Single-dish antenna at (sub)mm wavelengths

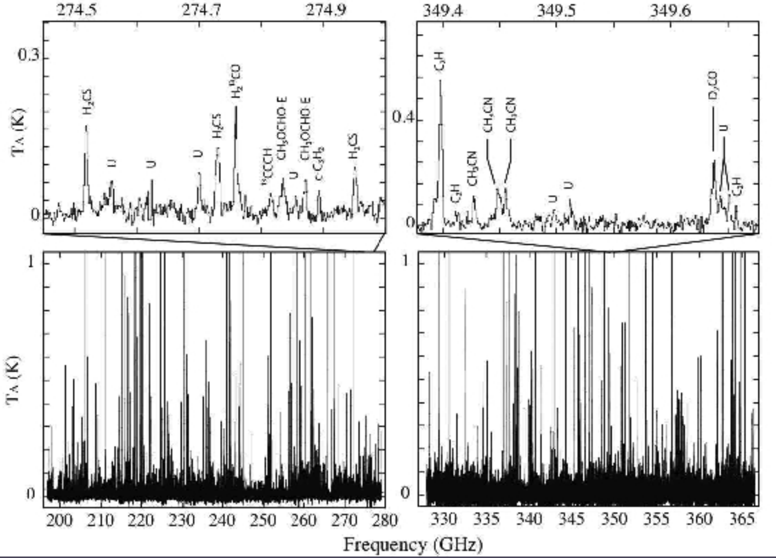
P. Hily-Blant
Institut de Planétologie et d'Astrophysique de Grenoble
Université Joseph Fourier

October 15, 2012

Introduction

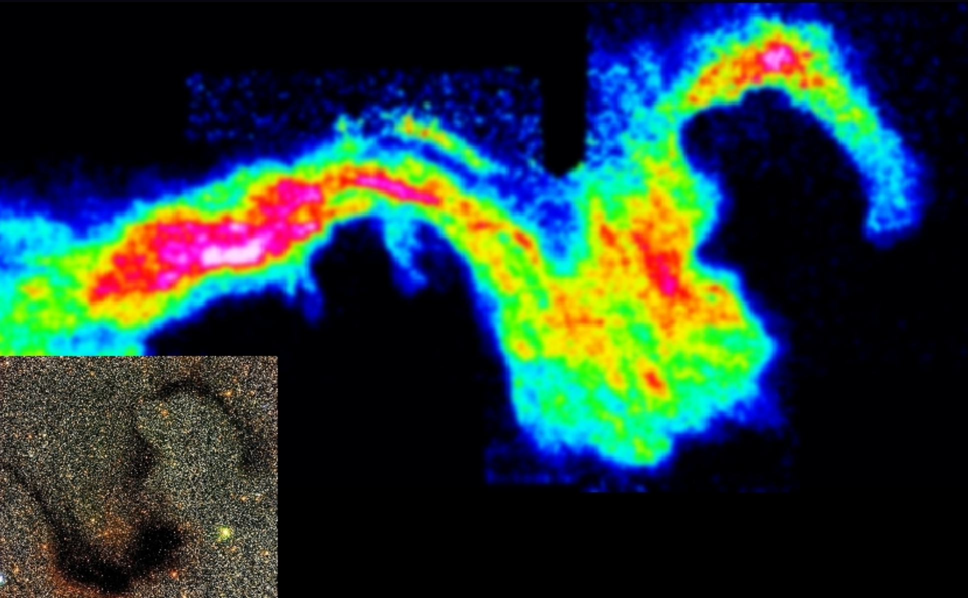
A single-dish antenna

Spectral surveys



Caux et al 2011 (IRAM spectral survey of I16293)

Spectro-imaging

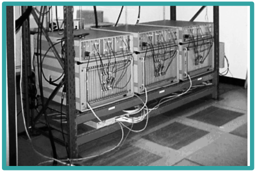


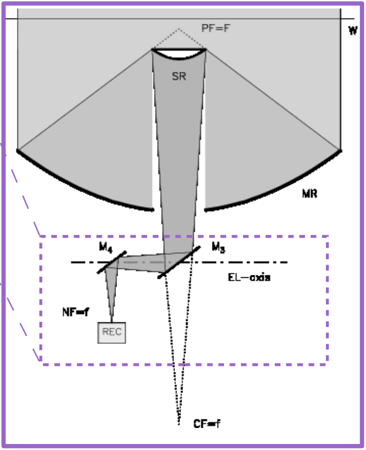
Few fW (1e-15 W)

Receiver cabin

Gain ~ 120 dB
Few mW

Backends





Frontends

Questions

Wishes

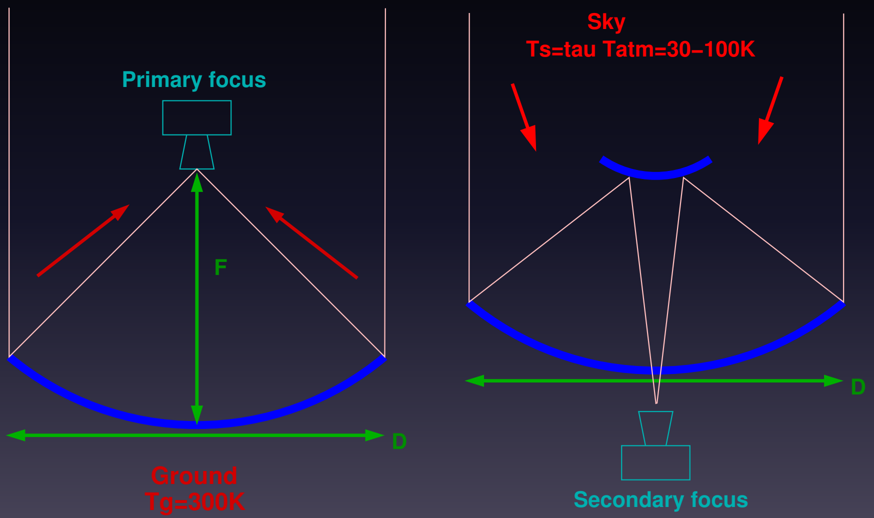
- Measure some power emitted in a (narrow) frequency range from a particular location
- Possibly want to make some (spectral/continuum) maps
- Eventually determine some chemical and/or physical properties

Questions

- Measurement fidelity ?
- Calibration (amplitude, frequency)
- Spatial resolution ?

