Operation of resonators in the nonlinear regime

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Resonator Workshop

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Caltech

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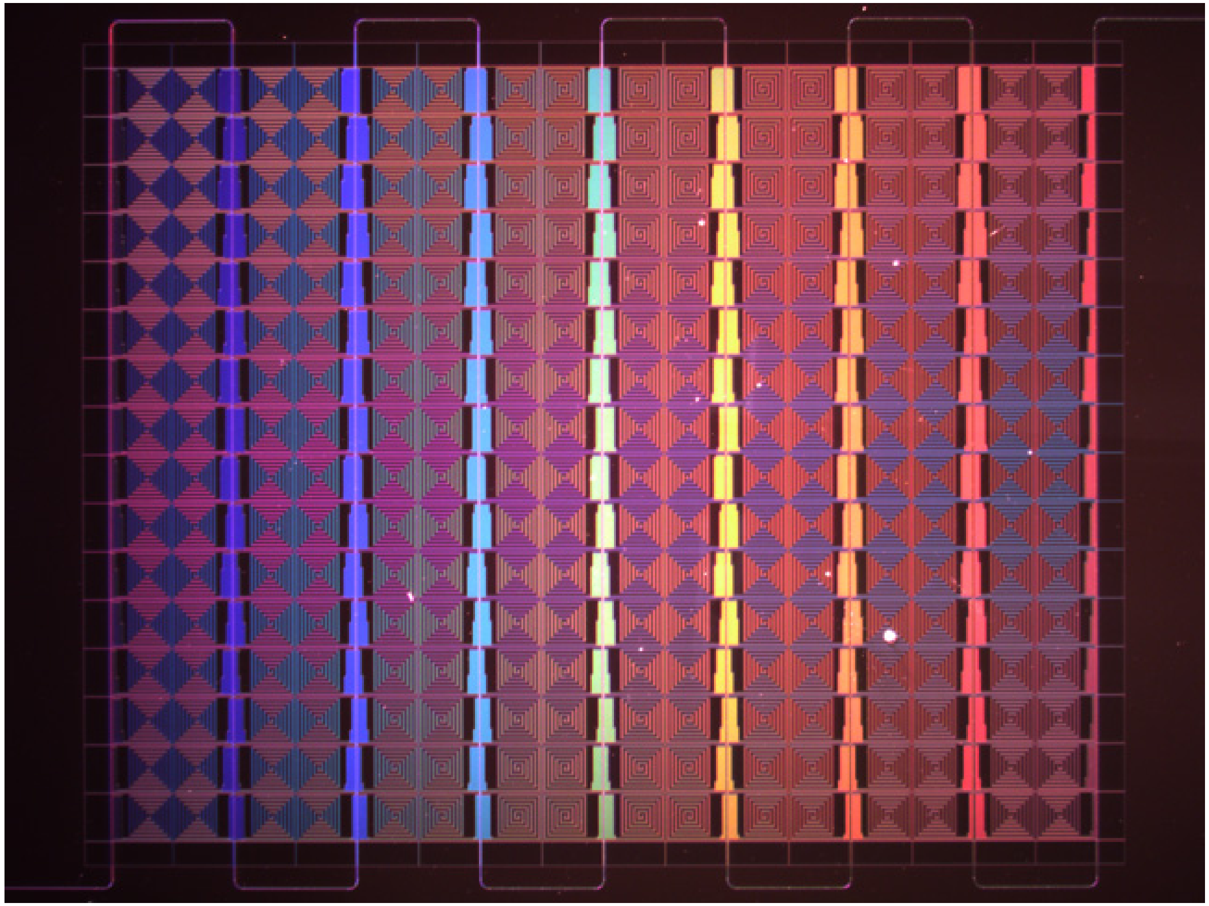
Motivation

TiN is a very promising material:

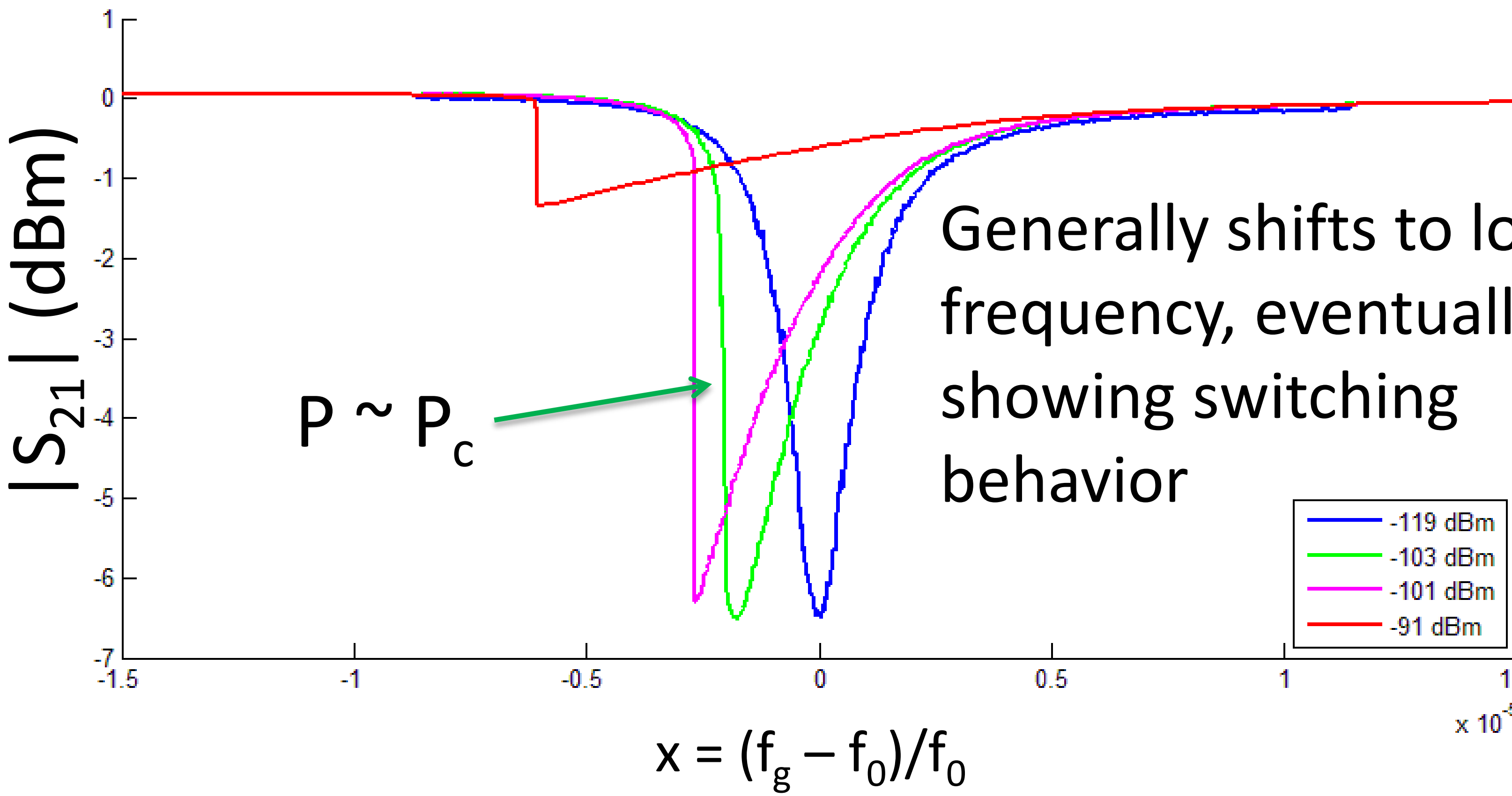
- High normal state resistivity (70-200 $\mu\Omega$–cm)
- Internal Qs exceeding 10^7
- Tunable Tc
- High kinetic inductance fraction



- 16x16 LEKID array. 100 nm TiN on Si with Tc ~ 2 K.
- Optimized for absorbing FIR radiation.
- Band-defining filters + Blackbody used to illuminate with ~ 5 pW radiation
- Qc ~ Qi = 800,000; Qr ~ 400,000

However: Onset of nonlinearity observed at low powers

Typical upward sweeping VNA scans

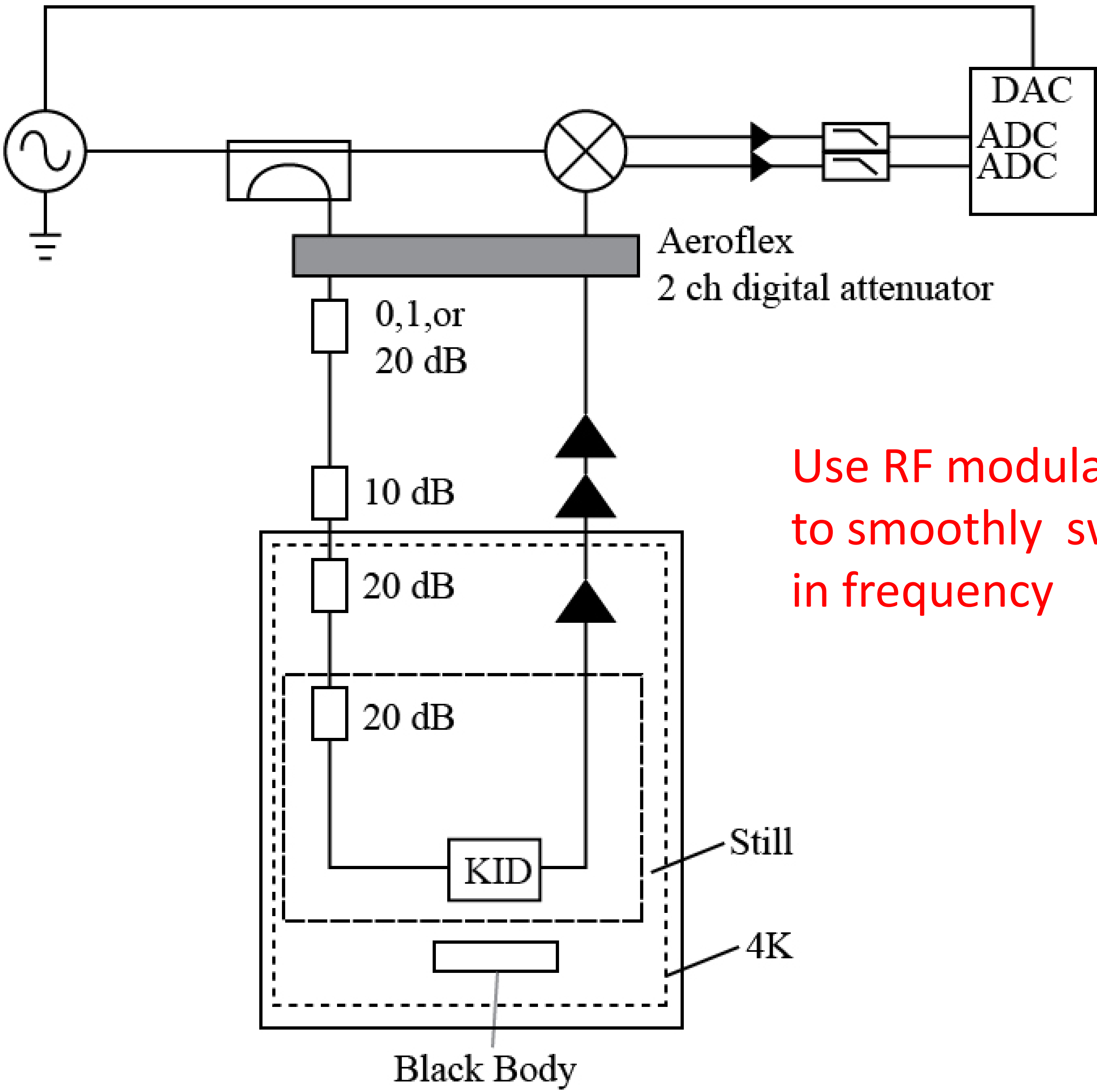


Generally shifts to lower frequency, eventually showing switching behavior

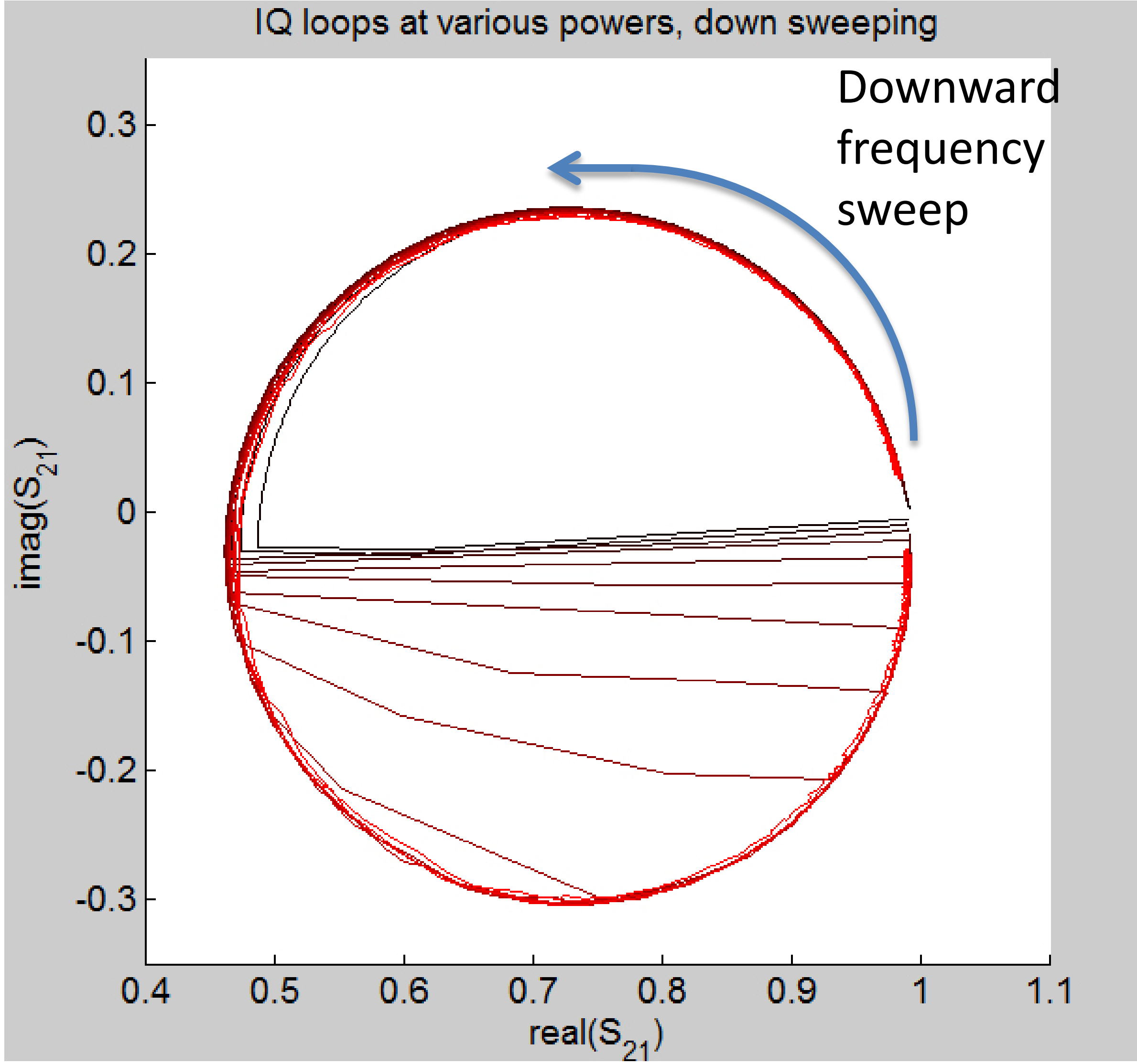
Higher power operation is desirable.

