Dealing with Noise

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I - Theory & Practice of noise

II – Low S/N analysis

Outline

1. Basic Theory
    1. Point source sensitivity
    2. Noise in images
    3. Extended source sensitivity
    4. Available Tools
2. Low S/N analysis
    1. Continuum data
    2. Line data
    3. Examples
    4. Advanced tricks: filtering & stacking

System Temperature

- The output power of the receiver is linked to the Antenna System Temperature by:

$$P_N = \gamma \, k \, T_{ant} \, \Delta \nu$$

- On source, the power is $P_N + P_a$ with

$$P_a = \gamma \, k \, T_a \, \Delta \nu$$

- T_a is called the antenna temperature of the source.
- This is not a purely conventional definition.

  It can be demonstrated that P_a is the power the receiver(+antenna) would deliver when observing a blackbody (filling its entire beam pattern) at the physical temperature T_a.
- Thus, T_{ant} is the temperature of the “equivalent’’ blackbody seen by the antenna (in the Rayleigh Jeans approximation)

System Temperature

- T_{ant} is given by (just summing powers…)

$T_{ant} = T_{bg}$ — cosmic background

$+ T_{sky} \approx \eta_f (1-\exp(-\tau_{atm})) T_{atm}$ — sky noise

$+ T_{spill} \approx (1-\eta_f-\eta_{loss}) T_{ground}$ — ground noise pickup

$+ T_{loss} \approx \eta_{loss} T_{cabin}$ — losses in receiver cabin

$+ T_{rec}$ — receiver noise

- This is a broad-band definition. It is a DSB (Double Side Band) noise temperature
- Many astronomical signals are narrow band. g being the image to signal band gain ratio, the equivalent DSB signal giving the same antenna temperature as a pure SSB signal is only

$$P_{DSB} = (1 \times P_{SSB} + g \times 0) / (1 + g)$$

System Temperature

- We usually refer the system temperature and antenna temperature to a perfect antenna (η_f = 1) located outside the atmosphere, and single sideband signal:

$$T_{sys} = (1+g)\ \exp(\tau_{atm}) T_{ant} / \eta_f$$

$$T_A^* = (1+g)\ \exp(\tau_{atm}) T_a / \eta_f$$

- This antenna temperature T_A^* is weather independent, and linked to the source flux S_ν by an antenna dependent quantity only

$$T_A^* = \eta_a\, A\, S_\nu / 2k$$

Noise Equation

- The noise power is T_{sys}, the signal is T_A^* , and there are $2\Delta\nu\ \Delta t$ independent samples to measure a correlation product in a time Δt, so the Signal to Noise is

$$R_{sn} = (2\Delta\nu\ \Delta t)^{1/2}\ T_A^*\ /\ T_{sys}$$

- On a single baseline, the noise is thus

$$\Delta S = \frac{\sqrt{2}kT_{sys}}{\eta_a A\sqrt{\Delta\nu\Delta t}}$$

- this is √2 less than that of a single antenna in total power
- but √2 worse than that of an antenna with the same total collecting area
- this sensitivity loss is because we ignore the autocorrelations

Noise Equation

- With quantization

$$\Delta S = \frac{\sqrt{2}kT_{sys}}{\eta_q \eta_a A\sqrt{\Delta\nu\Delta t}}$$

- With η_q the quantization efficiency
- Noise is uncorrelated from one baseline to another
- There are n(n-1)/2 baselines for n antennas
- So the point source sensitivity is

$$\Delta S = \frac{2kT_{sys}}{\eta_q \eta_a A\sqrt{n(n-1)\Delta\nu\Delta t}} = \frac{\mathcal{J}T_{sys}}{\eta_q \sqrt{n(n-1)\Delta\nu\Delta t}}$$

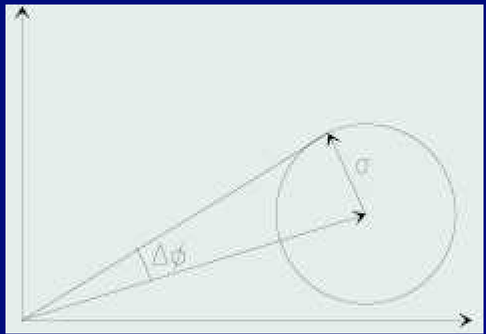
- Where $\mathcal{J} = \frac{2k}{\eta_a A}$ is the Jy/K conversion factor of one antenna

Noise on Amplitude and Phase

- For 1 baseline, this varies with Signal to Noise ratio
- On Amplitude

$$S \ll \sigma \quad \begin{cases} \sigma_A & \simeq \sigma\sqrt{2-\frac{\pi}{2}}\left(1+\left(\frac{S}{2\sigma}\right)^2\right) \\ \langle S \rangle & \simeq \sigma\sqrt{\frac{\pi}{2}}\left(1+\left(\frac{S}{2\sigma}\right)^2\right) \end{cases}$$

$$S \gg \sigma \quad \begin{cases} \sigma_A & \simeq \sigma \\ \langle S \rangle & \simeq S \end{cases}$$



$$S \ll \sigma \quad \left\{\sigma_\phi \simeq \frac{\pi}{\sqrt{3}}\left(1-\sqrt{\frac{9}{2\pi^3}\frac{S}{\sigma}}\right)\right.$$

$$S \gg \sigma \quad \left\{\sigma_\phi \simeq \frac{\sigma}{S}\right.$$

- On Phase
- Source detection is much easier on the phase than on the amplitude, since for S/N = 1, σ_ϕ = 1 radian = 60°.

Noise in Images

- The Fourier Transform is a linear combination of the visibilities with some rotation (phase factor) applied. How do we derive the noise in the image from that on the visibilities ?
- Noise on visibilities
  - the complex (or spectral) correlator gives the same variance on the real and imaginary part of the complex visibility $\langle \varepsilon_r^2 \rangle = \langle \varepsilon_i^2 \rangle = \langle \varepsilon^2 \rangle$
  - Real and Imaginary are uncorrelated $\langle \varepsilon_r \varepsilon_i \rangle = 0$
- So rotation (phase factor) has NO effect on noise

$$\varepsilon'_{\mathrm{R}} = \varepsilon_{\mathrm{R}} \cos(\phi) - \varepsilon_{\mathrm{I}} \sin(\phi)$$

$$\varepsilon'_{\mathrm{I}} = \varepsilon_{\mathrm{R}} \sin(\phi) + \varepsilon_{\mathrm{I}} \cos(\phi)$$

$$\langle {\varepsilon'_{\mathrm{R}}}^2 \rangle = \langle \varepsilon_{\mathrm{R}}^2 \rangle \cos^2(\phi) - 2\langle \varepsilon_{\mathrm{R}} \varepsilon_{\mathrm{I}} \rangle \cos(\phi) \sin(\phi) + \langle \varepsilon_{\mathrm{I}}^2 \rangle \sin^2(\phi) = \langle \varepsilon^2 \rangle$$

$$\langle \varepsilon'_{\mathrm{R}} \varepsilon'_{\mathrm{I}} \rangle = \langle \varepsilon_{\mathrm{R}}^2 \rangle \cos(\phi) \sin(\phi) - \langle \varepsilon_{\mathrm{I}}^2 \rangle \cos(\phi) \sin(\phi) = 0$$

