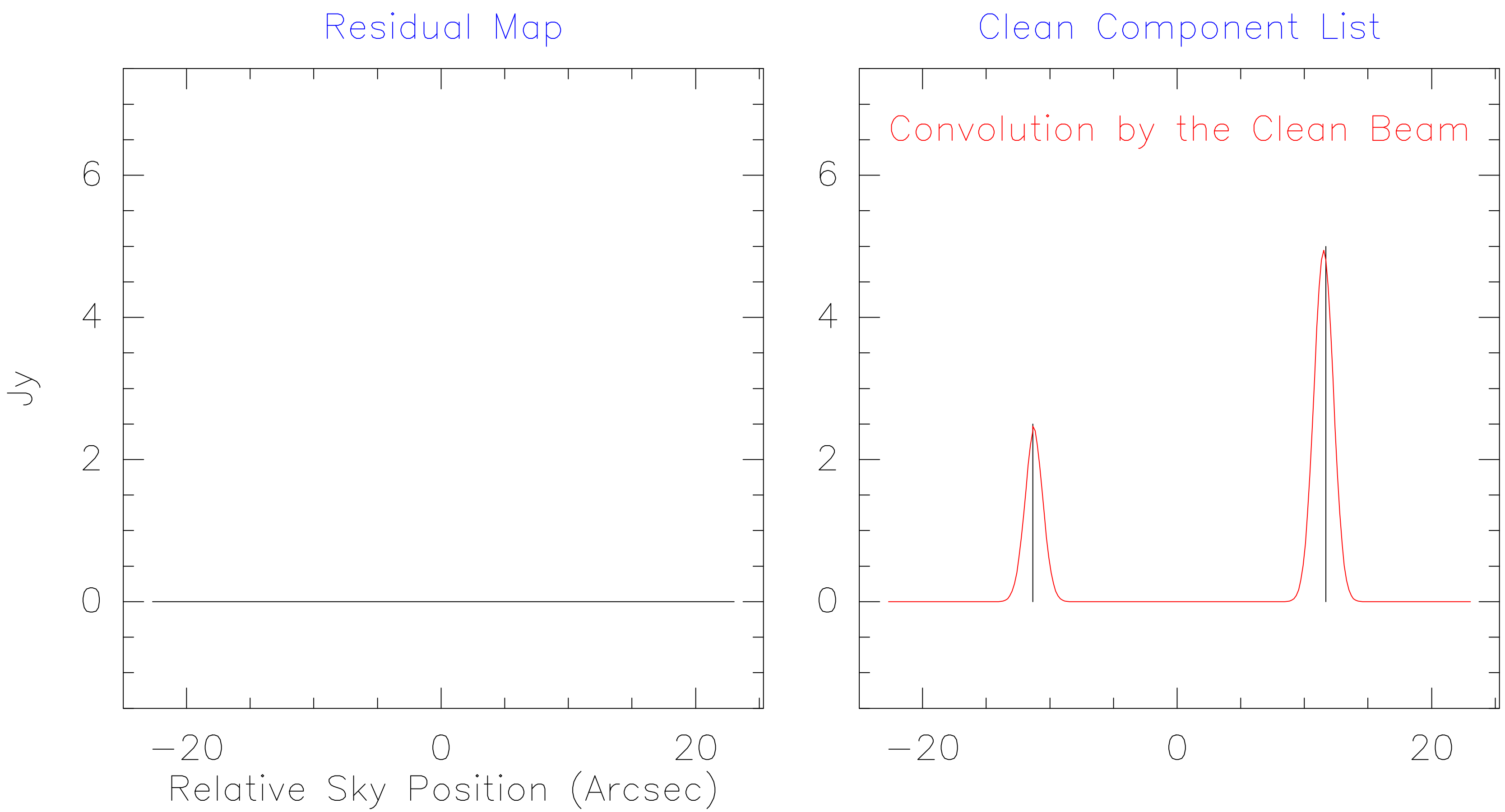
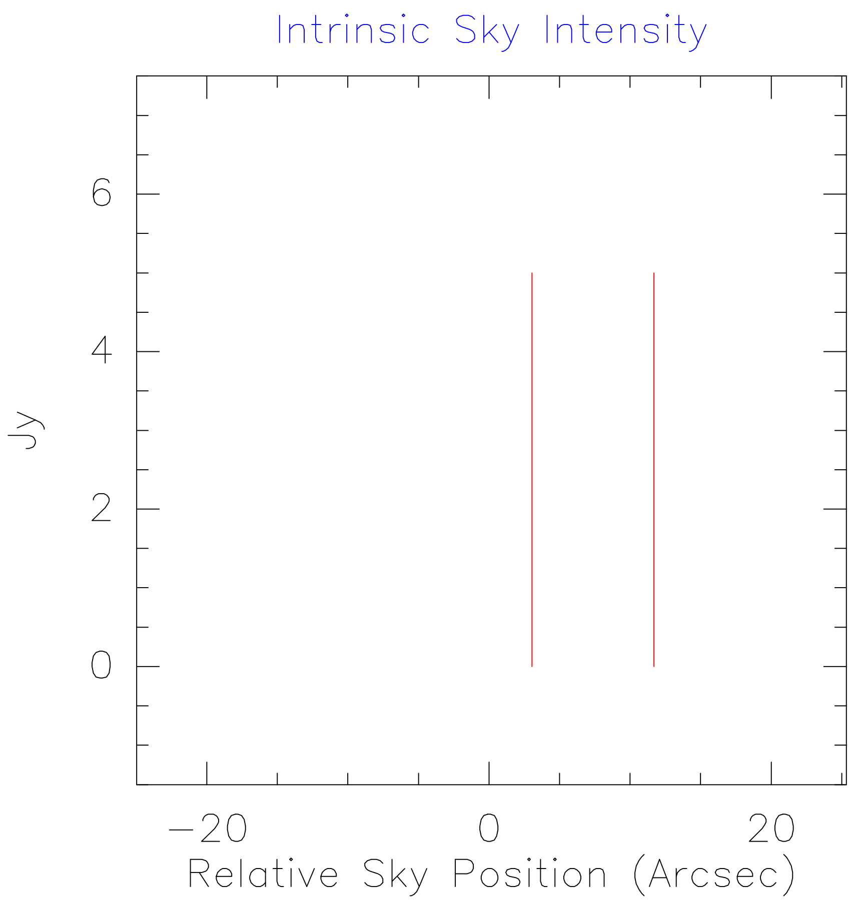


Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration



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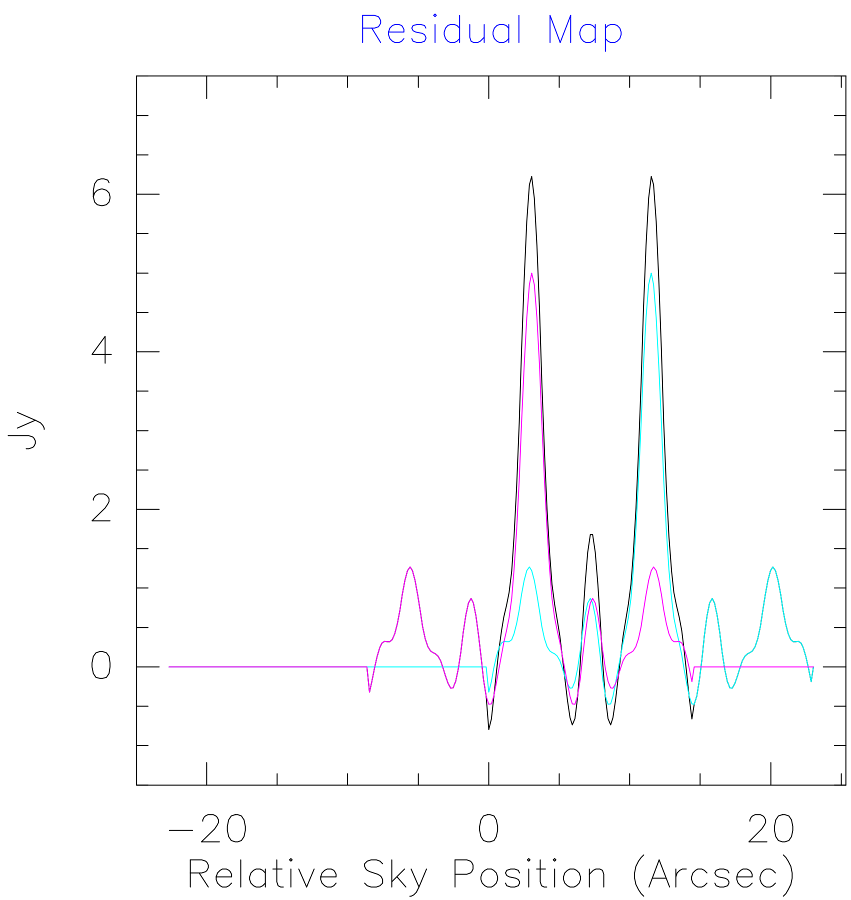

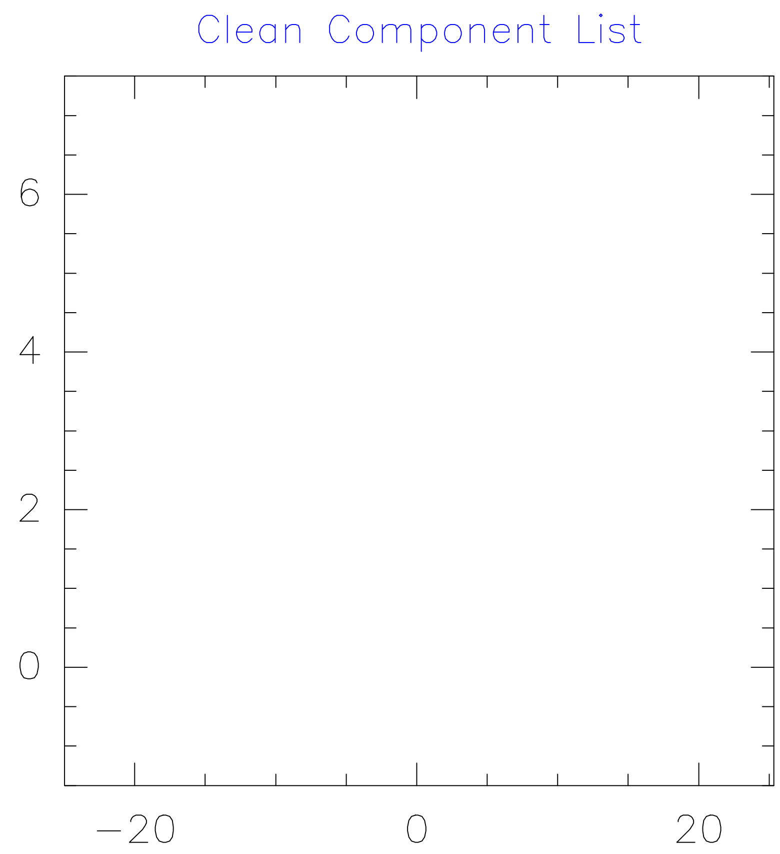

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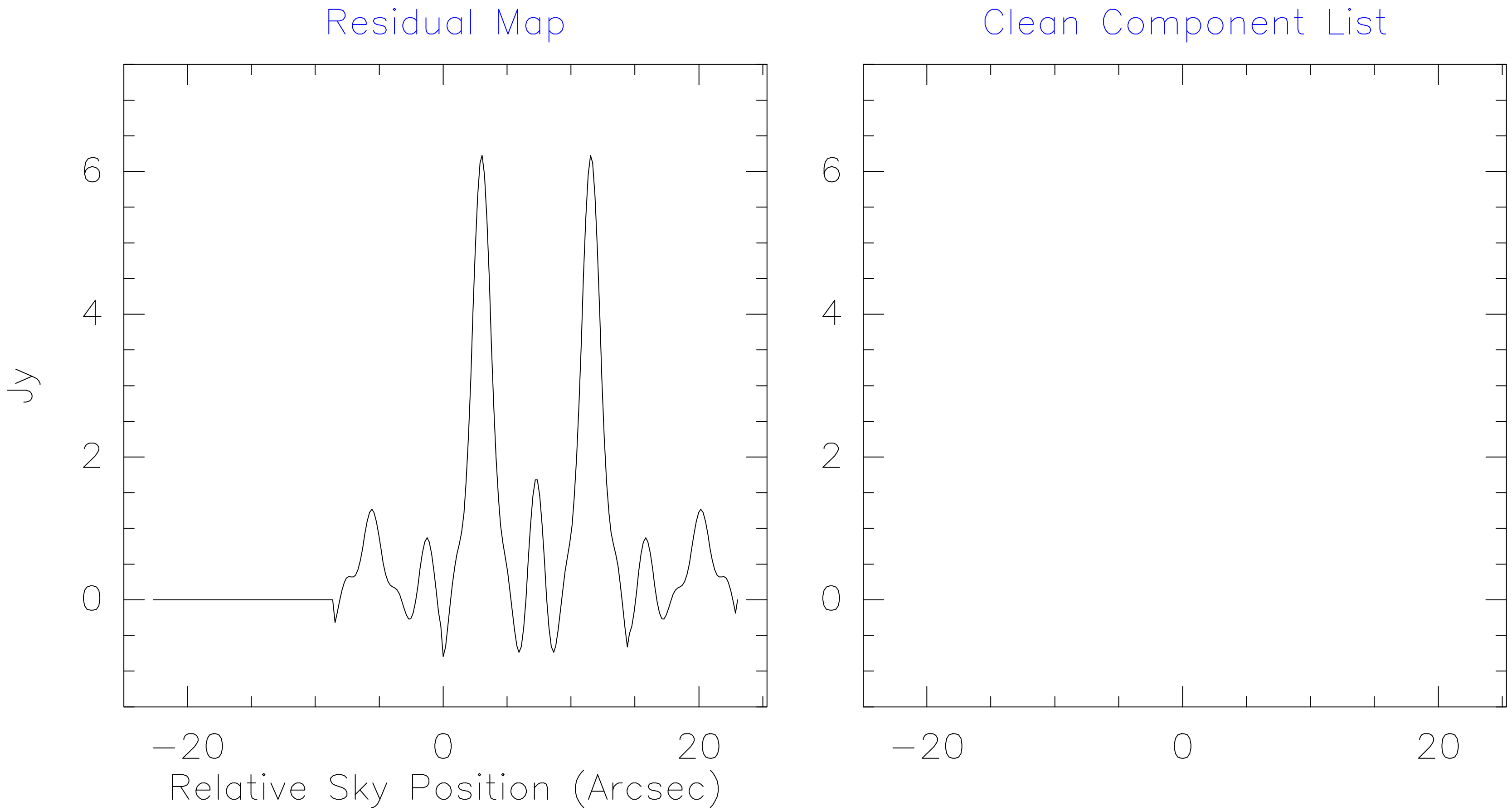

