Nonlinear kinetic inductance in TiN/NbTiN microresonators and Transmission lines

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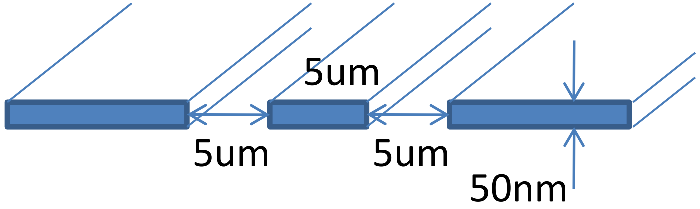
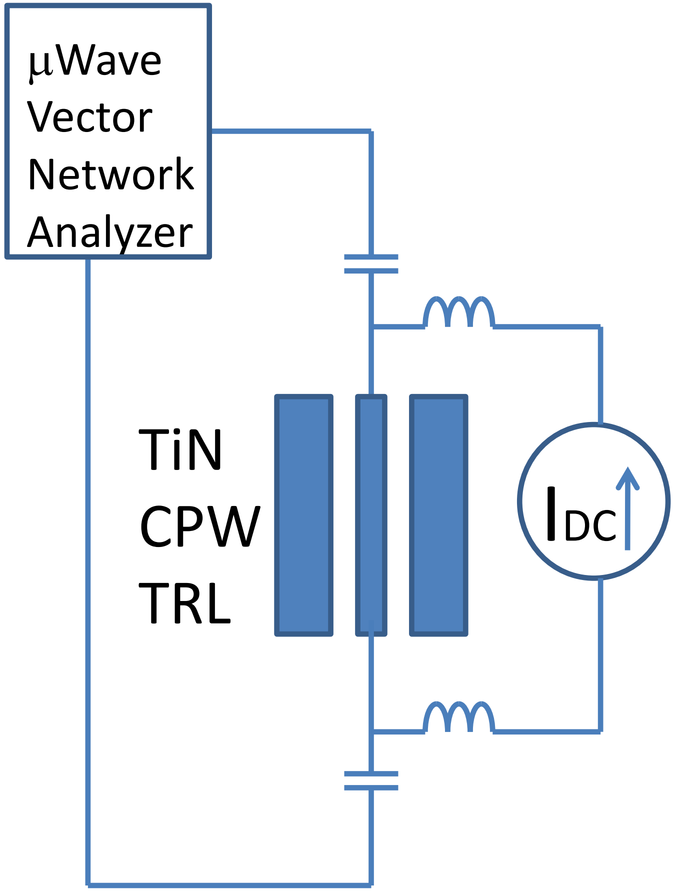
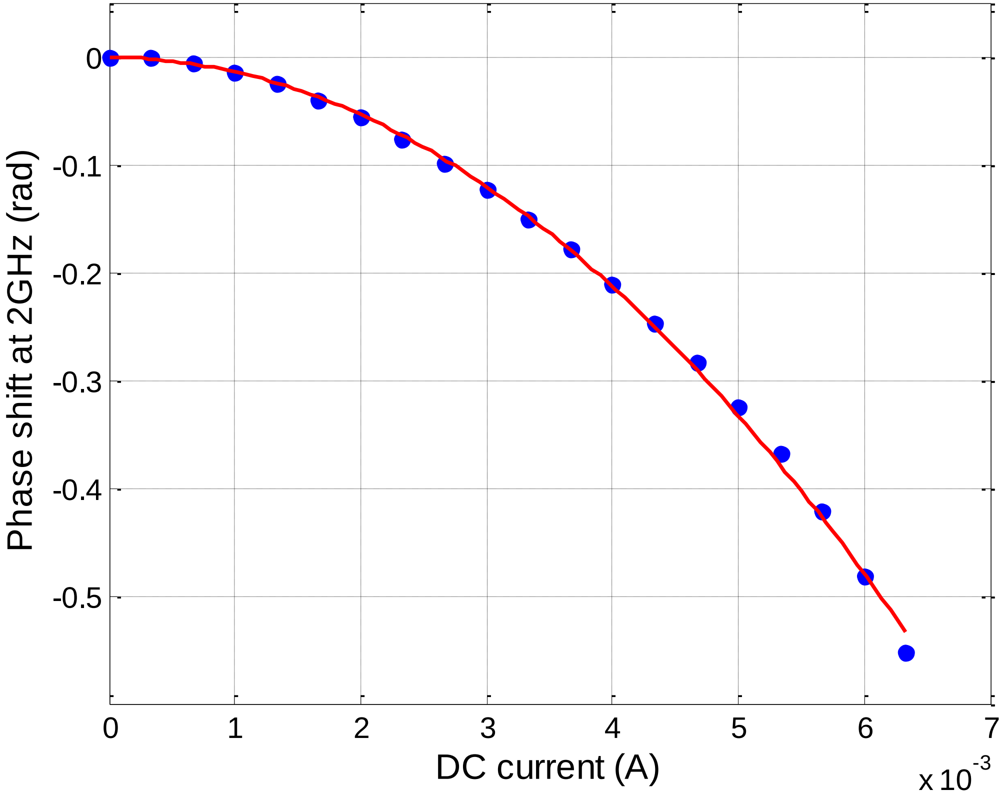
Nonlinear kinetic inductance

0.1 meter long CPW line





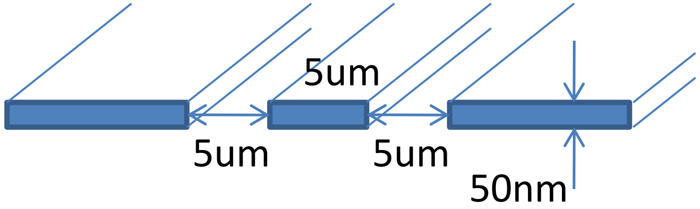


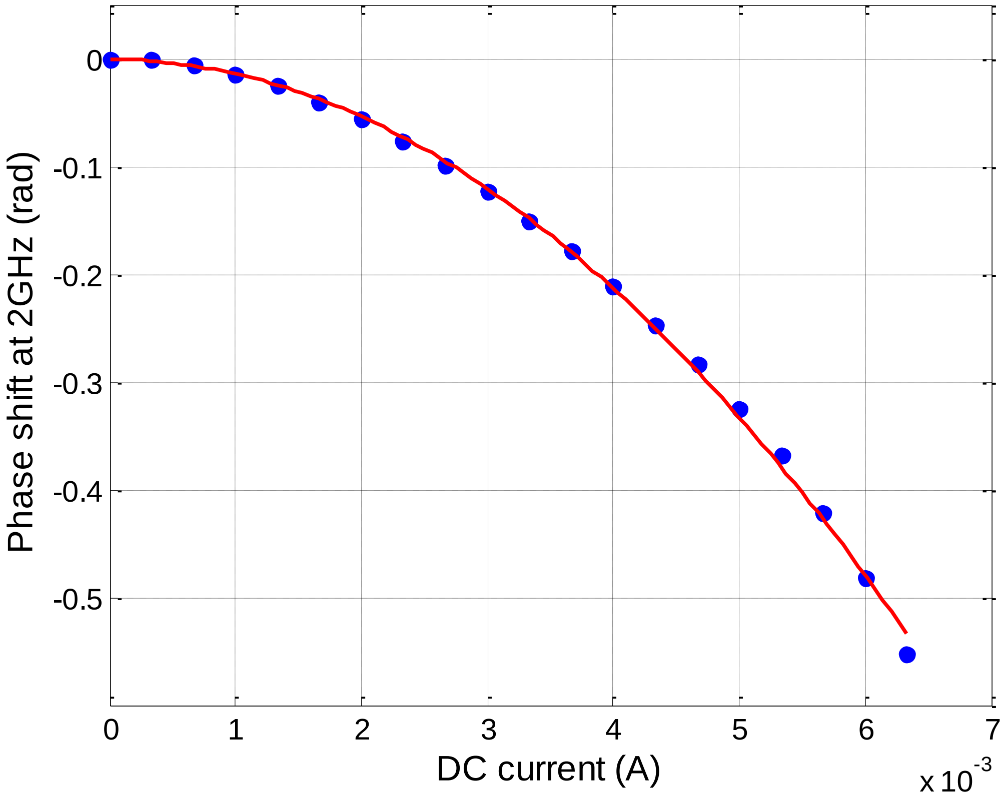
Nonlinear kinetic inductance

- $\Delta\theta \sim I^2$
- $L = L_0 (1 + I^2/I_*^2)$
- $\delta v_{ph} \sim I^2 \sim P$
- Kerr medium
- Line length =0.1m

-> 21 radians

- $\Delta\ell/\ell \sim 2.5\%$

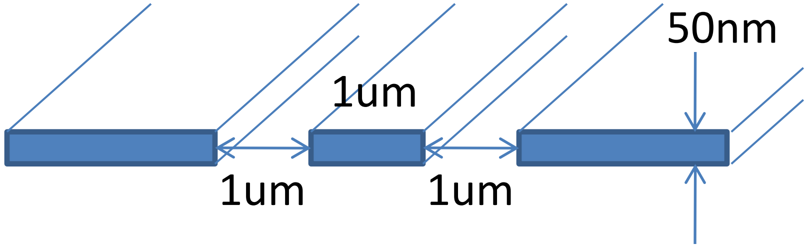




Superconducting Nitrides – large kinetic inductance

- $\lambda \approx 105\ \mathrm{nm} \times (\rho_n\ [\mu\Omega.\mathrm{cm}]\ /\ T_c\ [\mathrm{K}])^{1/2}$
  - Nitrides: $\rho_s \approx 100\ \mu\Omega.\mathrm{cm}$
  - λ (TiN, NbTiN) ≈ 500 nm