General properties

Primary *vs* Secondary focus



(dis)Advantages

- $f/D \longrightarrow f_e/D = m \times f/D$
  IRAM-30m, $m = 27.8$
  $f/D = 0.35$, $f_e/D \approx 10$ or 300 m
- Rx alignement easier: $1''$ on the sky $\leftrightarrow f_e/206265$ mm in focal plane
- increase effective area (or on-axis gain)
- decrease spillover
- *but* increase mechanical load
- obstruction by subreflector ($\varnothing = 2$ m at 30-m) $\Rightarrow$ wider main-beam

World map of radiotelescopes

Main single-dish antenna at λmm

Large aperture: f/D$\lesssim$ 1

Obs.	D (m)	ν (GHz)	λ (mm)	HPBW ($''$)	Latitude (deg)
IRAM	30	$70 - 345$	$4 - 0.7$	$35 - 7$	$+37$
APEX	12	$230 - 1200$	$1.3 - 0.3$	$30 - 6$	-22
JCMT†	15	$210 - 710$	$2 - 0.2$	$20 - 8$	$+20$
CSO†	10.4	$230 - 810$	$1.3 - 0.4$	$30 - 10$	$+20$
Herschel†	3.5	500–2000	$0.6 - 0.1$	$43 - 11$	space

Terminology (1): receivers

- Central frequency

$$\nu_0 = 80 - 2000\,\mathrm{GHz}$$

- Instantaneous bandwidth:

$$\Delta\nu = 1 - 32\,\mathrm{GHz}$$

- bolometers

$$\Delta\nu \approx 50\,\mathrm{GHz}$$

- one polarization (linear, circular)
- taper (apodization at the rim)

Terminology (2): backends

- Spectrometers:
  - digital: autocorrelators (AC), Fast Fourier Transform Spectrometer (FTS)
  - *(analogical: filter banks (FB), acousto-optic (AOS))*
- Spectral resolution

$$\delta\nu \approx 3 - 2000\,\mathrm{kHz}$$

- Large resolution power

$$R = \nu_0/\delta\nu \approx 10^5 - 10^8$$

Collected power from a source

Consider an unpolarized point source

- Monochromatic (and monomode) power collected by an area A_e:

$$p_\nu = \frac{1}{2} A_e \cdot S_\nu \qquad [\mathrm{W\,Hz^{-1}}]$$

- Flux density S_ν measured in Jy:

$$1\mathrm{Jy} = 10^{-26}\,\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$$

- Power in the bandwidth $\Delta\nu$:

$$p = \frac{1}{2} A_e \cdot S_\nu \cdot \Delta\nu \qquad [\mathrm{W}]$$

- Note: A 1 Jy source observed with a $10^3\ \mathrm{m}^{-2}$ radiotelescope during 40 yrs (*e.g.* Orion) with 50 MHz bandwidth $\Rightarrow \approx$ 40 GeV

Collected power from a source

Consider an unpolarized extended source

- Monochromatic (and monomode) power collected by an area A_e from solid angle $\delta\Omega$:

$$\delta p_\nu = \frac{1}{2} A_e \cdot I_\nu \cdot \delta\Omega \qquad [\mathrm{W\,Hz^{-1}}]$$

- Brightness I_ν measured in $\mathrm{Jy\,sr^{-1}}$:

$$1\,\mathrm{Jy\,sr^{-1}} = 10^{-26}\,\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}$$

Collected power from a source

Effective area of the antenna: $A_e = \eta A_{\mathrm{geom}}$

$\eta < 1$

Question: $\eta = ?$

Perfect antenna

Emitted power by an antenna

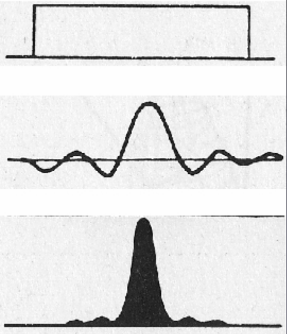
- Diffraction theory (Huygens-Fresnel, Fraunhoffer approx.):

$$E_{\mathrm{f-f}}(l,m) \propto \mathcal{F}[E_{\mathrm{ant}}(x,y)]$$

- $E_{\mathrm{ant}}(x,y)$ (grading): bounded on a finite domain Δr
  $\Rightarrow$ $E_{\mathrm{f-f}}(l,m)$ concentrated on a finite domain $\Delta\Omega$
  $(\Delta r \cdot \Delta\Omega \sim 1)$
- sharp cut of the antenna domain $\Rightarrow$ oscillations (side-lobes)
- apodization or taper: decrease the level of the sidelobes, to the cost of increasing $\Delta\Omega$

Antenna power pattern

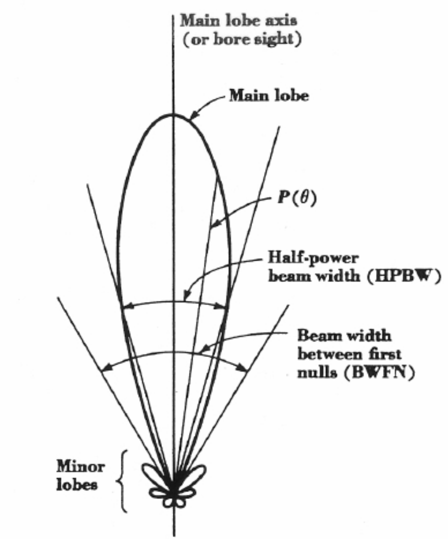
- Reciprocity: antenna in emission
- Distribution of electric field on the dish: $E_{\mathrm{ant}}(x,y)$
- Far-field radiated by the dish: $E_{\mathrm{f-f}}(l,m) \propto \mathcal{F}[E_{\mathrm{ant}}(x,y)]$
- Power emitted is a function of direction: $\propto |E_{\mathrm{f-f}}(l,m)|^2$