Noise in Imaging: first order

- In the imaging process, we combine (with some weights) the individual visibilities V_i. At the phase center:

$$I = (\Sigma w_i V_i) / \Sigma w_i$$

- for a point source at phase center, $V_i = V + \varepsilon_{Ri}$, ε_{Ri} being the real part of the noise

$$I = (\Sigma w_i (V + \varepsilon_{Ri})) / \Sigma w_i$$

- So its expectation is $I = V$, as $< \varepsilon_{Ri} > = 0$
- As $<\varepsilon_{Ri}\ \varepsilon_{Rj}> = 0$, its variance is

$$\sigma^2 = <I^2> - <I>^2 = = (\Sigma w_i^2 < \varepsilon_{Ri}^2 >) / (\Sigma w_i)^2$$

- Now using $< \varepsilon_{Ri}^2 > = \sigma_i^2$ and the natural weights $w_i = 1/\sigma_i^2$ we have

$$1/\sigma^2 = \Sigma (1/\sigma_i^2)$$

- Which is true anywhere else in the image by application of a phase shift

Weighting and Tapering

- When using non-natural weights (w_i # σ_i^2), either as a result of Uniform or Robust weighting, or due to Tapering, the noise (for point sources) increases by w_{rms} / w_{mean}

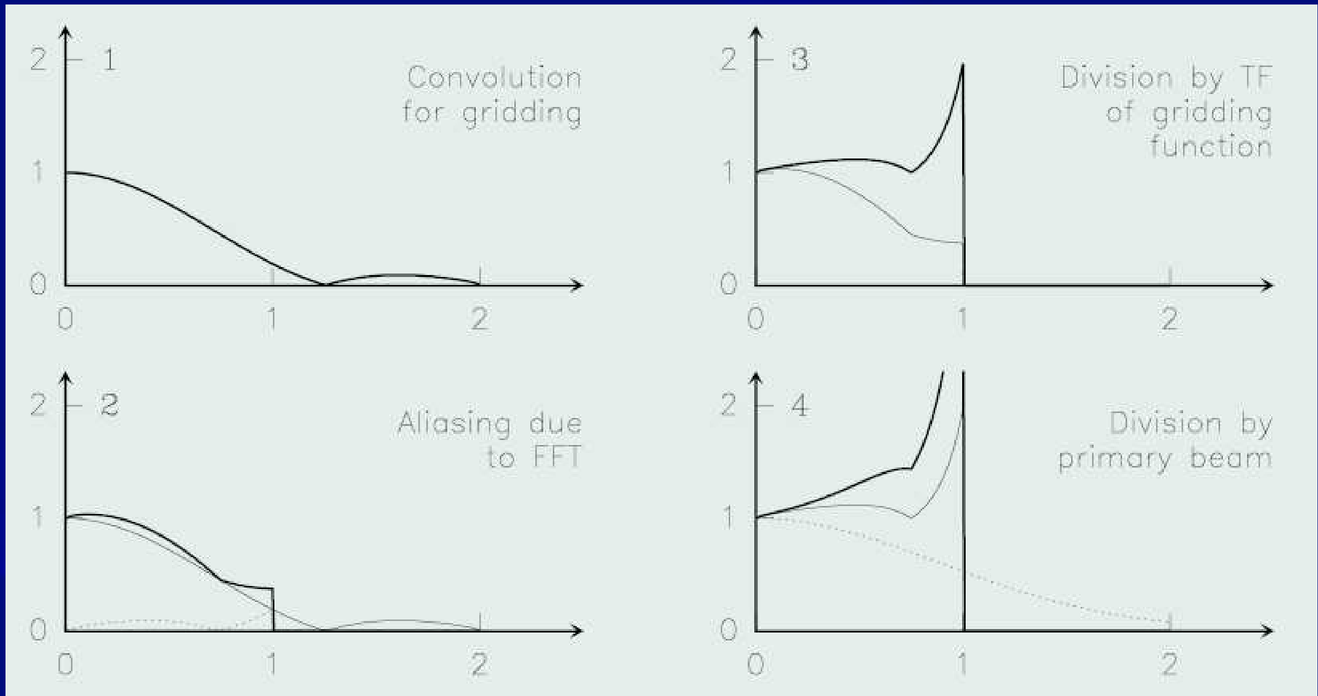
$$w_{rms} = ((\Sigma(WT)^2)/n)^{1/2}$$

$$w_{mean} = (\Sigma WT)/n$$

- Robust weighting improves angular resolution
- Tapering can be used to smooth data

Noise in Imaging

- Gridding introduces a convolution in UV plane, hence a multiplication in image plane
- Aliasing folds the noise back into the image
- Gridding Correction enhances the noise at edge
- Primary beam Correction even more...



Extended Source Sensitivity

- This is problematic. Here is the usual approach:
- We use brightness temperature for extended sources
- Use the flux to brightness conversion factor

$$S = \frac{2kT_b\Omega_s}{\lambda^2} = \frac{2kT_b\pi\theta_s^2}{4ln(2)\lambda^2}$$

for a synthesized beam of solid angle Ω_s (Gaussian of FWHM θ_s)

- Since from the antenna equation $\Omega_A A_{eff} = \lambda^2$, the flux noise equation

$$\Delta S = \frac{2kT_{sys}}{\eta_q A_{eff}\sqrt{n(n-1)\Delta\nu\Delta t}}$$

gives the brightness noise equation

$$\Delta T_b = \frac{\Omega_A}{\Omega_s}\frac{T_{sys}}{\eta_q\sqrt{n(n-1)\Delta\nu\Delta t}} = \left(\frac{\theta_p}{\theta_s}\right)^2 \frac{T_{sys}}{\eta_q\sqrt{n(n-1)\Delta\nu\Delta t}}$$

which is just a simple "beam dilution" formula applied to the standard noise for one antenna in total power, and accounting for n antennas.

Extended Source Sensitivity

$$\Delta T_b = \left(\frac{\theta_p}{\theta_s}\right)^2 \frac{T_{sys}}{\eta_q \sqrt{n(n-1)\Delta\nu\Delta t}}$$

- This is right only for sources just filling one synthesized beam θ_s.
- For more extended sources, it is not appropriate to count the number of synthesized beams n_b and divide by $\sqrt{n_b}$.
- This only gives a lower limit...
- Why ?
  - Averaging n_b beams is equivalent to smoothing
  - This is equivalent to tapering, i.e. to ignore the longest baselines...
  - This increases the noise ...
- Moreover, for very extended structures, missing flux may become a problem.

Bandwidth Effects

- The correlator channels have a non-square shape, i.e. their responses to narrow band and broad band signals differ.
- Hence the noise equivalent bandwidth $\Delta\nu_N$ is not the channel separation $\Delta\nu_C$, neither the effective resolution $\Delta\nu_R$
- These effects are of order 15-30 % on the noise.
- In practice, $\Delta\nu_N > \Delta\nu_C$, i.e. adjacent channels are correlated.
- Noise in one channel is less than predicted by the Noise Equation when using the channel separation as the bandwidth.
- But it does not average as $\sqrt{n_c}$ when using n_c channels...
- When averaging $n_c \gg 1$ i.e. many channels, the bandpass becomes more or less square: the effective bandwidth becomes $n_c \, \Delta\nu_C$.
- Consequence: There is no (simple) exact way to propagate the noise information when smoothing in frequency.
- Consequence: In GILDAS software, it is assumed $\Delta\nu_N = \Delta\nu_C = \Delta\nu_R$, and a $\sqrt{n_c}$ noise averaging when smoothing

Reweighting in Frequency ?