Example: TiN CPW line



$L_s / L_{tot} \approx 0.94$

$v_{ph} \approx 0.1\ c$

$Z0 \approx 220\ \Omega$

Kinetic Inductance Non-linearity

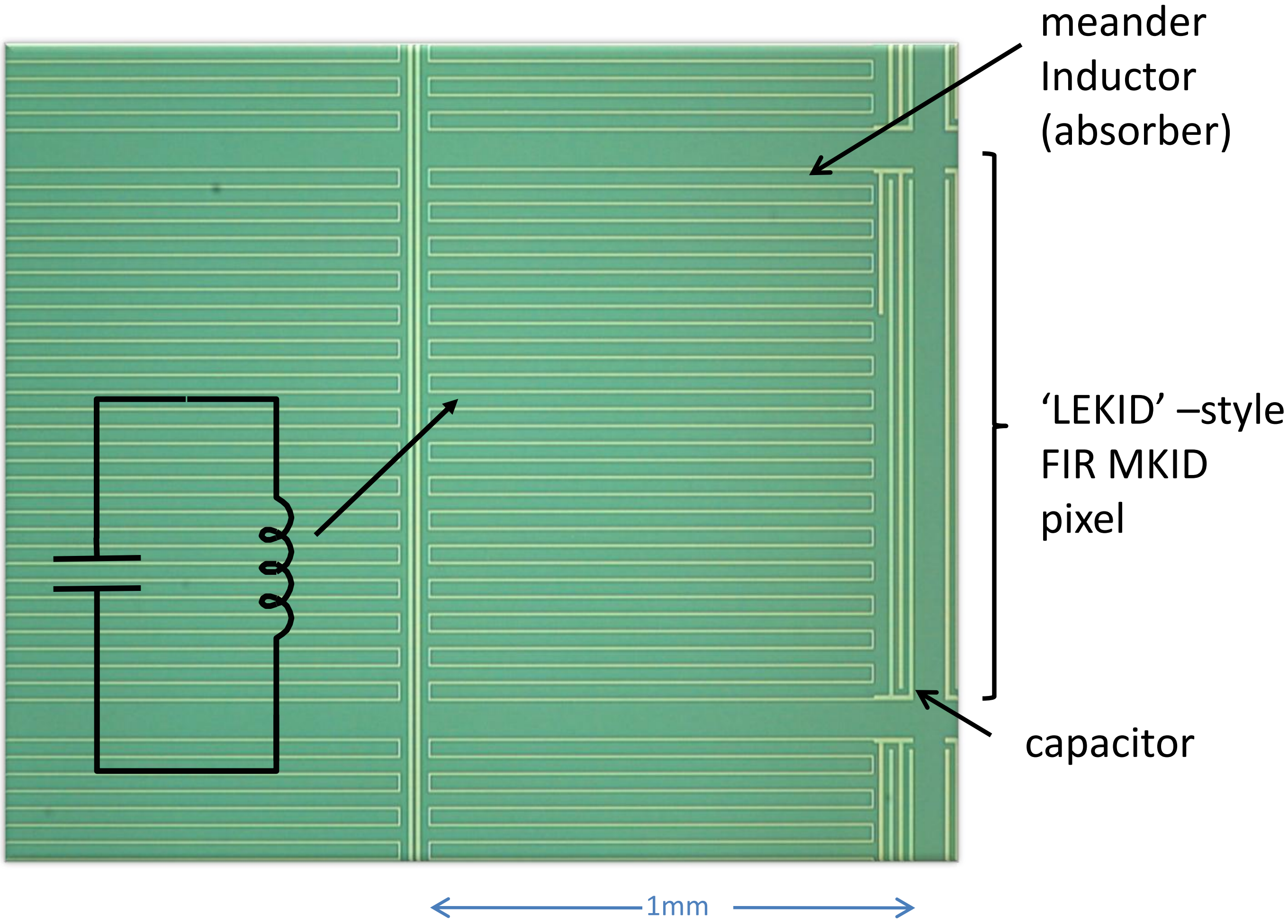
- Ginsberg-Landau theory
  - $L_s(I)$, $\lambda(I)$

$$\frac{\delta v_{ph}}{v_{ph}(I = 0)} = \frac{-L_{kin}}{4\,L_{tot}} \frac{\mu_0\,\lambda^2\,J_s^{\,2}}{\mu_0\,H_c^{\,2}}$$

Supercurrent kinetic energy

Condensation energy

TiN Resonator measurements



meander
Inductor
(absorber)

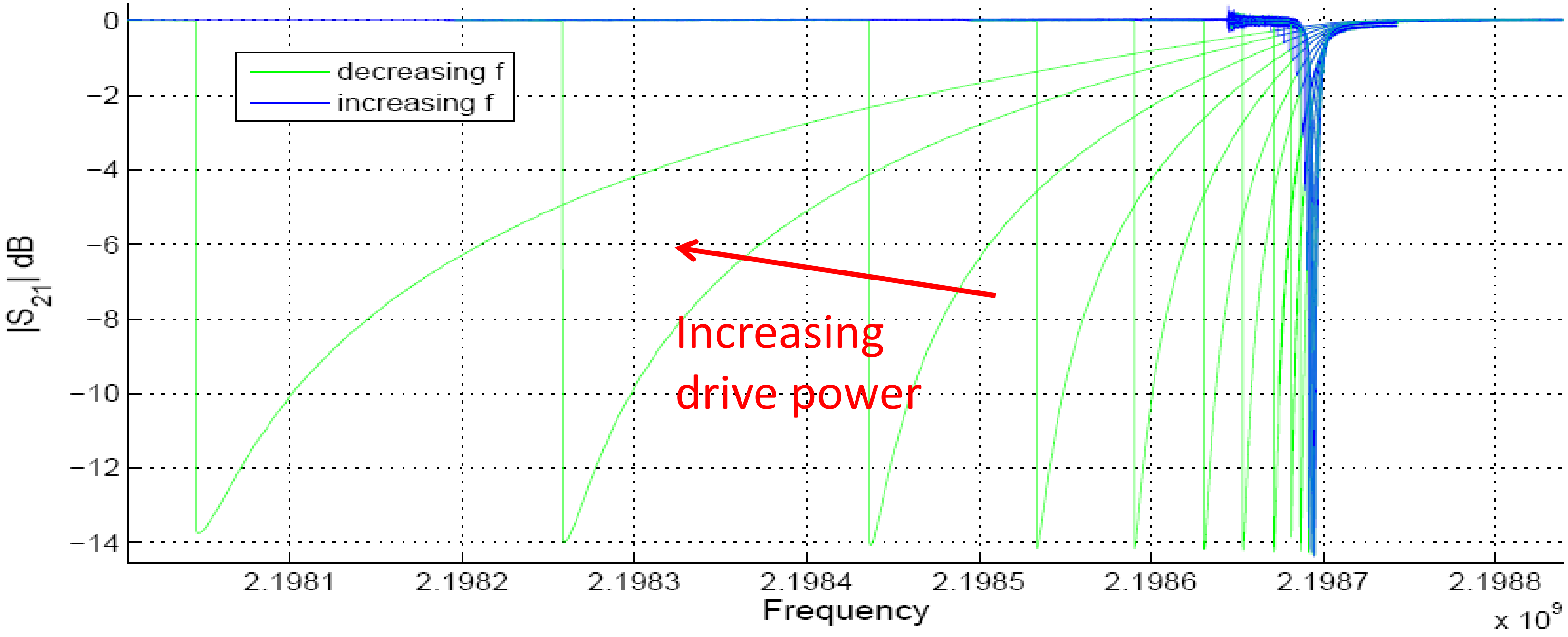
'LEKID' –style
FIR MKID
pixel

capacitor

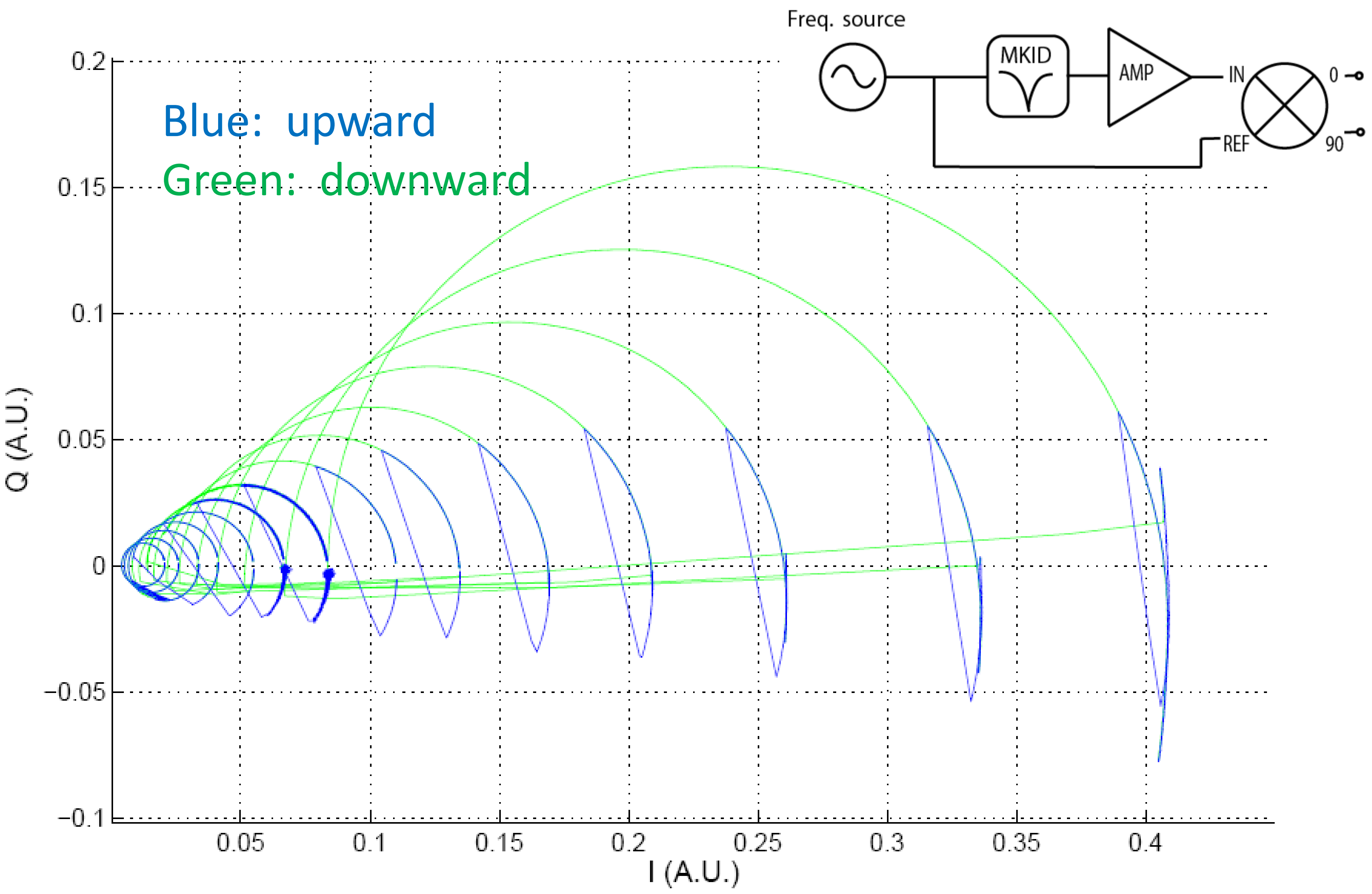
1mm

Nonlinear “Duffing” oscillator

- Resonance frequency depends on resonator current
- Hysteretic resonance curves:



Complex transmission



Non-linear resonator model

$$\begin{aligned} \delta f / f_0 &= -k_f I_{res}^2 \\ \delta Q_i^{-1} &= k_Q I_{res}^2 \end{aligned}$$