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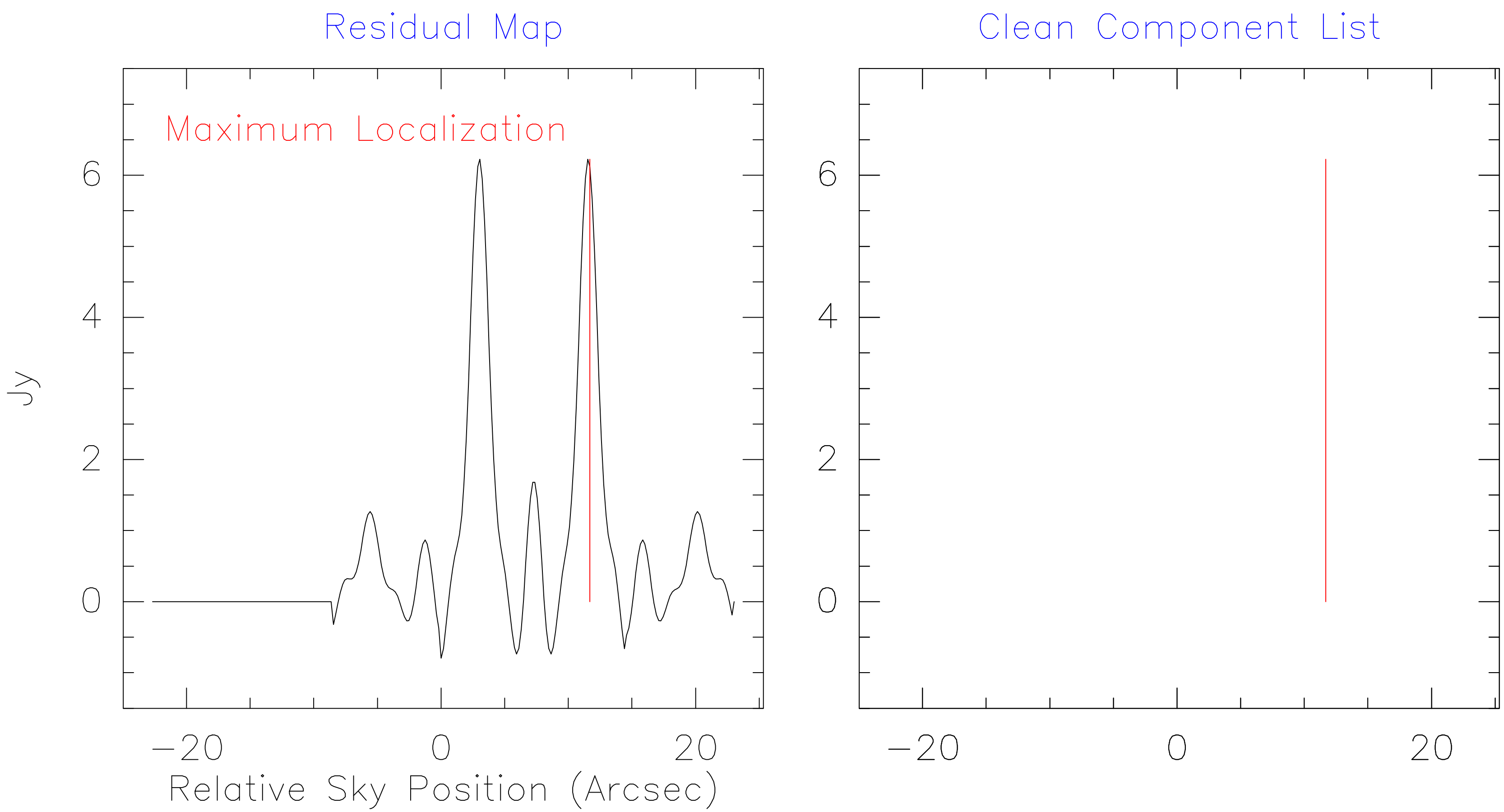

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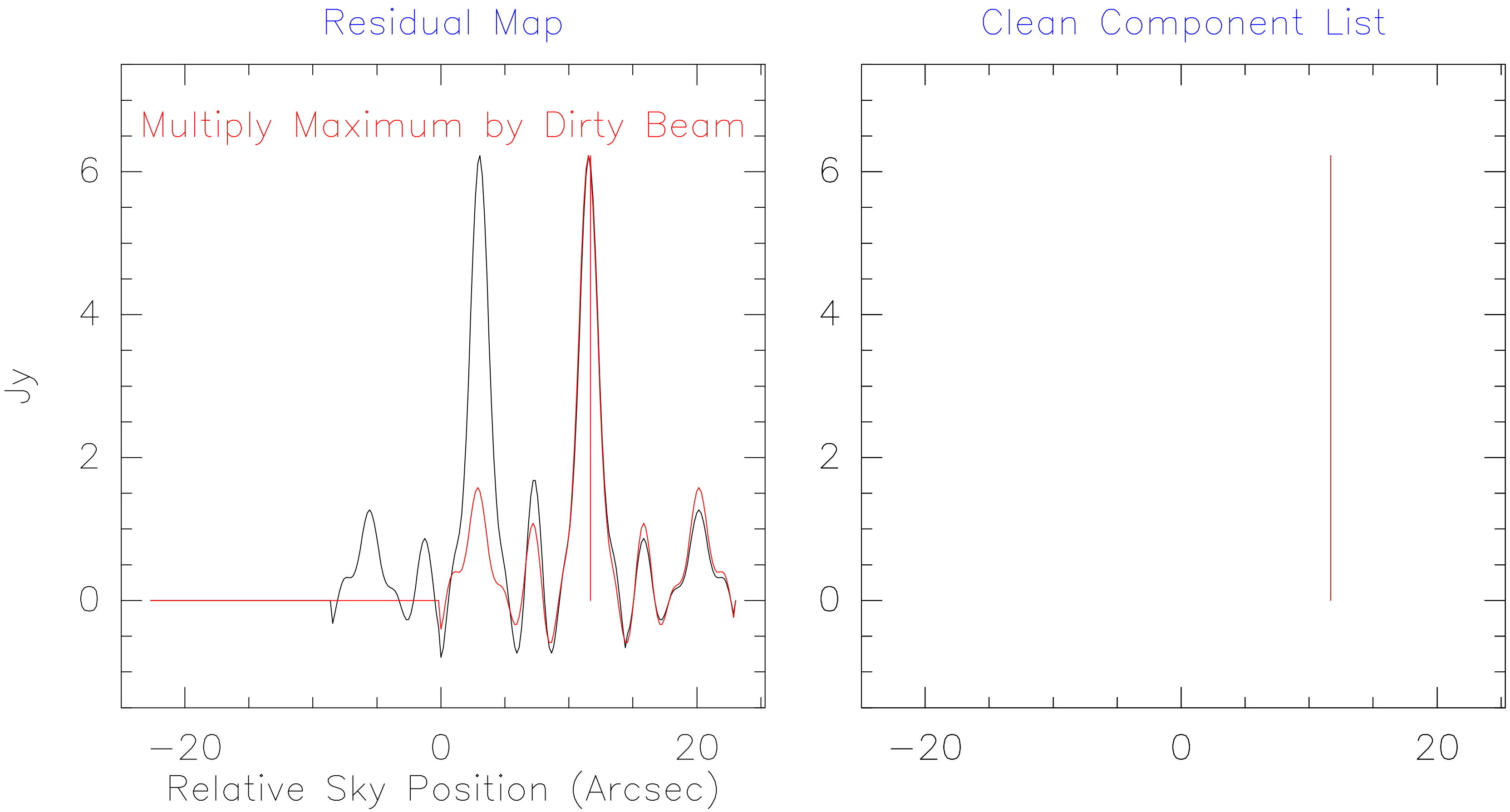

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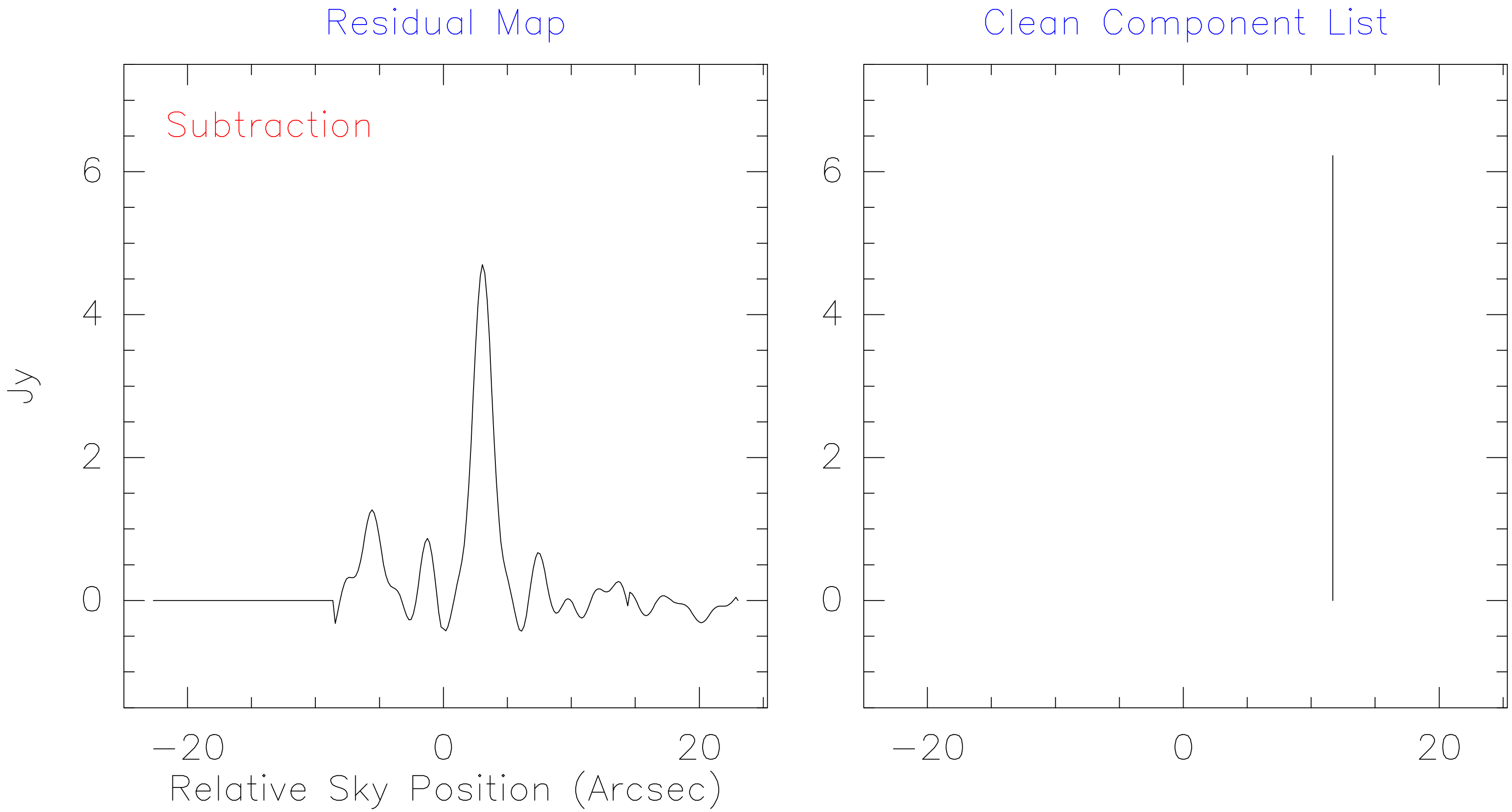

Deconvolution: III. The Basic Clean Algorithm
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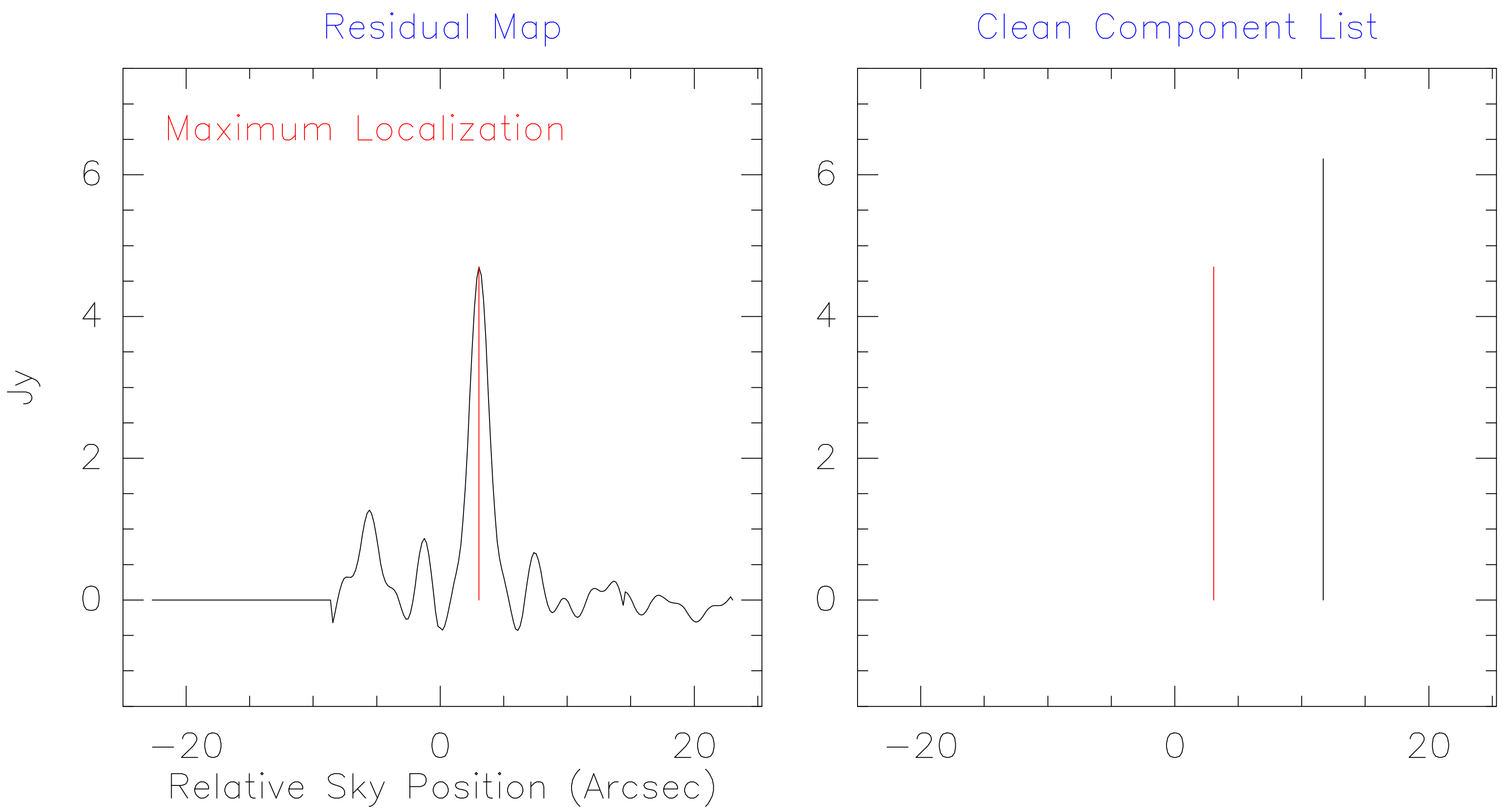