1) Suppress amplifier noise in dissipation and frequency direction.
2) Suppress TLS noise in the frequency direction.

Question: Can we understand the nonlinear behavior? Does this allow us to operate at higher powers?



Use RF modulated source
to smoothly sweep downward
in frequency



- Diameter of the IQ loop doesn't change significantly over broad power range (35 dB).
- Both well below and above onset of bifurcation.

- Qi observed to be fairly constant over large power range, above and below bifurcation.
- Kinetic inductance known to exhibit nonlinear behavior at large currents
- Due to symmetry considerations, leading term quadratic:

$$L_{kin}(I) = L_{kin}(0)[1+I^2/I_*^2]$$

Nonlinear scale factors

$E_* \propto LI_*^2$

$P_* = w_r E_*/Q_r$

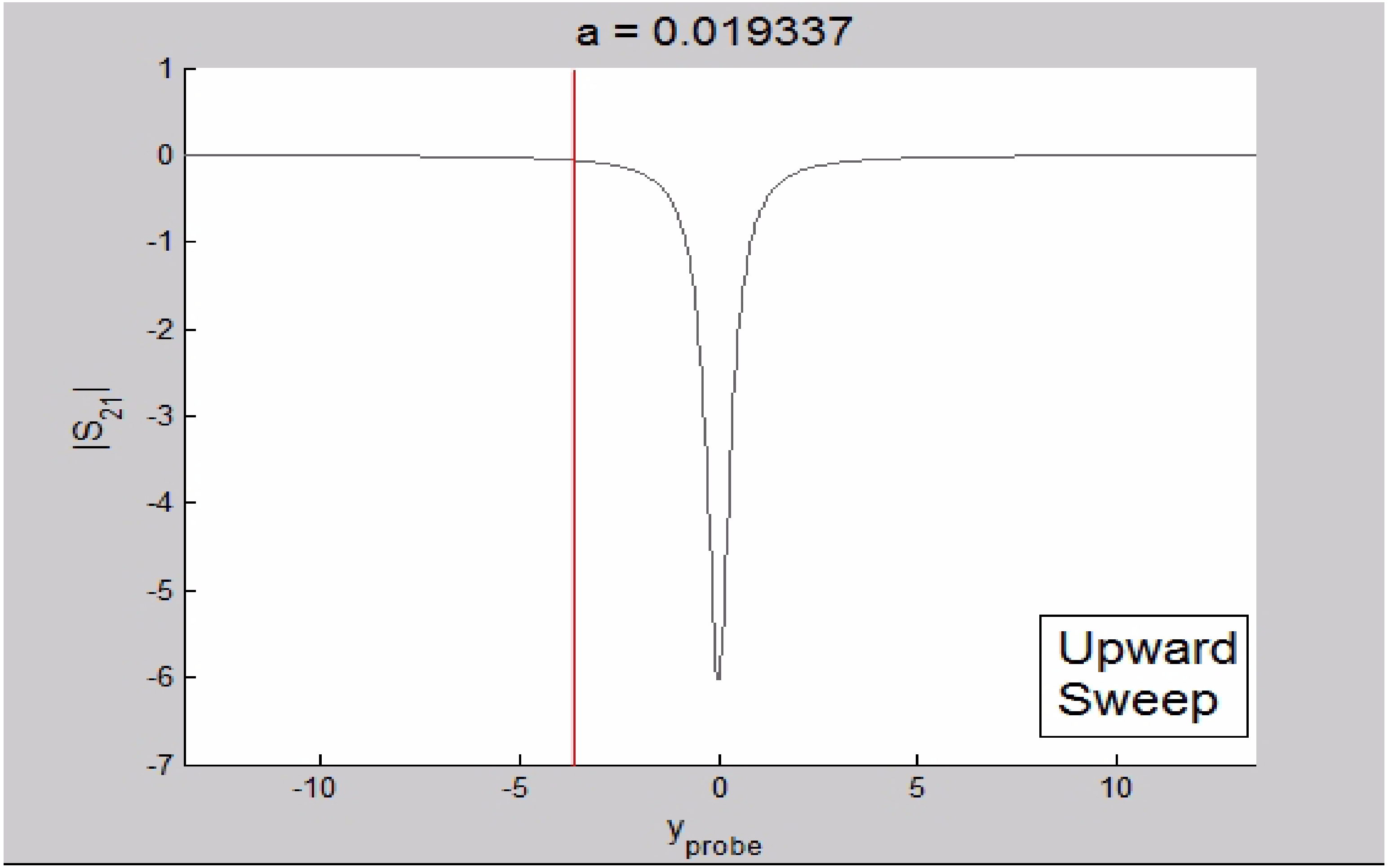
$a = (\chi_c Q_r/2)*(P/P_*)$

$a_c \approx .77$

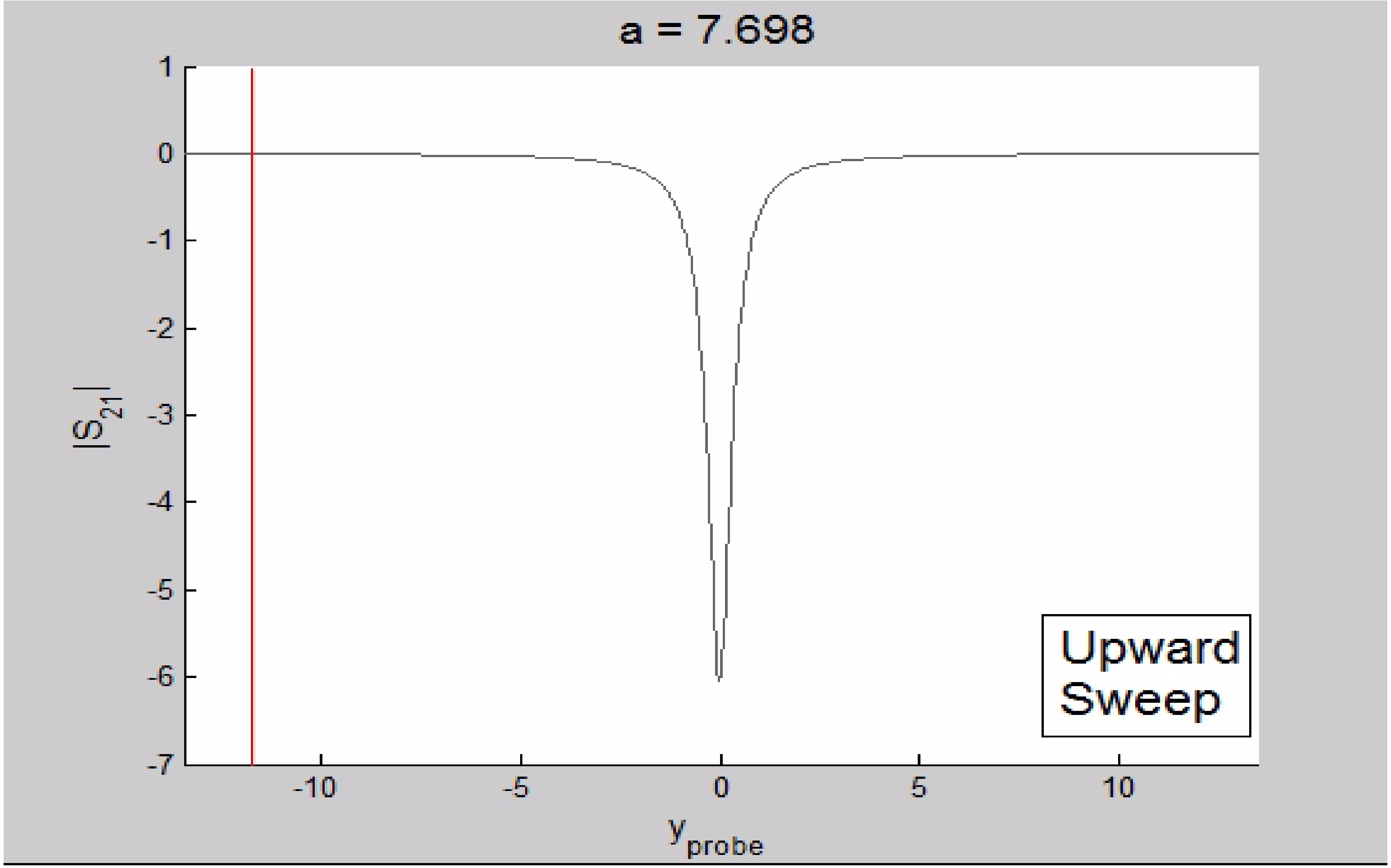
Fractional frequency shift measured in linewidths

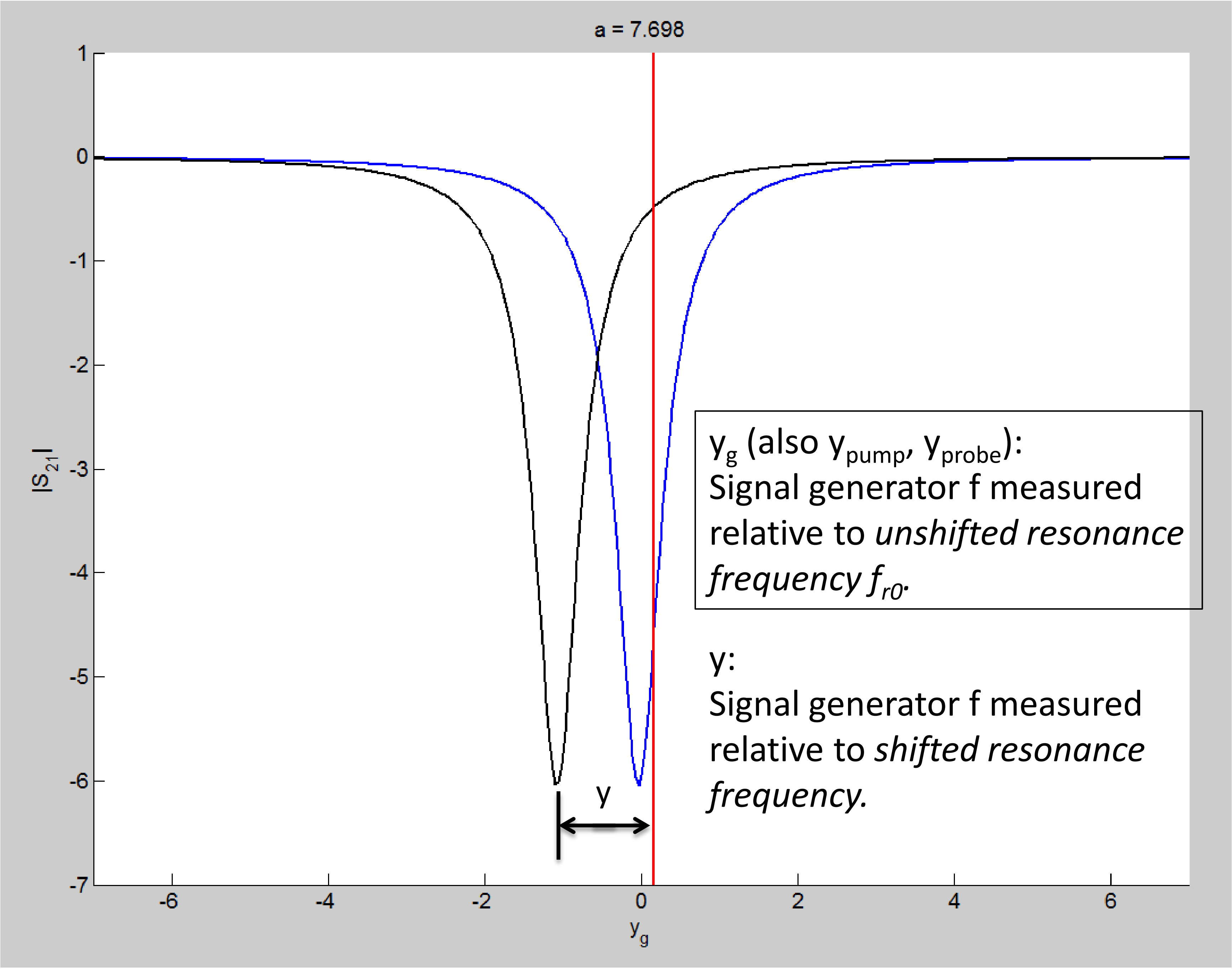
$y_g = (f_g - f_{r,0})/\ \Delta f$

VNA measurement:
Probe power well ***below*** the onset of nonlinearity



VNA measurement:
Probe power well ***above*** the onset of nonlinearity




a = 7.698
1
0
-1
-2
-3
-4
-5
-6
-7
|S21|
-6
-4
-2
0
2
4
6
yg
y
yg (also ypump, yprobe):
Signal generator f measured relative to *unshifted resonance frequency fr0*.
y:
Signal generator f measured relative to *shifted resonance frequency.*