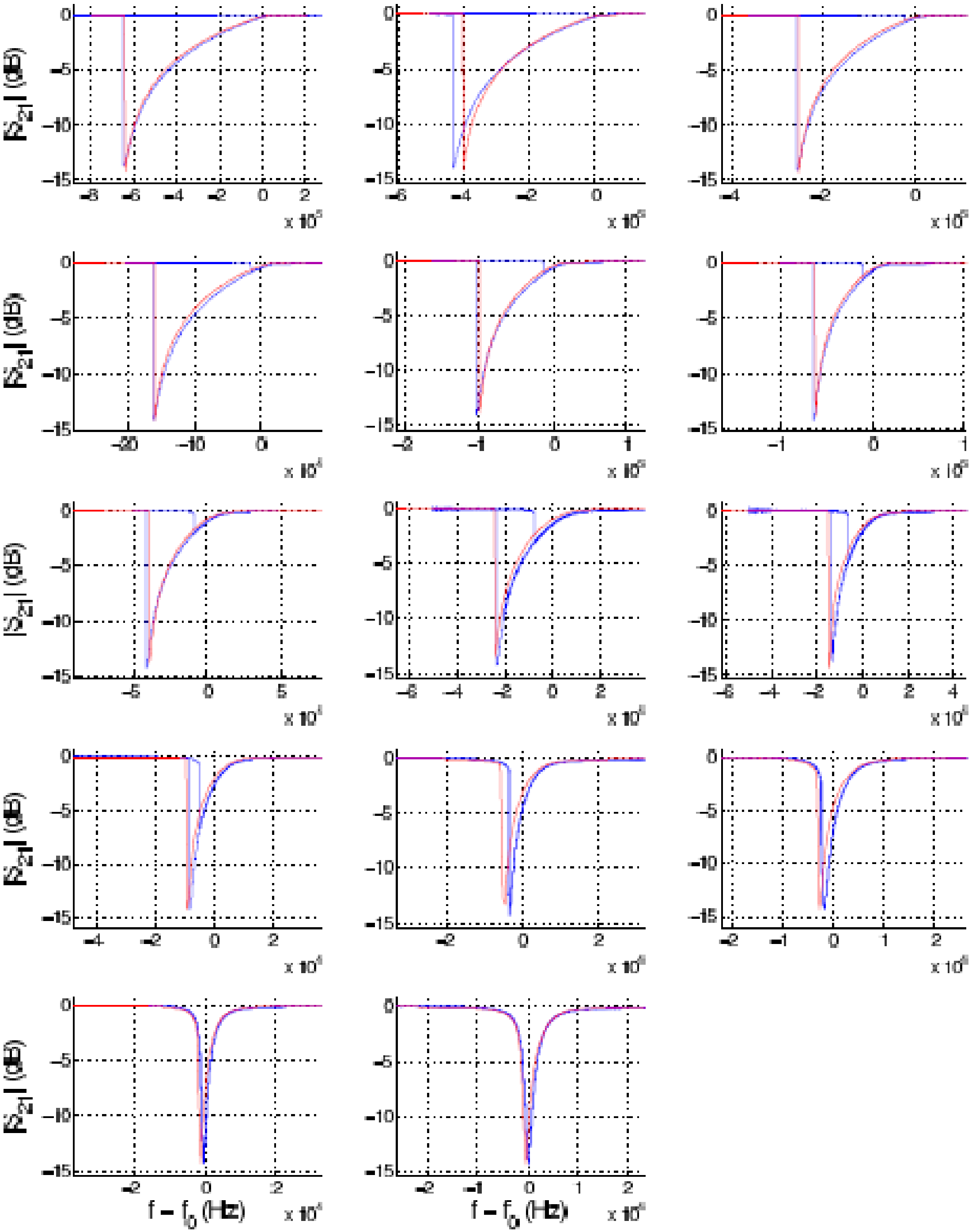
See Yurke and Buks (2008);
Dahm and Scalapino (1997)

$$S_{21} = 1 - \frac{Q_t / Q_c}{1 + 2iQ_t[f_0 + \delta f_0(f) - f]/f_0}$$

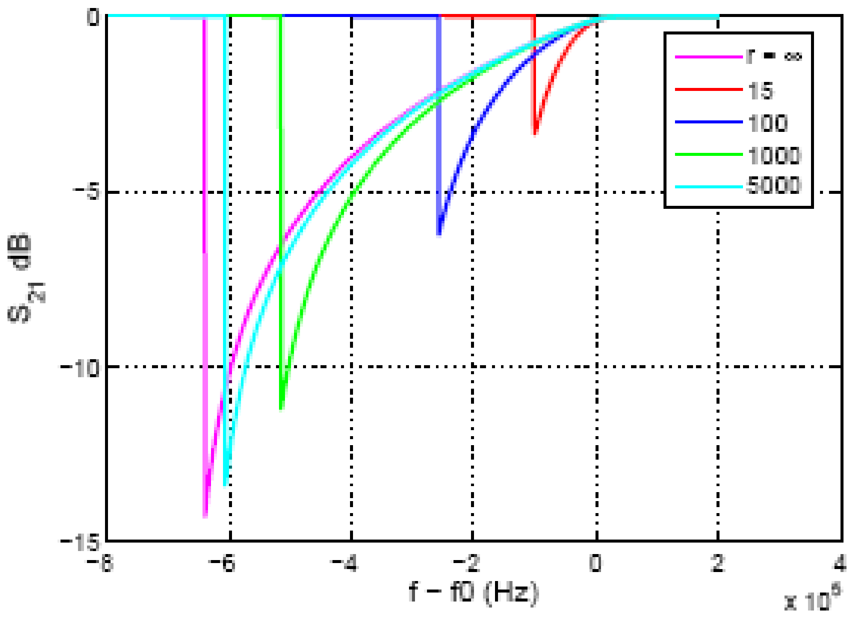
$$\frac{1}{Q_t} = \frac{1}{Q_c} + \frac{1}{Q_i}$$

$$I_{res}^2 = \frac{Q_c |1 - S_{21}|^2 P_{feedline}}{Z_0}$$

- Fits using NL resonator model
- $K_Q = 0$!

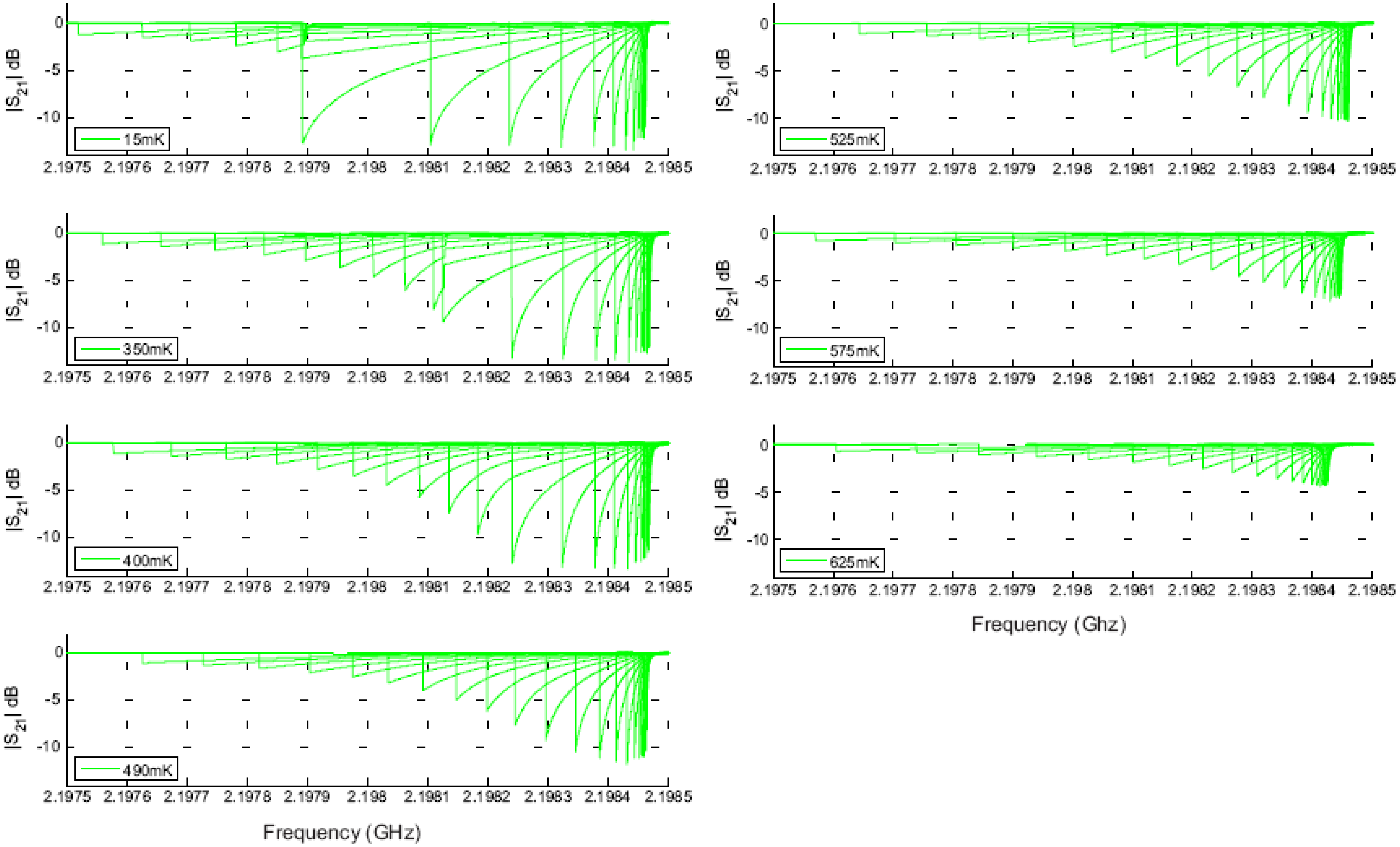


- Model results for different ratios of inductive to dissipative response

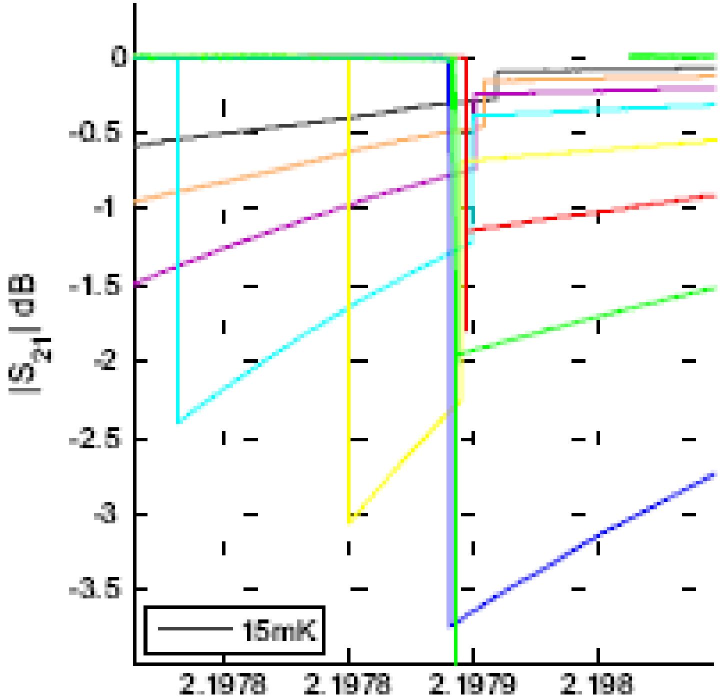


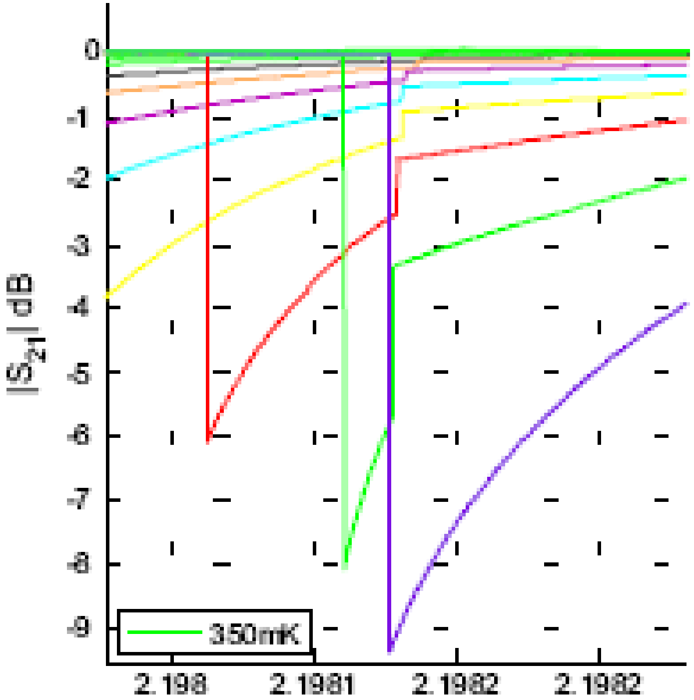
- δ resistance/ δ reactance < 2 x 10^{-4}