- The receiver bandpass is not flat: T_{sys} depends on ν
- Hence the weights depend on the channel number i
- When synthesizing broad band data, should we take the weights into account ?
- For pure continuum data
  - Yes: it improves S/N
  - But: ill-defined equivalent central frequency, and undefined equivalent detection bandwidth
  - so, may be: it depends on your scientific case...
  - Weighting could take into account a spectral index, for example…
- For line data
  - No: could degrade S/N if the line shape is not consistent with the weights
  - No: undefined bandwidth: does not allow to compute an integrated line flux
- In practice: not implemented in current GILDAS software. Could be useful for specific weak source searches. See "Optimal Filtering" later

Decorrelation

- Each visibility is affected by a random atmospheric phase
- Assuming a point source at the phase center, $V_i = V e^{i\phi_i} + \varepsilon_{Ri}$

$$I = (\sum w_i (V e^{i\phi_i} + \varepsilon_{Ri}))/(\sum w_i)$$

- the expectation of *I* is now only $V e^{-(\Delta\phi)^2/2}$
- The noise does not change,
- but the signal to noise is decreased.
- the Signal is spread around the source (seeing).
- So the effect is different for an extended source...
- This may limit the Dynamic range, and the effective noise level may be much higher than the thermal noise.
- The result depends on the source structure.
- There is so far no good simulation tool to evaluate the importance of this effect. It is not fully random at Plateau de Bure…

Estimating the Noise

- The weights are used to give a prediction of the noise level in the images.
- Predictions displayed by UV_MAP and UV_STAT
- Carried on in the image headers (aaa1%noise variable for an image displayed with GO MAP, GO NICE or GO BIT)
- but does not handle properly the noise equivalent bandwidth
- neither the effects of decorrelation...
- GO RMS will compute the rms level on the displayed image. May be biased by the source structure
- GO NOISE will plot an histogram of image values, and fit a Gaussian to it to determine the noise level. Will be less biased than GO RMS.
- Both GO NOISE and GO RMS will include dynamic range effects (i.e. give you the “true” noise of your image, rather than the theoretical).

Noise on Mosaics…

- GO NOISE does (yet) not work on mosaics…
- Because noise is NOT uniform on mosaics…
- $J = \Sigma B_i F_i / \Sigma B_i^2$
- Let us define $W = \Sigma B_i^2$
- If we instead use $L = J \; W^{1/2}$
  - The noise on L is uniform (provided all fields had similar noise) of value σ_L
  - It corresponds to the noise at the most sensitive place in the mosaic
  - L/σ_L is a signal-to-noise image
- Valid also for 1 field mosaic… L = F

Conclusions

- mm interferometry is not so difficult to understand
- even if you don't, the noise equation is all you need
- the noise equation

$$\Delta T_{\mathrm{b}} = \frac{T_{\mathrm{sys}}}{\eta n \sqrt{\Delta \nu t}} \left(\frac{\theta_{\mathrm{P}}}{\theta_{\mathrm{S}}} \right)^2$$

- allows you to check quickly if a source of given brightness T_b can be imaged at a given angular resolution θ_S and spectral resolution $\Delta \nu$ (n is the number of antennas, θ_P their primary beam width, and η an efficiency factor of order 0.5 – 0.8, and t the integration time…)
- T_{sys} is easy to guess: the simplistic value of 1 K per GHz of observing frequency is a good enough approximation in most cases.
- and *you* know T_b because you know the physics of your source!
- that is (almost) all you need to decide on the feasibility of an observation...

II – Low Signal to Noise

- When is a source detected ?
- What parameters can be derived ?

Low Signal to Noise

- A nice case
  - Observers advantage
    - You don't have to worry about bandpass & flux calibration…
  - Theorists advantage
    - The data is always compatible with your favorite model
- A necessary challenge
  - Mm interferometry is (almost) always sensitivity limited
  - But with proper analysis, you may still invalidate (falsify) some model/theory
- So let us see…

Low S/N -- Continuum

- Rule 1: do not resolve the source
- Rule 2: get the best absolute position before
- Rule 3: Use UV_FIT to determine the S/N ratio
- Rule 4: the 3-4-5 rule about position accuracy

< 1/10th of beam

- >3 σ signal for detection
- Fix the position
- Use an appropriate source size

About the beam

- >4 σ signal for detection
- Do not fix the position
- Use an appropriate source size

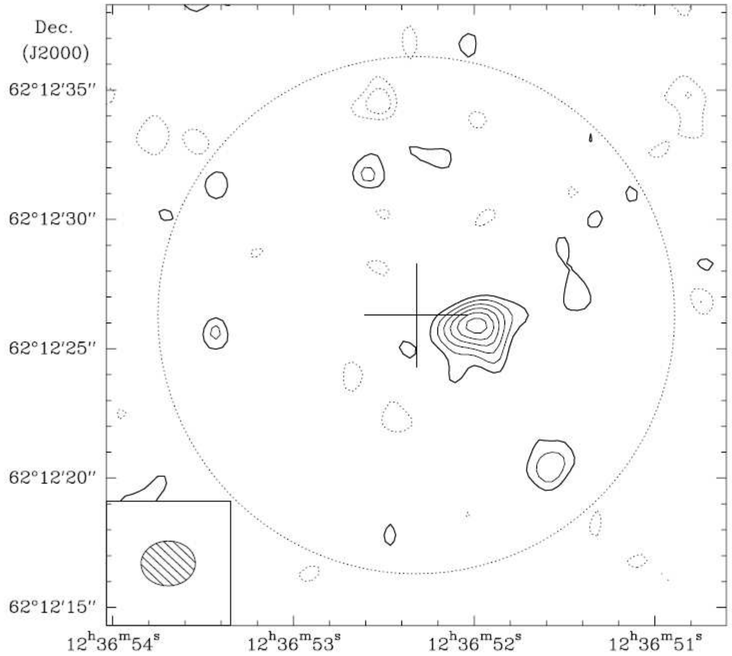
Unknown

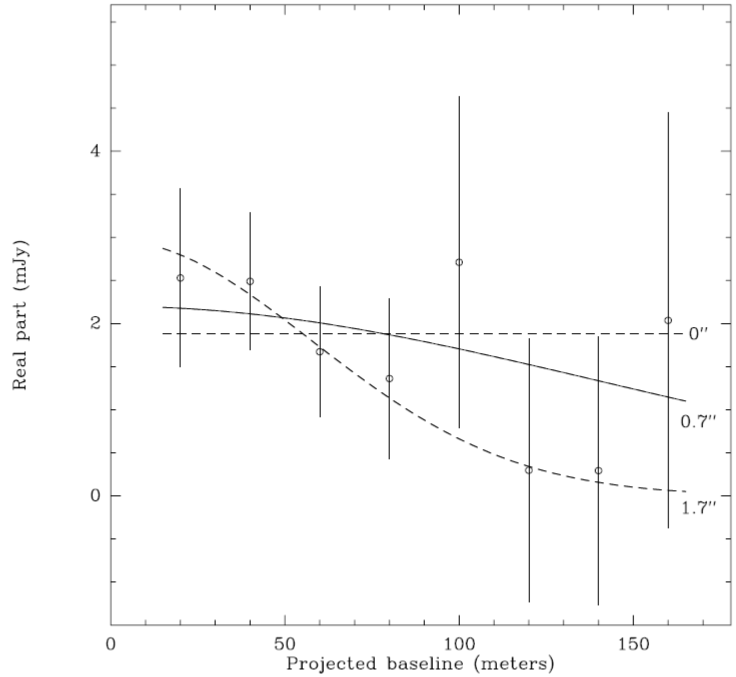
- 5 σ signal for detection
- make an image to locate
- Use as starting point
- Do not fix the position
- Use an appropriate source size

Continuum source parameters

- Sources of unknown positions have fluxes biased by 1 to 2 σ
- Free position 1 σ bias
- Position accuracy = beam/(S/N ratio)
- With < 6 σ , cannot measure any source size
  - divide data in two, shortest baselines on one side, longest on another. Each subset get a 4.2 σ error on mean flux.
  - Error on the difference is then just 3 σ, i.e. any difference must be larger than 33 % to be significant
  - Mean baseline length ratio for the subsets is at best 3.
  - No smooth source structure can give a visibility difference larger than 30 % on such a baseline range ratio.
- If size is free, error on flux increases quite significantly

Example: HDF source





- 7 σ detection of the strongest source in the Hubble Deep Field. Note that contours are *visually cheating* (start at 2 σ but with 1 σ steps).