Using conservation of energy, possible to derive the expression:

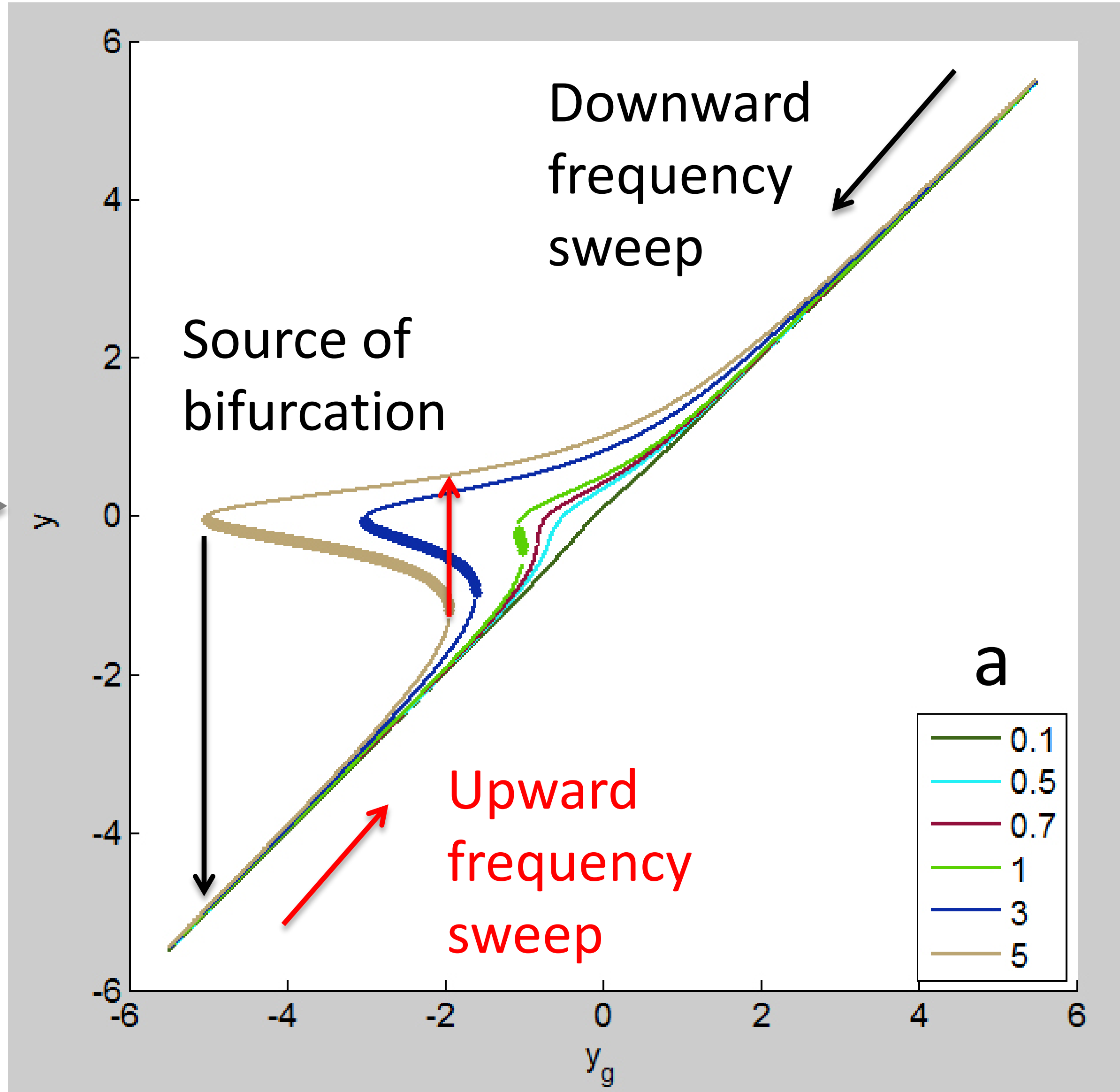
$y_g = \mathbf{y} - a/(1+4\mathbf{y}^2)$

Solve for y(yg)

Why this matters to everyone (even people operating at $P < P_c$):

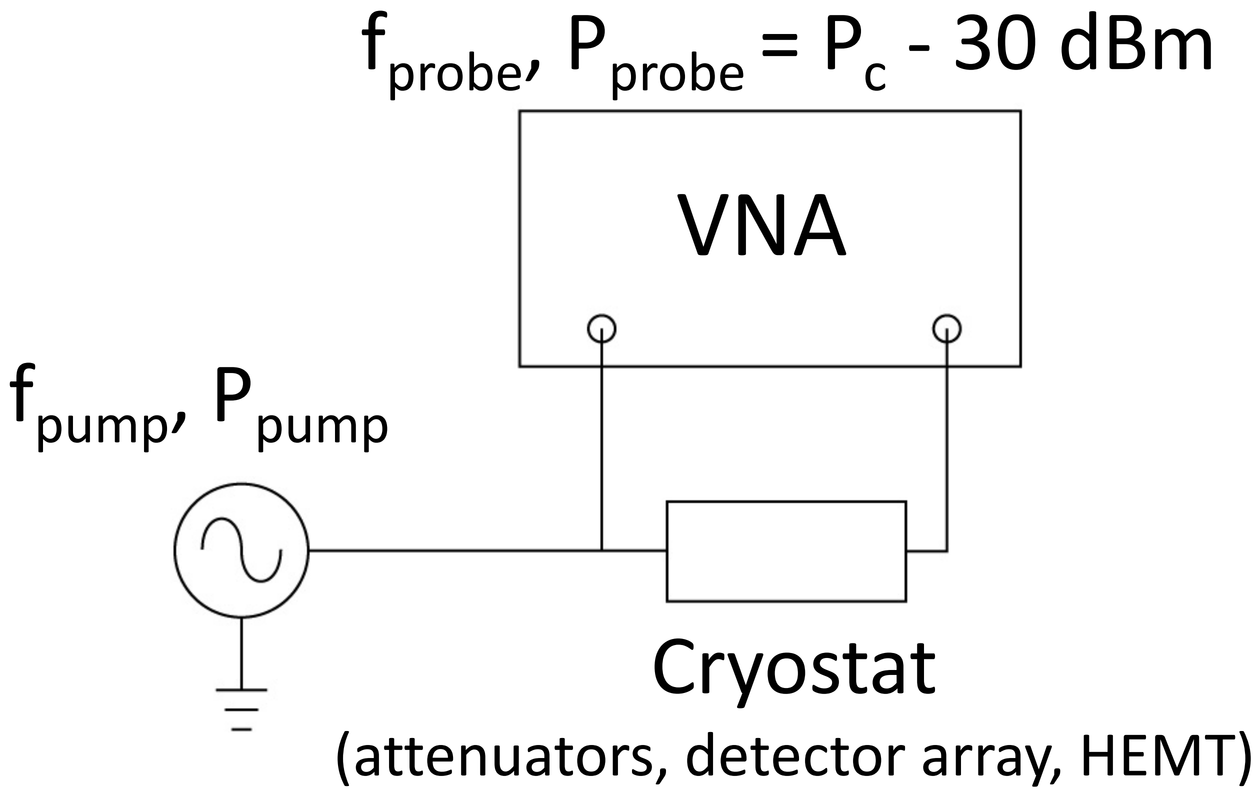
$S_{21} = 1 - Q_r/Q_c * [1/(1+2j\mathbf{y})]$

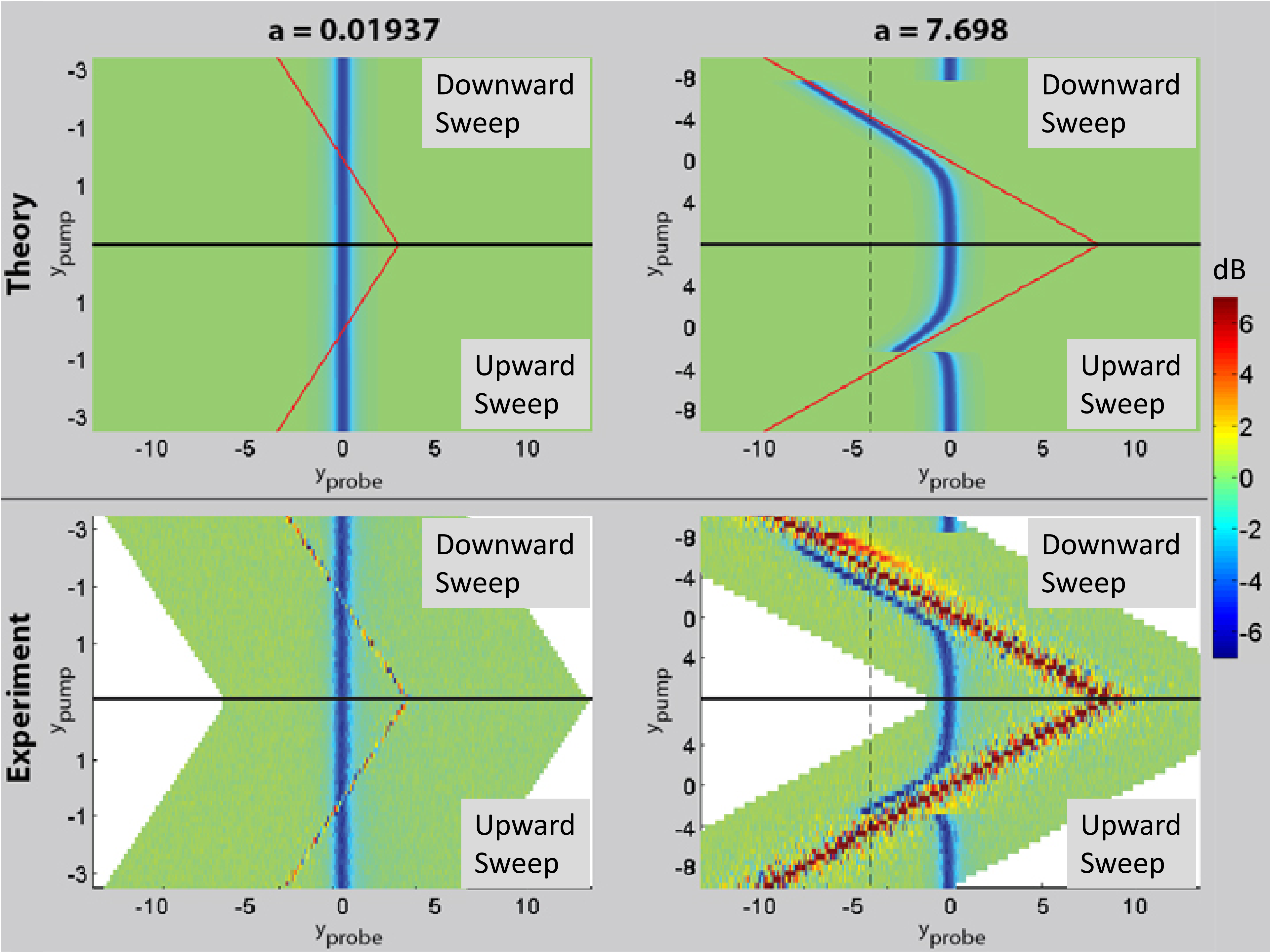
(We'll come back to this in a few slides ....)



Is this really what is going?

ie Can we measure this experimentally?