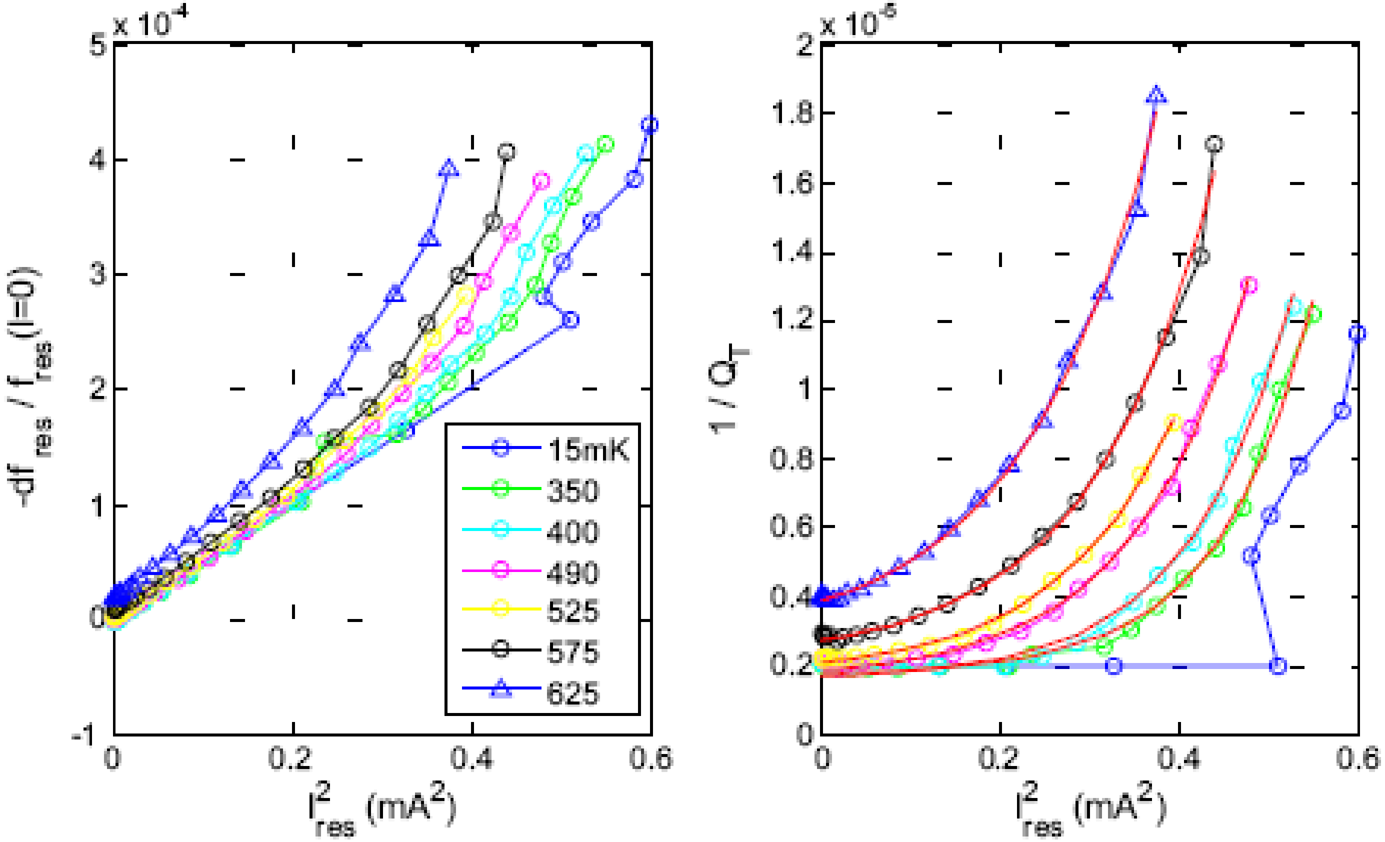
Measurements at elevated T



Transition to dissipative regime



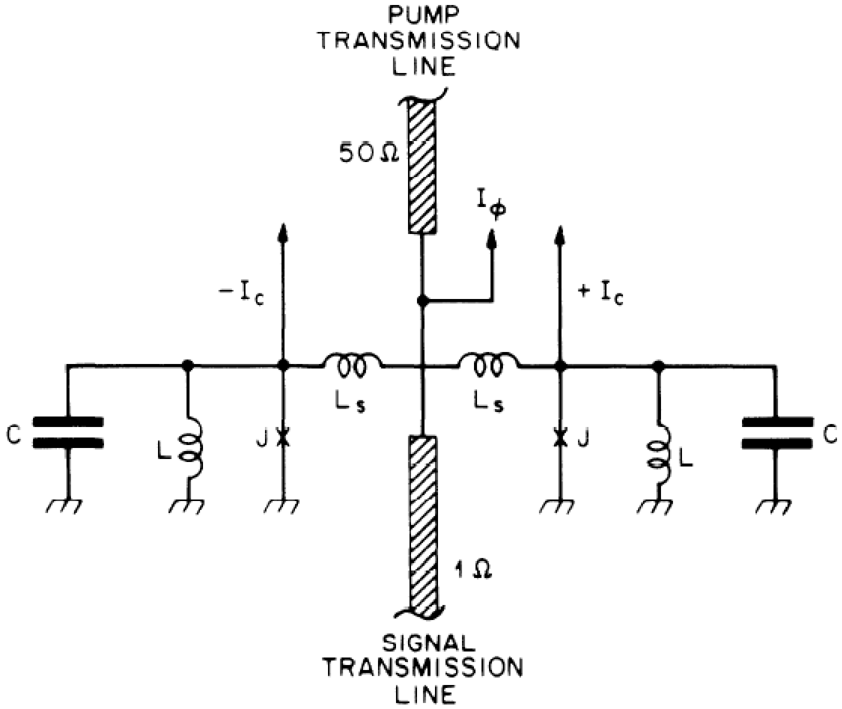




x 10^-4
5
4
3
2
1
0
-1
-df_res / f_res(I=0)
0
0.2
0.4
0.6
I^2_res (mA^2)
15mK
350
400
490
525
575
625
x 10^-6
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1 / Q_T
0
0.2
0.4
0.6
I^2_res (mA^2)

Josephson parametric amplifiers

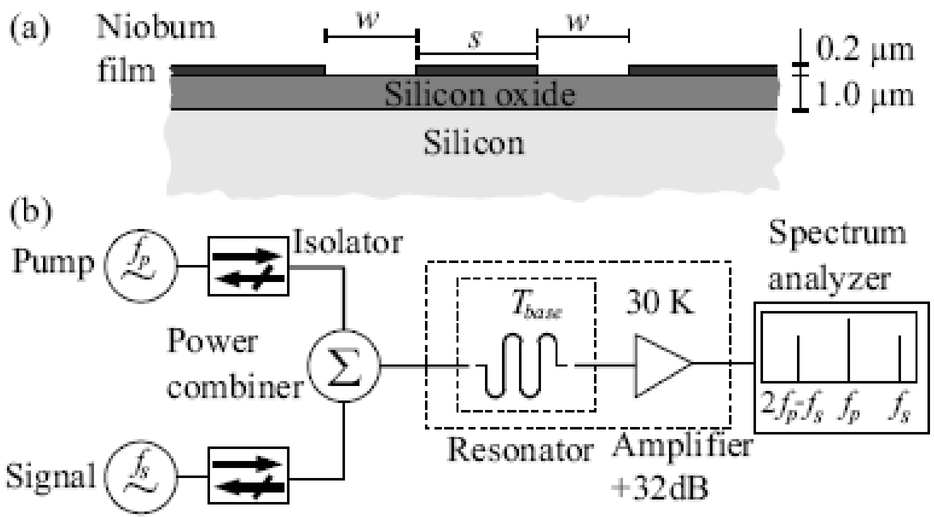
- Nonlinearity
  - Josephson inductance
  - Transfer of energy from strong pump to signal
  - Idler tone
- Purely reactive nonlinearity
  - May reach QL
- Resonant, narrow band

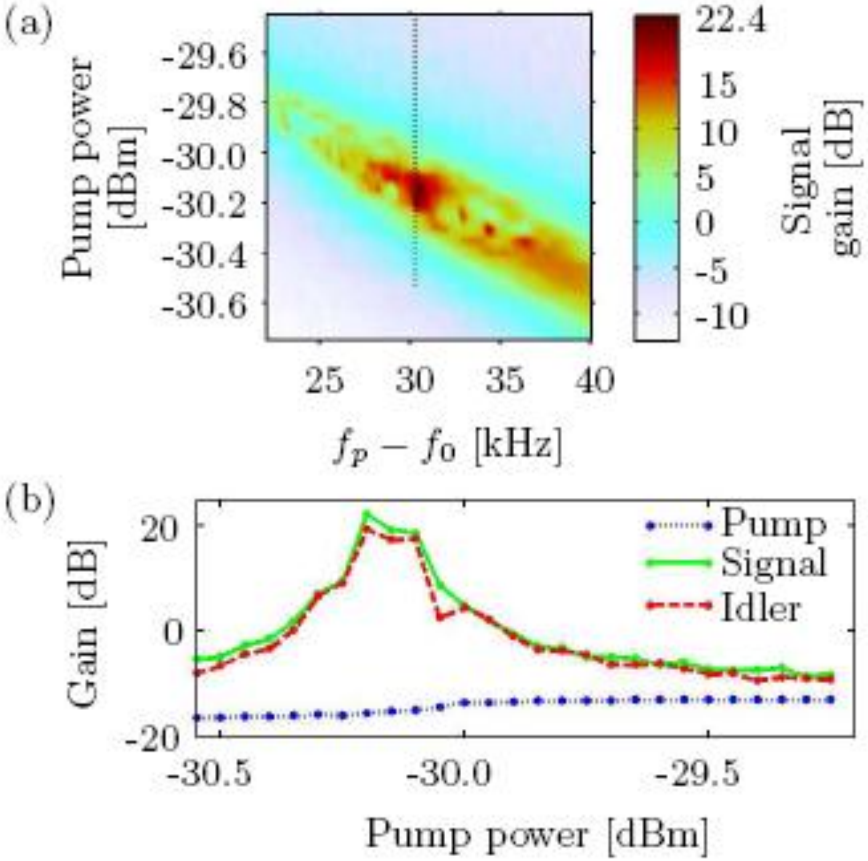


Yurke et al. (1988)

