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NOEMA sensitivity estimator

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Abstract

This memo describes the equations used in the NOEMA sensitivity estimator available in the `GILDAS/ASTRO` program.

Contents

1 Generalities **3**
1.1 The interferometric point source sensitivity . . . 3
1.2 The interferometric extended source sensitivity . . . 3
1.3 System temperature . . . 4
1.4 The number of polarizations . . . 5

2 Observing mode and elapsed telescope time **6**
2.1 Single-source, single-field observations . . . 7
2.2 Track-sharing, single-field observations . . . 7
2.3 Mosaicking . . . 8

1 Generalities

1.1 The interferometric point source sensitivity

The point source sensitivity for an interferometric measurement reads

$$\sigma_{\mathrm{Jy}} = \frac{J\,T_{\mathrm{sys}}}{\eta_{\mathrm{atm}}\sqrt{n_{\mathrm{ant}}\,(n_{\mathrm{ant}}-1)\,d\nu\,\Delta t}}, \tag{1}$$

where σ_{Jy} is the rms noise flux obtained by integration with an interferometer of n_{ant} identical antenna during the Δt integration time in a frequency resolution $d\nu$ with a system temperature given by T_{sys}. J is the Jy/K typical conversion factor of the interferometer. It reads

$$J = \eta_{\mathrm{spec}}\,J_{\mathrm{ant}}, \tag{2}$$

where η_{spec} is the spectrometer efficiency and J_{ant} the typical conversion factor of each interferometer antenna. J_{ant} is defined as

$$J_{\mathrm{ant}} = \frac{2k}{S}, \tag{3}$$

where k is the Boltzman constant, and S the effective antenna collecting area (eq. 3-113 in Kraus , 1982).

J characterizes the hardware, *i.e.* it assumes excellent atmospheric conditions. The atmospheric decorrelation is taken into account through an additional efficiency factor, η_{atm}, that is directly related to the atmospheric rms phase noise (ϕ_{rms}) through

$$\eta_{\mathrm{atm}} = e^{-\frac{\phi_{\mathrm{rms}}^2}{2}}. \tag{4}$$

Equation 1 is true only when the source is unresolved, *i.e.*, there is no effect of beam dilution. In practice this is rarely the case because the interferometer tries to resolve the source. Thus, this noise formula should be used with caution when preparing the observations. In practice, this formula is useful when one wishes to compare the sensitivity of two different interferometer. Indeed, this point source sensitivity is independent of the interferometer synthesized beam that depends on the details of the observations and, in particular, the interferometer configuration and the completeness of the Earth synthesis.

1.2 The interferometric extended source sensitivity

The sensitivity of an interferometer to an extended source reads

$$\sigma_{\mathrm{K}} = \frac{\theta_{\mathrm{prim}}^2}{\theta_{\mathrm{maj}}\,\theta_{\mathrm{min}}}\,\frac{T_{\mathrm{sys}}}{\eta_{\mathrm{atm}}\sqrt{n_{\mathrm{ant}}\,(n_{\mathrm{ant}}-1)\,d\nu\,\Delta t}}, \tag{5}$$

where σ_{K} is the rms noise brightness, θ_{prim} the half primary beam width, and θ_{maj} and θ_{min} the half beamwidth along the major and minor axes of the synthesized beam.

This formula clearly states that the sensitivity to extended sources depends on the dilution of the synthesized beam in the primary beam. For a given interferometer, the primary beamwidth is a fixed quantity while the synthesized beam is to first order proportional to the longest baseline in the current interferometer configuration. Hence, doubling the largest baseline will multiply σ_{K} by a factor $4(=2^2)$ for the same integration time or it will multiply the integration time by a factor $16(=2^4)$ in order to reach the same sensitivity. This just reflects that while the interferometer tries to mimic a single-dish antenna of same diameter as the largest baseline, all the antenna of the interferometer only fill a fraction of the total collecting area of the single-dish, this fractions decreasing with a power of two as the baseline linearly increases.

It is easy to show that σ_{K} and σ_{Jy} are linked through

$$\sigma_{\mathrm{K}} = \frac{4\,\ln 2\,\lambda^2}{2\pi\,k\,\theta_{\mathrm{maj}}\theta_{\mathrm{min}}}\,\sigma_{\mathrm{Jy}}, \tag{6}$$

where λ is the observed wavelength. In practice, time/sensitivity estimator usually computes the relationship between Δt and σ_{Jy}, and then the relationship between σ_{K} and σ_{Jy}.