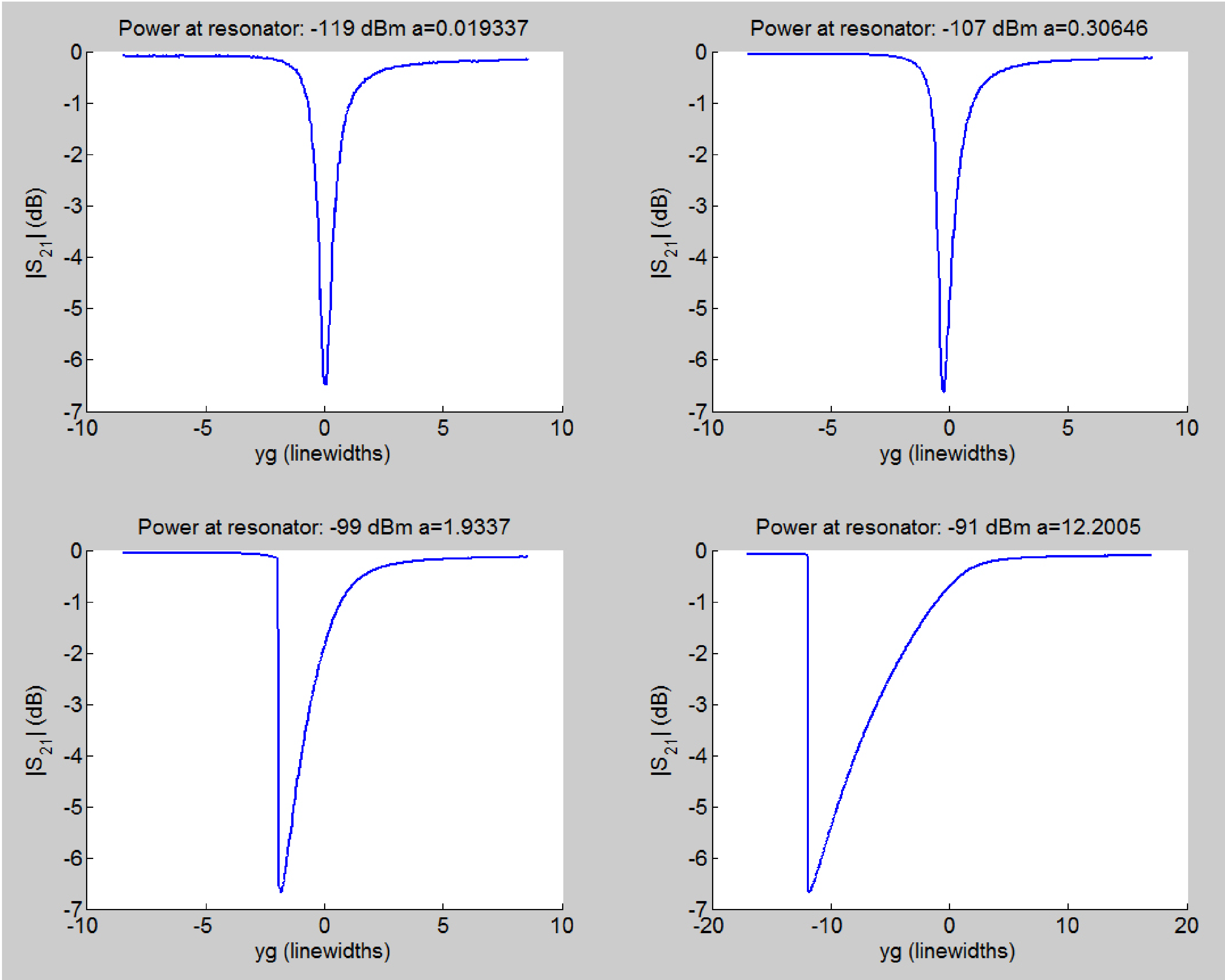




a = 0.01937
a = 7.698
Theory
Experiment
Downward Sweep
Upward Sweep
Downward Sweep
Upward Sweep
Downward Sweep
Upward Sweep
Downward Sweep
Upward Sweep
y pump
y probe
dB
6
4
2
0
-2
-4
-6
-3
-1
1
-10
-5
0
5
10
-8
-4
0
4

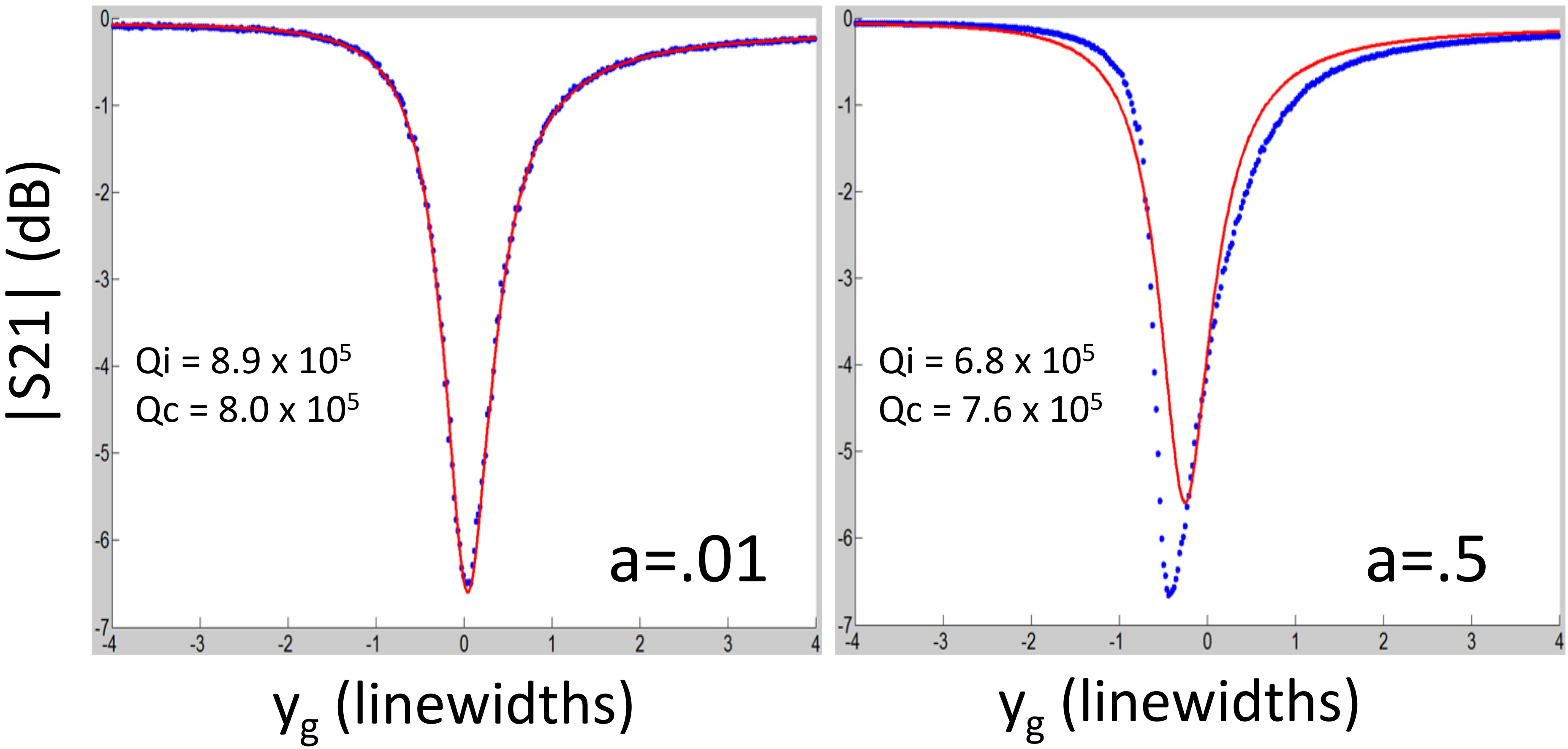
Fitting nonlinear resonances

$$S_{21} = 1 - Q_r/Q_c * [1/(1+2jQ_r x)]$$



Fitting single-tone curves

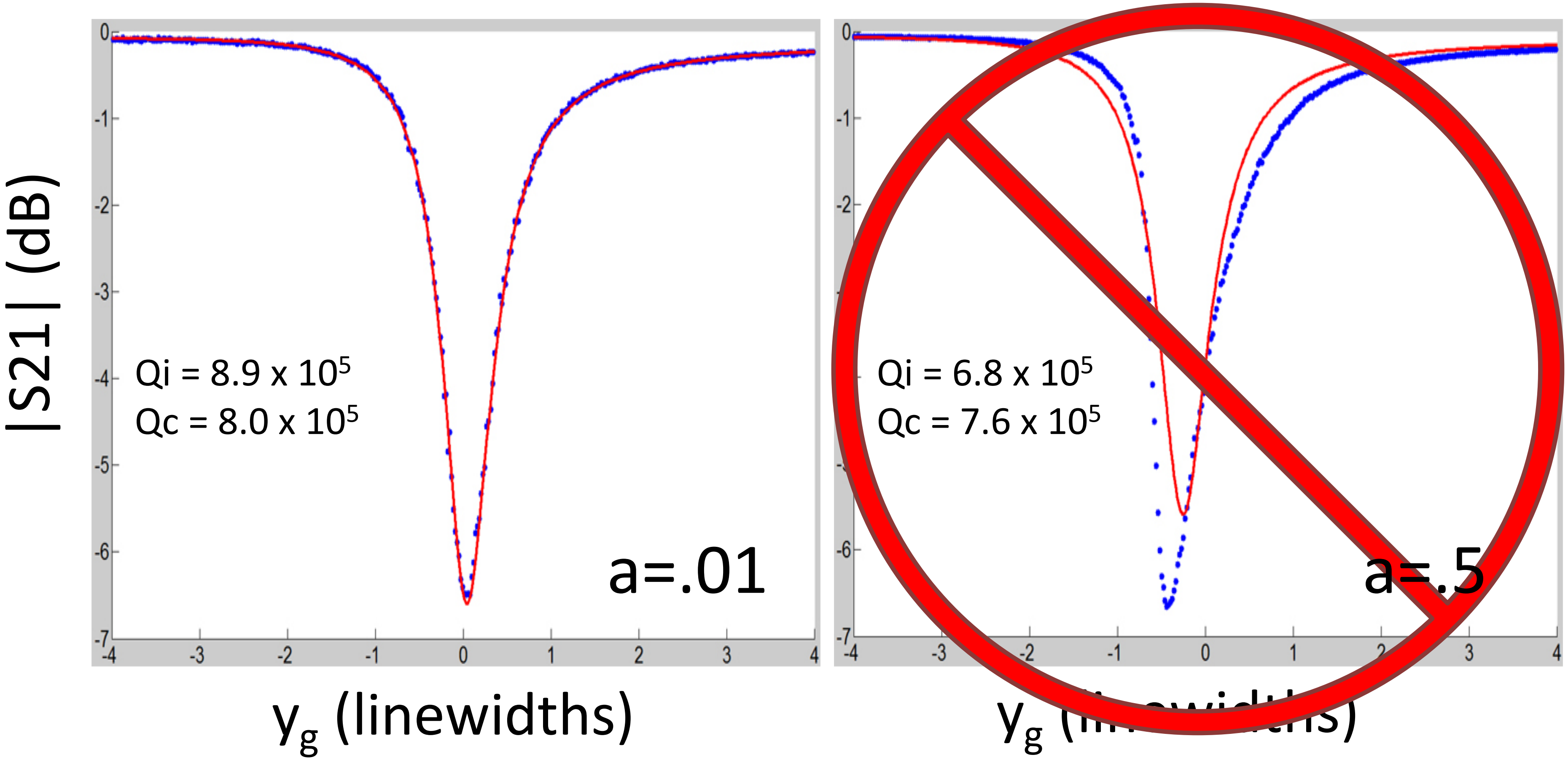
$$t_{21}(f) = ae^{-2\pi j f \tau}\left[1 - \frac{Q_r/Q_c e^{j\phi_0}}{1 + 2jQ(\frac{f-f_r}{f_r})}\right]$$



* Jiansong Gao's Thesis, Caltech, 2008

Fitting single-tone curves

$$t_{21}(f) = a e^{-2\pi j f \tau} \left[1 - \frac{Q_r / Q_c e^{j\phi_0}}{1 + 2jQ(\frac{f-f_r}{f_r})} \right]$$



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Fitting single-tone curves

$$t_{21}(f) = a e^{-2\pi j f \tau}\left[1 - \frac{Q_r/Q_c e^{j\phi_0}}{1+2j\ y}\right]$$

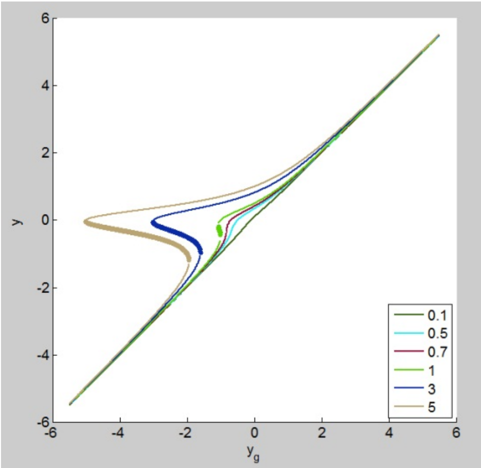
$$y_g = y - a/(1+4y^2)$$

High power:

1) Fix everything except Qi by fitting low power curve (a ~ .01)

2) Fit downward sweeping curves.