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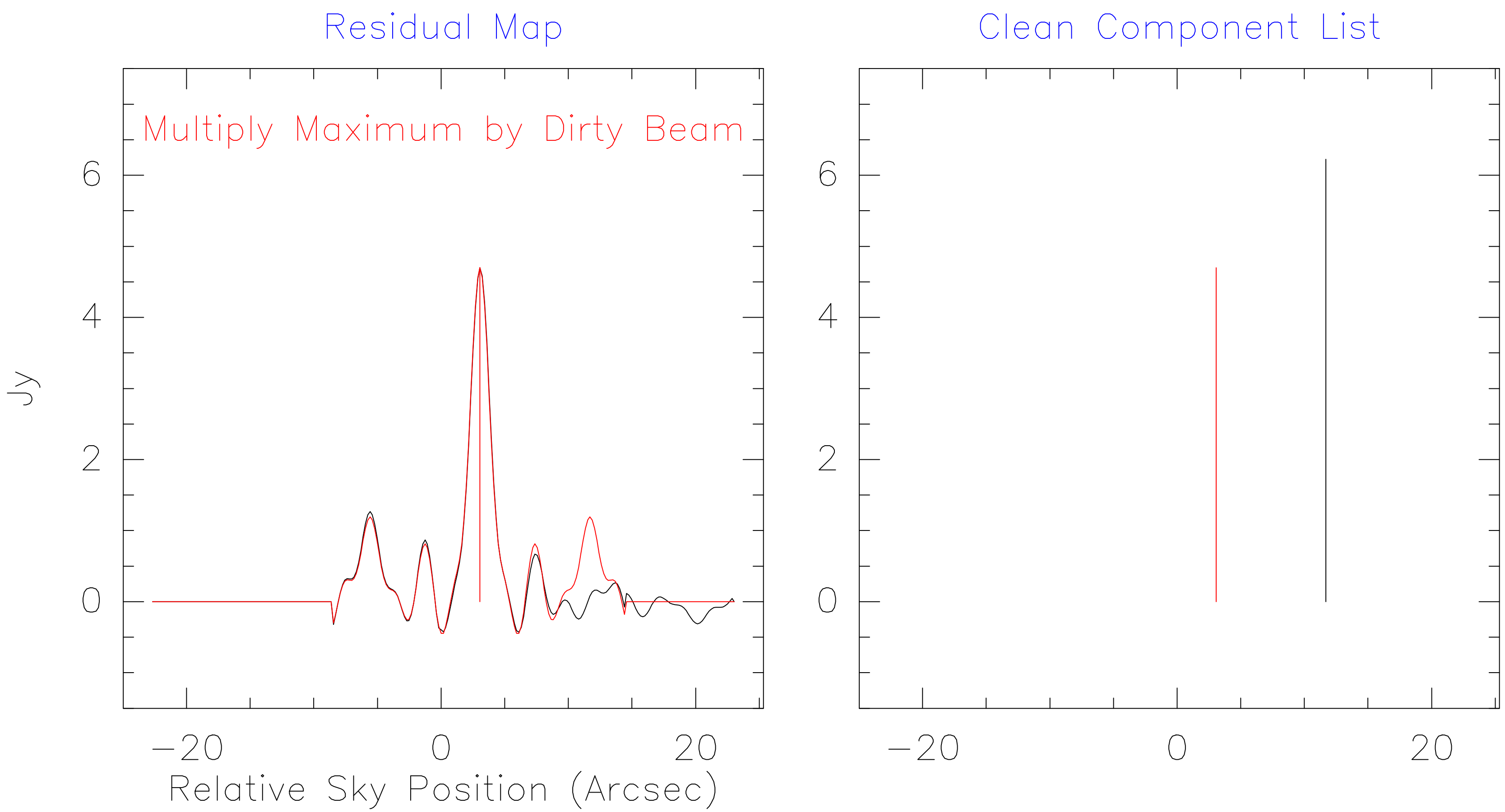
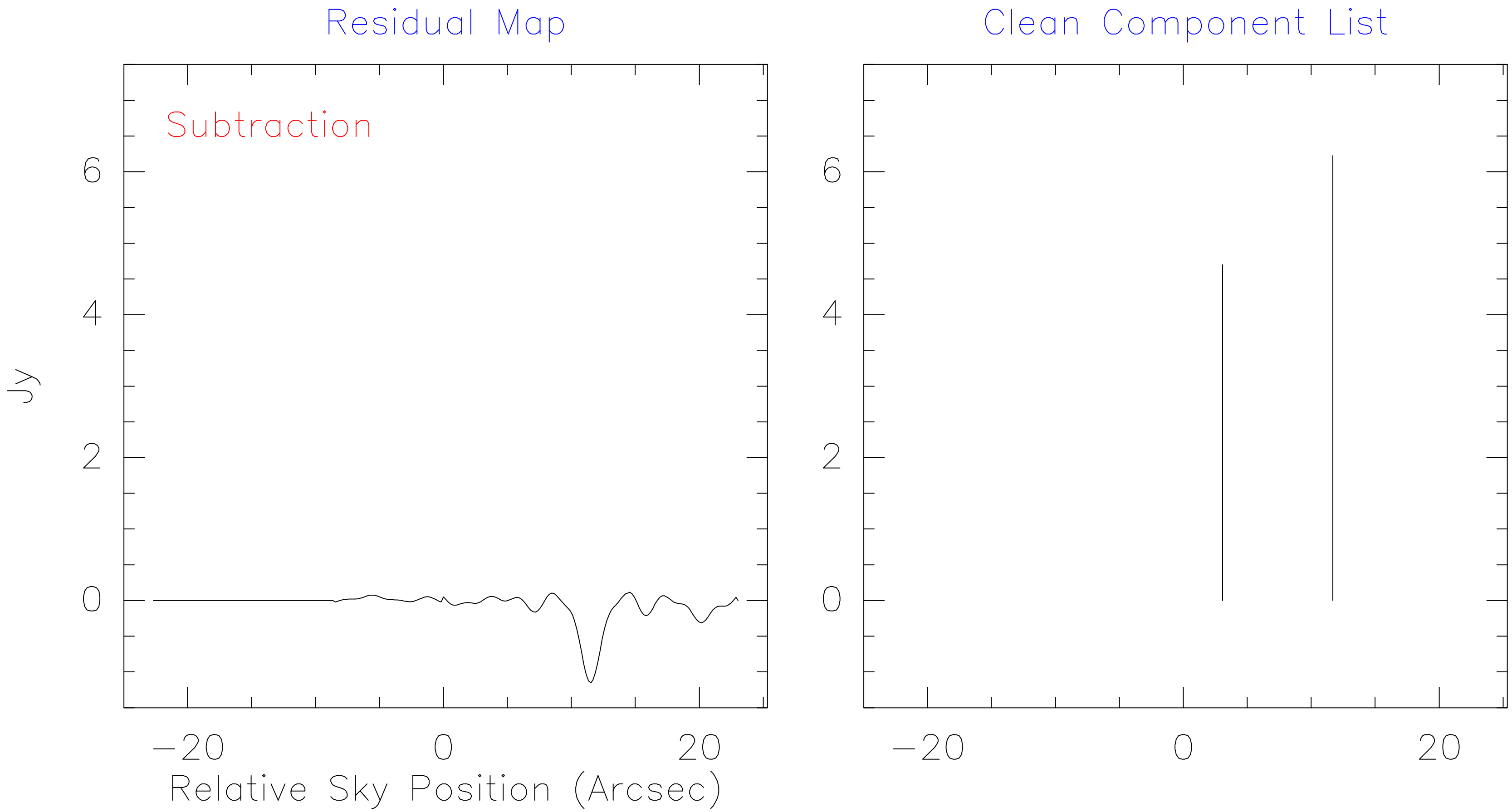
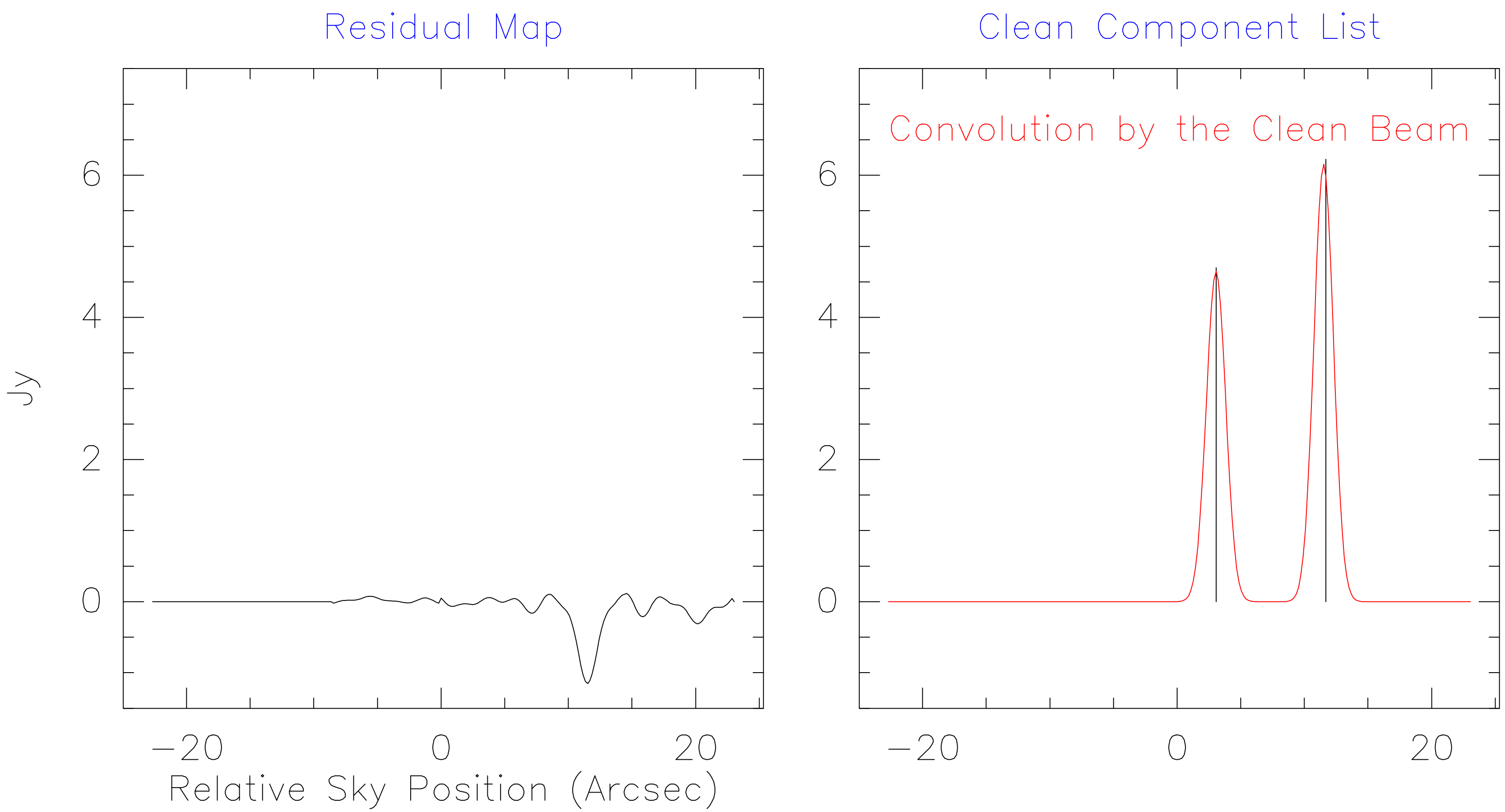


Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration



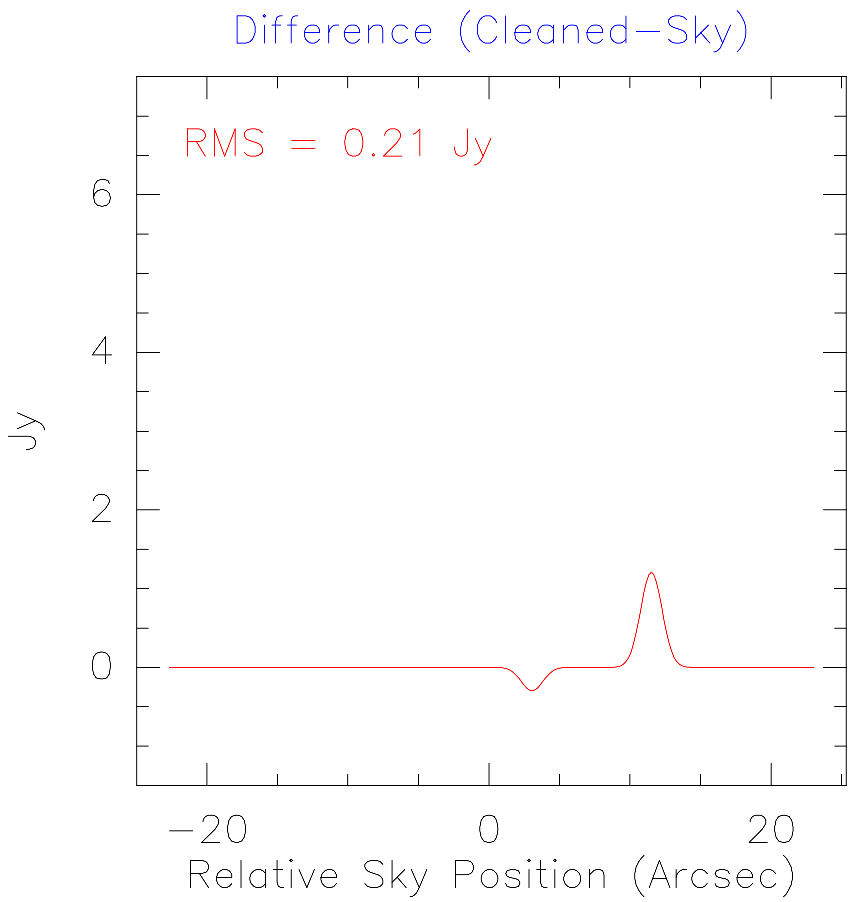
Deconvolution: III. The Basic Clean Algorithm
2. Second Illustration



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

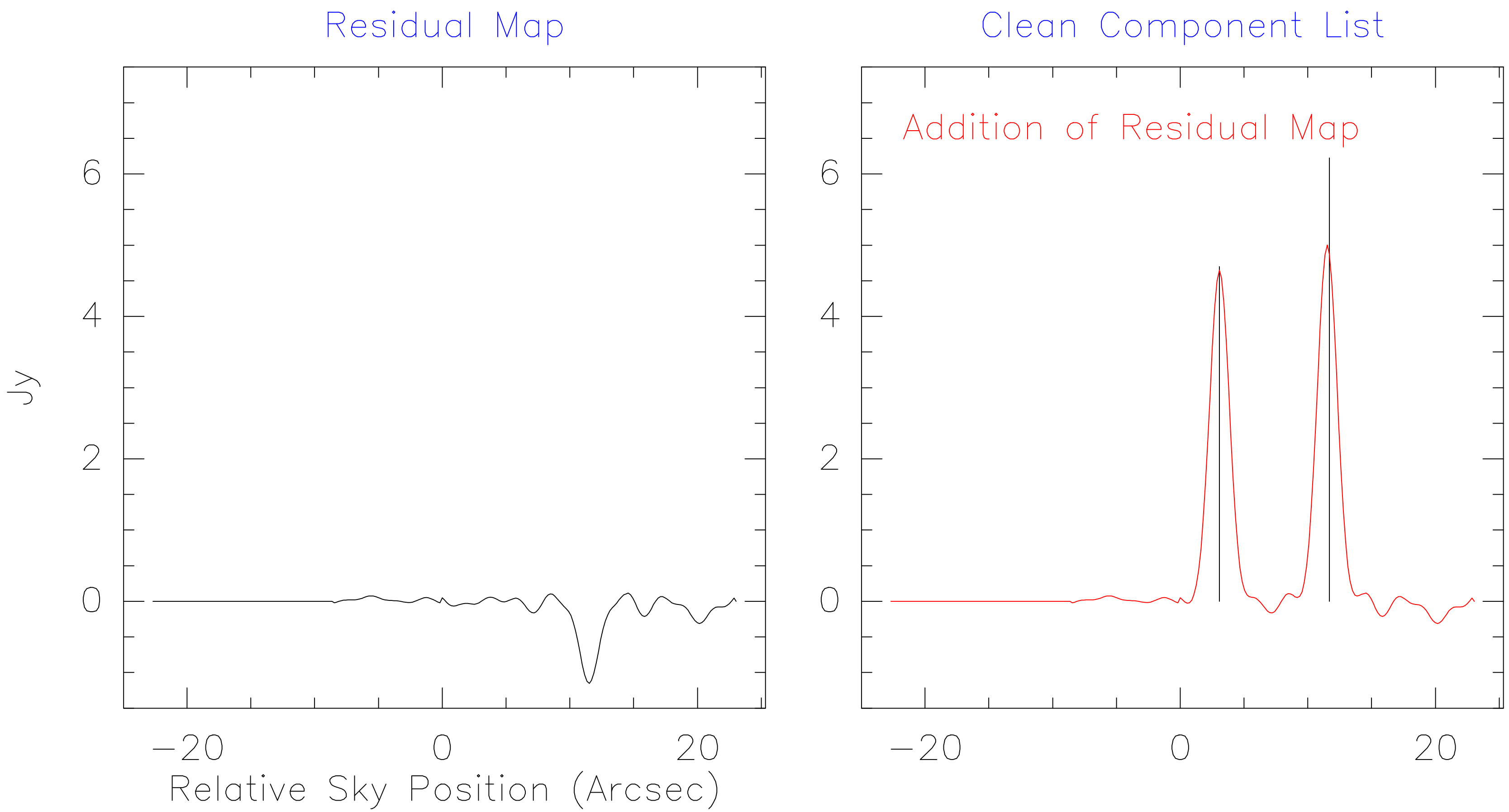
Convergence:

Too superficial cleaning $\Rightarrow$ Approximate results.
Too deep cleaning $\Rightarrow$ Divergence.