Attempt to derive a size. Size can be as large as the synthesized beam... Note that the integrated flux increases with the source size.

Line sources: things get worse…

- Line velocity unknown: observer will select the brightest part of the spectrum ➔ bias
- Line width unknown: observer may limit the width to brightest part of the spectrum ➔ another bias
- If position is unknown, it is determined from the integrated area map (or visibilities) made from the tailored line window specified by the astronomer. This gives a biased total flux !.
- All these biases are positive (noise is added to signal).
- Any speculated extension will increase the total flux, by enlarging the selected image region (same effect as the tailored line window).
- Net result 1 to 2 σ positive bias on integrated line flux.
- *Things get really messy if a continuum is superposed to the weak line...*

Line sources: How ?

- Point source or unresolved source (< 1/3rd of the beam)
  - Determine position (e.g. from 1.3 mm continuum if available, or from integrated line map if not, or from other data)
  - Derive line profile by fitting point or small (fixed size), fixed position, source into UV spectral data
  - Gives you a flux as function of velocity/frequency
  - Fit this spectrum by Gaussian (with or without constant baseline offset, depending on whether the continuum flux is known or not)

Line sources: How ?

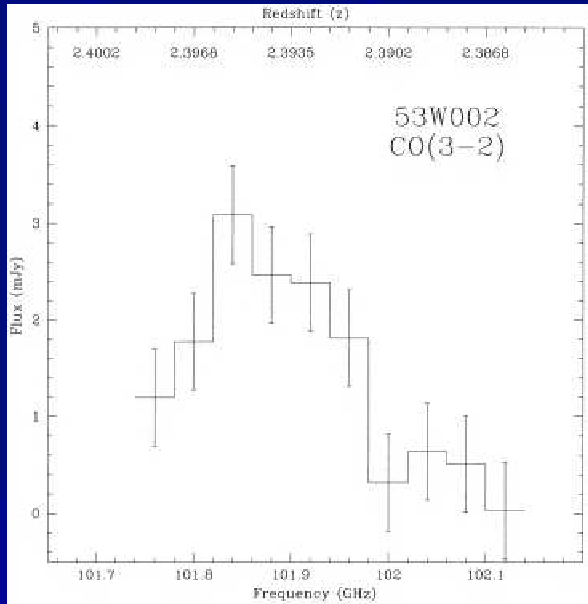
- Extended sources, and/or velocity gradient
  - Fit multi-parameter (6 for an elliptical gaussian) source model for each spectral channel into UV data
  - Consequence : signal in each channel should be >6 σ to derive any meaningful information.
  - Strict minimum is 4 σ (per line channel...) to get flux and position for a fixed size Gaussian
  - Velocity gradients not believable unless even better signal to noise is obtained per line channel...

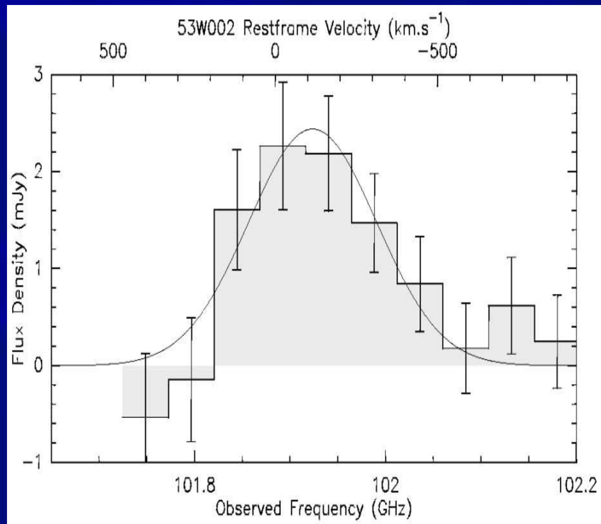
Line sources: Conclusions

- Do not believe velocity gradient unless proven at a 5 σ level. Requires a S/N larger than 6 in each channel. Remember that position accuracy per channel is the beamwidth divided by the signal-to-noise ratio...
- Do not believe source size unless S/N > 10 (or better)
- Expect line widths to be very inaccurate
- Expect integrated line intensity to be positively biased by 1 to 2 σ
- even more biased if source is extended
- These biases are the analogous of the Malmquist bias

Examples

- Examples are numerous, specially for high redshift CO.
- e.g. 53 W002 :
  - OVRO (Scoville et al. 1997) claims an extended source, with velocity gradient. Yet the total line flux is 1.5 $\pm$ 0.2 Jy.km/s i.e. (at best) only 7 σ .
  - PdBI (Alloin et al. 2000) finds a line flux of 1.20 $\pm$ 0.15 Jy.km/s, no source extension, no velocity gradient, different line width and redshift.
  - Note that the line fluxes agree within the errors...





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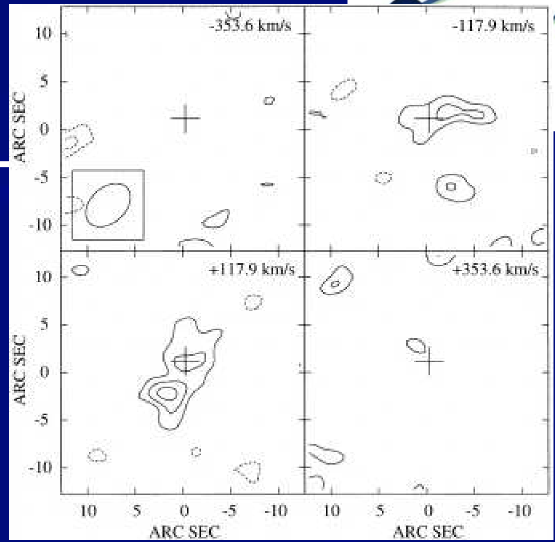
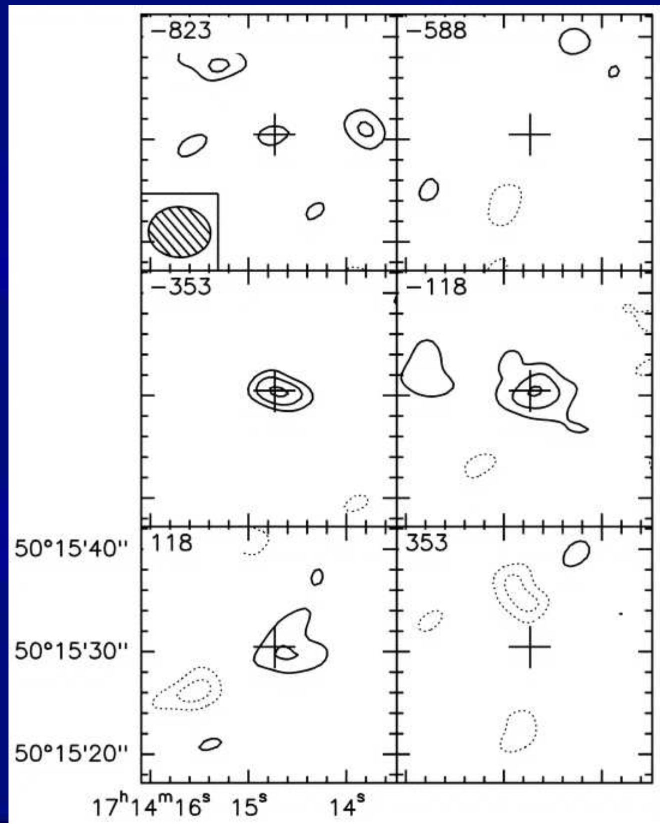
  - But the images (contours) look convincing !
  - Answer : beware of visually confusing contours which start at 2 σ (sometimes even 3), but are spaced by 1 σ