- Power pattern: $\mathcal{P}(l,m) \propto |E_{\mathrm{f-f}}(l,m)|^2$
- Beam solid angle: $\Omega_A = \int_{4\pi} \mathcal{P}(\Omega)\,\mathrm{d}\Omega \leq 4\pi$
- Effective area: $A(l,m) = A_e \cdot \mathcal{P}(l,m) \leq A_e$
- Fundamental relation:

$$\boxed{A_e \Omega_A = \lambda^2}$$

Power pattern $\mathcal{P}(l, m)$

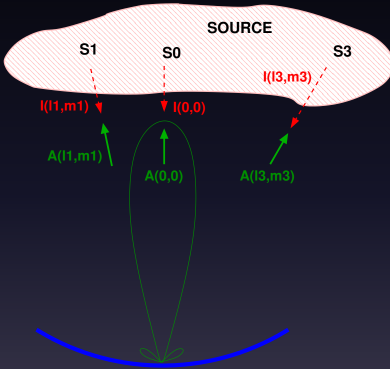


Power collected by an antenna (2)

Given a source of brightness $I_\nu(l,m) = I_\nu(\Omega)$

- Source flux density:
  $S_\nu = \int_{\Omega_s} I_\nu(\Omega)\, \mathrm{d}\Omega$
- Observed flux density:

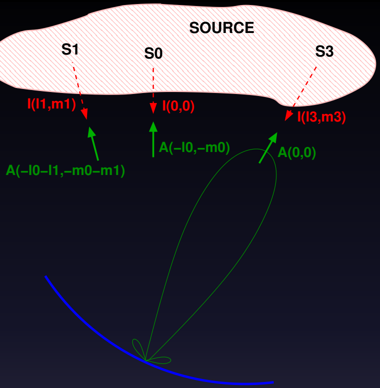
$$\boxed{S_{\mathrm{obs}} = \int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega < S_\nu}$$



- Power received from $\mathrm{d}\Omega_i$:
  $\mathrm{d}p_\nu = \frac{1}{2} A(\Omega_i)\, I_\nu(\Omega_i)\, \mathrm{d}\Omega_i$
- Incoherent emission: add intensities

Pointing towards a fixed position of the source at a fixed position

$$\boxed{p_\nu(l=0, m=0) = \frac{A_e}{2} \int_{\Omega_S} \mathcal{P}(\Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega = \frac{1}{2} A_e S_{\mathrm{obs}}}$$



- Antenna tilted towards $\Omega_0 = (l_0, m_0)$
- Power received from the direction Ω_i

$$\mathrm{d}p_\nu(\Omega_i) = \frac{1}{2} A(\Omega_0 - \Omega_i)\, I_\nu(\Omega_i)\, \mathrm{d}\Omega_i$$

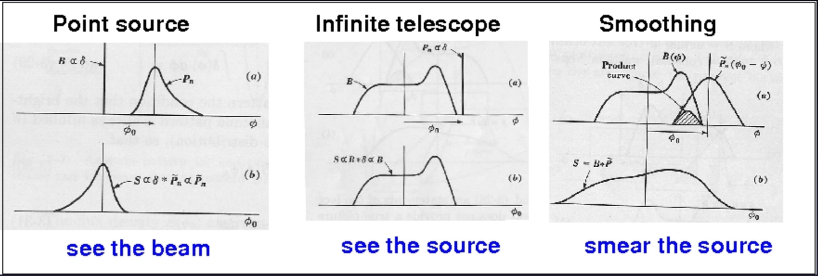
- Incoherent emission: add intensities

Scanning a source leads to a convolution

$$\boxed{S_{\rm obs}(\Omega_0) = \int_{\Omega_S} \mathcal{P}(\Omega - \Omega_0) I_\nu(\Omega)\, \mathrm{d}\Omega}$$

$$\boxed{p_\nu(\Omega_0) = \frac{A_e}{2} \int_{\Omega_S} \mathcal{P}(\Omega_0 - \Omega)\, I_\nu(\Omega)\, \mathrm{d}\Omega = \frac{1}{2} A_e S_{\rm obs}(\Omega_0)}$$

Convolution: consequences



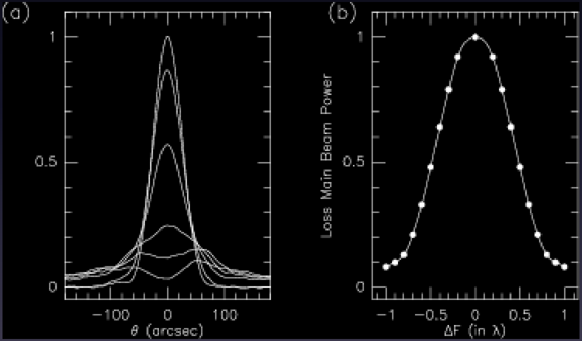
$$\boxed{\theta_{\rm obs} = \sqrt{\theta^2_{\rm mb} + \theta^2_{\rm sou}}}$$

Note: When quoting sizes from observations, must quote the deconvolved size (when needed).

Real antenna

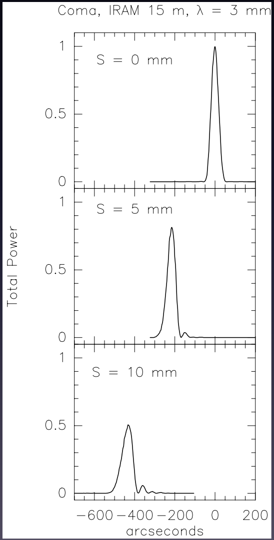
Systematic deformations (1)

Defocus



Systematic deformations (2)

Coma: misaligned subref.