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:
Improvement when convergence not reached;
Noise estimation.



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Addition of residual map:
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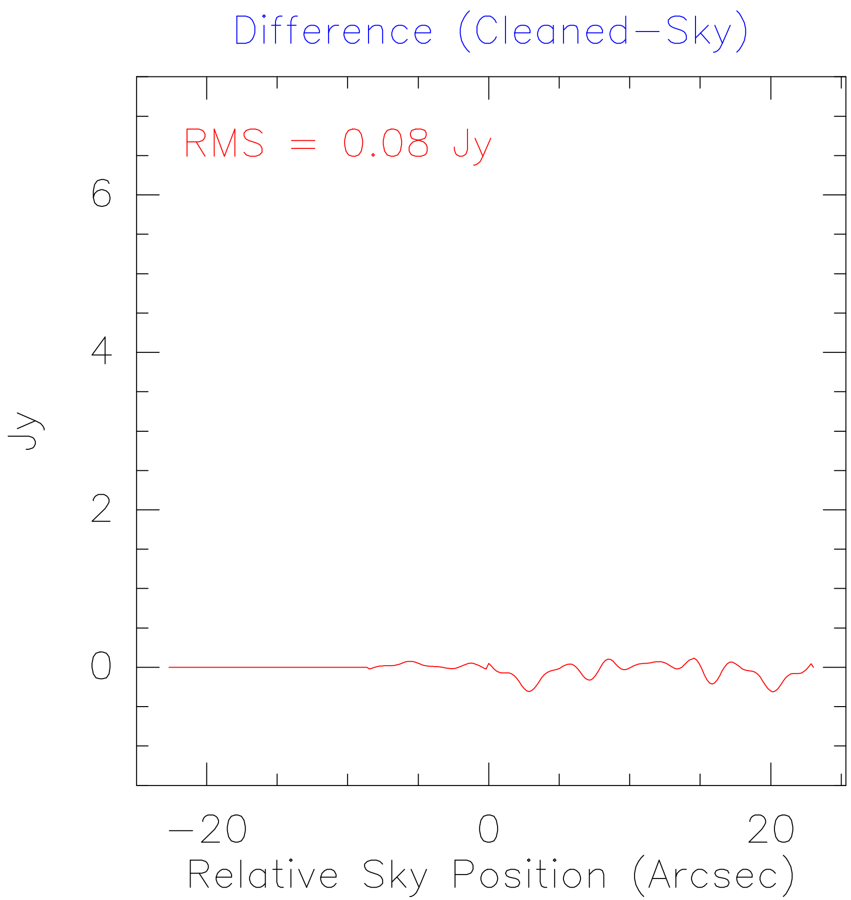
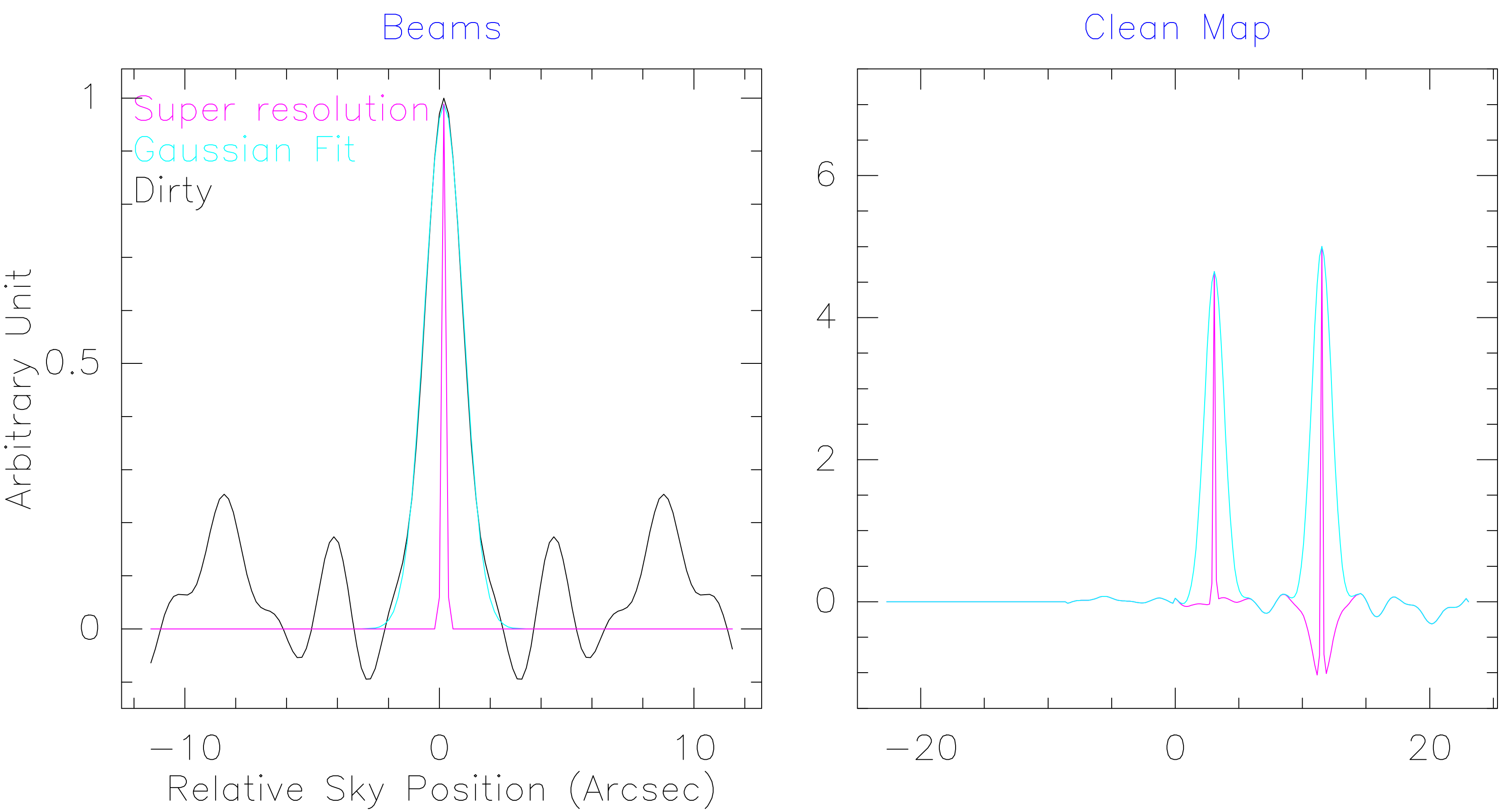


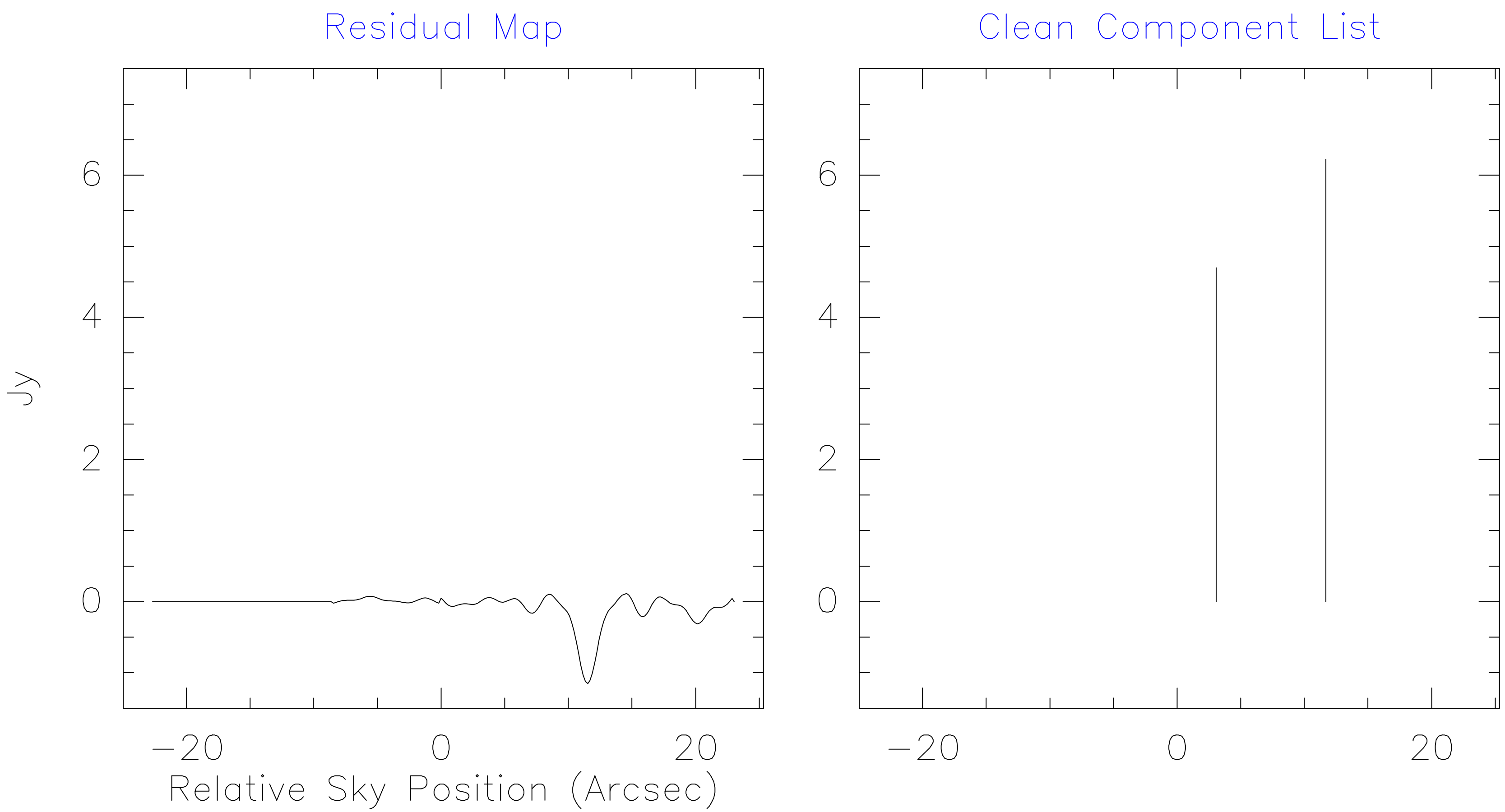
Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Choice of clean beam:
Gaussian of FWHM matching the synthesized beam size.
$\Rightarrow$ Super resolution strongly discouraged.



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

Negative clean components are mandatory.



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

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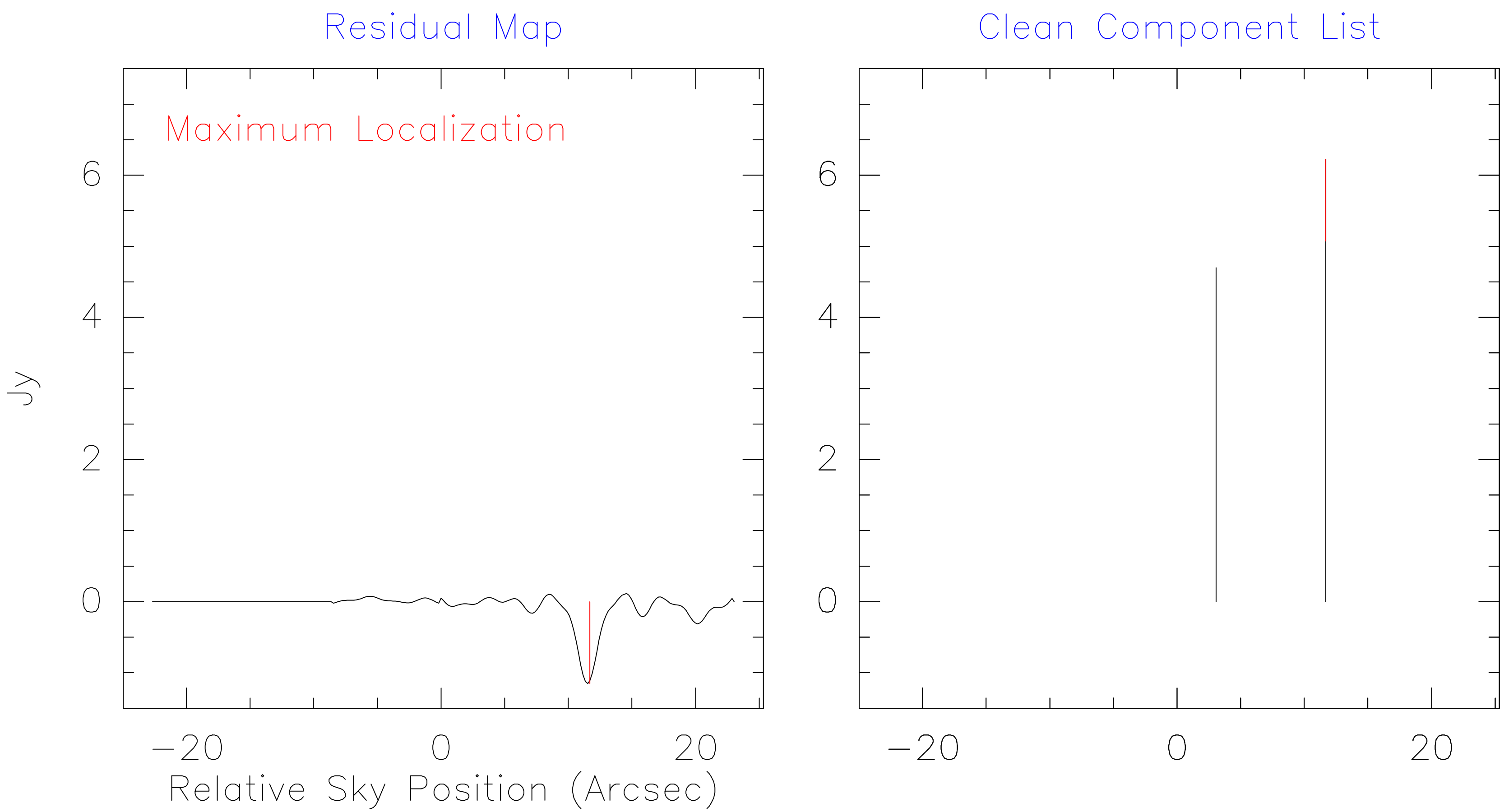