Kinetic inductance cavity para-amp

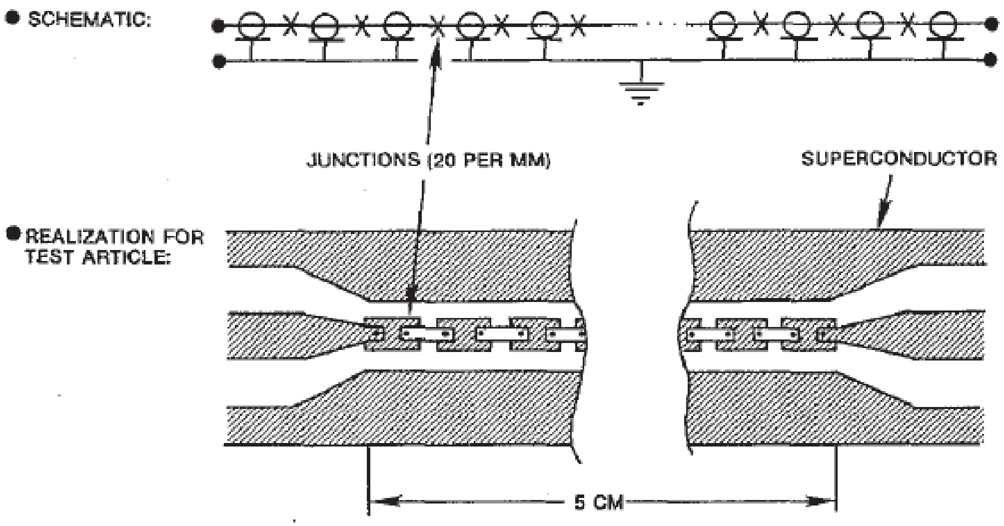
- Tholen et al. (2007)
- Nb CPW resonator

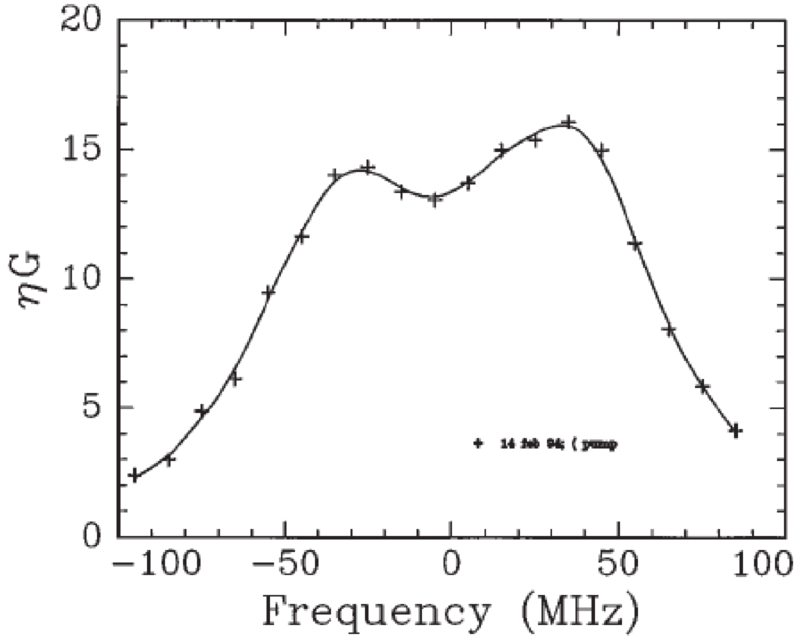




Traveling wave Josephson paramp

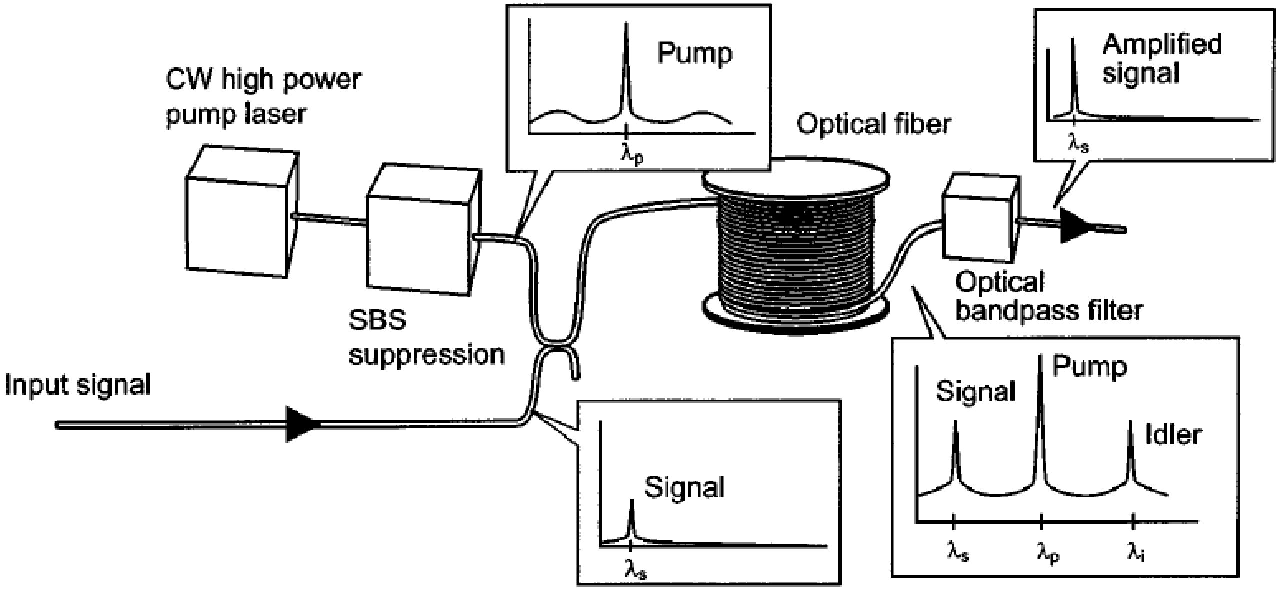
- Proposed by Sweeny and Mahler (1985)
- Experimentally realized by Yurke et al. (1996)
- Gain x bandwidth ~ 500MHz





Optical fiber paramp

- Intensity dependent index: $n \sim E^2$
- Kerr medium

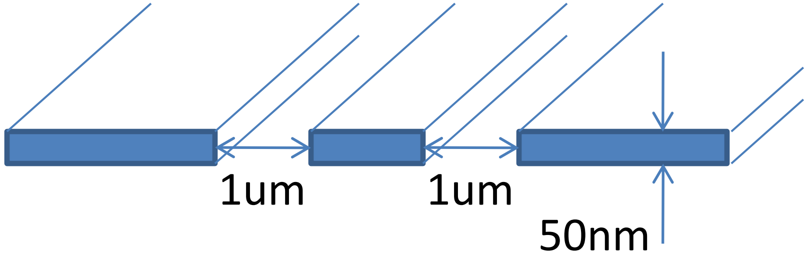


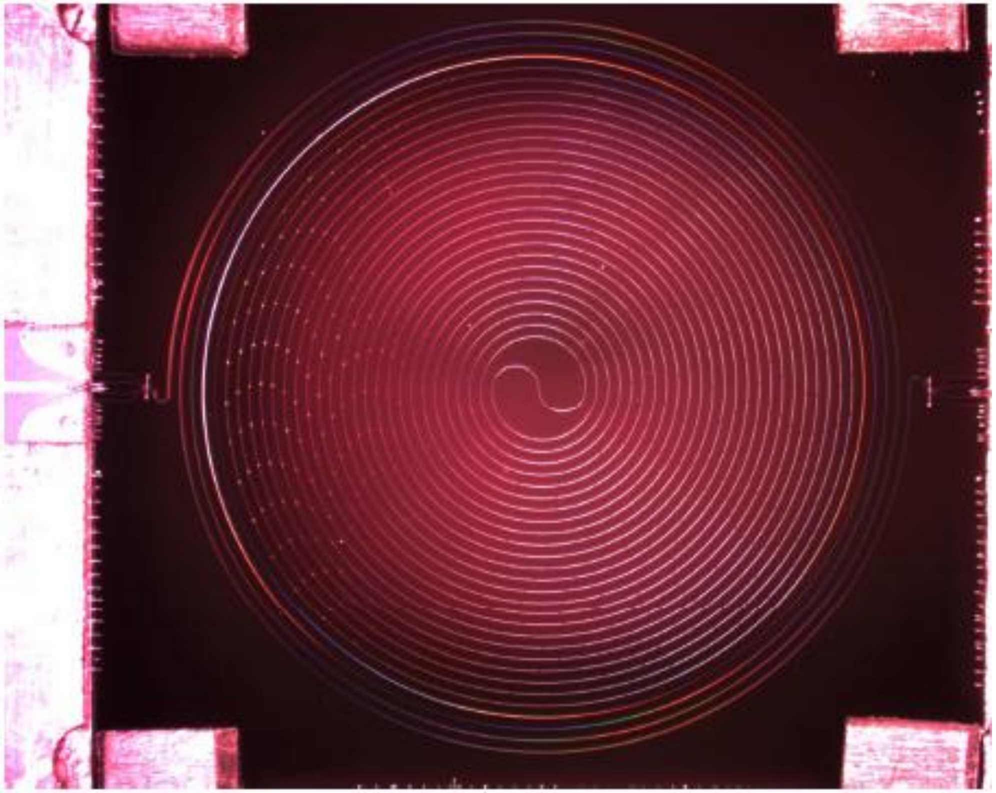
from Hansryd et al. (2002)

- Yurke’s device an analogue of this

DTWKI paramp ver 1.0

- DTWKI = Dispersion-engineered Traveling Wave Kinetic Inductance
- Single layer TiN or NbTiN
- 0.8 m CPW line
  - Tapers at input, output match 50 ohms





Coupled mode equations

- From the optical fiber paramp: (Stohlen et al.)

$$
\begin{aligned}
\frac{dA_p}{dz} &= i\gamma[(|A_p|^2 + 2(|A_s|^2 + |A_i|^2))A_p \\
&\quad + 2A_s A_i A_p^* \exp(i\Delta\beta z)], \\
\frac{dA_s}{dz} &= i\gamma[(|A_s|^2 + 2(|A_i|^2 + |A_p|^2))A_s \quad \longleftarrow \text{SPM, XPM} \\
&\quad + A_i^* A_p^2 \exp(-i\Delta\beta z)], \quad \longleftarrow \text{Energy transfer} \\
\frac{dA_i}{dz} &= i\gamma[(|A_i|^2 + 2(|A_s|^2 + |A_p|^2))A_i \\
&\quad + A_s^* A_p^2 \exp(-i\Delta\beta z)].
\end{aligned}
$$

- $A_{p,s,i}$: slowly varying (complex) amplitudes
- γ: non-linearity parameter
- $\Delta\beta = \beta(\omega_s) + \beta(\omega_i) - 2\beta(\omega_p)$

Coupled mode equations

- From the optical fiber paramp: (Stohlen et al.)