Systematic deformations (3)

Astigmatism

$\Rightarrow$Beam deformation (no pointing error)

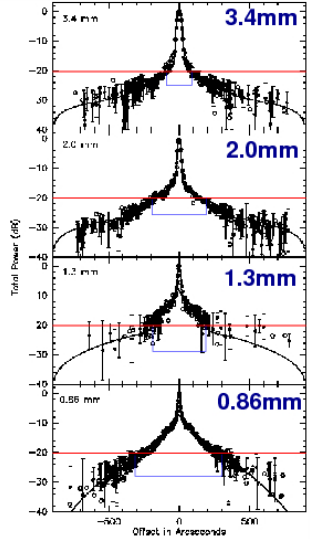
Beam pattern

- Main lobe
- Secondary lobes (finite surface antenna)
- Error lobes (surface irregularities)
- main-beam collects less power
- if correlation length ℓ
  $\Rightarrow$ Gaussian error-beam
  $\Theta_{\rm EB} \approx \lambda/\ell$

  real beam = main-beam + error-beam(s)
- Questions:

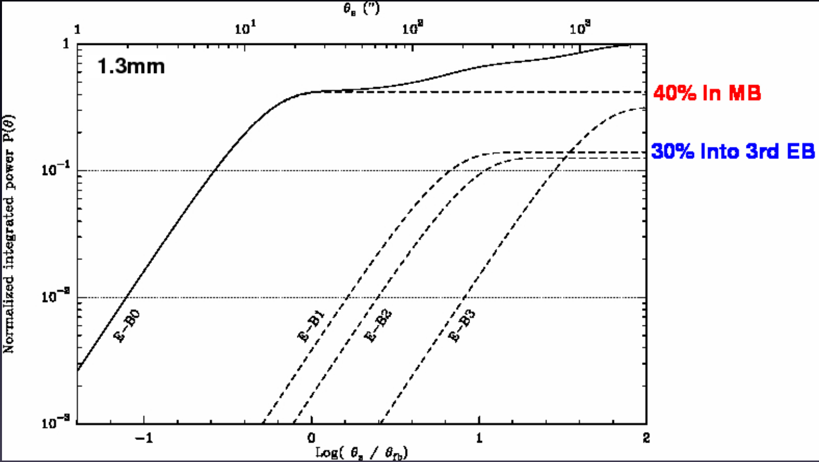
  Power collected in each e-beam ?

  FWHMs of the e-beams ?



Greve et al 1998

Error-Beams at IRAM-30m



Temperature scales

THE ASTROPHYSICAL JOURNAL, 161: L43–L44, July 1970

CARBON MONOXIDE IN THE ORION NEBULA

R. W. WILSON, K. B. JEFFERTS, AND A. A. PENZIAS
Bell Telephone Laboratories, Inc., Holmdel, New Jersey, and
Crawford Hill Laboratory, Murray Hill, New Jersey
Received 1970 June 5

ABSTRACT

We have found intense 2.6-mm line radiation from nine galactic sources which we attribute to carbon monoxide.

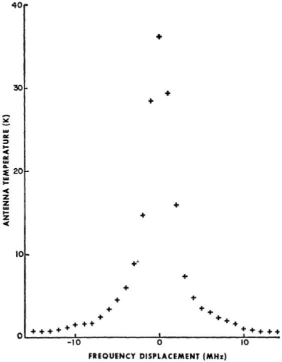


FIG. 1

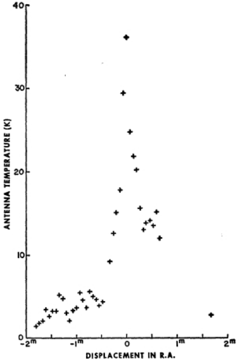


FIG. 2

FIG. 1.—Spectrum of CO radiation in the Orion Nebula made with the NRAO forty-channel line receiver. The center frequency is 115, 267.2 MHz.

FIG. 2.—Distribution in right ascension of the peak antenna temperature of CO radiation at a declination of $-5°24'21''$.

Antenna temperature: T_A

- Johnson noise in terms of an equivalent temperature: the average power transferred from a conductor (in thermal equilibrium) to a line within $\delta\nu$: $\delta p = k\, T\, \delta\nu$
- Antenna temperature defined by

$$p_\nu = kT_A \qquad [\mathrm{W \cdot Hz^{-1}}] = [\mathrm{J}] = [\mathrm{J \cdot K^{-1}}][\mathrm{K}]$$

- On the other hand: $p_\nu = \frac{A_e}{2}\left(\mathcal{P} * I_\nu\right) = \frac{\lambda^2}{2\Omega_A}\left(\mathcal{P} * I_\nu\right)$
- Antenna temperature:

$$\boxed{T_A(\Omega_0) = \frac{A_e}{2k} \int_{\Omega_S} I_\nu(\Omega)\mathcal{P}(\Omega - \Omega_0)\,\mathrm{d}\Omega}$$

- Using $A_e\Omega_A = \lambda^2$, we may write:

$$\boxed{T_A(\Omega_0) = \frac{1}{\Omega_A} \int_{\Omega_S} \frac{\lambda^2}{2k} I_\nu(\Omega)\mathcal{P}(\Omega - \Omega_0)\,\mathrm{d}\Omega}$$

Speaking in terms of temperatures

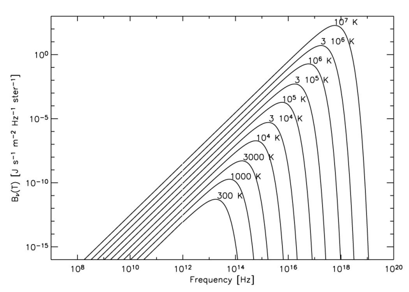
Black-body radiation at temperature T: $B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$