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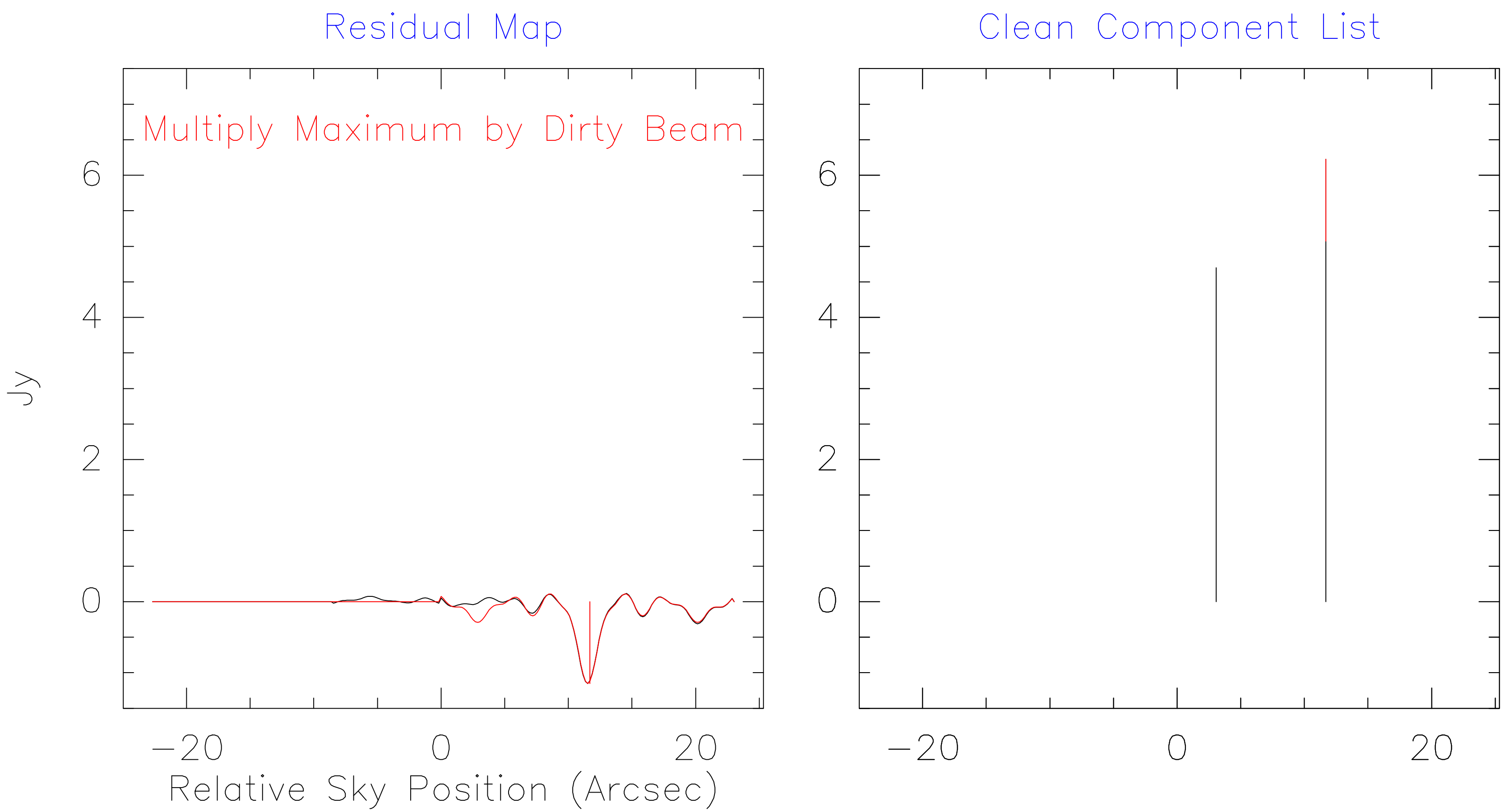

Deconvolution: III. The Basic Clean Algorithm
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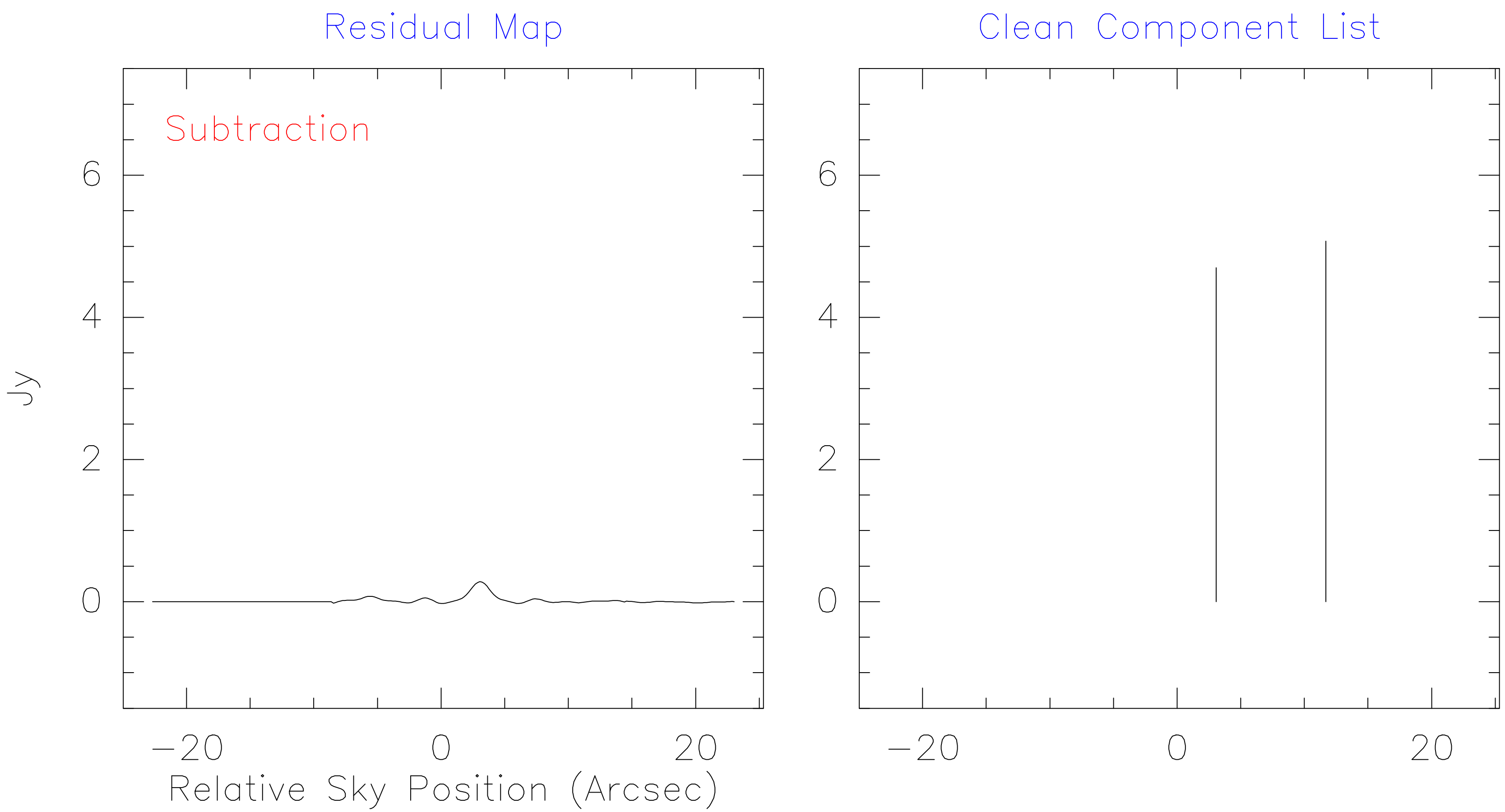
Negative clean components are mandatory.



Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

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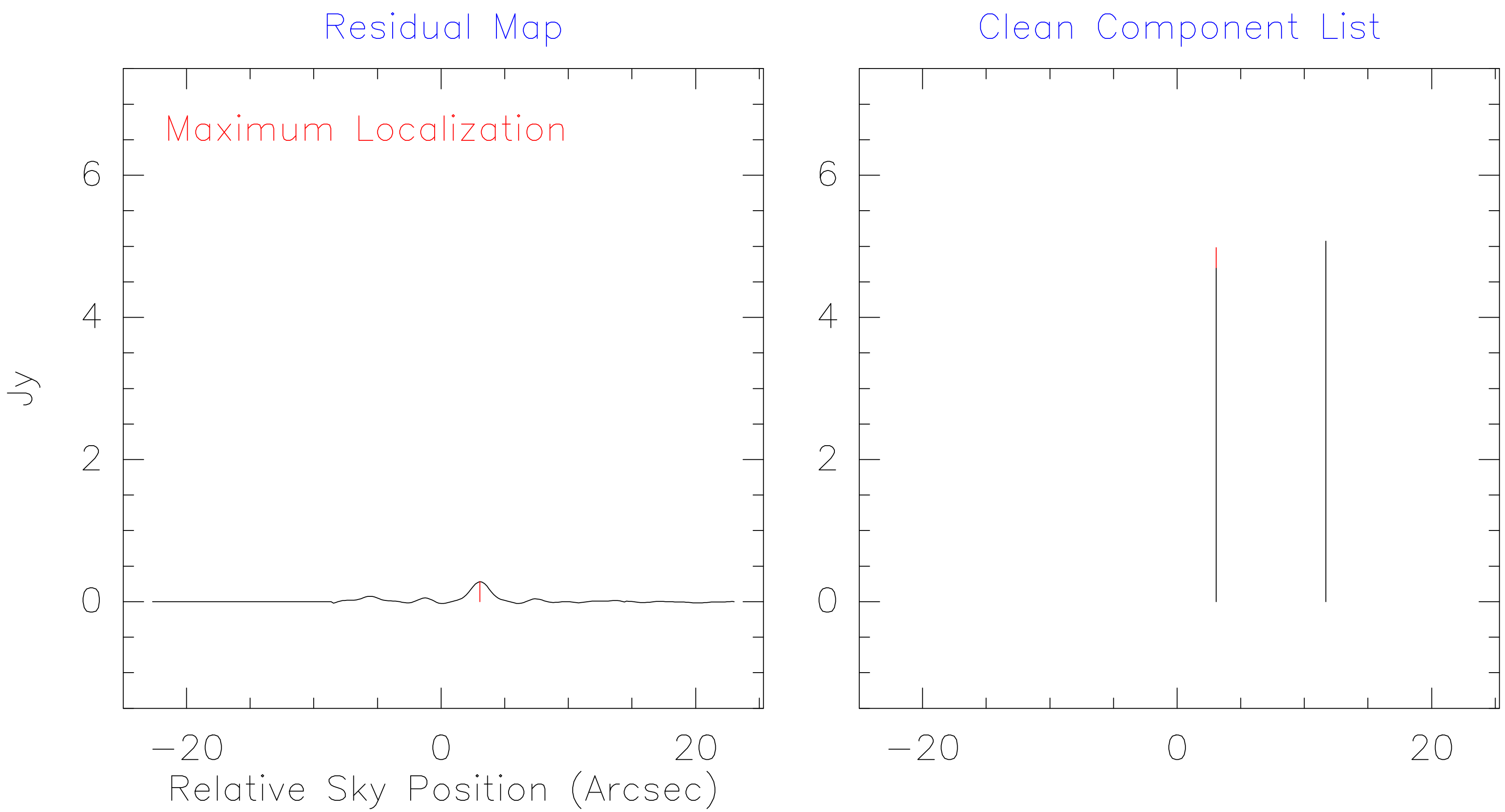

Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

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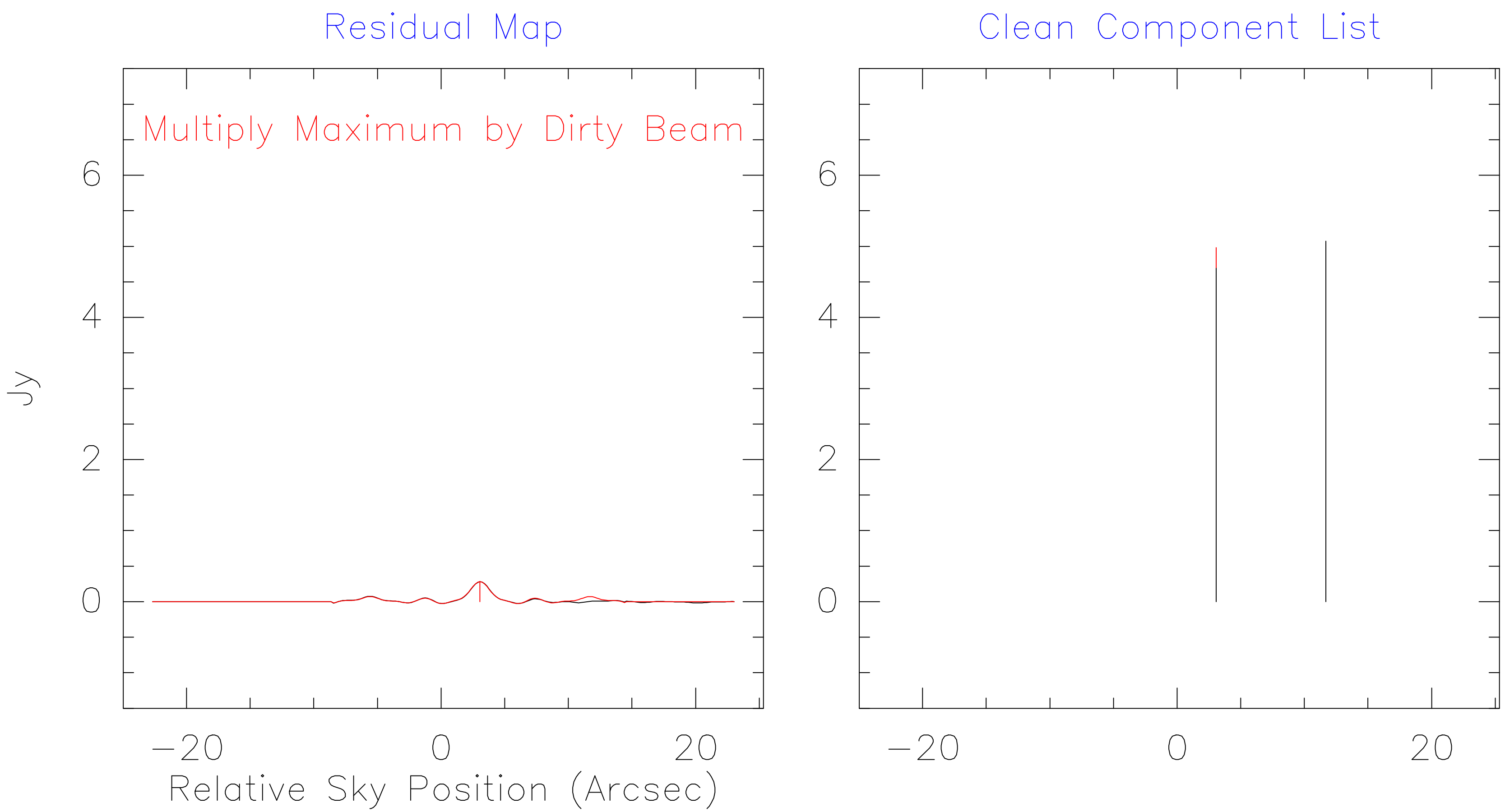