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&\quad + A_s^* A_p^2 \exp(-i\Delta\beta z)].
\end{aligned}
$$

SPM, XPM

Energy transfer

Without dispersion ($\Delta\beta = 0$):

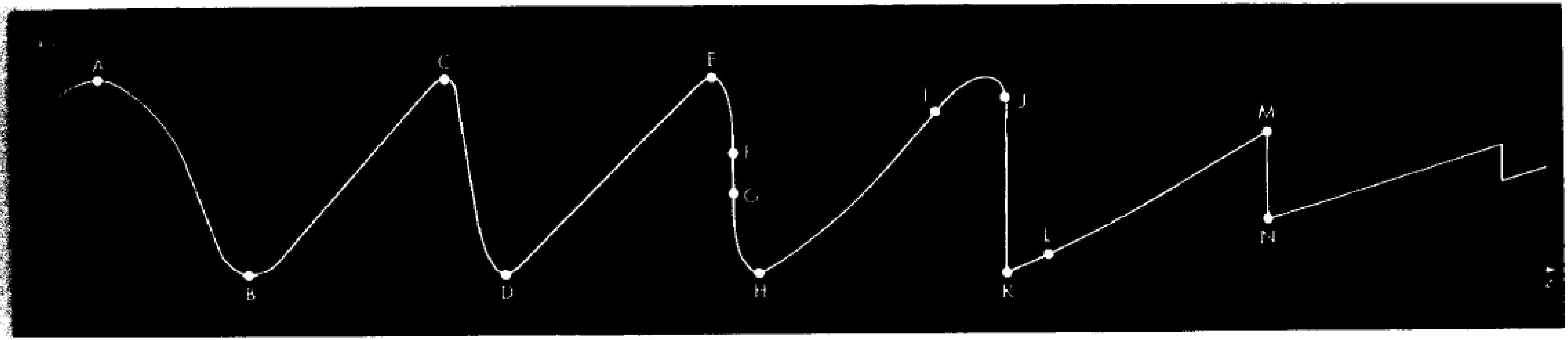
$$G = 1 + (\gamma P_p L)^2 = 1 + (\Delta\theta)^2$$

Total phase shift in radians in response to pump

Shock wave formation

- Superconducting TRLs are virtually dispersionless
- Harmonic generation is efficient

Figure 3 **Voltage as a function of distance along the line, due to a periodic voltage applied at $z=0$. *FG* is a shock wave that has just formed. The shock wave *JL* is near its maximum amplitude. *MN* is a shock wave decreasing in amplitude.**



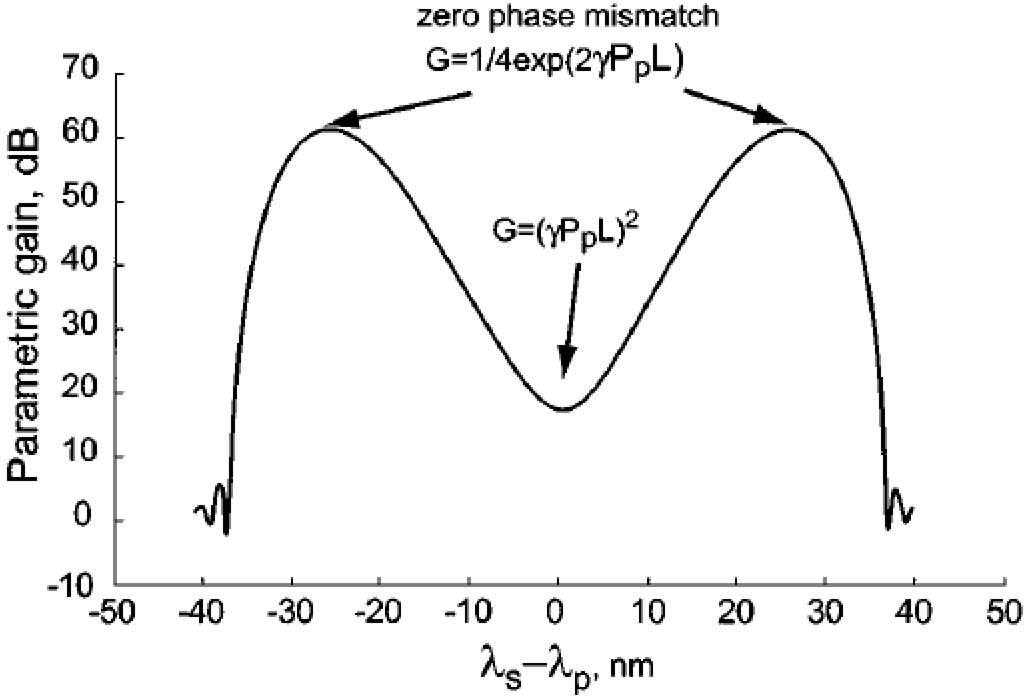
Landauer (1960)

$$L = L_0(1 + I^2/I_*^2)$$

$$V = L dI/dt \qquad I^2 dI/dt \longrightarrow 3f_P \longrightarrow 5f_P,\ 7f_P,\ \text{etc.}$$

Adding dispersion

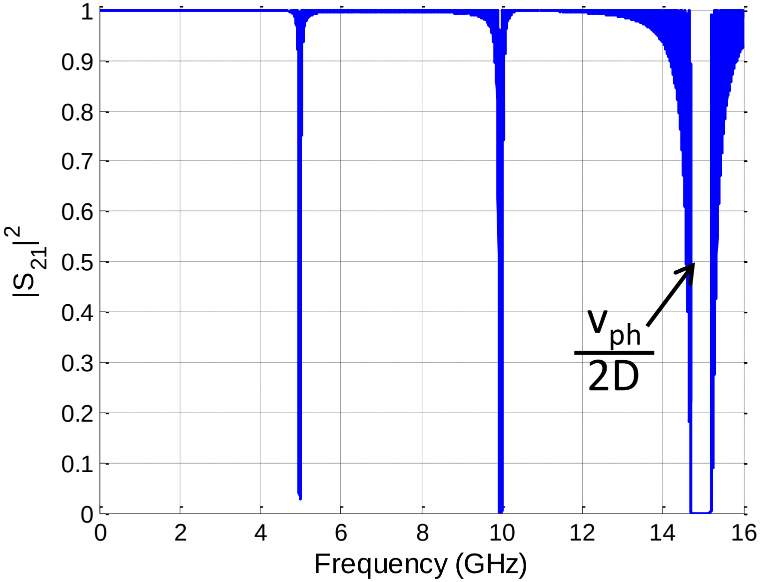
from Hansryd et al. (2002)

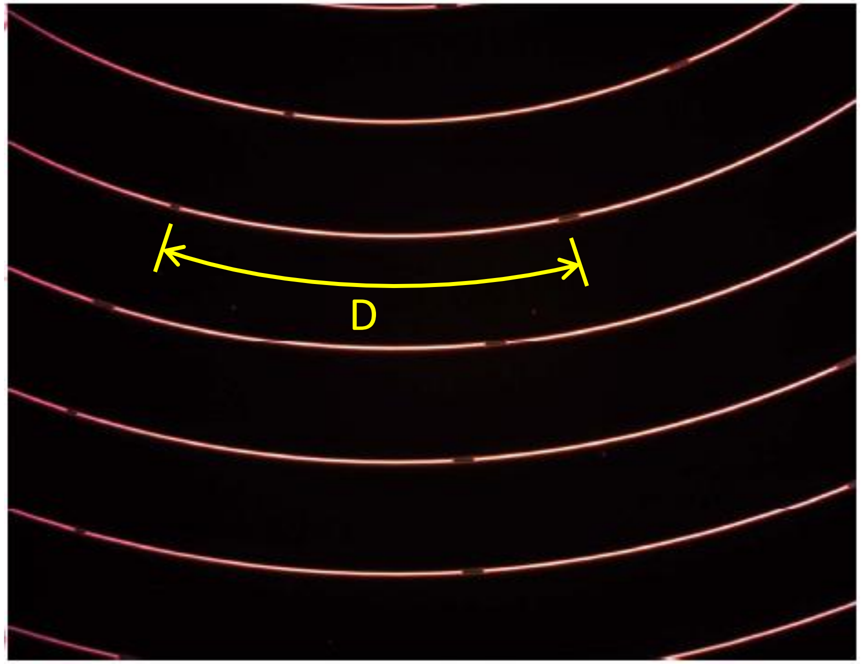


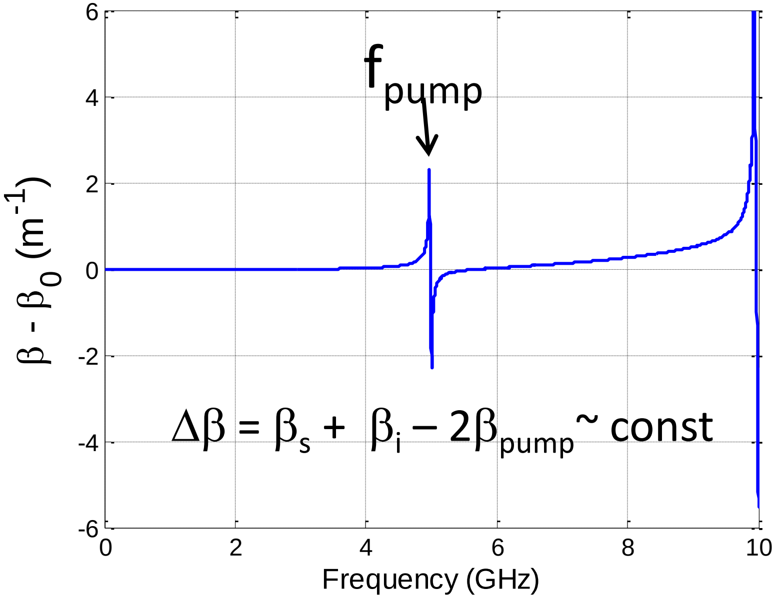
- Phase mismatch
  - Limited bandwith
- Optimal dispersion compensates nonlinear phase missmatch
  - Exponential gain regime