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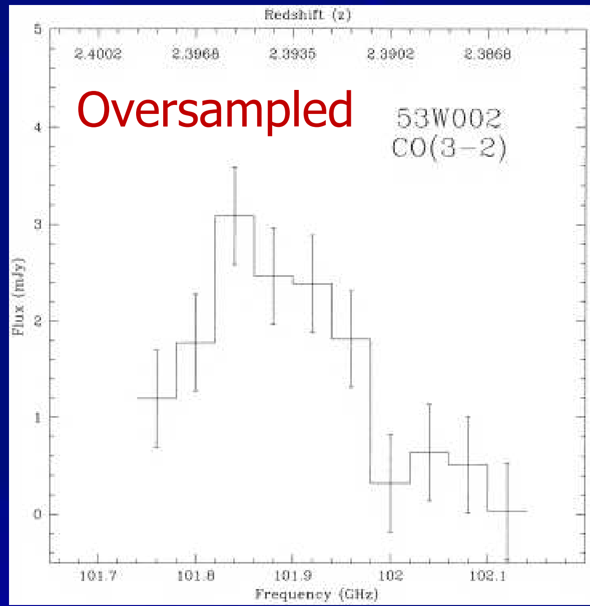

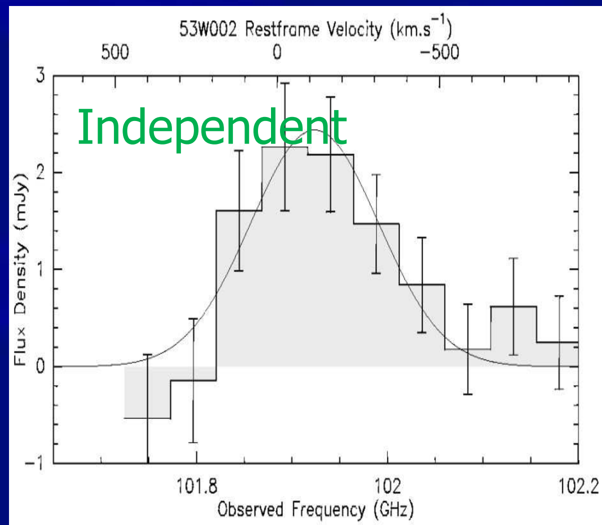

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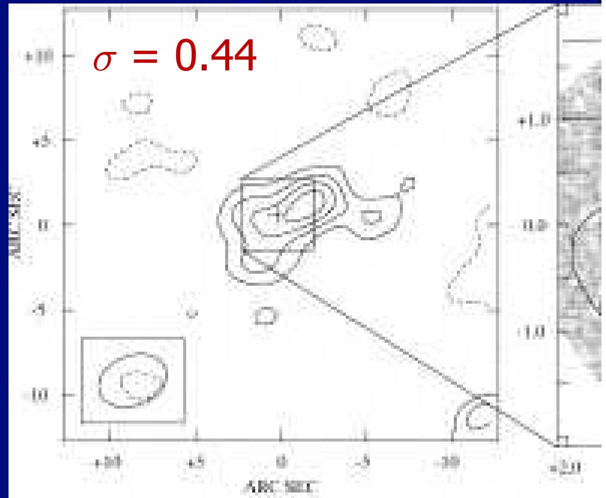
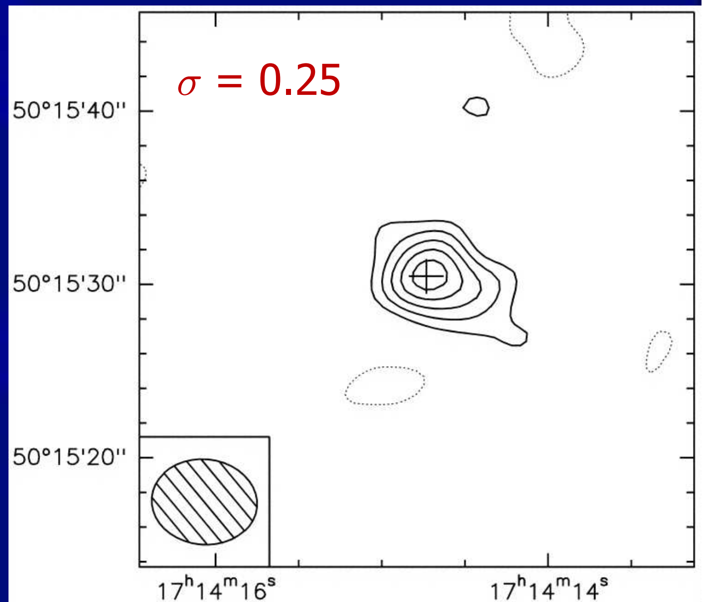
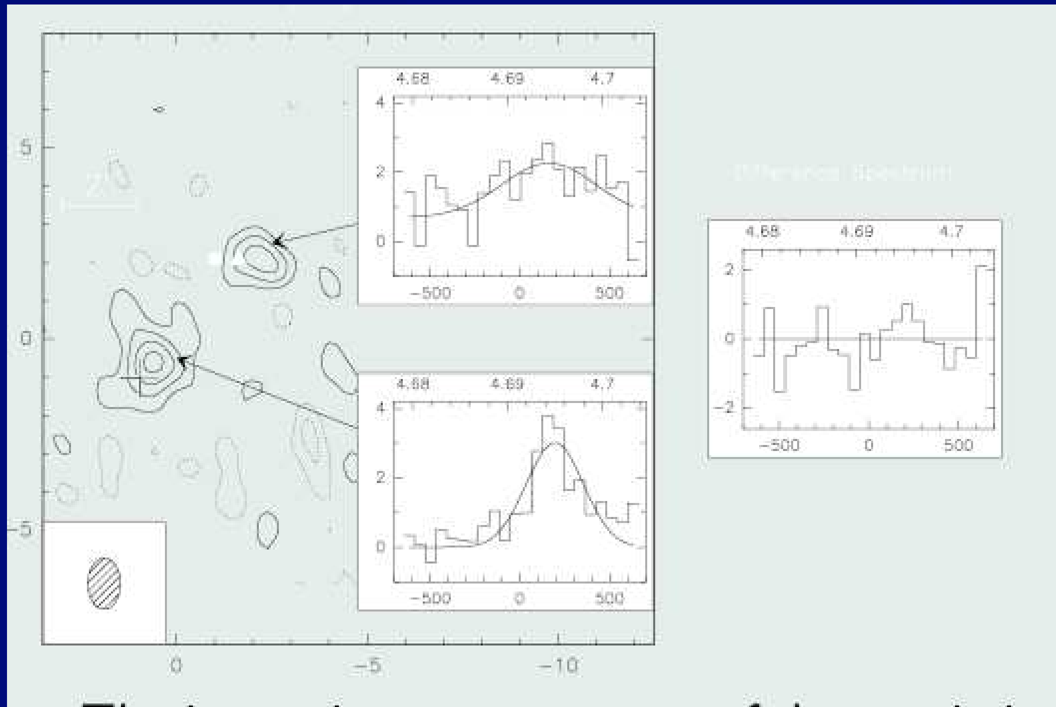