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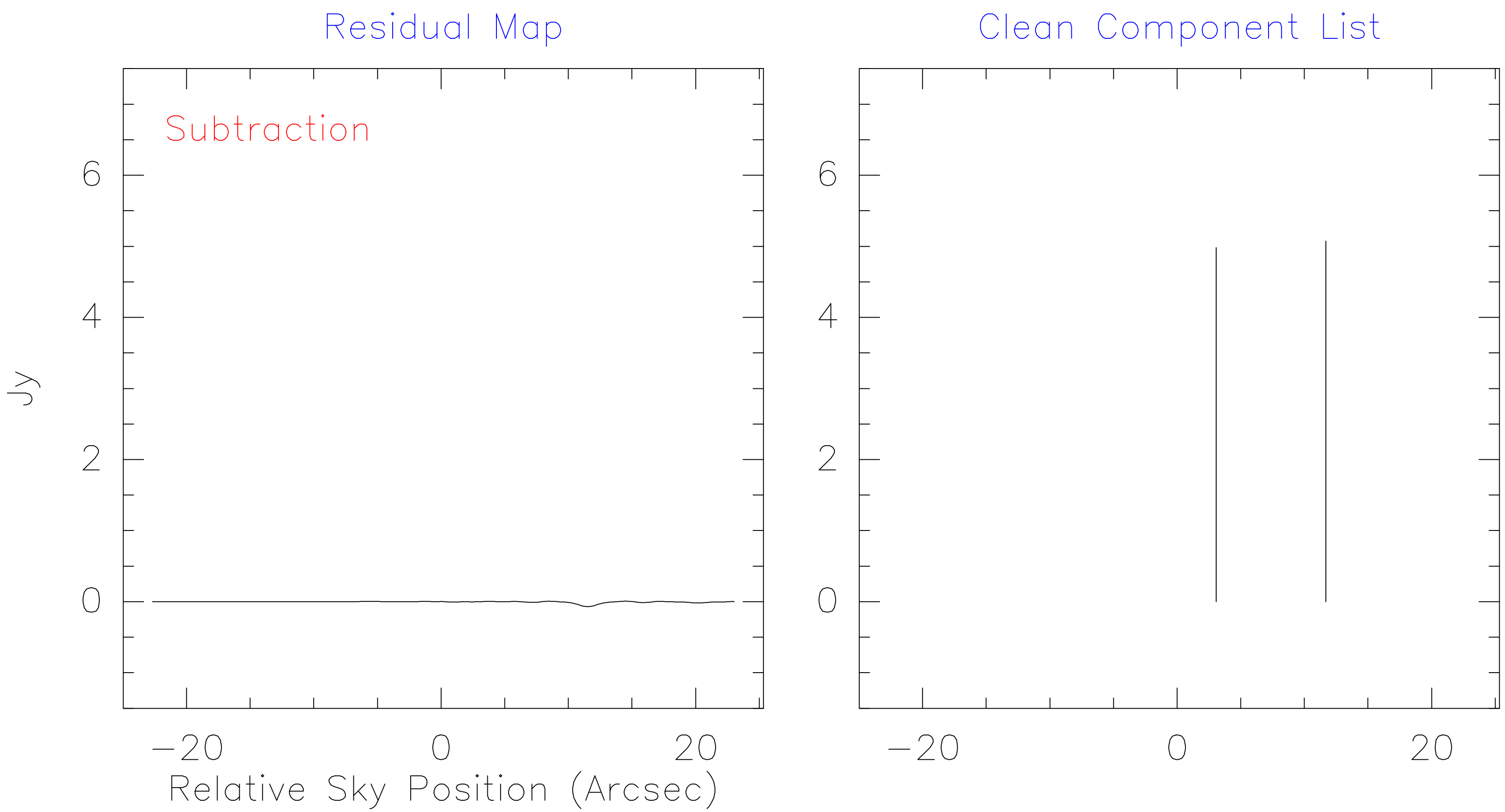

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3. Little Secrets

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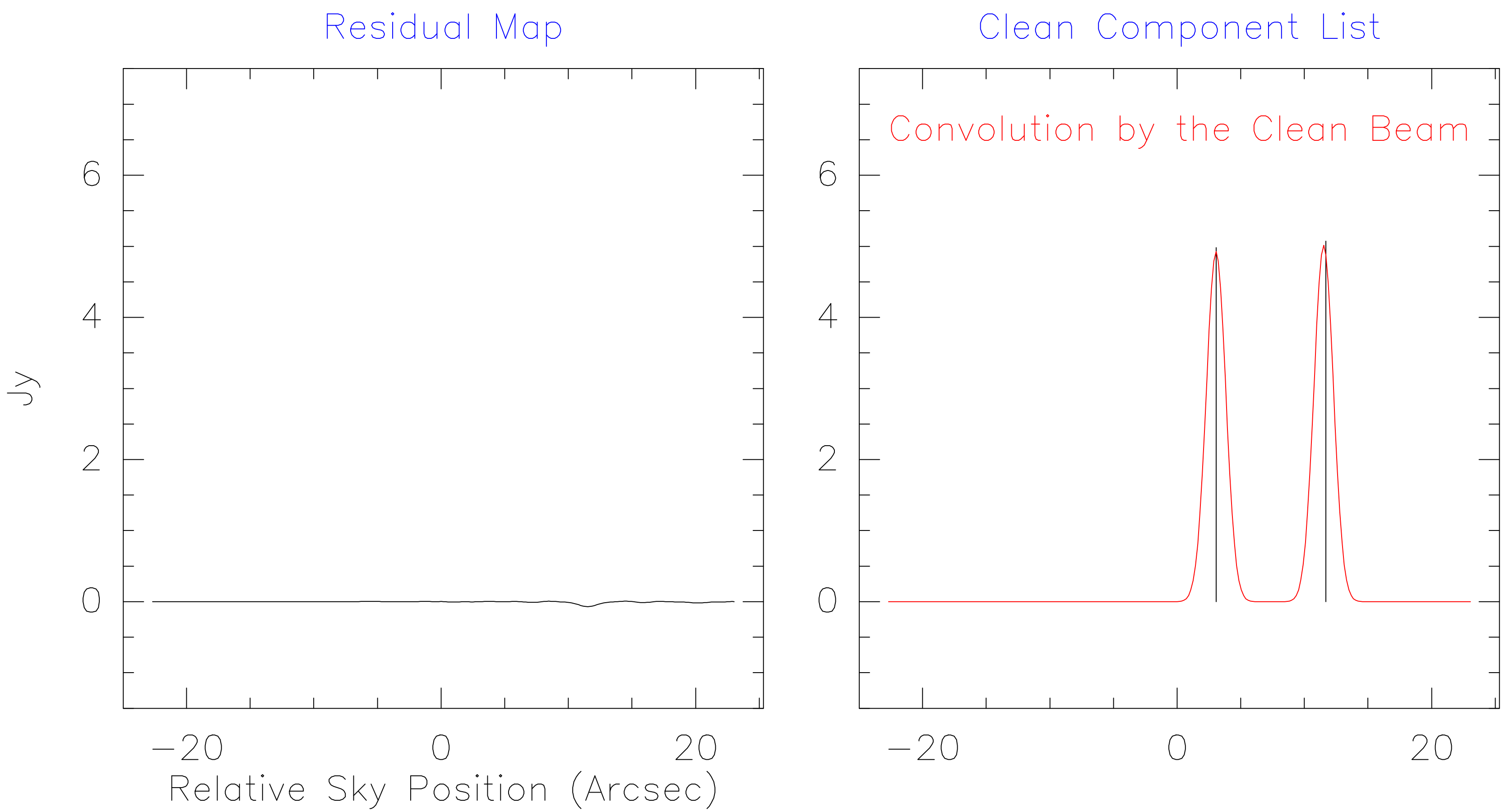

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3. Little Secrets

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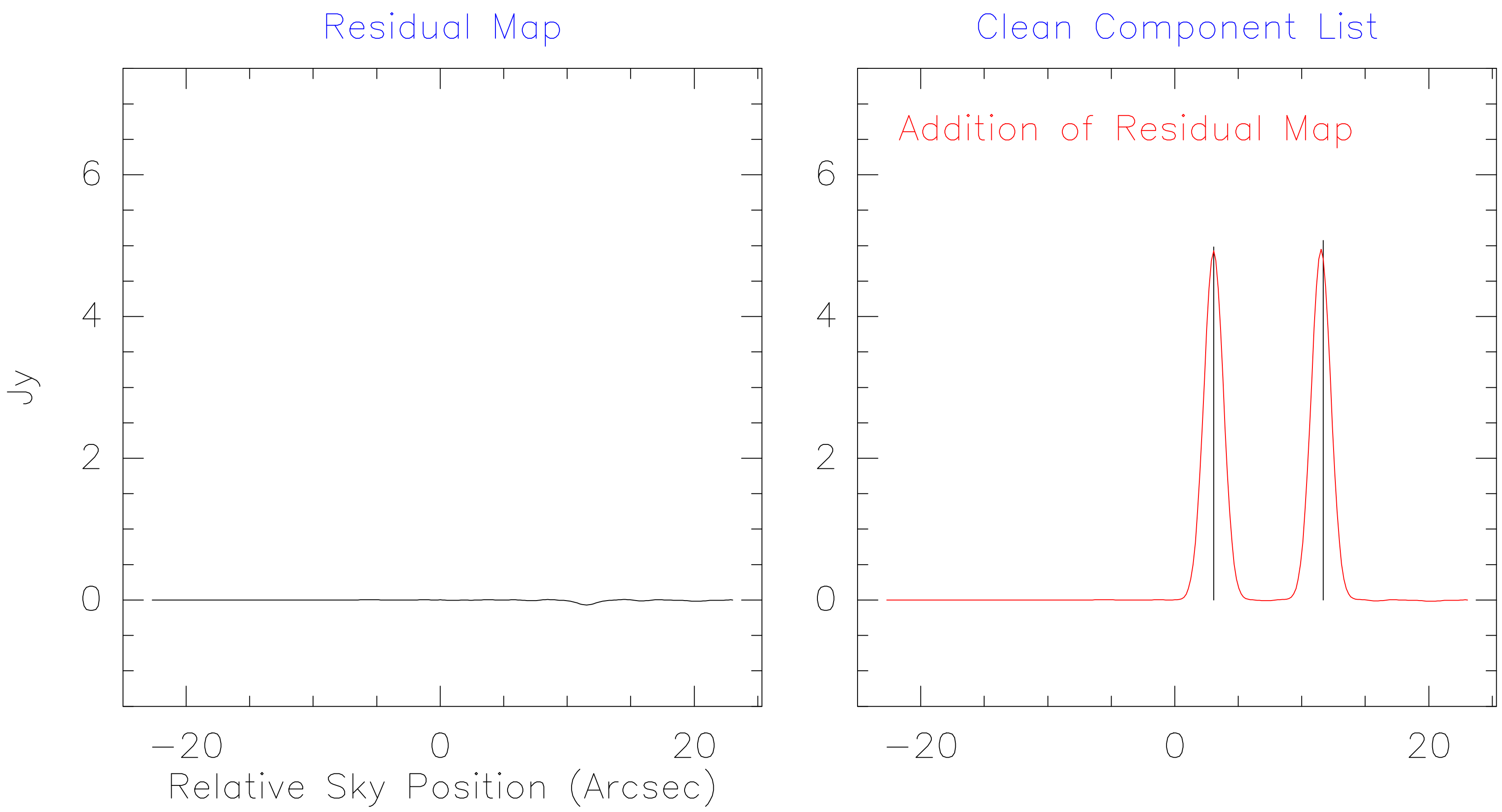
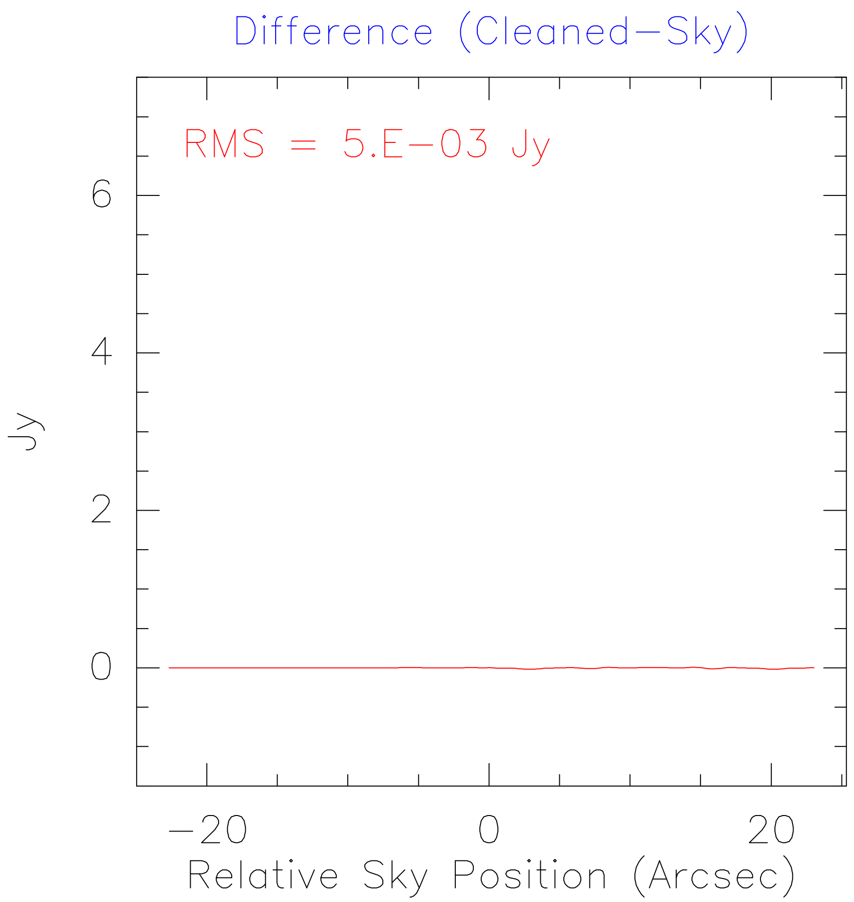