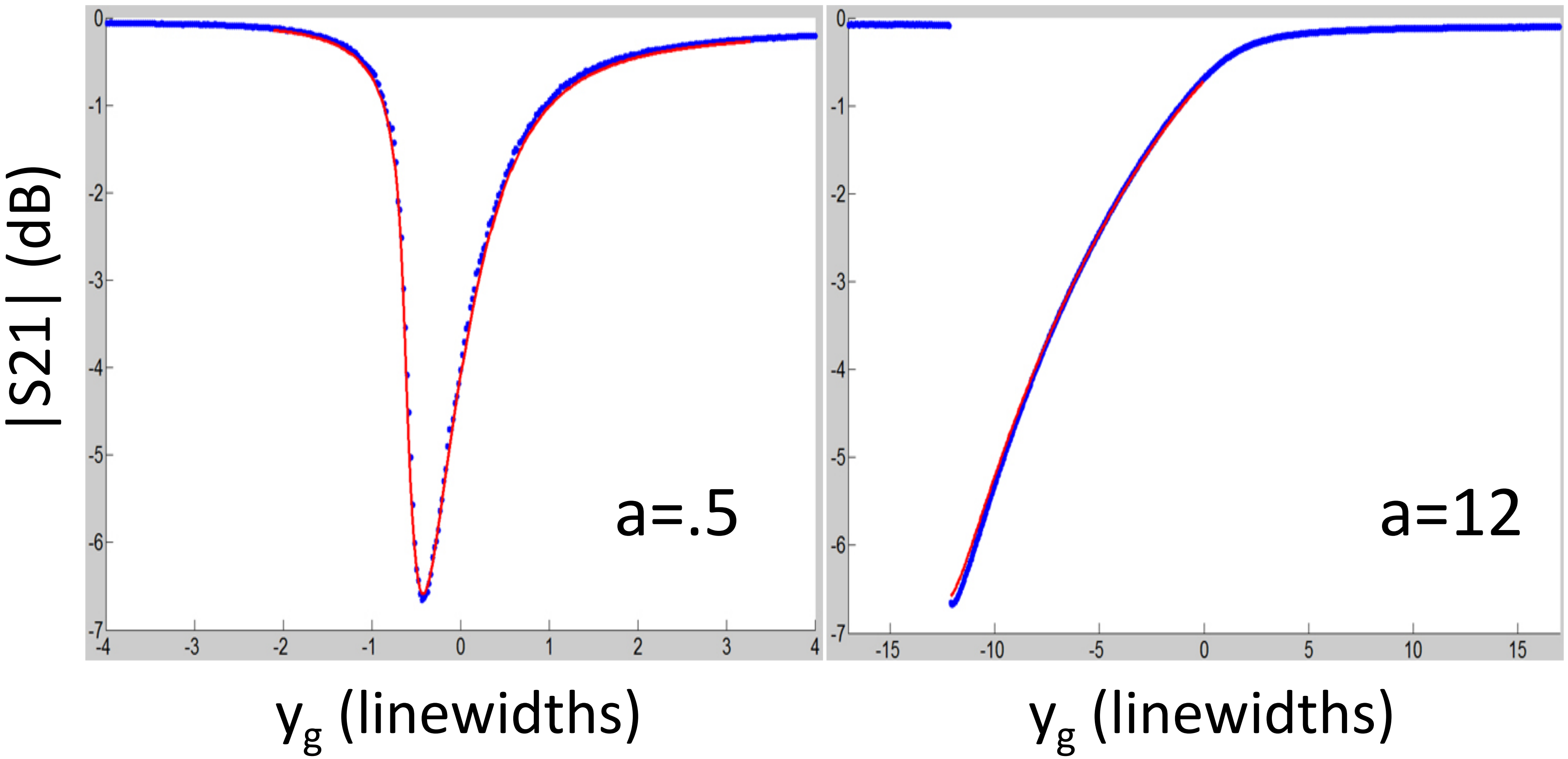
y

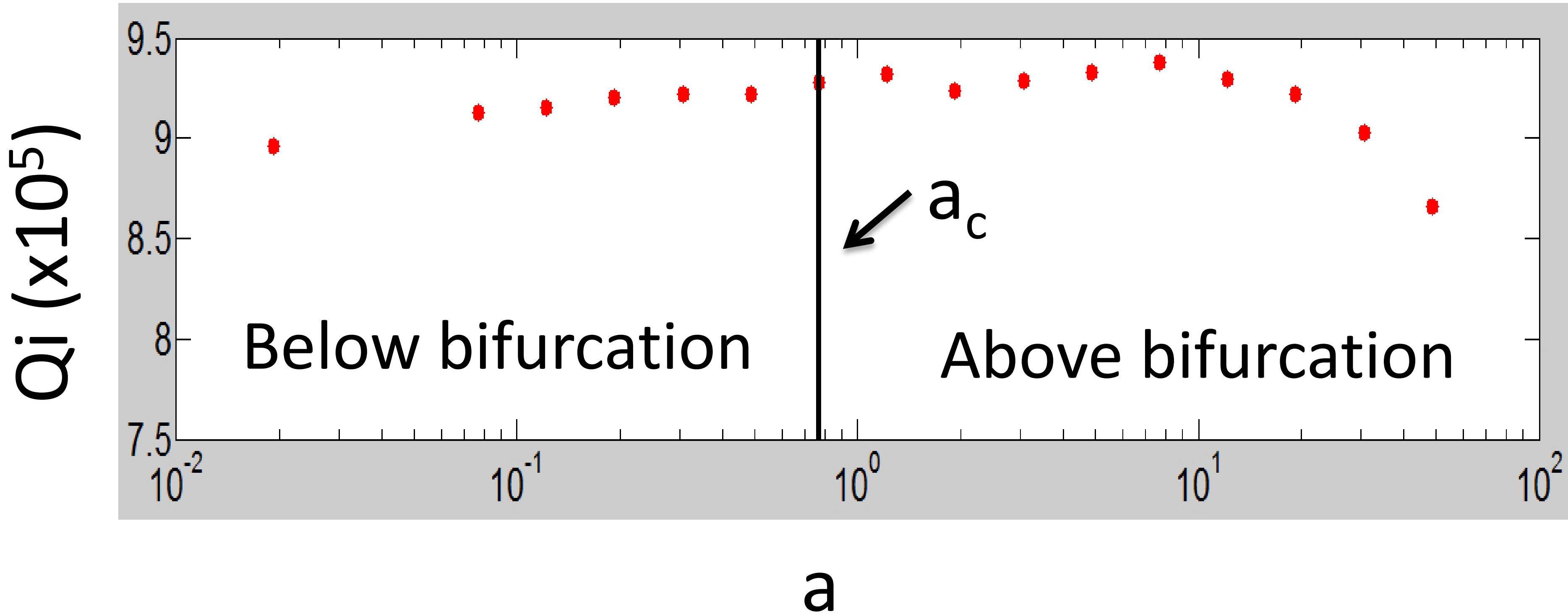


yg

Fitting single-tone curves

$$t_{21}(f) = ae^{-2\pi jf\tau}\left[1 - \frac{Q_r/Q_c e^{j\phi_0}}{1+2j\ y}\right]$$

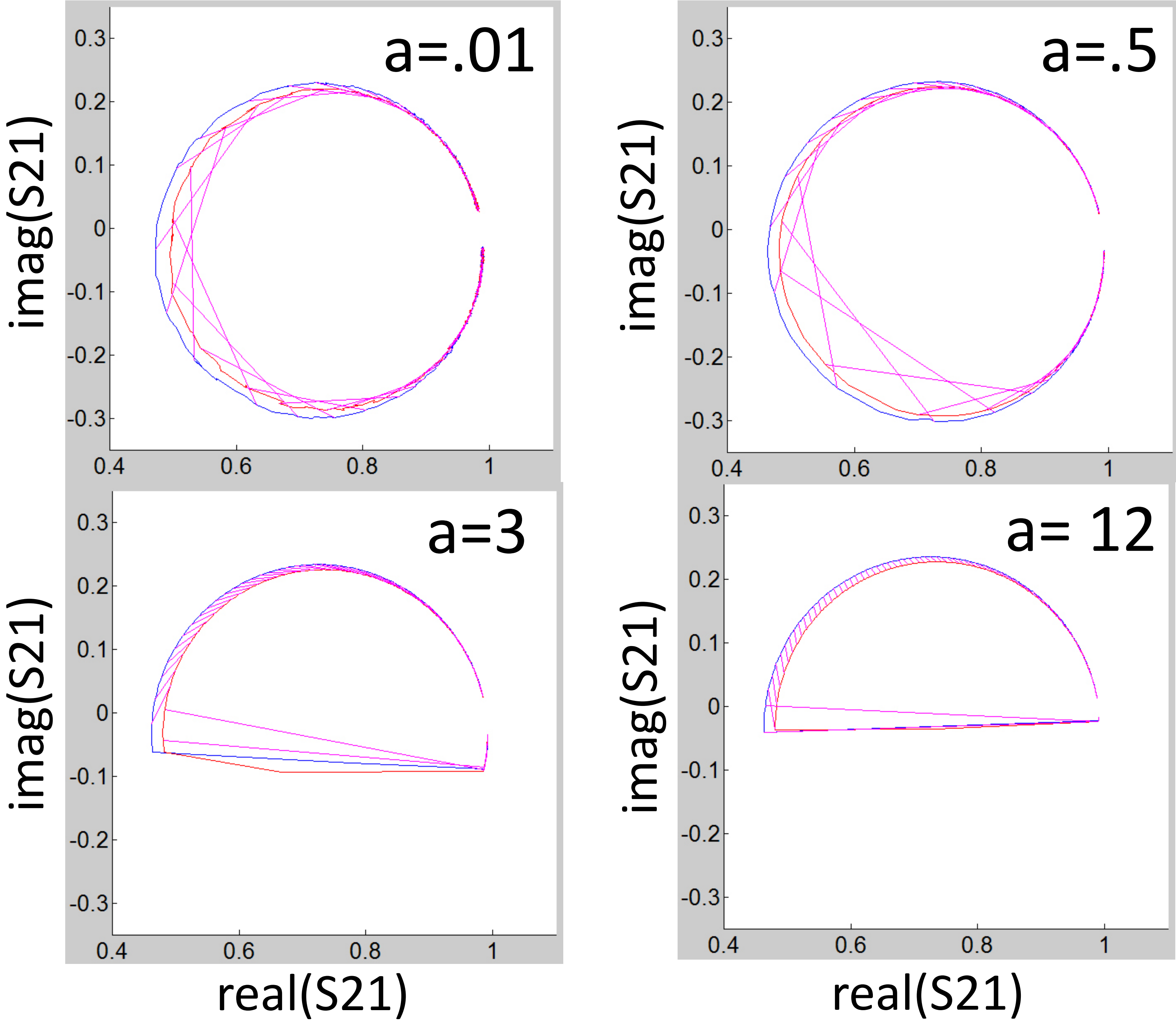




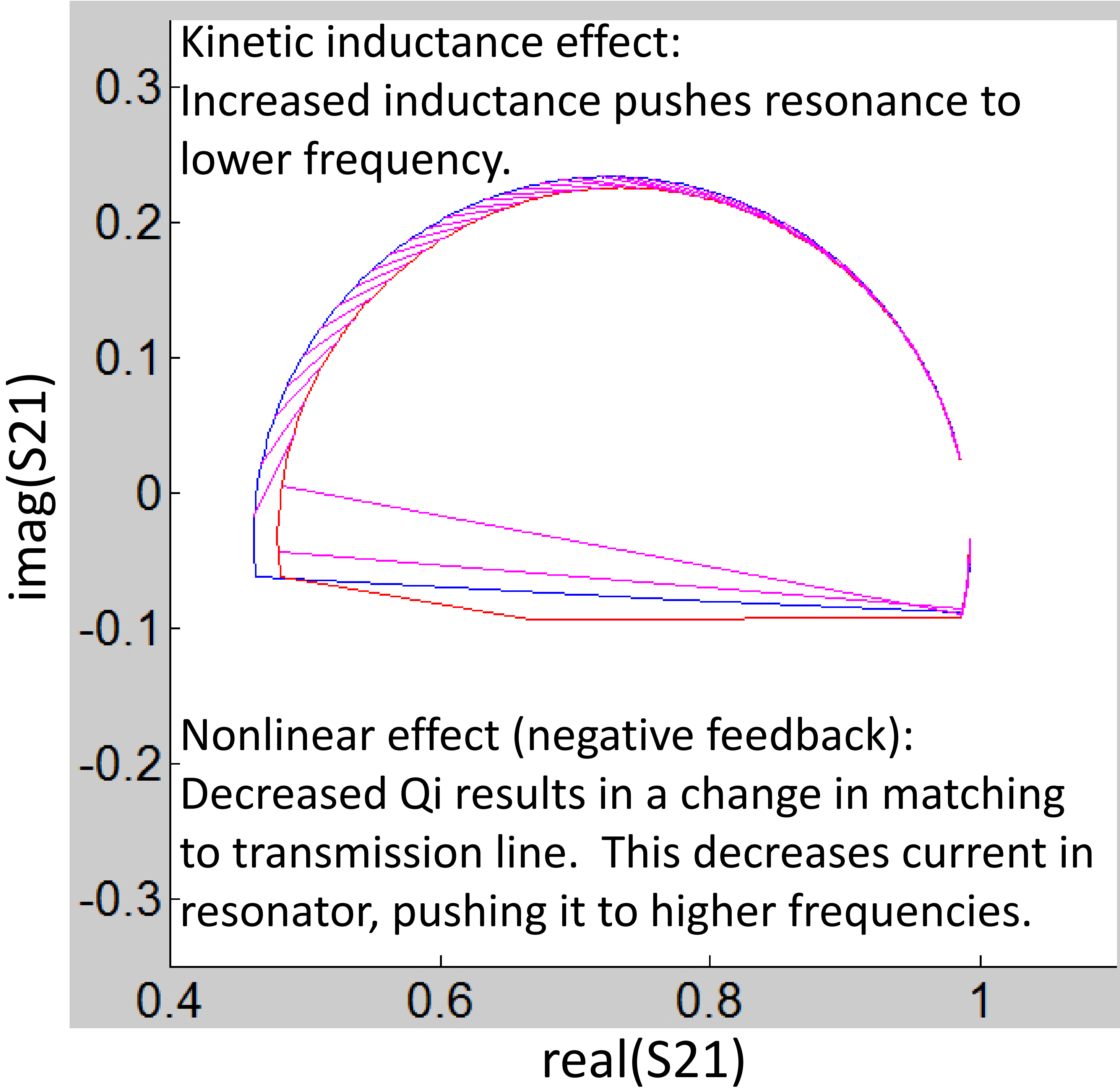
Qi (x10^5)
9.5
9
8.5
8
7.5
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
a_c
Below bifurcation
Above bifurcation
a