Example: (no) Velocity Gradients

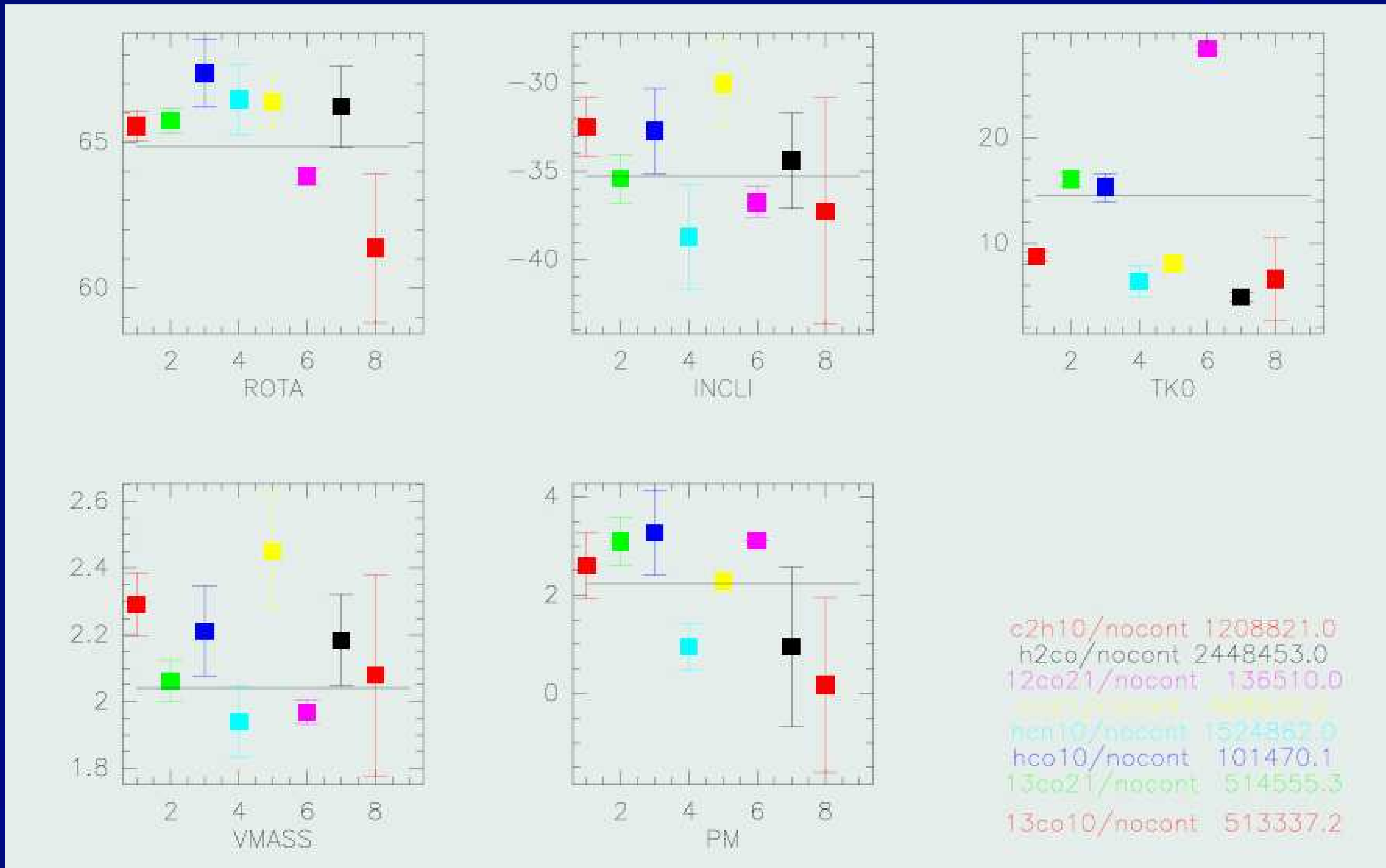


- Contour map of dust emission at 1.3 mm, with 2 σ contours
- The inserts are redshifted CO(5-4) spectra from the indicated directions
- A weak continuum (measured independently) exist on the Northern source
- The rightmost insert is a difference spectrum (with a scale factor applied, and continuum offset removed): No SIGNIFICANT PROFILE DIFFERENCE!
- i.e. No Velocity Gradient measured.

How to analyze weak lines ?

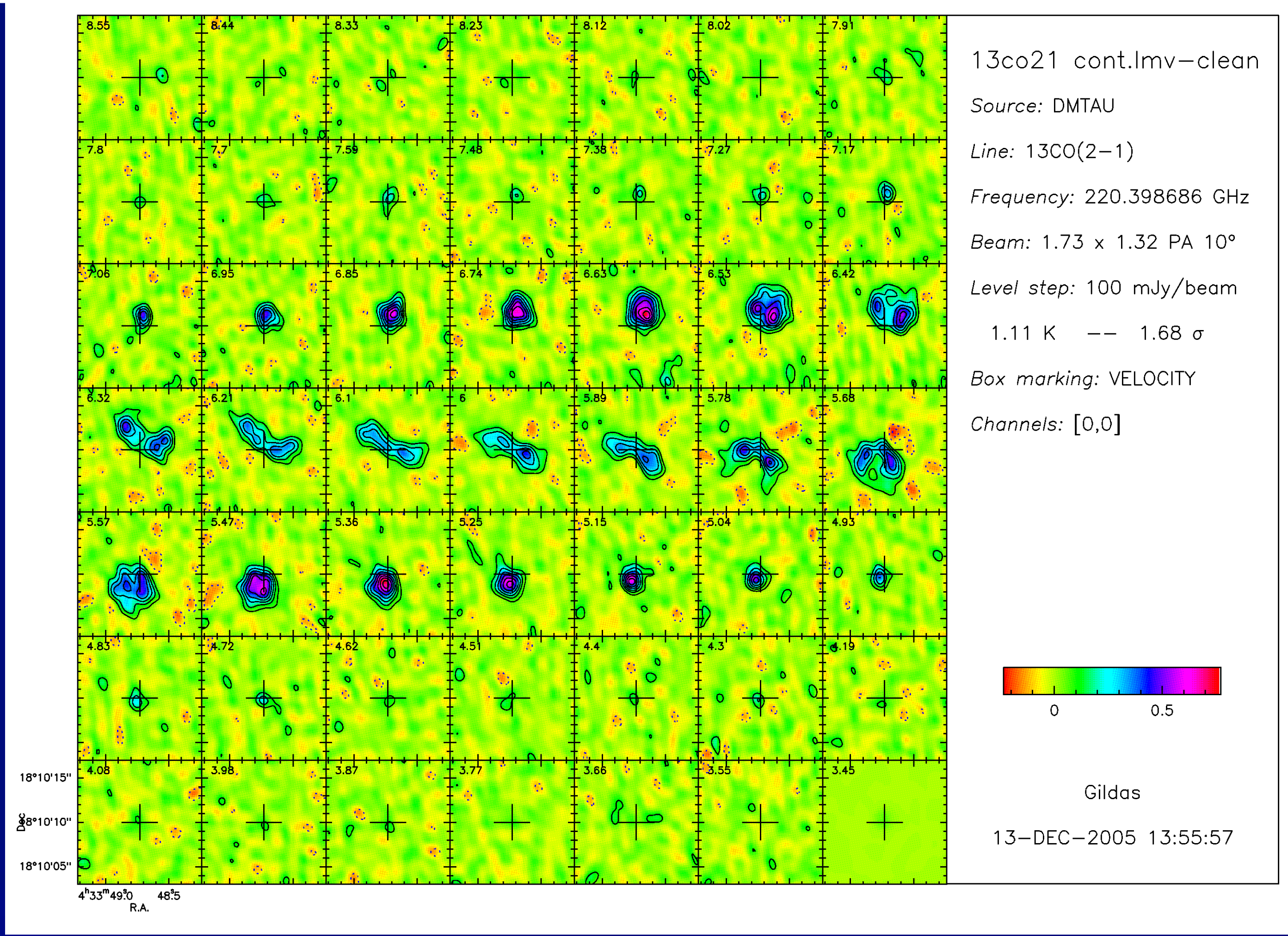
- Perform a statistical analysis (e.g. χ^2, or other statistical test) comparing model prediction to observations, i.e. *VISIBILITIES*
- The GILDAS software offer tools to compute visibilities from an image / data cube (task UV_FMODEL)
- Beware that (original) channels are correlated ($\Delta\nu_N > \Delta\nu_C$)
- Appropriate statistical tests can actually provide a better estimate of the noise level than the prediction given by the weights.
- Up to you to develop the model adapted to your science case (and select the proper statistical tool for your measurement).
- GILDAS even provides minimization tools: the ADJUST command (but with no guarantee of suitability to your case, though. Expertise recommended !).