Deconvolution: III. The Basic Clean Algorithm
3. Little Secrets

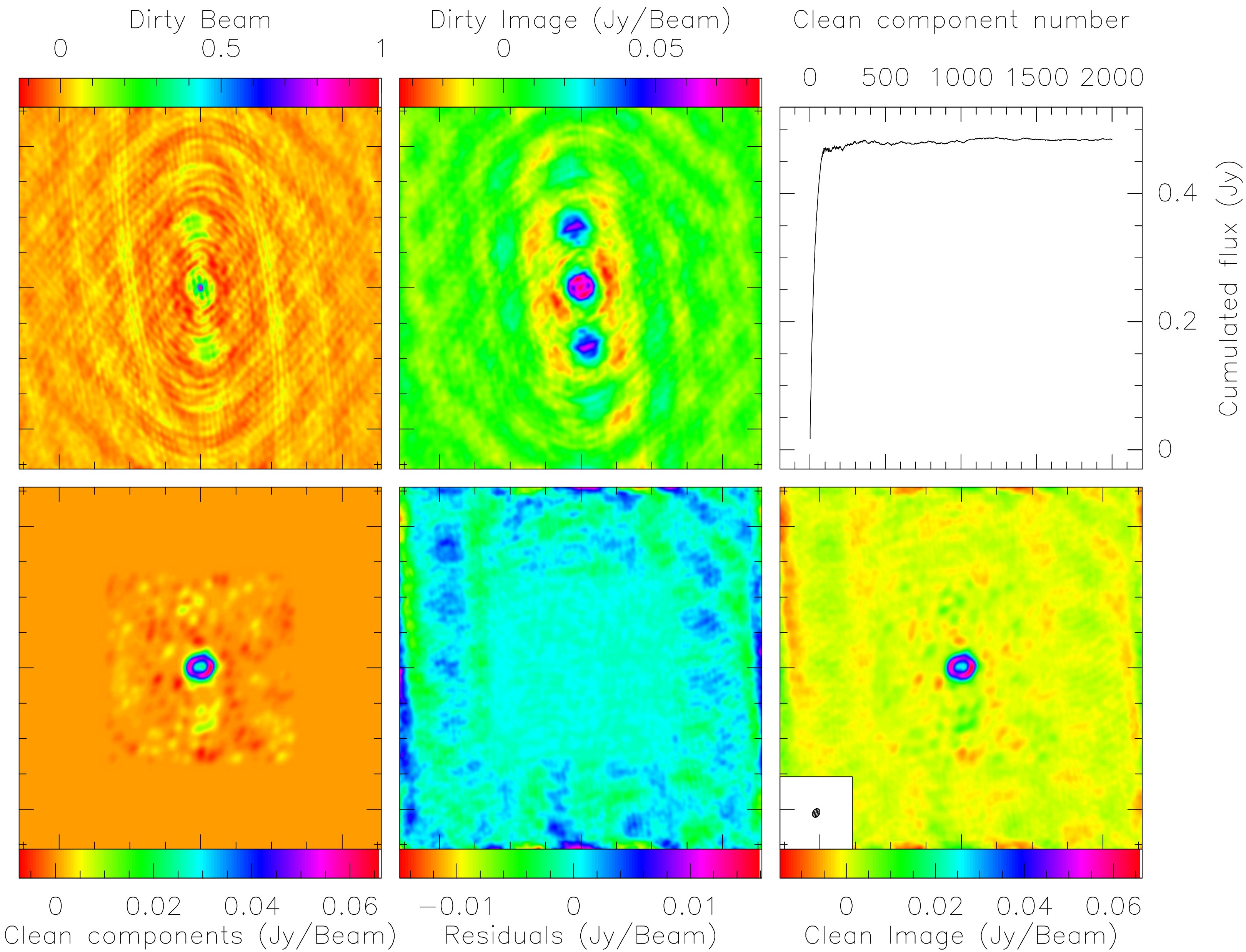
Negative clean components are mandatory.



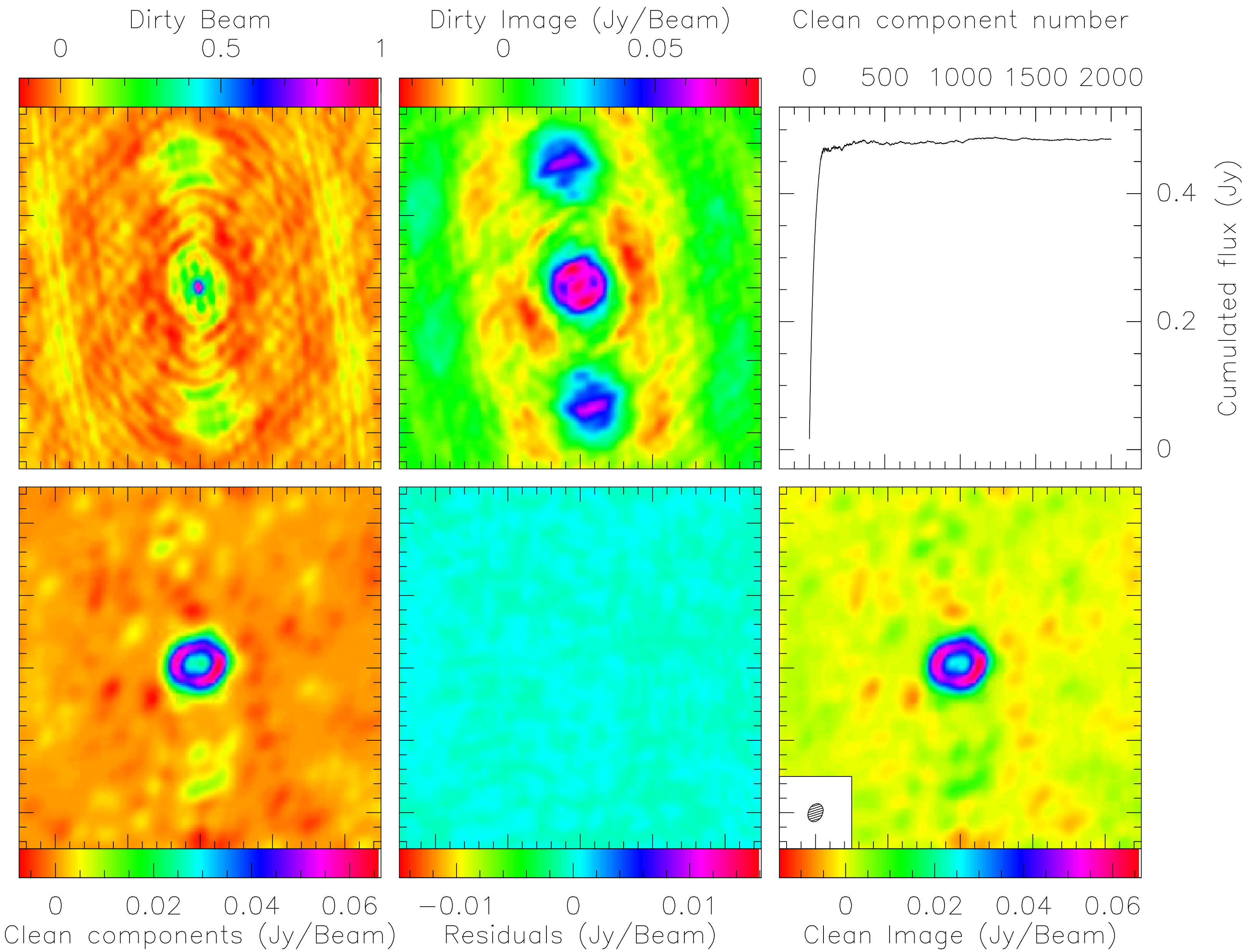
Deconvolution: III. The Basic Clean Algorithm
4. Other Little Secrets

- Stopping criterions:
  - Total number of Clean components;
  - $|I_{\text{max}}| <$ fraction of noise (when noise limited);
  - $|I_{\text{max}}| <$ fraction of dirty map max (when dynamic limited).
- Loop gain: Good results when $\gamma \sim 0.1 - 0.3$.
- Cleaned region: Only the inner quarter of the dirty image.
- Support: Definition of a region where CLEAN components are searched.
  - *A priori* information $\Rightarrow$ Help CLEAN convergence.
  - But bias if support excludes signal regions
    $\Rightarrow$ Be wise!

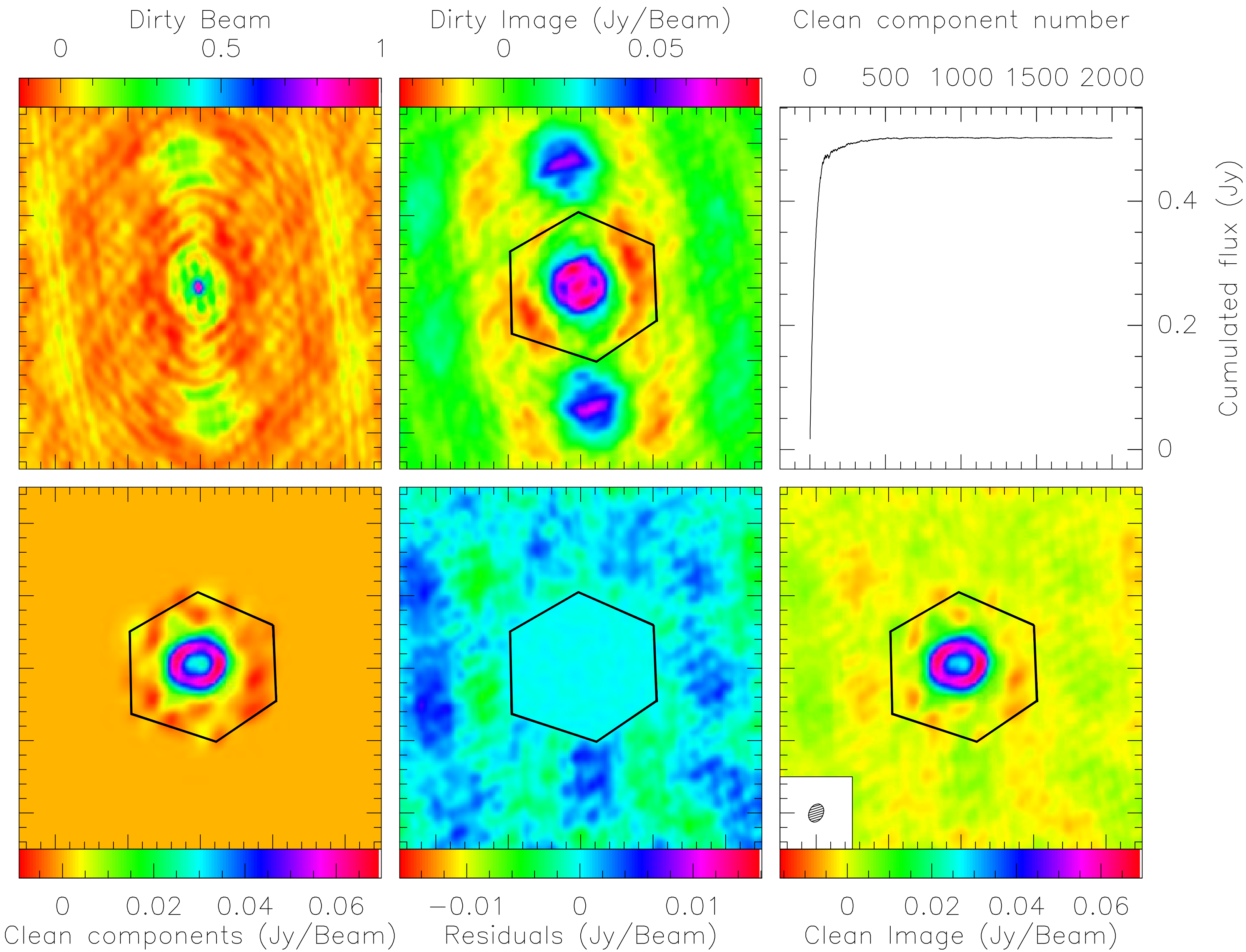
Deconvolution: III. The Basic Clean Algorithm
5. A True Example without support



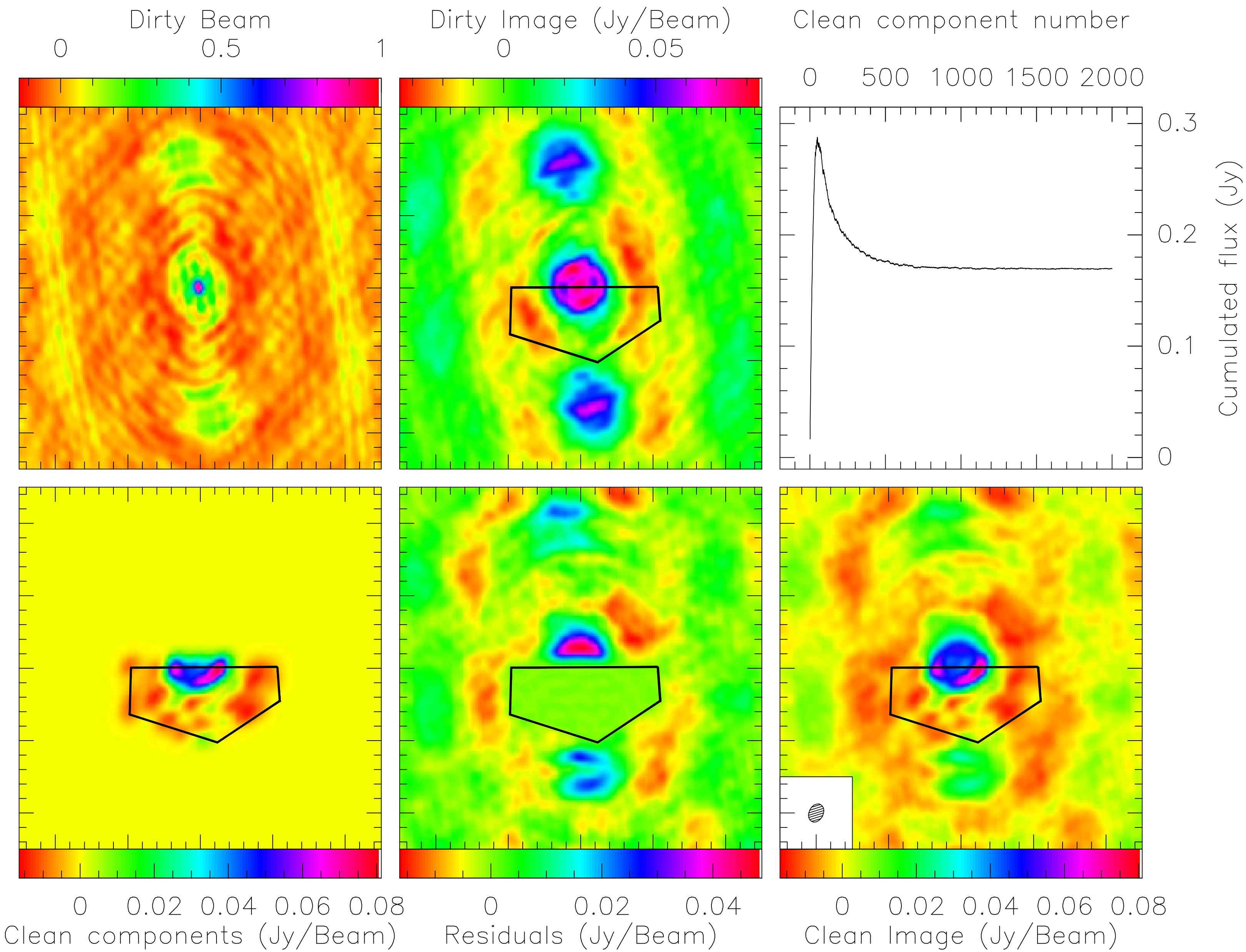
Deconvolution: III. The Basic Clean Algorithm
5. A True Example without support (zoom)



Deconvolution: III. The Basic Clean Algorithm
5. A True Example with right support



Deconvolution: III. The Basic Clean Algorithm
5. A True Example with wrong support



Deconvolution: IV. CLEAN Variants

Basic:

- `HOGBOM` (Hogböm 1974)
  Robust but slow.

Faster Search Algorithms:

- `CLARK` (Clark 1980)
  Fast but instable (when sidelobes are high).
- `MX` (Cotton& Schwab 1984)
  Better accuracy (Source removal in the uv plane), but slower (gridding steps repeated).

Better Handling of Extended Sources:

- `MULTI` (Multi-Scale Clean by Cornwell 1998)
  Multi-resolution approach.