- Dimensions of brightness: $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}$
- Rayleigh-Jeans approximation: $h\nu \ll kT$

$$\frac{\lambda^2}{2k} B_\nu(T) \approx T$$

- Flux density of a black-body:

$$S_\nu = \int_{\Omega_S} B_\nu(T,\Omega)\,\mathrm{d}\Omega = 4\pi B_\nu(T)$$



Definitions: T_B, T_R

- *Brightness temperature* T_B of source brightness I_ν:

$$\boxed{I_\nu(\Omega) = B_\nu(T_\mathrm{B})}$$

- *Radiation temperature*, T_R, in the Rayleigh-Jeans regime approximation

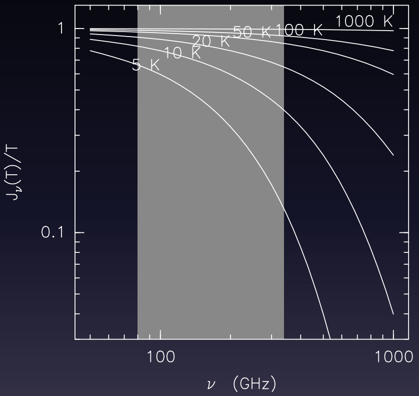
$$I_\nu(\Omega) = \frac{2k\nu^2}{c^2} T_\mathrm{R}(\Omega) = \frac{2k}{\lambda^2} T_\mathrm{R} \qquad [\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}]$$

- Relationship between T_B and T_R:

$$T_\mathrm{R} = J_\nu(T_\mathrm{B}) = \frac{h\nu}{k} \frac{1}{\exp(h\nu/kT_\mathrm{B}) - 1} = \frac{T_0}{\exp(T_0/T_\mathrm{B}) - 1}$$

- In the following: $I_\nu(\Omega) \rightarrow T_\mathrm{R}(\Omega)$

R-J approximation in the (sub)mm



$$\frac{2k}{\lambda^2} J_\nu(T_\mathrm{B}) = B_\nu(T_\mathrm{B})$$

Consequences

- Monochromatic power received by the antenna:

$$\boxed{p_\nu(\Omega_0) = \frac{k}{\Omega_A} \int_{\Omega_S} \mathcal{P}(\Omega)\, T_\mathrm{R}(\Omega_0 - \Omega)\, \mathrm{d}\Omega}$$

- Observed flux density ($p_\nu = 1/2 A_e S_\nu$)

$$S_\mathrm{obs}(\Omega_0) = \frac{2k}{\lambda^2} \int_{\Omega_S} \mathcal{P}(\Omega)\, T_\mathrm{R}(\Omega_0 - \Omega)\, \mathrm{d}\Omega$$

Antenna temperature: T_A

- Antenna temperature:

$$\boxed{T_A(\Omega_0) = \frac{A_e}{\lambda^2} \int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{R}}(\Omega - \Omega_0)\, \mathrm{d}\Omega}$$

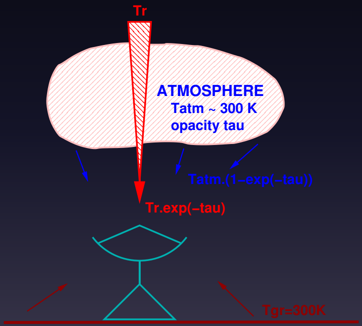
- Using $A_e \Omega_A = \lambda^2$, we may write:

$$\boxed{T_A(\Omega_0) = \frac{1}{\Omega_A} \int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{R}}(\Omega - \Omega_0)\, \mathrm{d}\Omega}$$

- Note that:

$$T_A(\Omega_0) = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{R}}(\Omega - \Omega_0)\, \mathrm{d}\Omega}{\int_{4\pi} \mathcal{P}(\Omega)\, \mathrm{d}\Omega}$$

From T_A to T'_A



- At frequency ν:

$$\begin{aligned} T_A &= \eta_s \left\{ T_\mathrm{R} e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) T_\mathrm{atm} \right\} \\ &+ (1 - \eta_s) T_\mathrm{gr} \end{aligned}$$

- Correct for **atmospheric attenuation**:

$$\boxed{T'_A = T_A e^{\tau_\nu}}$$

- *Note: for space-based telescopes (e.g. HIFI/Herschel):* $T'_\mathrm{A} = T_\mathrm{A}$

From T'_{A} to T^*_{A}

- Correct for rear-sidelobes: measure the monochromatic power recieved from the forward 2π sr. Hence, $\Omega_A = \mathcal{P}_{4\pi} \to \mathcal{P}_{2\pi} = \int_{2\pi} \mathcal{P}(\Omega)\,\mathrm{d}\Omega$:

$$T^*_{\mathrm{A}}(\Omega_0) = \frac{T'_{\mathrm{A}}}{F_{\mathrm{eff}}} = \frac{1}{\mathcal{P}_{2\pi}} \int_{\Omega_S} \mathcal{P}(\Omega)\, T_{\mathrm{R}}(\Omega_0 - \Omega)\,\mathrm{d}\Omega$$

$$\boxed{T^*_{\mathrm{A}} = \frac{e^{-\tau_\nu}}{F_{\mathrm{eff}}} T_A}$$

- Forward efficiency: $F_{\mathrm{eff}} = \mathcal{P}_{2\pi}/\mathcal{P}_{4\pi}$

From T'_A to T_mb

- Take into account main-beam and error-lobes
- Same as T^*_A but in Ω_mb instead of 2π. Hence, $\Omega_A \rightarrow \mathcal{P}_\mathrm{mb} = \int_\mathrm{mb} \mathcal{P}(\Omega)\,\mathrm{d}\Omega$:

$$T_\mathrm{mb}(\Omega_0) = \frac{T'_\mathrm{A}}{B_\mathrm{eff}} = \frac{\int_{\Omega_S} \mathcal{P}(\Omega)\; T_\mathrm{R}(\Omega_0 - \Omega)\,\mathrm{d}\Omega}{\mathcal{P}_\mathrm{mb}},$$

- Beam efficiency: $B_\mathrm{eff} = \mathcal{P}_\mathrm{mb}/\mathcal{P}_{4\pi}$
- Useful relation for a Gaussian beam:

$$\Omega_\mathrm{mb} = \int_\mathrm{mb} \mathcal{P}(\Omega)\,\mathrm{d}\Omega = 1.133\,\theta^2_\mathrm{mb}$$