Example of Analysis



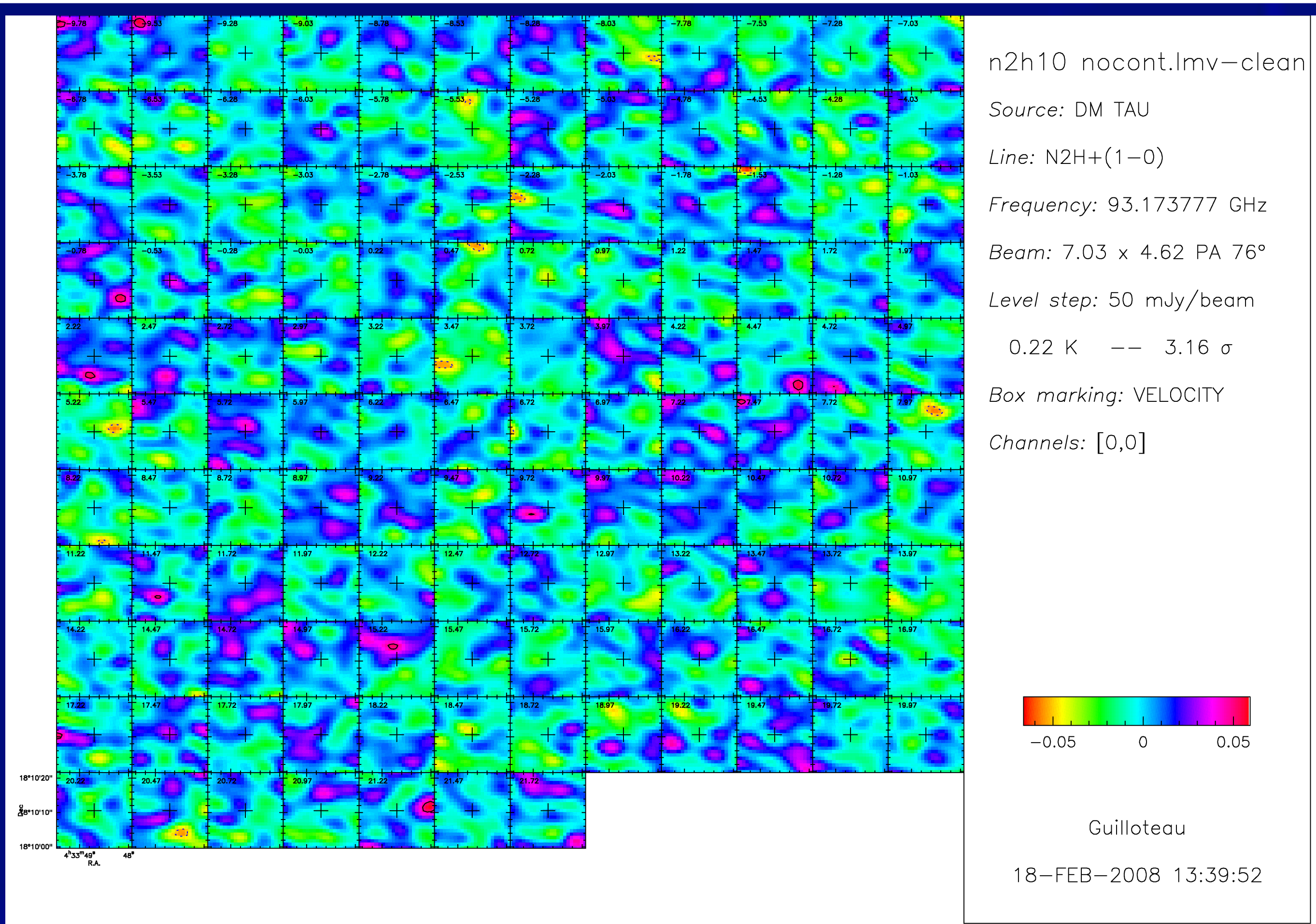
- Error bars derived from a χ^2 analysis in the UV plane, using a line radiative transfer model for proto-planetary disks.

Example of Analysis



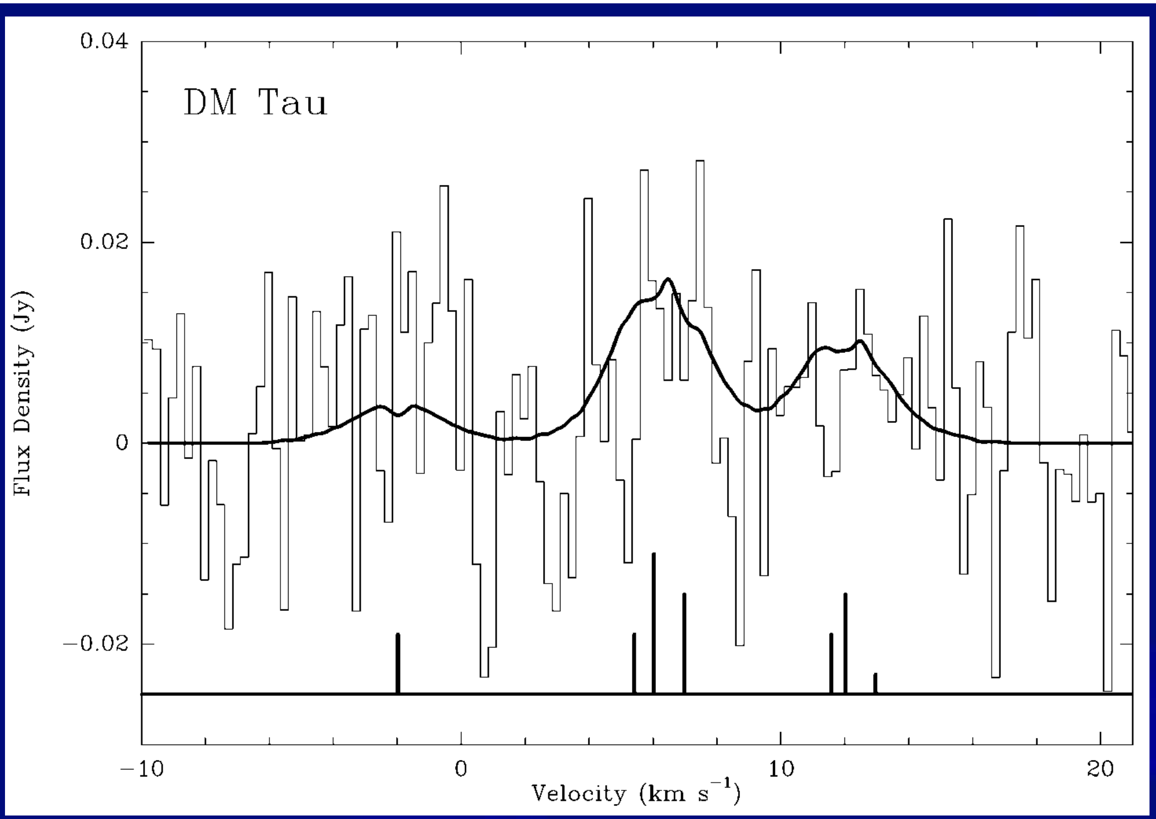
- A typical data cube from which the previous parameters were derived. It has quite decent S/N, and one can recognize the rotation pattern of a Keplerian disk