NEP calculations: Signal

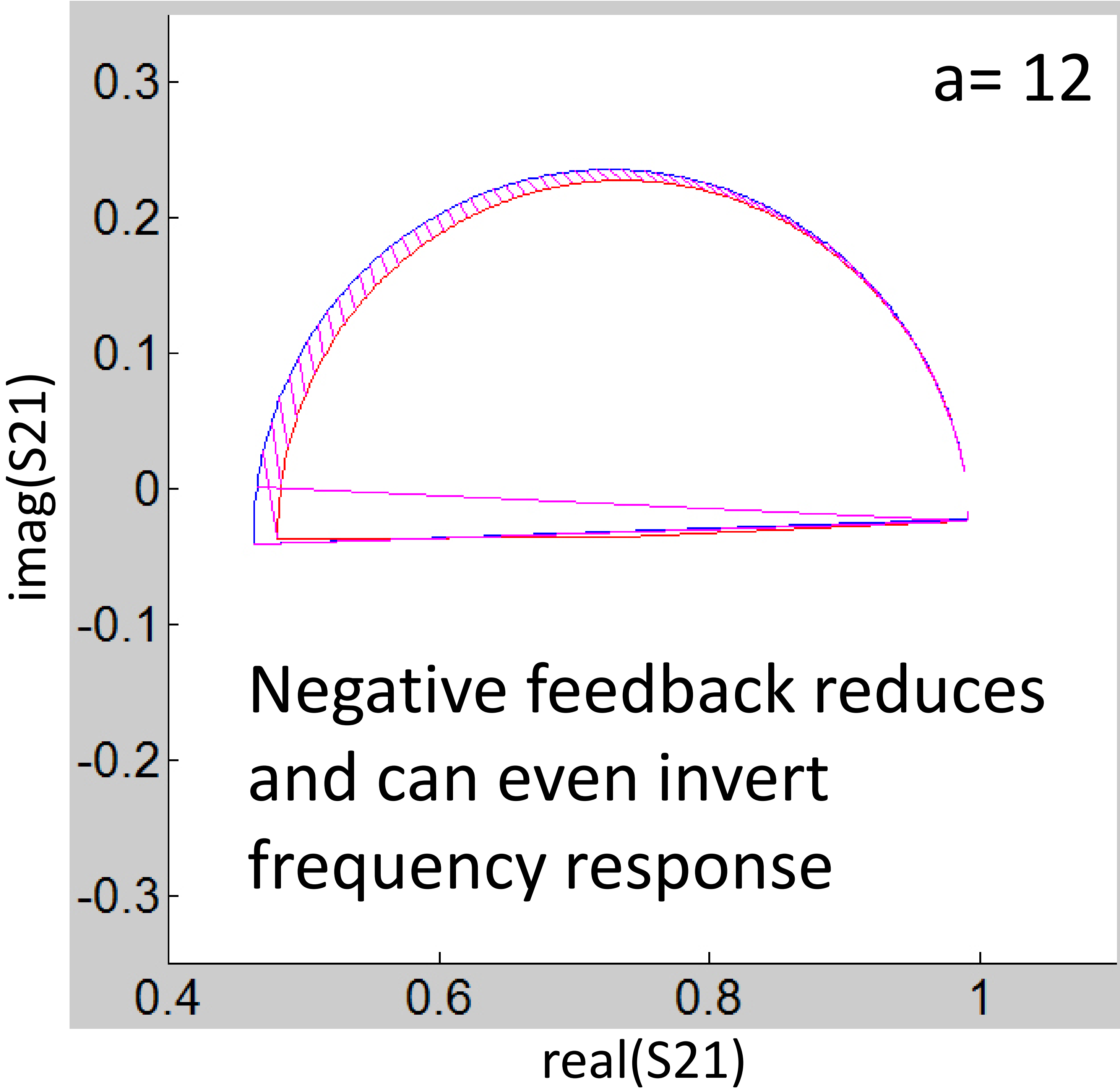
Signal: Response to .5 pW while under 5 pW loading



a=.01
a=.5
a=3
a= 12
imag(S21)
real(S21)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.4
0.6
0.8
1

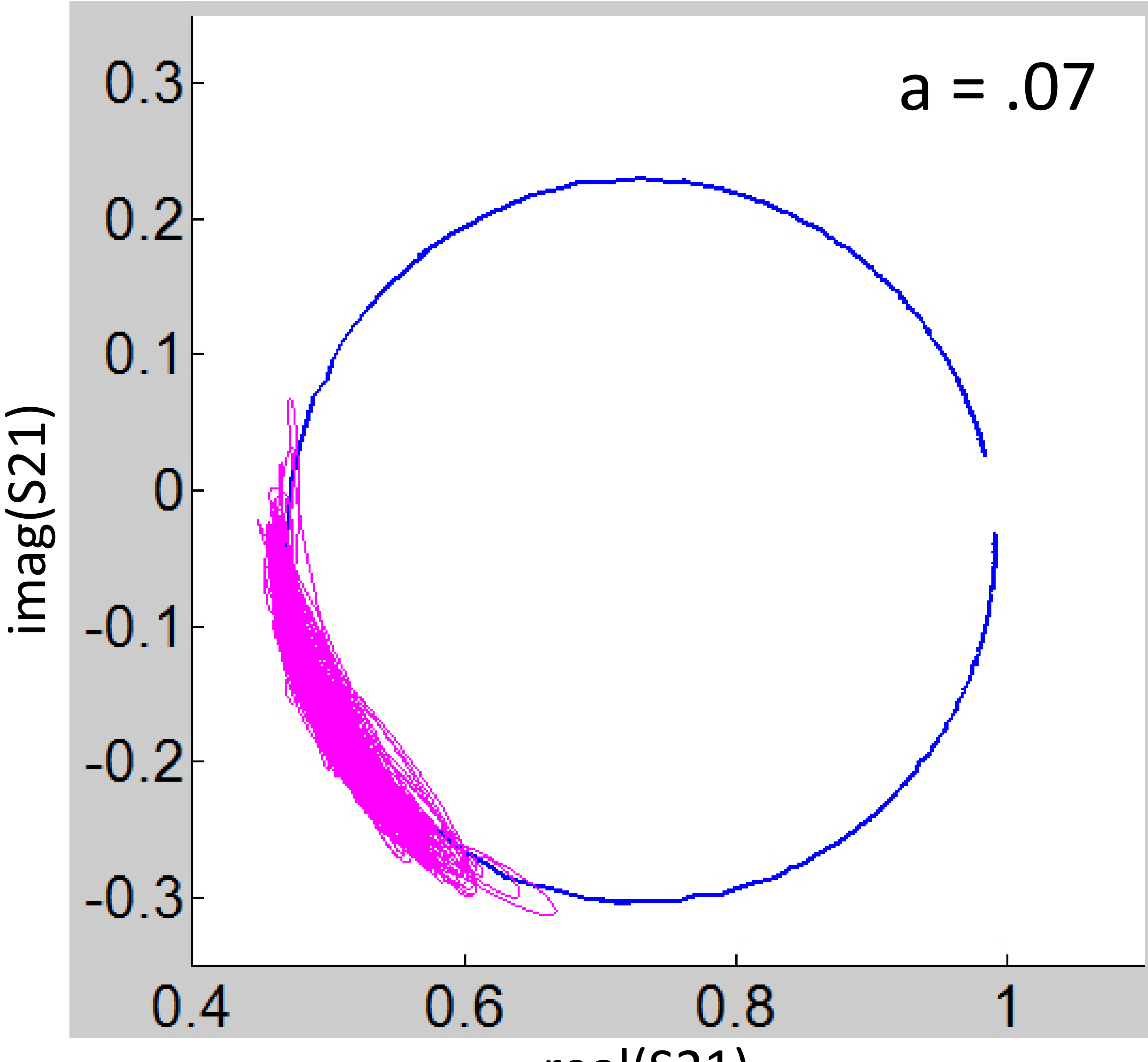

Kinetic inductance effect:
Increased inductance pushes resonance to lower frequency.
Nonlinear effect (negative feedback):
Decreased Qi results in a change in matching to transmission line. This decreases current in resonator, pushing it to higher frequencies.
imag(S21)
real(S21)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.4
0.6
0.8
1

a=3



a= 12
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.4
0.6
0.8
1
imag(S21)
real(S21)
Negative feedback reduces and can even invert frequency response

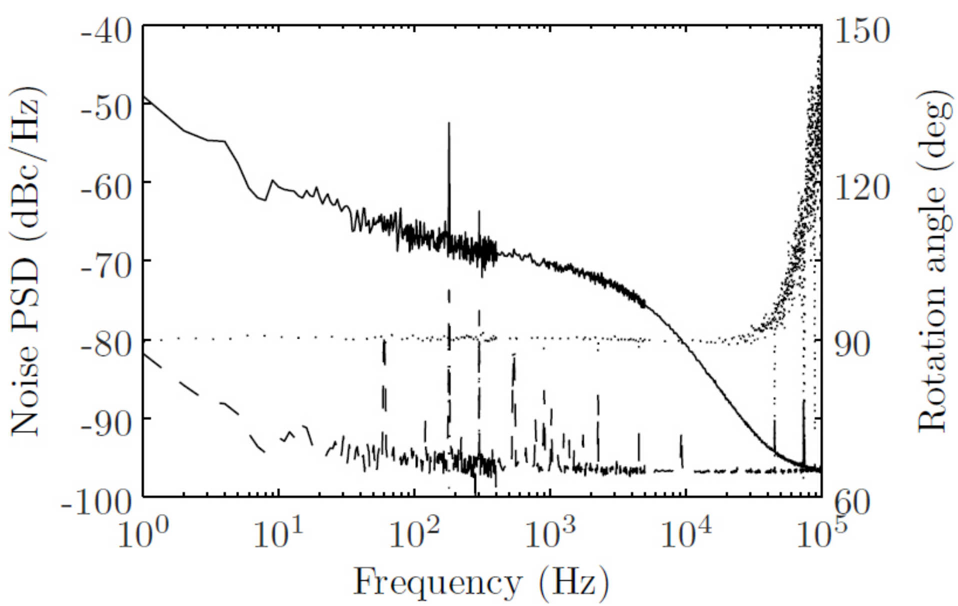
NEP calculations: Noise


a = .07
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
imag(S21)
0.4
0.6
0.8
1
real(S21)

Jiansong Gao Method*: Diagonalize Spectral Density Matrix

$$\langle \delta\xi(\nu)\delta\xi^\dagger(\nu')\rangle = S(\nu)\delta(\nu-\nu'), \quad S(\nu) = \begin{pmatrix} S_{II}(\nu) & S_{IQ}(\nu) \\ S^*_{IQ}(\nu) & S_{QQ}(\nu) \end{pmatrix} \tag{5.2}$$

$$O^T(\nu)\,\mathrm{Re}\,S(\nu)\,O(\nu) = \begin{pmatrix} S_{aa}(\nu) & 0 \\ 0 & S_{bb}(\nu) \end{pmatrix} \tag{5.3}$$



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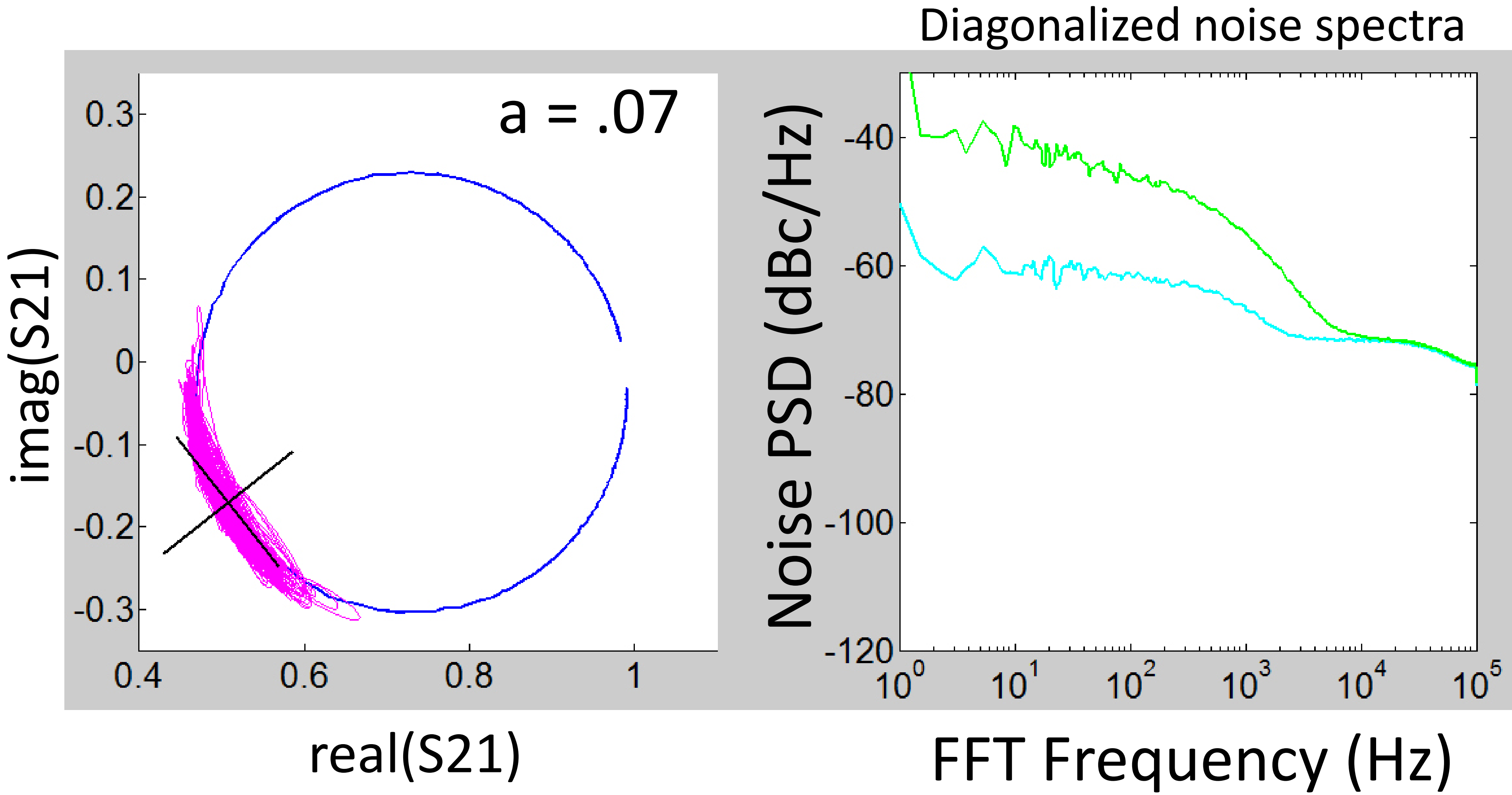