Deconvolution: IV. CLEAN Variants (continued)

Exotic use at PdBI:

- `SDI` (Steer, Dewdney, Ito 1984)
  Created to minimize stripes.
- `MRC` (Multi-Resolution Clean by Wakker & Schwarz 1988)
  Too simple multi-resolution approach.

Deconvolution: V. Recommended Practices

- Method: Start with `CLARK` and turn to `HOGBOM` in case of high side-lobes.
- Support:
  - Start without one.
  - Define one on your first clean image if really needed (*i.e.* difficulties of convergence).
- Stopping criterion:
  - Use a large enough number of iterations to ensure convergence.
  - Clean down to the noise level unless a very strong source is present.
- Misc: Consult an expert until you become one.

Visualization and Image Analysis

Fourier Transform and Deconvolution:
The two key issues in imaging.

Stage	Implementation
Calibrated Visibilities	
$\Downarrow$ Fourier Transform	`GO UVSTAT, GO UVMAP`
Dirty beam & image	
$\Downarrow$ Deconvolution	`GO CLEAN`
Clean beam & image	
$\Downarrow$ Visualization	`GO BIT, GO VIEW`
$\Downarrow$ Image analysis	`GO NOISE, GO FLUX, GO MOMENTS`
Physical information on your source	

Photometry: I Generalities

- Brightness = Intensity (e.g. Power $= I_\nu(\alpha, \beta) dA d\Omega d\nu$)
- Flux unit: 1 Jy = 10^{-26} W m^{-2} Hz^{-1}.
- Source flux measured by a single–dish antenna:
  $F_\nu = B * I_\nu$ with B the antenna beam.
- Relationship between measured flux and temperature scales:
  $T_A = \frac{\lambda^2}{2k\Omega_A} F_\nu$, $T_A^\star = \frac{\lambda^2}{2k\Omega_{2\pi}} F_\nu$ and $T_{mb} = \frac{\lambda^2}{2k\Omega_{mb}} F_\nu$ because
  - $P_\nu = \frac{1}{2} A_e F_\nu$ Power detected by the single–dish antenna.
  - $P'_\nu = kT$ Power emitted by a resistor at temperature T.
  - $P_\nu = P'_\nu \Rightarrow T_A = \frac{A_e}{2k} F_\nu$.
  - $\lambda^2 = A_e \Omega_A$ (diffraction).
  - $\Omega_{2\pi} = F_{\mathrm{eff}} \Omega_A$ or $F_{\mathrm{eff}} = \frac{\text{Forward beam}}{\text{Total beam}}$.
  - $\Omega_{mb} = B_{\mathrm{eff}} \Omega_A$ or $B_{\mathrm{eff}} = \frac{\text{Main beam}}{\text{Total beam}}$.

Photometry: II Visibilities

Visibility unit: Jy because:

$$V = \text{2D FT}\left\{B_{\text{primary}}.I_{\text{source}}\right\}$$
$$= \iint B_{\text{primary}}(\sigma).I_{\text{source}}(\sigma)\,\exp(-i2\pi\mathbf{b}.\sigma/c)d\Omega.$$

Effect of flux calibration errors on your image:

- Multiplicative factor if uniform in uv plane.
- Convolution (*i.e.* distorsion) else.

Photometry: III Dirty map

Ill–defined because:

- $S(u = 0, v = 0) = 0 \Rightarrow$ Area of the dirty beam is 0!
- $V(u = 0, v = 0) = 0 \Rightarrow$ Total flux of the dirty image is 0!
  $\Rightarrow$ A source of constant intensity will be fully filtered out.
- A single point source of 1 Jy appears with peak intensity of 1.
- Several close-by point sources of 1 Jy appears with peak intensities different of 1.

Photometry: IV Clean map
(my dream: Don't take it seriously)

$I_{\text{clean}} = \frac{1}{\Omega_{\text{clean}}}\left(B_{\text{clean}} * I_{\text{point}}\right)$: *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* 1 steradian).

Unit: Jy/sr

Consequences:

- Source flux computation by integration inside a support:

$$\underset{[\text{Jy}]}{\text{Flux}} = \sum_{ij \,\in\, \mathcal{S}} \underset{[\text{Jy/sr}]}{I_{\text{clean}}} \; \underset{[\text{sr}]}{d\Omega}$$

  with $d\Omega$ the image pixel surface.

- From Brightness to temperature: $T_{\text{clean}} = \frac{\lambda^2}{2k} I_{\text{clean}}$

Photometry: IV Clean map (reality)

$I_{\text{clean}} = B_{\text{clean}} * I_{\text{point}}$: *i.e.* convolution of a set of point sources (mimicking the sky intensity distribution) by the clean beam.

Behavior: Brightness, *i.e.* Source flux measured in a given solid angle (*i.e.* clean beam).

Unit: Jy/beam with 1 beam = Ω_{clean} sr.

Consequences:

- Source flux computation by integration inside a support:

$$\underset{[\text{Jy}]}{\text{Flux}} = \sum_{ij \,\in\, \mathcal{S}} \underset{[\text{Jy/beam}]}{I_{\text{clean}}} \cdot \underset{[\text{beam}]}{\frac{d\Omega}{\Omega_{\text{clean}}}}$$

  with $\frac{d\Omega}{\Omega_{\text{clean}}}$ the nb of beams in the surface of an image pixel.

- From Brightness to temperature: $T_{\text{clean}} = \frac{\lambda^2}{2k\Omega_{\text{clean}}} I_{\text{clean}}$

Photometry: IV Clean map

Consequences of a Gaussian clean beam shape:

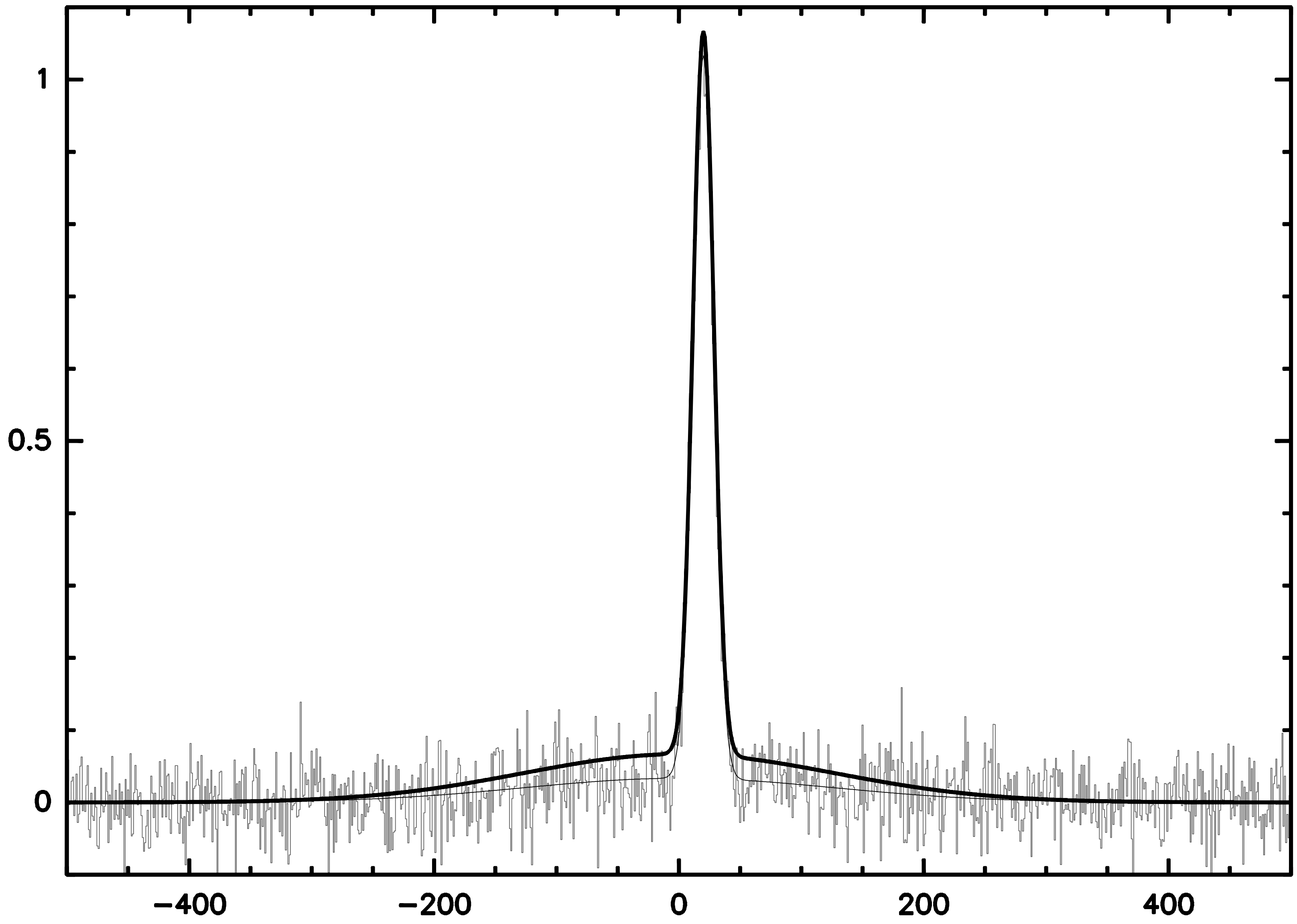
- No error beams, no secondary beams.
- T_{clean} is a main beam temperature.

Natural choice of clean beam size: Synthesized beam size
(*i.e.* fit of the central peak of the dirty beam).
$\Rightarrow$ Minimize unit problems when adding the dirty map residuals.

Caveats of flux measurements:

- CLEAN does not conserve flux
  (*i.e.* CLEAN extrapolates unmeasured short spacings).
- Large scales are filtered out (source size $>$ 1/3 primary beam size $\Rightarrow$ need of short spacings, cf. lecture by F. Gueth).
- $I_{\text{clean}} = B_{\text{primary}}.I_{\text{source}} + N$
  $\Rightarrow$ Primary beam correction may be needed:
  $I_{\text{clean}}/B_{\text{primary}} = I_{\text{source}} + N/B_{\text{primary}}$ $\Rightarrow$ Varying noise!
- Seeing scatters flux.

Photometry: V Importance of Extended, Low Level Intensity



Noise: I. Formula

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\rm sys}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\rm ant}(N_{\rm ant}-1)A}}$$

- δT Brightness noise [K].
- λ Wavelenght.
- k Boltzmann constant.
- Ω Synthesized beam solid angle.
- A Antenna area.
- σ Flux noise [Jy].
- $T_{\rm sys}$ System temperature.
- Δt On-source integration time.
- $\Delta\nu$ Channel bandwidth.
- $N_{\rm ant}$ Number of antennas.

and η Global efficiency (= Quantum x Antenna x Atm. Decorrelation).

Noise: II. σ to compare instruments

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)}A}$$

Wavelenght: 1 mm. $T_{\text{sys}} = 150$ K. Decorrelation $= 0.8$.

Instrument	Bandwidth	σ	On-source time
PdBI 2009	8 GHz	1.0 mJy/Beam	3 min
ALMA 2012	16 GHz	1.0 mJy/Beam	3 sec
ALMA 2012	16 GHz	0.12 mJy/Beam	3 min

One order of magnitude ($\sim 8\times$) sensitivity increase in continuum.

Noise: III. δT to prepare observations: 1. Continuum

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)A}}$$

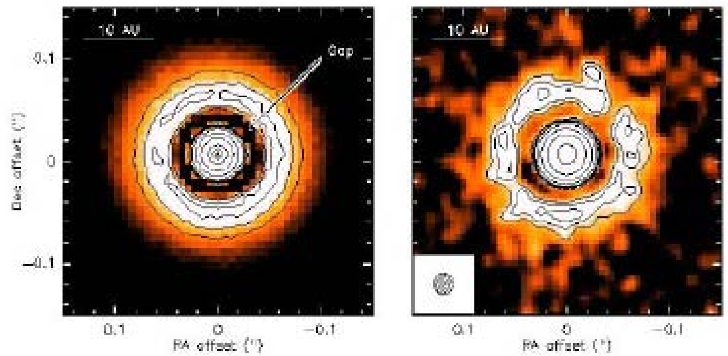
Wavelenght: 1 mm. $T_{\text{sys}} = 150$ K. Decorrelation $= 0.8$.

Instrument	Bandwidth	Resol.	δT	On time	Comment
PdBI 2009	8 GHz	0.30″	30 mK	3 hrs	
ALMA 2012	16 GHz	0.30″	30 mK	3 min	Low contrast, many objects
ALMA 2012	16 GHz	0.30″	4 mK	3 hrs	High contrast, same object
ALMA 2012	16 GHz	0.03″	30 mK	500 hrs	5.7% of a civil year
ALMA 2012	16 GHz	0.03″	400 mK	3 hrs	Intermediate sensitivity
ALMA 2012	16 GHz	0.10″	30 mK	3 hrs	Intermediate resolution

Almost one order of magnitude ($\sim 8\times$) sensitivity increase

$\Rightarrow$ A factor ~ 3 resolution increase (same integration time, same noise level).

Wolf et al. 2002, 0.02″ in 3 hrs.



Noise: III. δT to prepare observations: 2. Line

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)A}}$$

Channel width: $0.8\,\mathrm{km\,s^{-1}}$. Wavelenght: 1 mm. Decorrelation = 0.8.

Instrument	Resolution	δT	On-source time	Comment
PdBI now	$1''$	0.3 K	2 hrs	
ALMA 2012	$1''$	0.3 K	3.5 min	Same line, many objects
ALMA 2012	$1''$	0.05 K	2 hrs	Fainter lines, same objec
ALMA 2012	$0.1''$	0.3 K	575 hrs	6.5% of a civil year!
ALMA 2012	$0.1''$	5 K	2 hrs	Intermediate sensitivity
ALMA 2012	$0.4''$	0.3 K	2 hrs	Intermediate resolution

A factor $\sim$ 6 sensitivity increase
$\Rightarrow$ A factor $\sim$ 2.4 resolution increase
(same integration time, same noise level).

Noise: IV. Advices

$$\delta T = \frac{\lambda^2}{2k}\frac{\sigma}{\Omega} \qquad \text{with} \qquad \sigma = \frac{2k}{\eta}\frac{T_{\text{sys}}}{\sqrt{\Delta t \Delta \nu}\sqrt{N_{\text{ant}}(N_{\text{ant}}-1)}A}$$

- For your estimation:
  - Use a sensitivity estimator!
    http://www.eso.org/sci/facilities/alma/observing/tools/etc/
  - The estimator is probably optimistic!
  - Use δT not σ.

Writing the Paper: Your job!

Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} \text{and} F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_2}), \text{ then } F_1.F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} * \tilde{F_2}.$$

2 Sampling size $\overset{\text{FT}}{\rightleftharpoons}$ Image size.

3 Bandwidth size $\overset{\text{FT}}{\rightleftharpoons}$ Pixel size.

4 Finite support $\overset{\text{FT}}{\rightleftharpoons}$ Infinite support.

5 Fourier transform evaluated at zero spacial frequency
= Integral of your function.

$$V(u=0, v=0) \overset{\text{FT}}{\rightleftharpoons} \sum_{ij \,\in\, \text{image}} I_{ij}.$$

Photographic Credits and References

- R. N. Bracewell, "The Fourier Transform and its Applications".
- J. D. Kraus, "Radio Astronomy".
- R. Narayan and R. Nityananda, Ann. Rev. Astron. Astrophys., 1986, 24, 127–170, "Maximum Entropy Image Restoration in Astronomy"