Example of Analysis



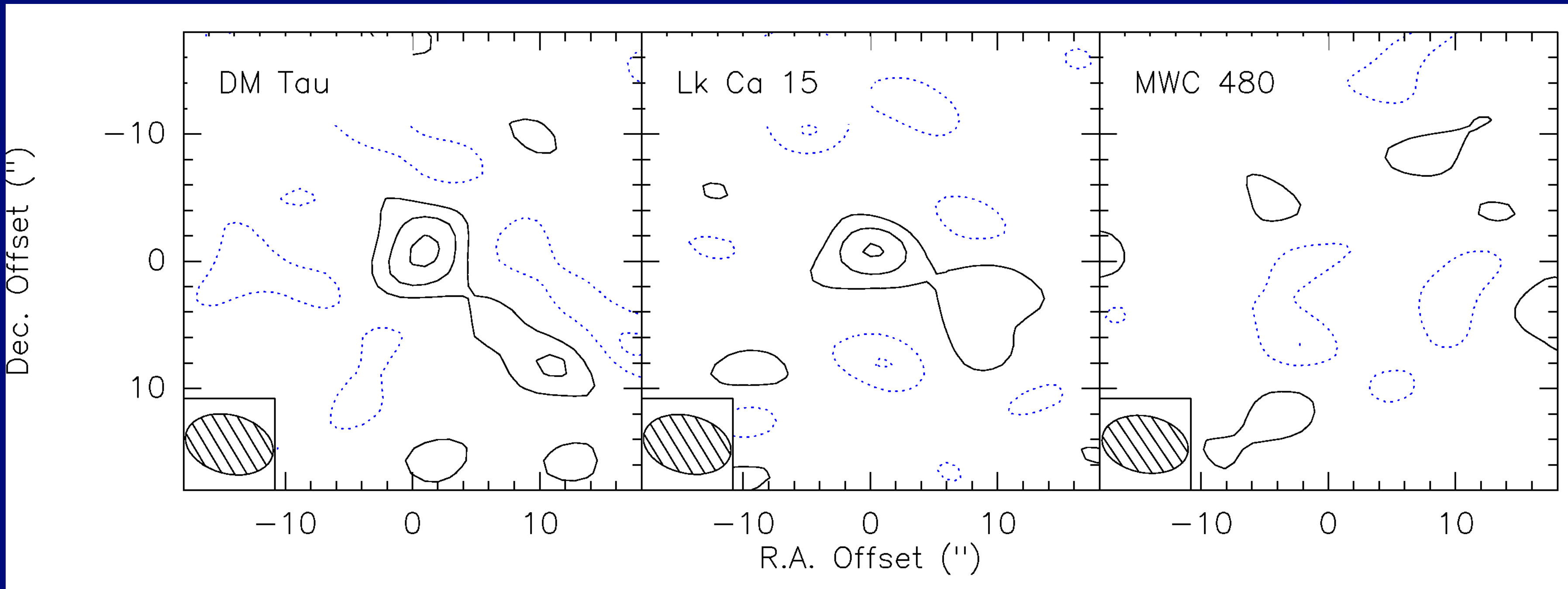
- A (really) low Signal to Noise image of the protoplanetary disk of DM Tau in the main group of hyperfine components of the N_2H^+ 1-0 transition.
- It really looks like absolute nothing... but a treasure is hidden inside the noise!

Example of Analysis



- Best fit integrated profile for the N_2H^+ 1-0 line, derived from a χ^2 analysis in the UV plane, using a line radiative transfer model for proto-planetary disks, assuming power law distributions, and taking into account the hyperfine structure.
- The observed spectrum is the integrated spectrum over a 6x6" area (from the Clean or Dirty image, does not really matter). The noise is about 11 mJy.

Example of Analysis



- Signal-to-noise maps of the integrated N_2H^+ 1-0 line emission, using the best profile derived from the χ^2 analysis in the UV plane as a (velocity) smoothing kernel (optimal filtering).
- 7 σ detection for DM Tau, 6 σ detection for LkCa 15
- Nothing for MWC 480

ALMA won’t (always) save you !

- ALMA is only 7 times more sensitive than PdB (at 3mm, better ratio at higher frequencies)
- on the N_2H^+ case, it will (in a mere 8 hours), obtain a peak 10 σ detection per channel, which is quite good, but will barely "see" the weakest hyperfine components.
- but if the resolution is increased just to 2”, the S/N will drop by a factor 3 (in this favorable case, as the structure remain unresolved in one direction...)
- and a search for the ^{15}N substitute remain beyond (reasonable) reach !.
- This is a simple molecule. Things a little more complex, e.g. HCOOH, HC_3N will be tough
- you can transpose this example for extragalactic studies

Optimal Filtering

- Changing the frequency dependence of weights and signal to adjust for a continuum spectral index
- Convolve by expected line profile for blind line search
- If line profile unknown, convolve by several possible ones, and see if one convolution leads to a significant signal

Stacking on weak sources

- Idea: you have N sources of known positions in your field
- hope to get √N improvement in S/N if all are identical
- « Shift and Add » in image plane
- But you do not deconvolve each source correctly (each has low S/N)
- So sidelobes may reduce the √N improvement
- To what extent ?
- Depends on
  - Source distribution
  - UV coverage
- E.g. extreme case 1 baseline, 2 sources just separated by the interfrange ➔ destructive interference, no signal at all !

Stacking on weak sources

- Equivalent to « Phase Rotate and Accumulate » in UV plane
  - For each source,
    - phase-shift the original UV table to the source position
    - Append the resulting visibilities to a common UV table
  - At the end, image that common UV table
- N times more visibilities ➔ $\sqrt{N}$ gain ?
- NO: they are linearly correlated (just a phase factor)
- Just a linear regression problem (even for mosaics)
  - Generate a model UV table
    - For each source and each field
    - Apply primary beam attenuation
    - Compute source visibility
    - Accumulate into model UV table
  - Linear fit to find the best scale factor to match the observations.
- This process gives the correct error estimate given the source distribution and UV coverage