Temperature scales

Definitions

$$\text{Forward efficiency:} \quad F_{\text{eff}} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{4\pi}}$$

$$\text{Beam efficiency:} \quad B_{\text{eff}} = \frac{\mathcal{P}_{\text{mb}}}{\mathcal{P}_{4\pi}}$$

Consequences

$$T_{\text{mb}} = \frac{F_{\text{eff}}}{B_{\text{eff}}} \; T_{\text{A}}^{*} = \frac{\mathcal{P}_{2\pi}}{\mathcal{P}_{\text{mb}}} \; T_{\text{A}}^{*}$$

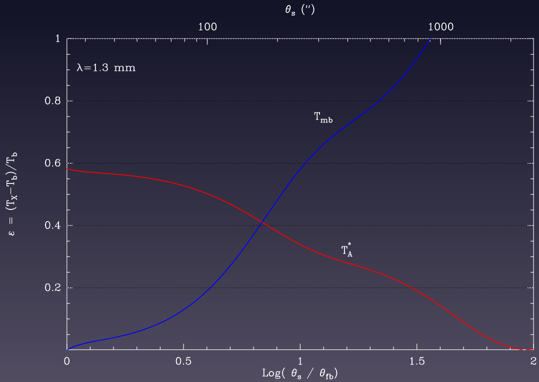
What you measure is T_{A}^{*} or T_{mb} (usually $\neq T_{\text{R}}$)

Limiting cases

- Small sources: $\Omega_S \ll \Omega_{\rm mb}$
  - $T_{\rm mb} \approx \frac{P(0)\Omega_S T_{\rm R}}{\mathcal{P}_{\rm mb}} = T_{\rm R}\frac{\Omega_S}{\mathcal{P}_{\rm mb}}$
  - Gaussian sources & beam: $\int_{\Omega_S} \mathcal{P}(\Omega)\, T_R(\Omega_0 - \Omega)\,\mathrm{d}\Omega = 1.133(\theta^2_{\rm sou} + \theta^2_{\rm mb})$ and $\mathcal{P}_{\rm mb} = 1.133\theta^2_{\rm mb}$ hence $T_{\rm R} = T_{\rm mb}\frac{\theta^2_{\rm mb}}{\theta^2_{\rm sou}+\theta^2_{\rm mb}}$: beam dilution
- Large sources: $\Omega_S \gg \mathcal{P}_{\rm mb}$
  - $T^*_{\rm A} \approx T_{\rm R}\frac{\int_{2\pi} P(\Omega)\,\mathrm{d}\Omega}{\mathcal{P}_{2\pi}} \approx T_{\rm R}$
- Special case: $\Omega_S = \mathcal{P}_{\rm mb}$
  - $T_{\rm mb} = T_{\rm R}\frac{\int_{\Omega_S} P(\Omega)\,\mathrm{d}\Omega}{\mathcal{P}_{\rm mb}} = T_{\rm R}$
  - Main-beam temperature gives the source brightness
- General (worse) case: $\Omega_S \sim \mathcal{P}_{\rm mb}$
  - $T^*_{\rm A} = \frac{T_{\rm R}}{\mathcal{P}_{2\pi}} \int_{\Omega_S} P(\Omega)\,\mathrm{d}\Omega$
  - Main-beam temperature usually quoted
  - If source of uniform brightness and beam pattern known, feasible, but in real life... Which scale to use: $T^*_{\rm A}$, $T_{\rm mb}$?

Which temperature scale ?

Source size	Temperature scales
$\Omega_S = 2\pi$	$T_\mathrm{R} = T_\mathrm{A}^*$
$\Omega_S = \Omega_\mathrm{mb}$	$T_\mathrm{R} = T_\mathrm{mb}$
$2\pi < \Omega_S$	$T_\mathrm{R} < T_\mathrm{A}^*$
$\Omega_\mathrm{mb} < \Omega_S < 2\pi$	$T_\mathrm{A}^* < T_\mathrm{R} < T_\mathrm{mb}$
$\Omega_\mathrm{mb} > \Omega_S$	$T_\mathrm{mb} < T_\mathrm{R}$



Calibration

Goals of the calibration

Overview

- Atmospheric calibration:
  - atmospheric emission/abs at frequency ν: radiative transfer
  - turbulence affects the intensity through phase shifts
- Full detection chain calibration (antenna, receiver,)
  - Antenna-sky coupling: $F_{\rm eff}$
  - Receiver: gain, noise, stability
  - Cables, backends (*e.g.* dark currents)

Goals

- Input is an e-m field and output at backends are counts
- Question 1: how to convert from counts to power in physical units ?
- Question 2: how to correct for the atmospheric contribution ?

Notations

Telescope pointing at a source receives

$$C_{\rm sou} = \chi \left\{ T_{\rm rec} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} + T_{\rm sky} \right\}$$

where

$$T_{\rm sky} = F_{\rm eff}\,(1 - e^{-\tau_\nu})\,T_{\rm atm} + (1 - F_{\rm eff})\,T_{\rm gr}$$

- $T_{\rm rec}$: noise contribution from the receiver
- $F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou}$: signal from the scientific target after propagation through the atmosphere
- $T_{\rm sky}$: signal emitted by the atmosphere ($T_{\rm atm}$) and the ground ($T_{\rm gr}$)
- Orders of magnitude (IRAM-30m):
  - $T_{\rm atm} \approx T_{\rm gr} \approx 290$ K
  - $T_{\rm rec} \approx 50 - 70$ K at $100 - 350$ GHz at the IRAM-30m
  - $\nu_{\rm GHz} = 100 - 350$: $F_{\rm eff}(\nu) \approx 90 - 80\%$, $B_{\rm eff}(\nu) \approx 80 - 35\%$,
  - $T_{\rm sky} \approx 30 - 100$ K at $100 - 230$ GHz
  - Note: if $\tau_\nu \ll 1$ (good weather), $T_{\rm sky} \approx F_{\rm eff}\,\tau_\nu\, T_{\rm atm}$