At low powers, noise has significant curvature

Transform to remove curvature

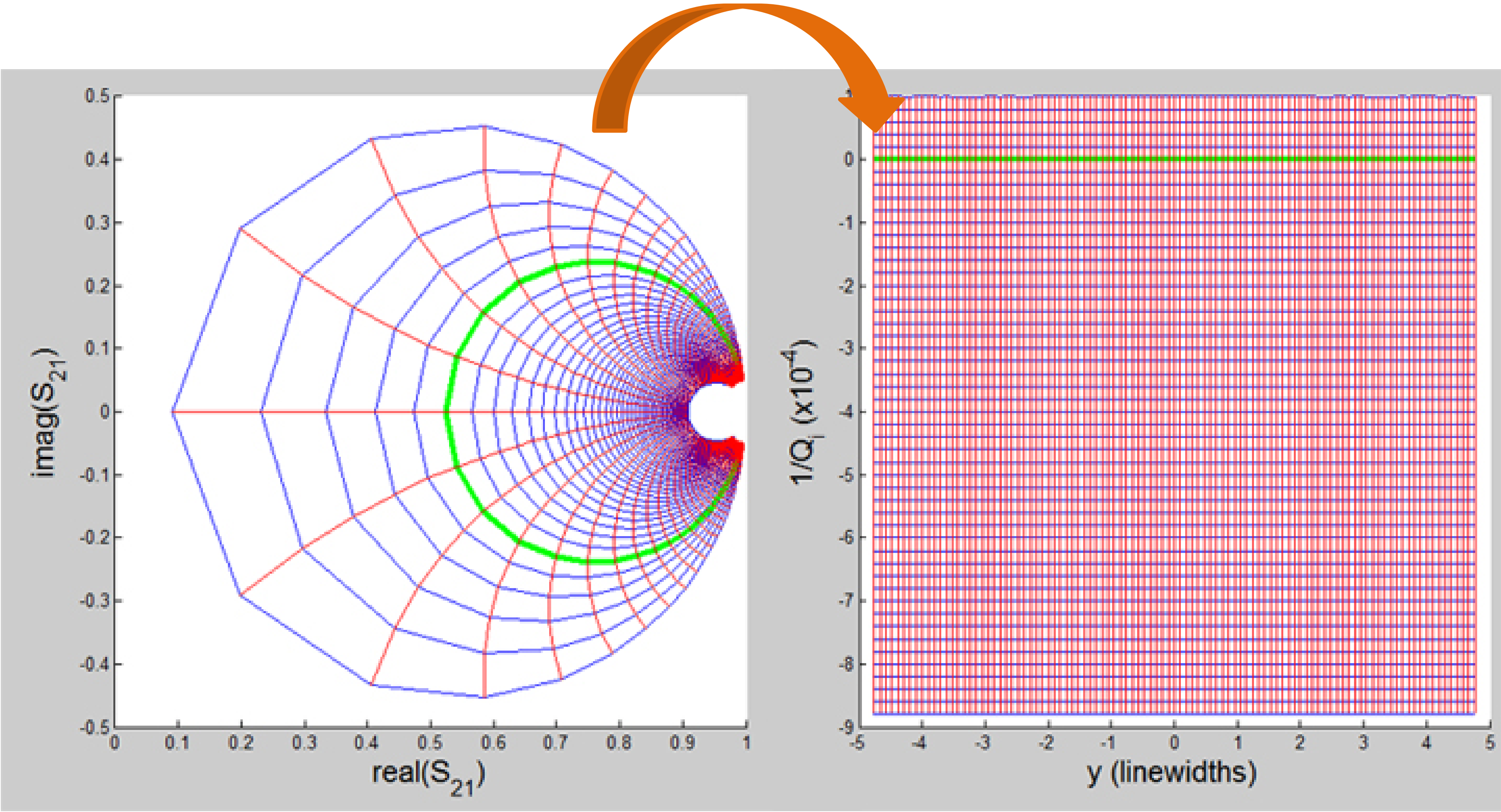
$$S_{21} = 1- (Q_r/Q_c)[1/(1+2jQ_r x)]$$

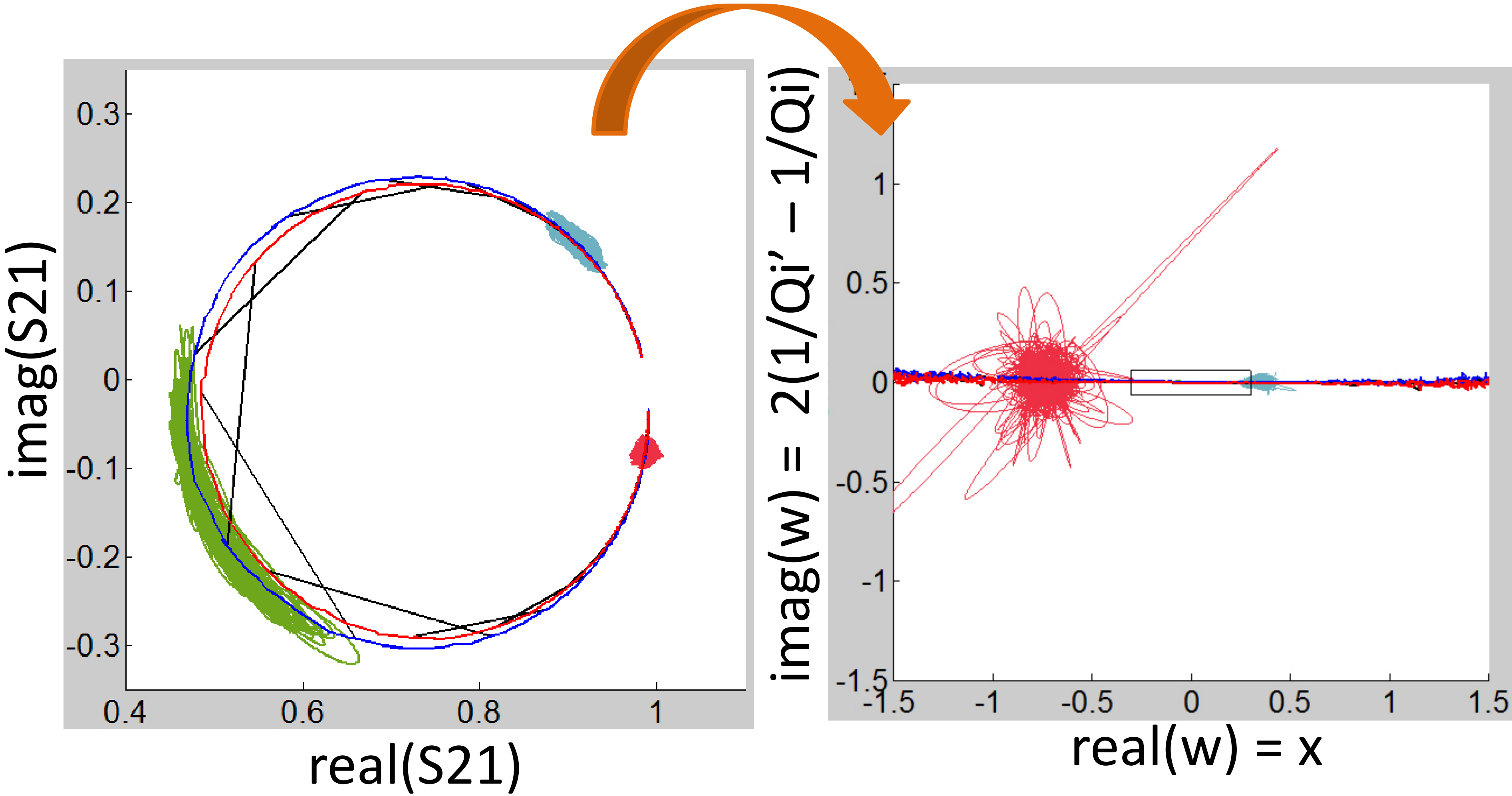
$$S_{21} = 1- (Q_r/Q_c)[1/(1+2jQ_r \mathbf{w})]$$

$\mathbf{w} = 1/(2jQ_c)\ (1/(1- S_{21}) - 1/(2jQ_r)$; fixed Qi'

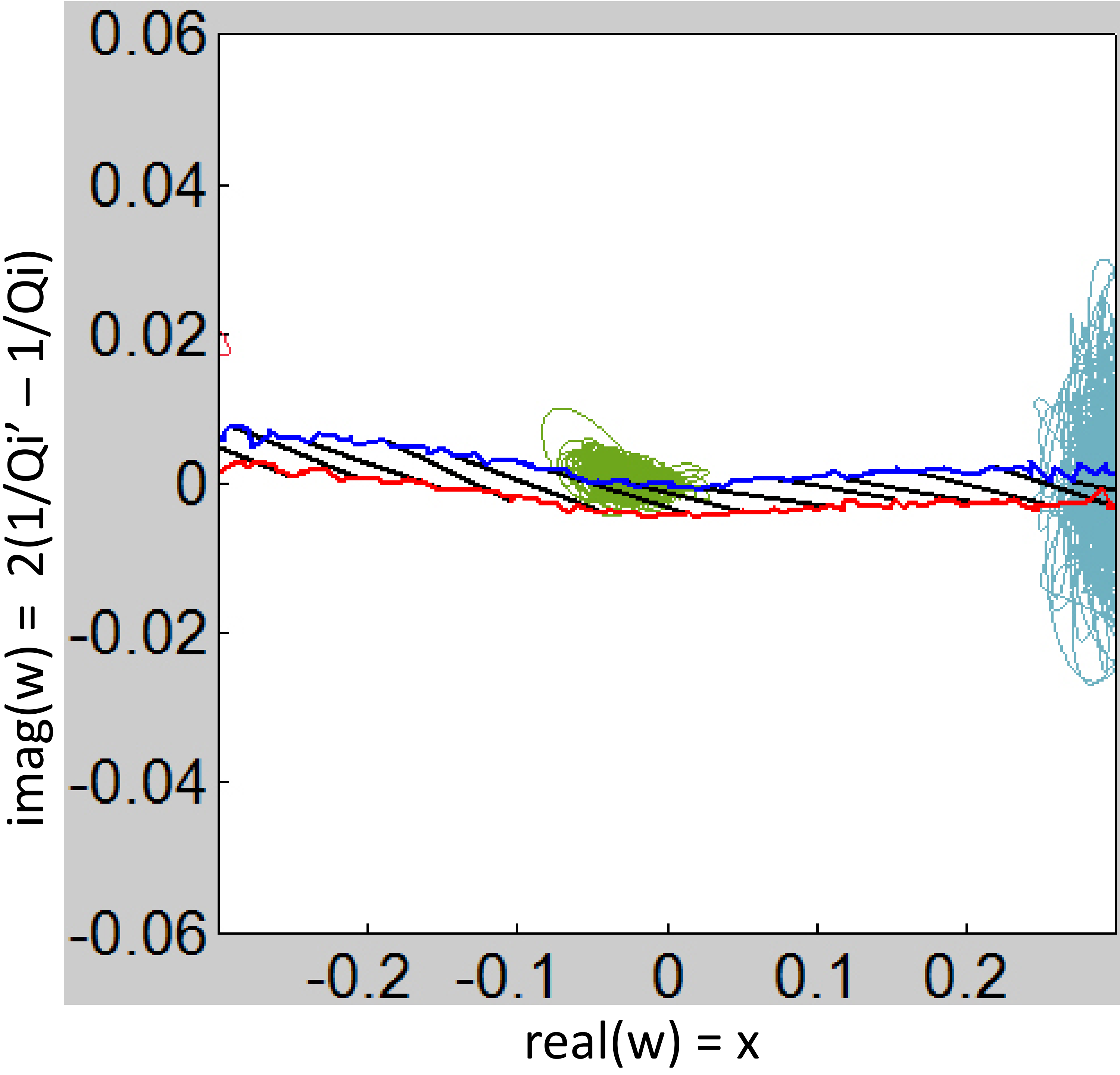
real($\mathbf{w}$) = x; imag($\mathbf{w}$) = 2(1/Qi' – 1/Qi)

Transform

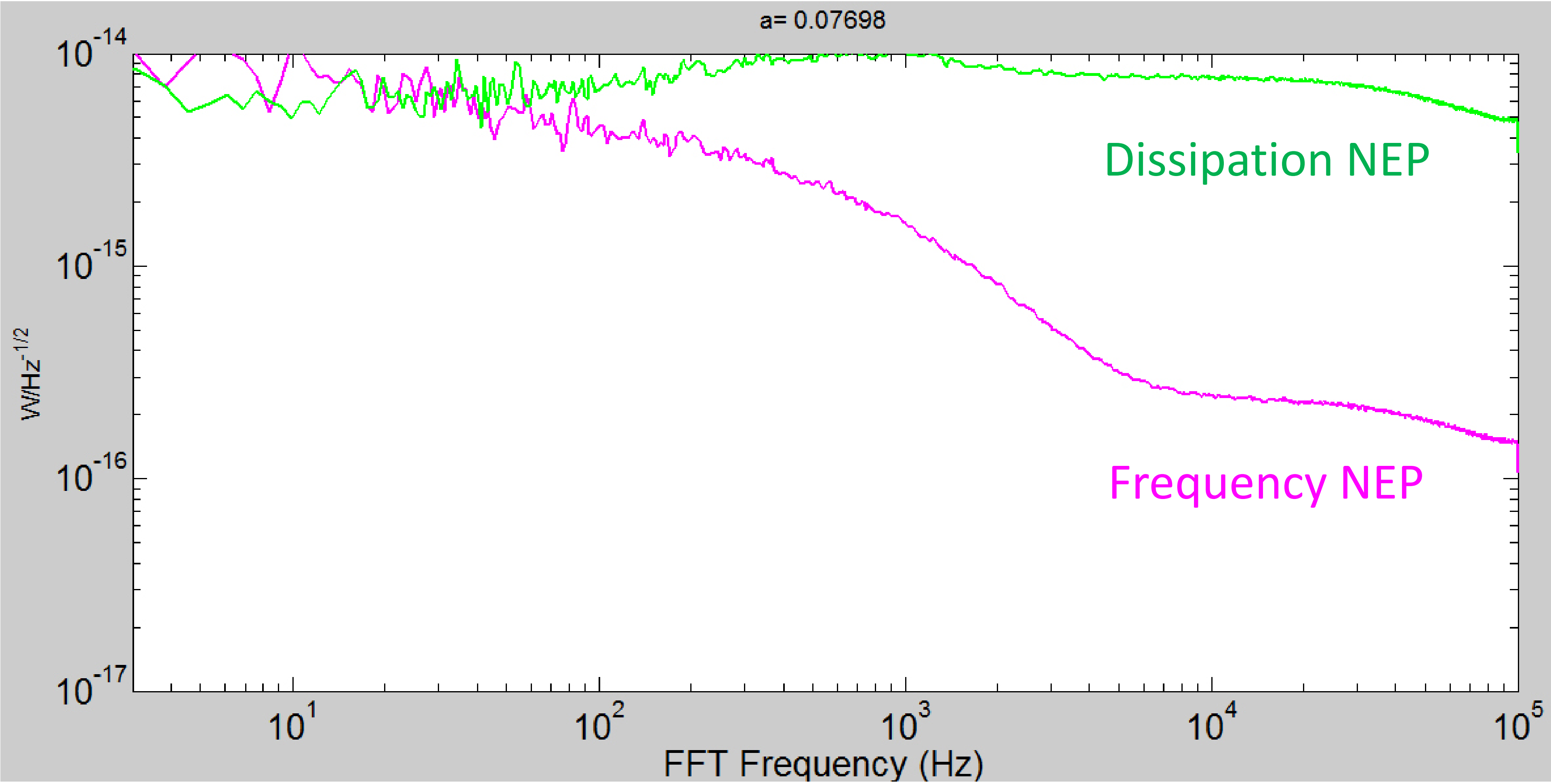




0.3
0.2
0.1
0
-0.1
-0.2
-0.3
imag(S21)
0.4
0.6
0.8
1
real(S21)
imag(w) = 2(1/Qi' − 1/Qi)
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
real(w) = x