Dispersion-engineered traveling wave kinetic inductance (DTWKI) amplifier

- Periodically load TRL
  - Produce stop band at 3 x f_{pump}
  - Dispersion around f_{pump}

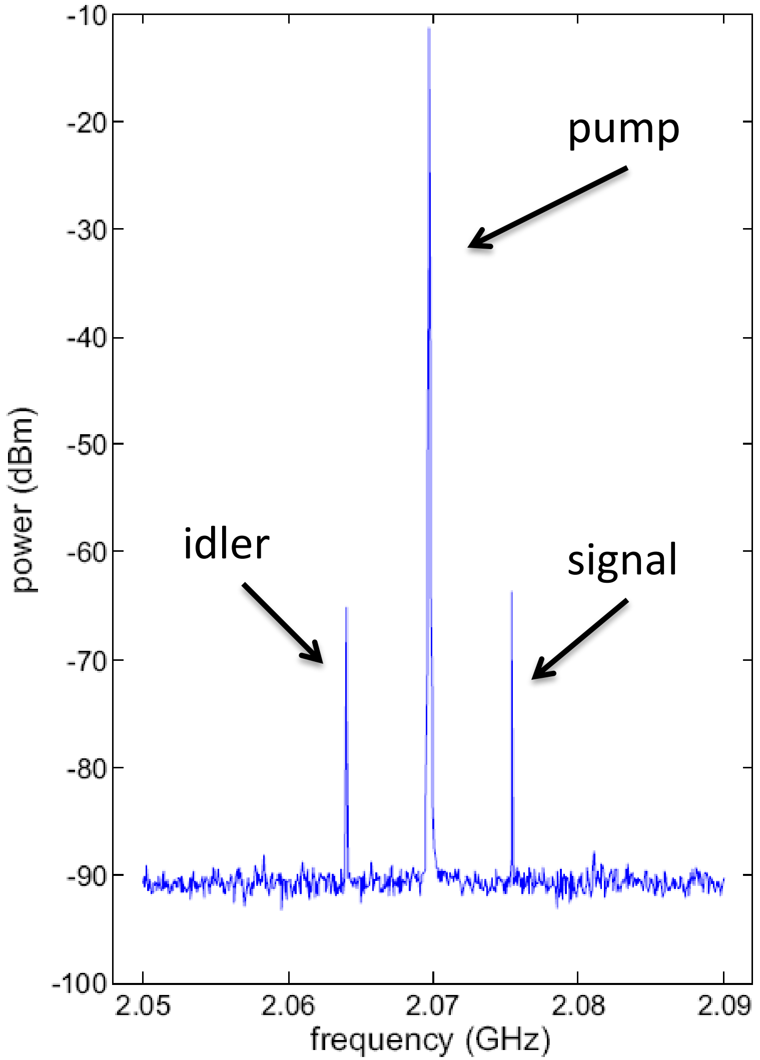




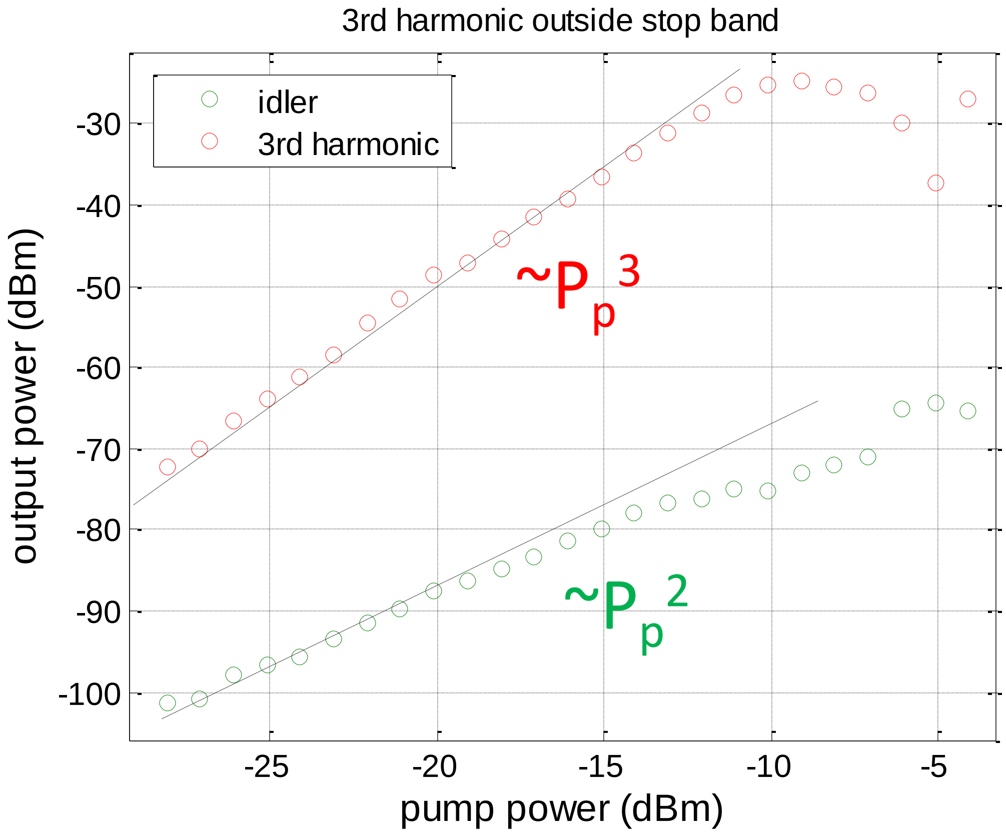


$\Delta\beta = \beta_s + \beta_i - 2\beta_{pump} \sim \text{const}$

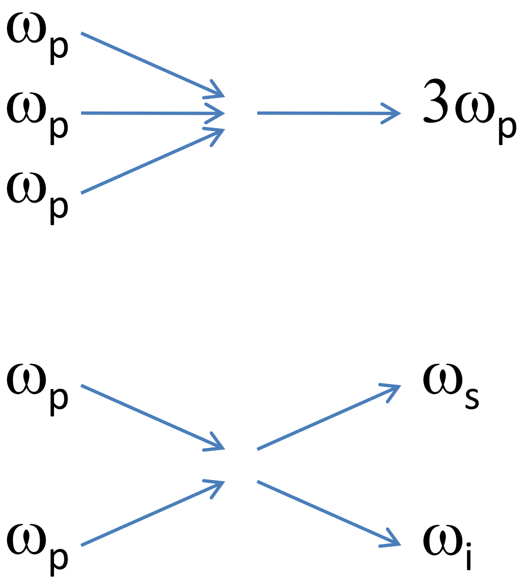
Idler generation



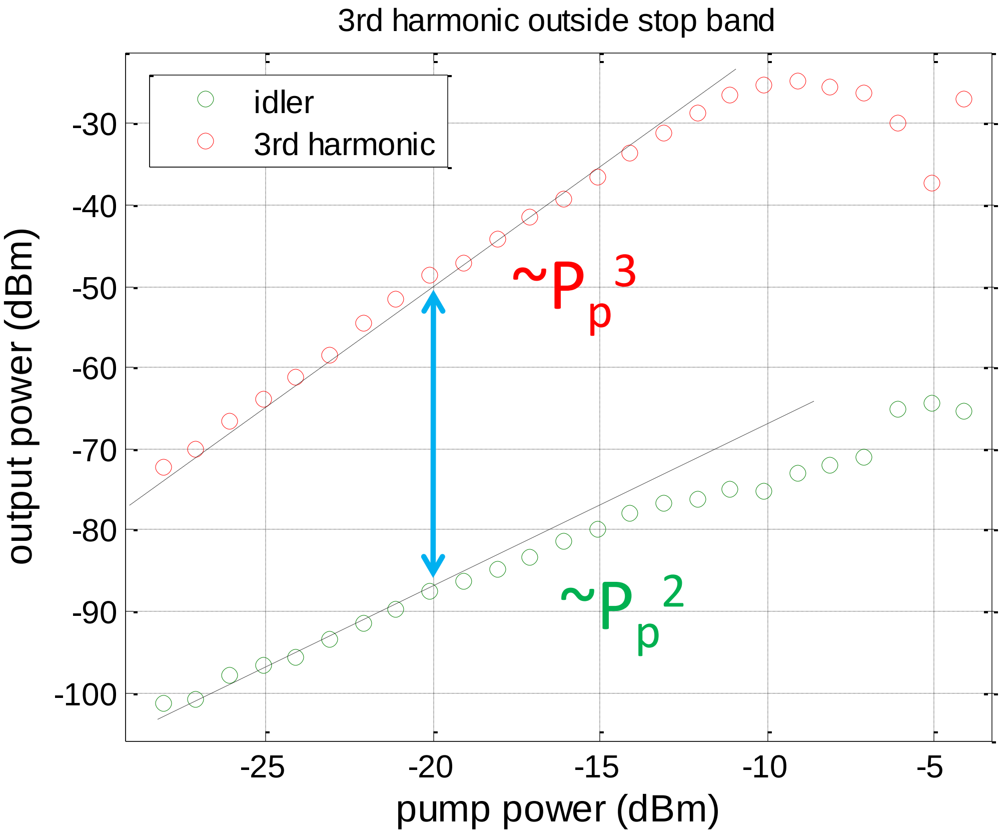
3rd Harmonic/Idler Generation



(signal power = -60dBm)



3rd Harmonic/Idler Generation



(signal power = -60dBm)

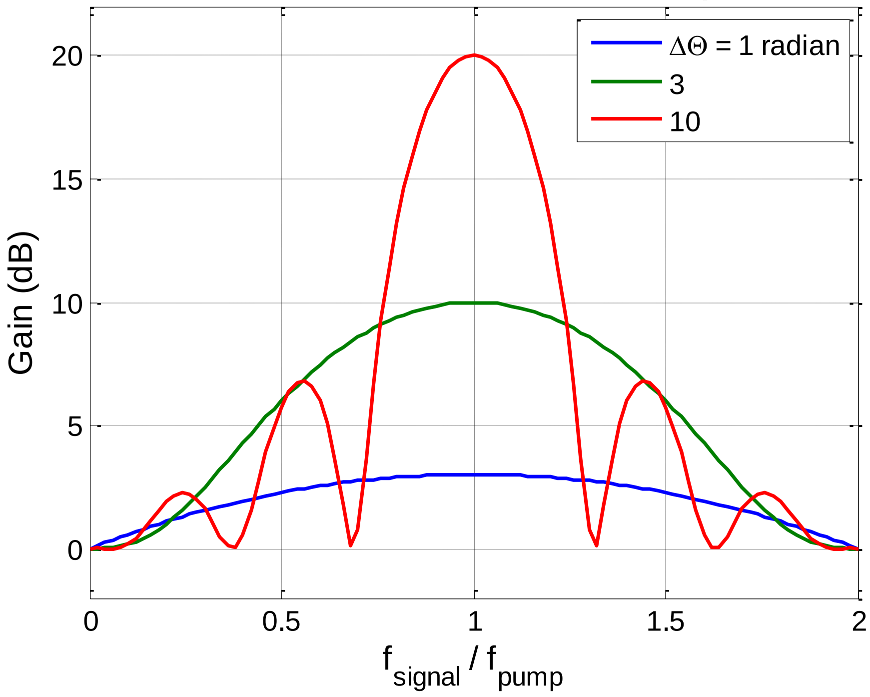
$$\frac{P^{out}(3\omega_p)}{P^{out}(\omega_i)} = \frac{P_p}{P_s}$$