The "Chopper Wheel" method

Perform 3 measurements: hot, empty sky, source

$$
\begin{aligned}
C_{\rm sou} &= \chi \left\{ T_{\rm rec} + T_{\rm sky} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} \right\} \\
C_{\rm atm} &= \chi \left\{ T_{\rm rec} + T_{\rm sky} \right\} \\
C_{\rm hot} &= \chi \left\{ T_{\rm rec} + T_{\rm hot} \right\}
\end{aligned}
$$

Making differences:

$$
\begin{aligned}
\Delta C_{\rm sig} &= C_{\rm sou} - C_{\rm atm} = \chi F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou} \\
\Delta C_{\rm cal} &= C_{\rm hot} - C_{\rm atm} = \chi (T_{\rm hot} - T_{\rm sky})
\end{aligned}
$$

$$\boxed{T_{\rm sou} = T_{\rm cal} \frac{\Delta C_{\rm sig}}{\Delta C_{\rm cal}}}$$

Definition of $T_{\rm cal}$ (correcting for atm contrib. and spillover)

$$\boxed{T_{\rm cal} = (T_{\rm hot} - T_{\rm sky}) \frac{e^{\tau_\nu}}{F_{\rm eff}}}$$

Calibration outputs (1): $T_{\rm cal}$

Rewrite $T_{\rm sky}$ and $\Delta C_{\rm cal}$:

$$\begin{aligned} T_{\rm sky} &= T_{\rm gr} + F_{\rm eff}(T_{\rm atm} - T_{\rm gr}) - F_{\rm eff}\, e^{-\tau_\nu} T_{\rm atm} \\ T_{\rm cal} &= T_{\rm gr} + e^{\tau_\nu}[T_{\rm gr} - T_{\rm atm}] + e^{\tau_\nu}/F_{\rm eff}[T_{\rm hot} - T_{\rm gr}] \\ \Delta C_{\rm cal} &= \chi\{\,(T_{\rm hot} - T_{\rm gr}) + F_{\rm eff}(T_{\rm gr} - T_{\rm atm}) + F_{\rm eff}\, e^{-\tau_\nu} T_{\rm atm}\} \end{aligned}$$

- Assume $T_{\rm hot} = T_{\rm atm} = T_{\rm gr}$.
- Then $T_{\rm cal} = T_{\rm hot}$

$$T_{\rm sou} = T_{\rm hot}\frac{\Delta C_{\rm sig}}{\Delta C_{\rm cal}}, \quad T_{\rm cal} = T_{\rm atm} = T_{\rm gr} = T_{\rm hot}$$

- No need to know $e^{-\tau_\nu}$ and $F_{\rm eff}$ (Penzias & Burrus ARAA 1973):
- But $T_{\rm rec}$ not known

The refined “Chopper Wheel” method

- General case: different $T_{\rm atm}$, $T_{\rm hot}$ and $T_{\rm gr}$
  $\Rightarrow$ must solve for $e^{-\tau_\nu}$ and $F_{\rm eff}$.
- $e^{-\tau_\nu}$: model of the atmosphere trying to reproduce $T_{\rm sky}$ varying the amount of dominant species in the model (hence providing us with pwv).
- $F_{\rm eff}$: skydips (measure atm. at several elevations)
- Perform $3+1$ measurements: hot, cold, empty sky, source

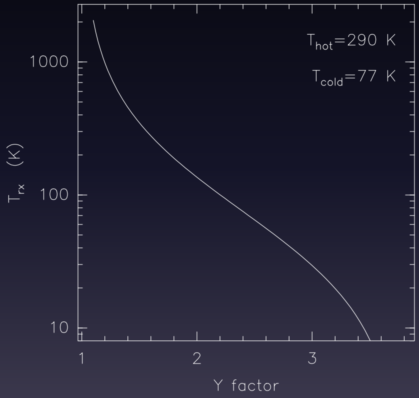
$$\begin{aligned}
C_{\rm sou} &= \chi\left\{T_{\rm rec} + T_{\rm sky} + F_{\rm eff}\, e^{-\tau_\nu}\, T_{\rm sou}\right\} \\
C_{\rm atm} &= \chi\left\{T_{\rm rec} + T_{\rm sky}\right\} \\
C_{\rm hot} &= \chi\left\{T_{\rm rec} + T_{\rm hot}\right\} \\
C_{\rm col} &= \chi\left\{T_{\rm rec} + T_{\rm col}\right\}
\end{aligned}$$

Calibration outputs (2): T_{rec}

Using hot & cold loads measurements lead to T_{rec}:

$$T_{\mathrm{rec}} = \frac{T_{\mathrm{hot}} - Y T_{\mathrm{col}}}{Y - 1}$$

$$Y = \frac{C_{\mathrm{hot}}}{C_{\mathrm{col}}} = \frac{T_{\mathrm{rec}} + T_{\mathrm{hot}}}{T_{\mathrm{rec}} + T_{\mathrm{col}}}$$



Calibration outputs (3): T_{sys}

System temperature: describes the noise including all sources from the sky down to the backends

- $T_{\mathrm{sys}} = T_{\mathrm{cal}}\, C_{\mathrm{off}} / \Delta C_{\mathrm{cal}}$
- used to determine the total statistical noise. For heterodyne receivers, noise is given by the "radiometer formula":

$$\boxed{\sigma_T = \frac{\kappa \cdot T_{\mathrm{sys}}}{\sqrt{\delta\nu\, \Delta t}}}$$

- δ_ν: spectral resolution
- Δt: total integration time
- κ depends on the observing mode:
- example: position switching, $\mathrm{ON\text{-}OFF} \Rightarrow \sqrt{2}$, $t_{\mathrm{ON}} = t_{\mathrm{OFF}} \Rightarrow \Delta t = 2t_{\mathrm{ON}} \Rightarrow \sqrt{2}, \Rightarrow \kappa = 2$