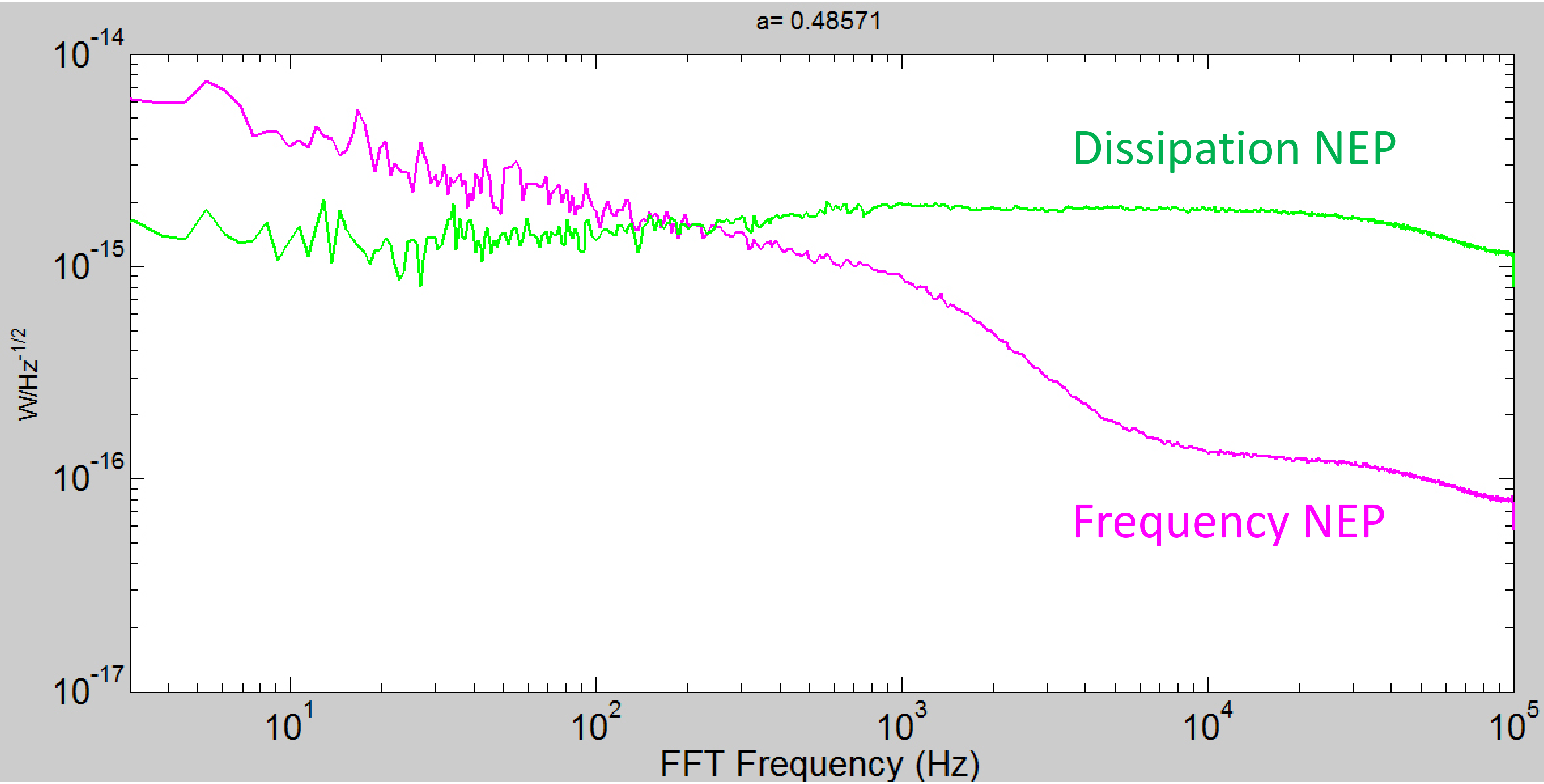

0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.2
-0.1
0
0.1
0.2
imag(w) = 2(1/Qi' − 1/Qi)
real(w) = x

a = .08



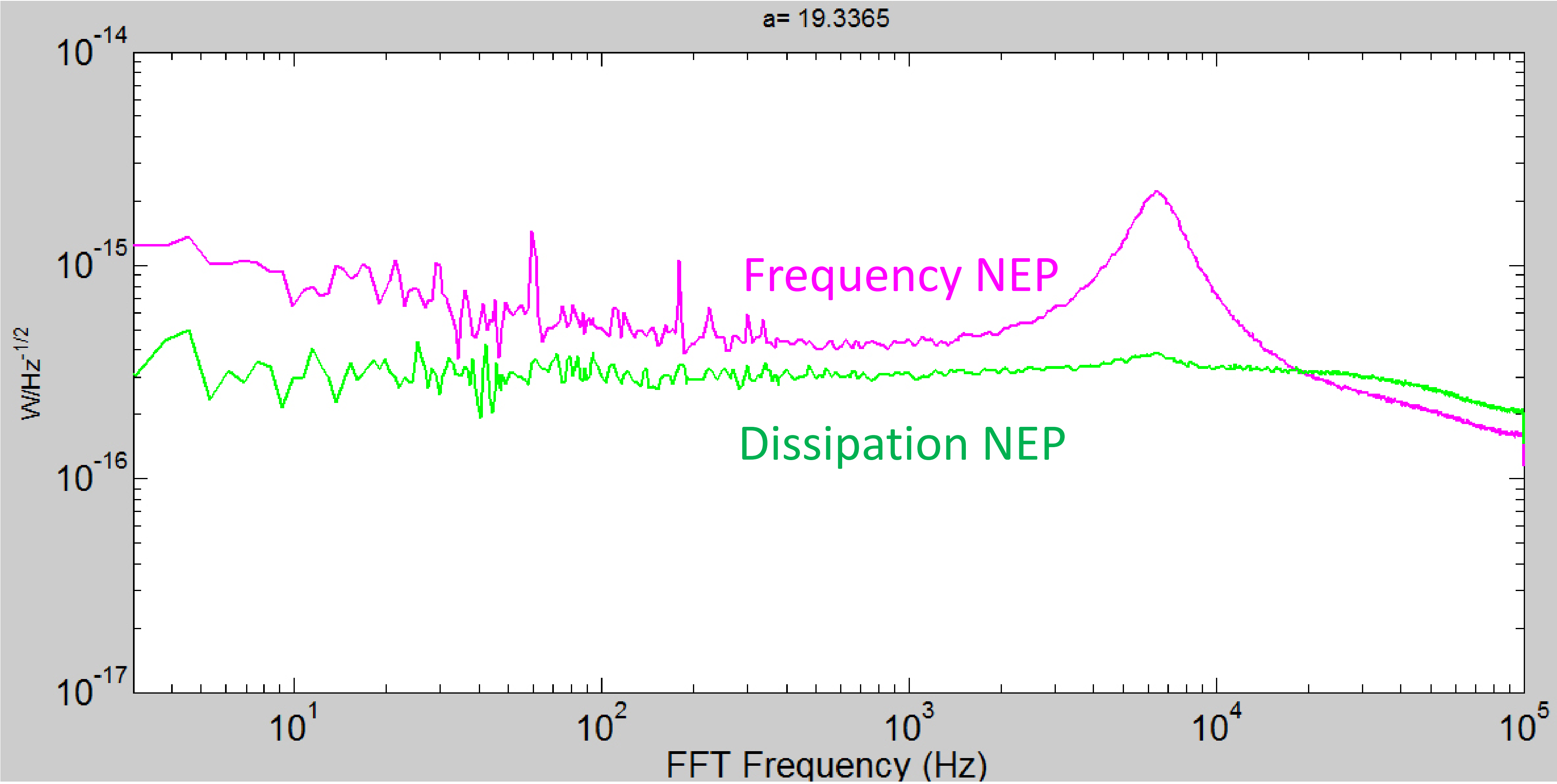
NEP = 6 x 10^{-15} W/Hz^1/2

a = .5



NEP = 1 x 10^{-15} W/Hz^1/2

a = 19



NEP = 3 x 10^{-16} W/Hz^1/2

Is this useful?

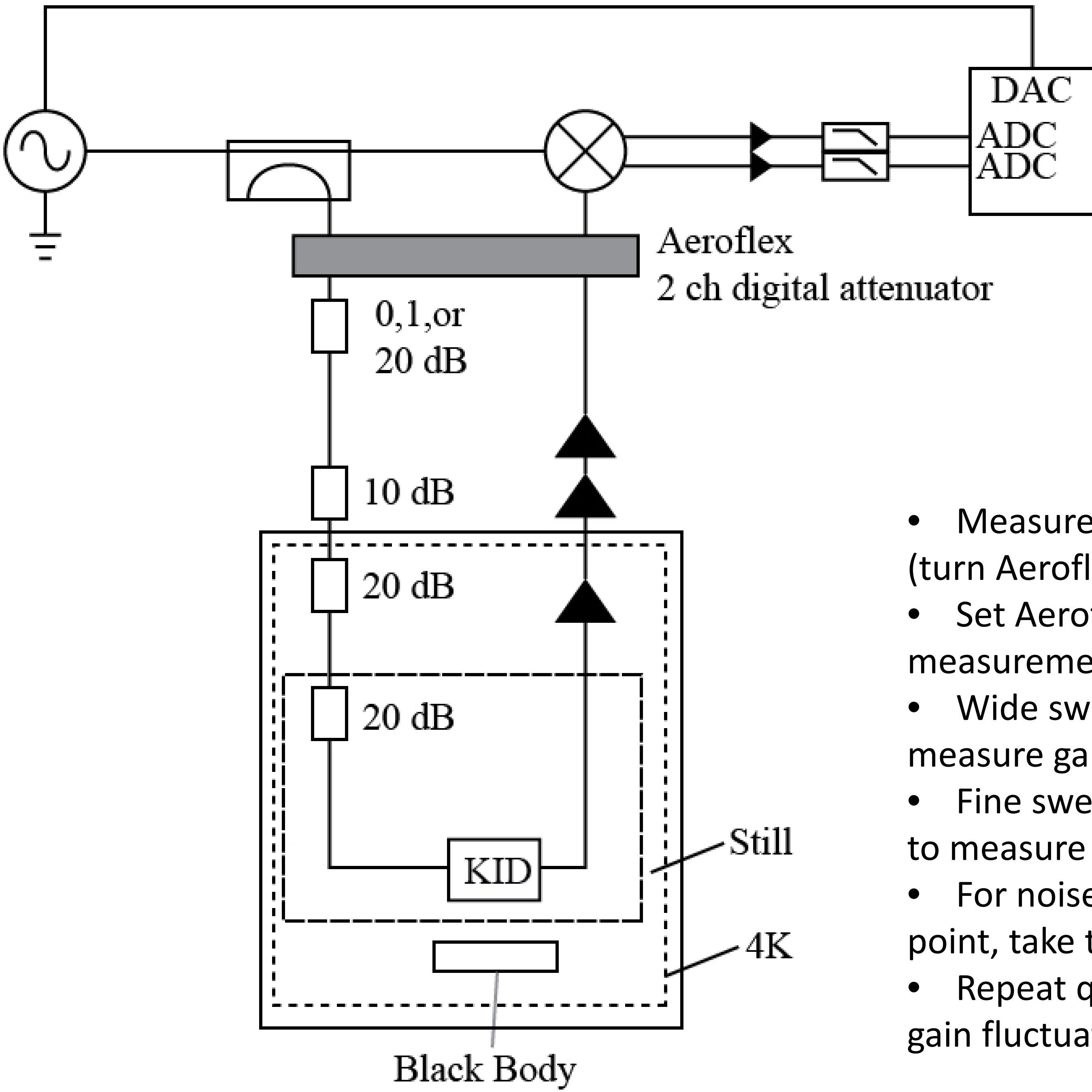
- Possible to access higher powers by downward frequency sweeping.
- Results in reduced noise, improved NEP.
- In order to operate above bifurcation, it is necessary to downward frequency sweep to operating point. -> Significantly increases electronic complexity, but feasible.
- Reduced dynamic range?

  -> avoid jumps

  -> may be difficult to operate with large sky noise

Thank you!

- Peter Day
- Byeong Ho Eom
- Rick Leduc
- Chris McKenney
- Omid Noroozian
- Jonas Zmuidzinas



- Measure DC offset
(turn Aeroflex to max attenuation)
- Set Aeroflex to appropriate measurement value
- Wide sweep the resonance to measure gain, cable delay
- Fine sweep the resonance to measure signal
- For noise, fine sweep to noise point, take time trace
- Repeat quickly (minimize gain fluctuations, etc)