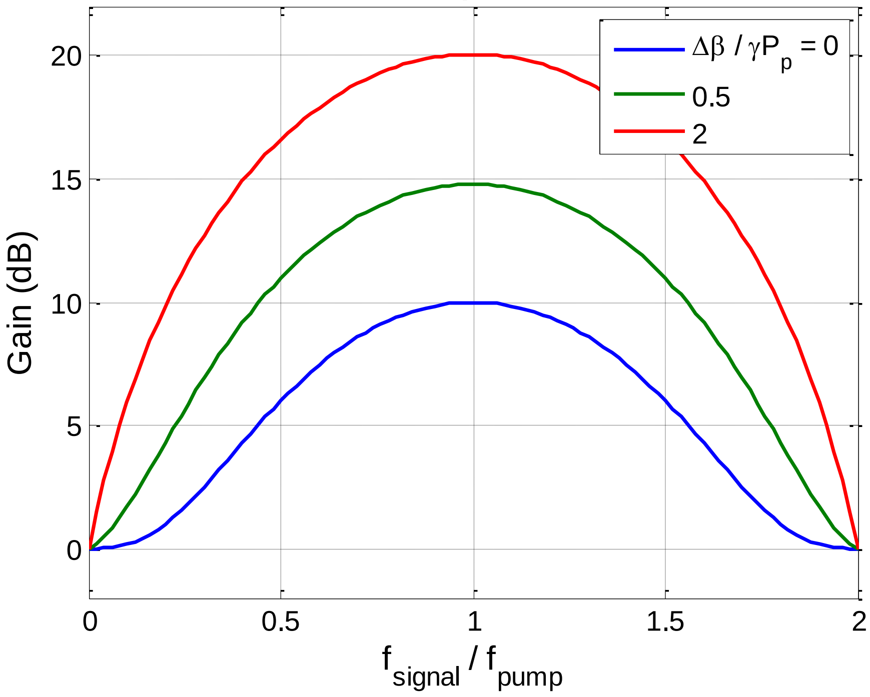
eg. @ P_p = -20 dBm,
P_s = -60 dBm

$$\frac{P^{out}(3\omega_p)}{P^{out}(\omega_i)} = 40 \text{ dB}$$

Coupled-mode prediction

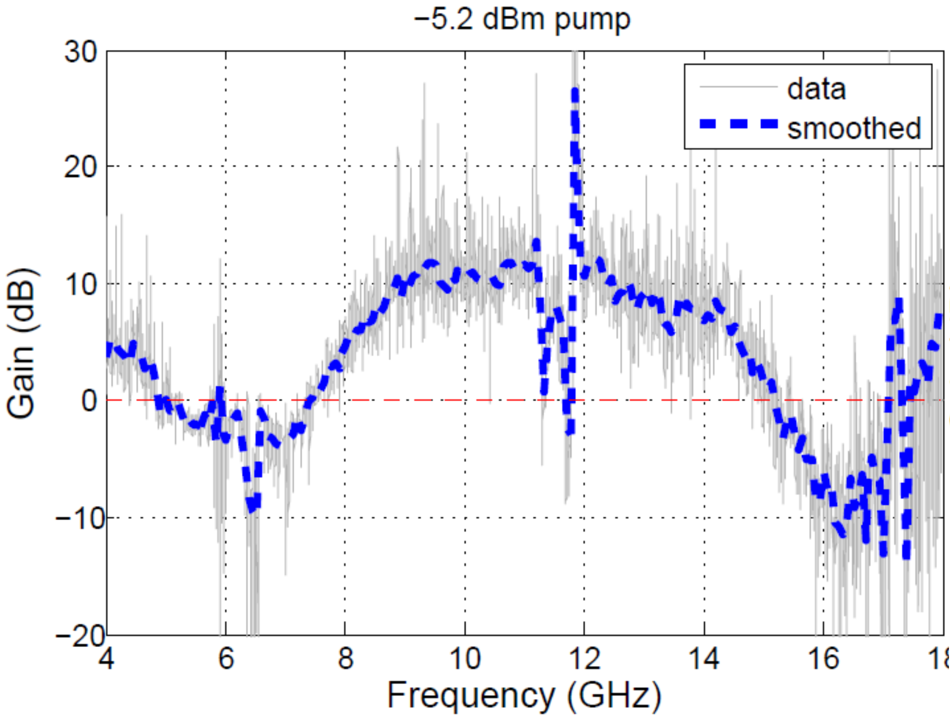
- Assume harmonic generation blocked

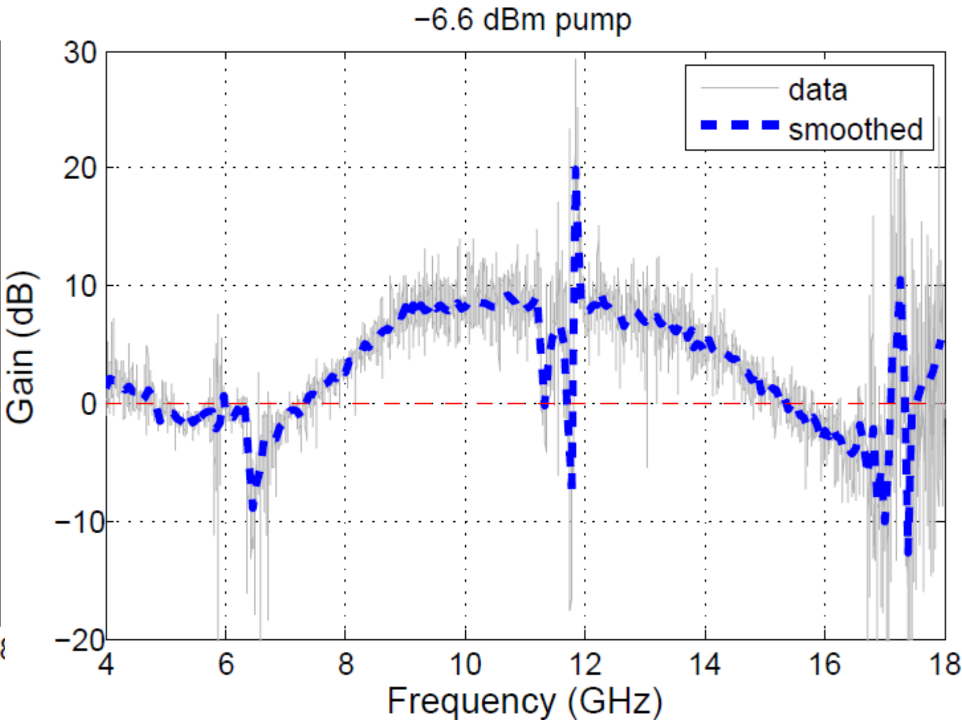

Dispersionless around pump ($\Delta\beta$ = 0)


$\Delta\Theta = 3$, $\Delta\beta$ = Constant

Paramp gain

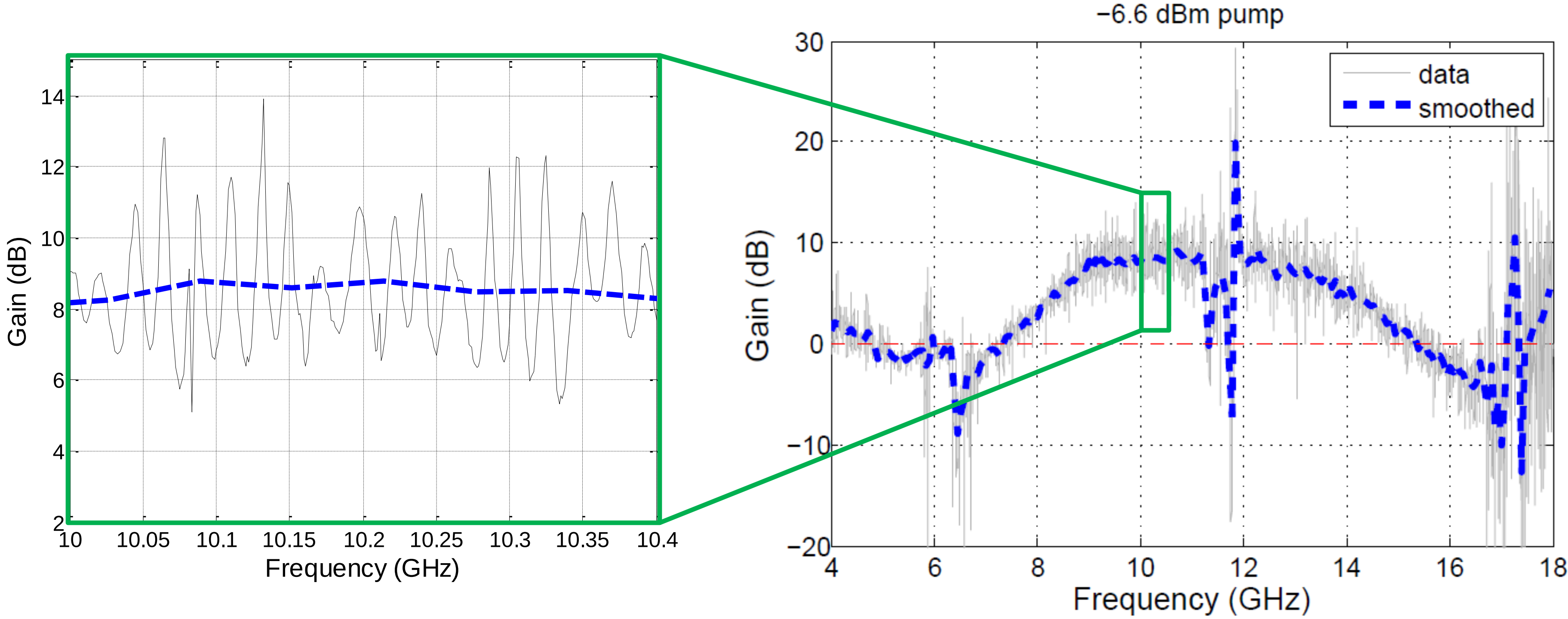
- NbTiN device, pump at 11.56 GHz
  - Near (2nd) engineered dispersion feature





Paramp gain

- NbTiN device, pump at 11.56 GHz
  - Near (2nd) engineered dispersion feature
- Ripple due to reflections, amplifying medium



Conclusion

- Nonlinearity, low dissipation of superconducting nitride films enables a new broad-band paramp
- Measured gain roughly consistent with expectations
  - Noise measurements are underway
- Applications
  - Microwave amp for detector array readout
    - Potential of QL performance
  - Radio astronomy (mm – submm)
    - NbTiN gap frequency > 1 THz
  - Quantum information
  - Axion search