From $T_{\rm mb}$ to S_ν, from Kelvin to Jansky

- flux density: $S_\nu = \int_{\Omega_S} I_\nu(\Omega)\,\mathrm{d}\Omega = \frac{2k}{\lambda^2}\int_{\Omega_S} T_{\rm R}\,\mathrm{d}\Omega$
- power received by the antenna: $kT'_A = k\frac{T^*_{\rm A}}{F_{\rm eff}} = \frac{1}{2}S_\nu A_e$

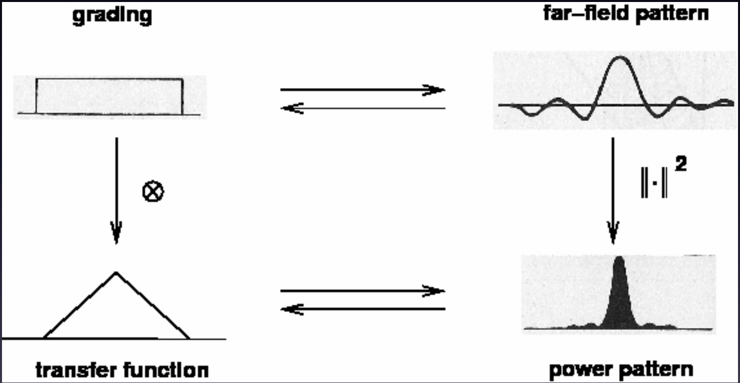
$$\frac{S_\nu}{T^*_{\rm A}} = \frac{2k}{A}\frac{F_{\rm eff}}{\eta_A} \qquad \mathrm{Jy\,K^{-1}}$$

- 1 Jy = $10^{-26}\ \mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$
- values of $S_\nu/T^*_{\rm A}$ are tabulated *e.g.* on IRAM-30m web page ($\approx$ 6 @ 100 GHz, $\approx$ 11 @ 340 GHz)
- How to convert the temperatures into $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}\,sr^{-1}}$?

$$I_\nu(\Omega) = \frac{2k}{\lambda^2}\,T_R(\Omega)$$

Summary

Image formation: total power telescope



- antenna scans the source
- image: convolution of I_0 by beam pattern $I'_\nu = \mathcal{P} * I_{0,\nu}$
- measure directly the brightness distribution I_0

Interferometer field of view

$$F = D * (\mathcal{P} \times I) + N$$

F = dirty map = FT of observed visibilities
D = dirty beam ($\longrightarrow$ deconvolution)
$\mathcal{P}$ = power pattern of single-dish
(primary beam B in the following)
I = sky brightness distribution
N = noise distribution

- **An interferometer measures the product** $\mathcal{P} \times I$
- $\mathcal{P}$ has a finite support $\longrightarrow$ limits the size of the field of view

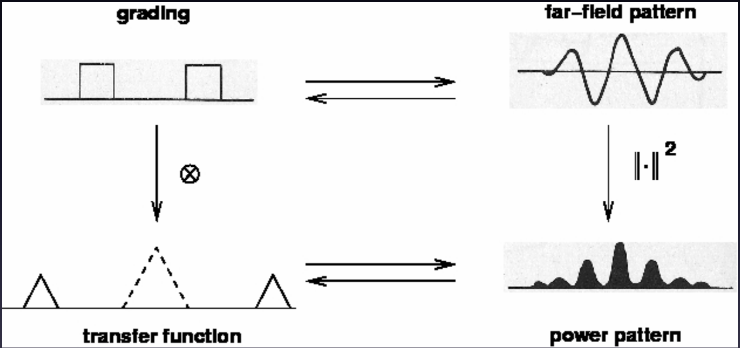
Physical parameters

- Chose an adapted temperature scale (T_{A}^{*}, T_{mb})
- Correct for error-beam pick-up when needed
- Amplitude calibration: often enough $\rightarrow$ 10% accuracy

Summary

- full-aperture antenna: $\mathcal{P} * \mathbf{I}$
- interferometry sensitive to $\mathcal{P} \times \mathbf{I}$
- **amplitude calibration:**
  - converts counts into temperatures
  - corrects for atmospheric absorption
  - corrects for spillover
- **lobe = main-lobe + error-lobes** (*e.g.* as much as 50% in error-lobes at 230GHz for the 30m)
- Pay attention to the **temperature scale** to use (T_A^*, T_{mb},...)

Image formation: correlation telescope



- antennas fixed w.r.t. the source
- correlation temperature: $\mathcal{T}(0,0)$ Fourier transform of $I_0 \times \mathcal{P}$
- measure the Fourier transform of the brightness distribution I_0
- image built afterwards

Interferometer field of view

Measurement equation of an interferometric observation:

$$F = D * (B \times I) + N$$

F	=	dirty map = FT of observed visibilities
D	=	dirty beam ($\longrightarrow$ deconvolution)
B	=	primary beam
I	=	sky brightness distribution
N	=	noise distribution

- **An interferometer measures the product $B \times I$**
- B has a finite support $\longrightarrow$ limits the size of the field of view
- B is a Gaussian $\longrightarrow$ primary beam correction possible (proper estimate of the fluxes) but strong increase of the noise

Primary beam width

$$\begin{array}{ccc} \text{Aperture function} & \rightleftharpoons & \text{Voltage pattern} \\ \star\downarrow & & \downarrow|\cdot|^2 \\ \text{Transfert function } T(u,v) & \rightleftharpoons & \text{Power pattern } B(\ell,m) \\ & & = \textbf{Primary beam} \end{array}$$

Gaussian illumination $\Longrightarrow$ to a good approximation, B is a Gaussian of $1.2\,\lambda/D$ FWHM

Plateau de Bure

$D = 15$ m

Frequency	Wavelength	Field of View
85 GHz	3.5 mm	$58''$
100 GHz	3.0 mm	$50''$
115 GHz	2.6 mm	$43''$
215 GHz	1.4 mm	$23''$
230 GHz	1.3 mm	$22''$
245 GHz	1.2 mm	$20''$