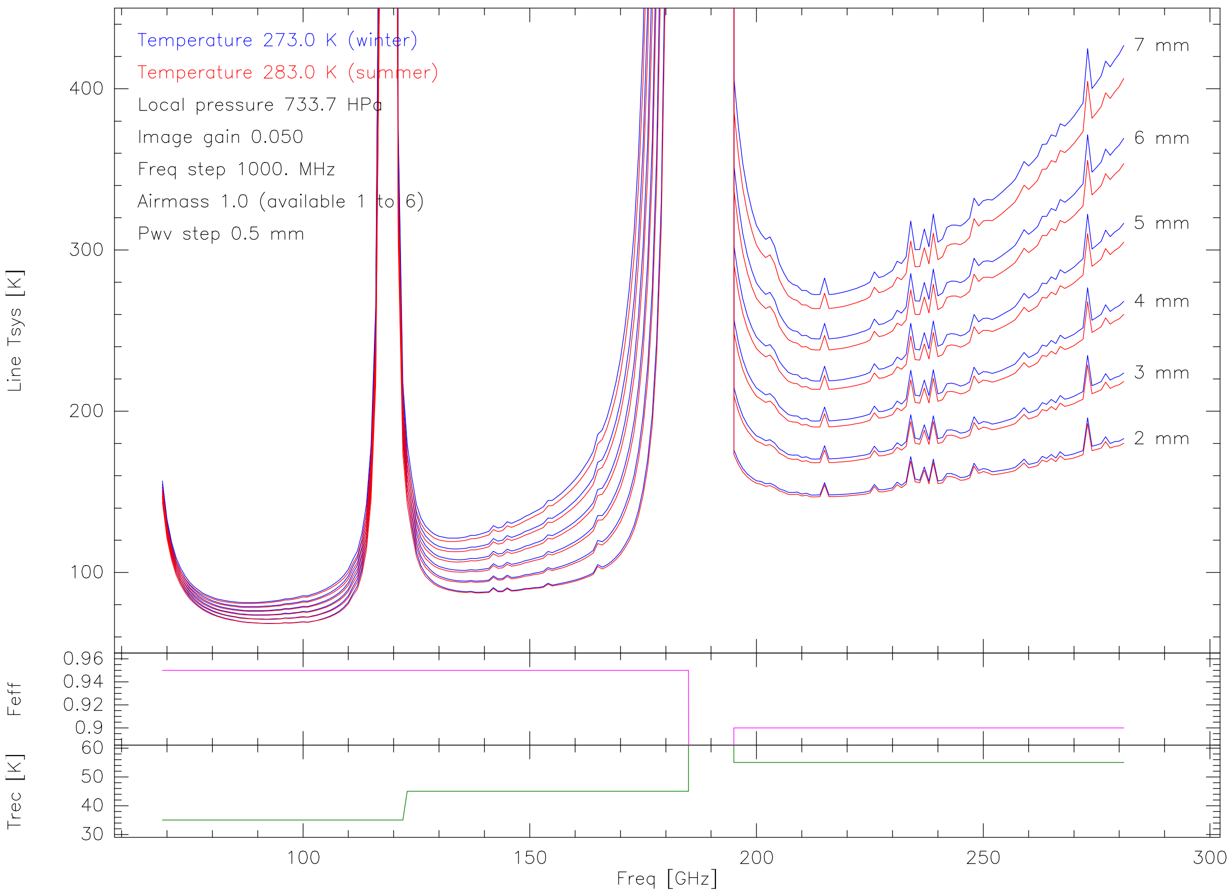

Figure 1: **Top:** Summer (red) and Winter (blue) semester $T_{\rm sys}$ for different precipitable water vapor (PWV) amount and for a source at zenith. The numbers indicate PWV values assumed in the computation. **Middle:** Assumed forward effiencies in the computation. **Bottom:** Assumed receiver temperatures in the computation.

1.3 System temperature

The system temperature is a summary of the noise added by the system. This noise comes from 1) the receiver and the optics, 2) the emission of the sky, and 3) the emission picked up by the secondary side lobes of the telescope. It is usual to approximate it (in the $T_{\rm a}^{\star}$ scale) with

$$T_{\rm sys} = \frac{(1+G_{\rm im})\exp\{\tau_{\rm s}\,A\}}{F_{\rm eff}}\left[F_{\rm eff}\,T_{\rm atm}\,\left(1-\exp\{-\tau_{\rm s}\,A\}\right)+(1-F_{\rm eff})\,T_{\rm cab}+T_{\rm rec}\right], \quad (7)$$

where $G_{\rm im}$ is the receiver image gain, $F_{\rm eff}$ the telescope forward efficiency, $A = 1/\sin(\text{elevation})$ the airmass, $\tau_{\rm s}$ the atmospheric opacity in the signal band, $T_{\rm atm}$ the mean physical atmospheric temperature, $T_{\rm cab}$ the ambient temperature in the receiver cabine and $T_{\rm rec}$ the noise equivalent temperature of the receiver and the optics. All those parameters are easily measured, except $\tau_{\rm s}$, which depends on the amount of water vapor in the atmosphere and which is estimated by complex atmospheric models.

In the ASTRO sensitivity estimator in detailed mode, the system temperature is computed using an atmospheric model (ATMOSPHERE command). In proposal mode, the $T_{\rm sys}$ is interpolated in frequency and airmass from tabulated values (see Fig. 1). The airmass is estimated using the maximum elevation of a source at the chosen Declination. The values are different for summer and winter due to the different atmospheric characteristics. Moreover, the chosen amount of precipitable water vapor depends on the

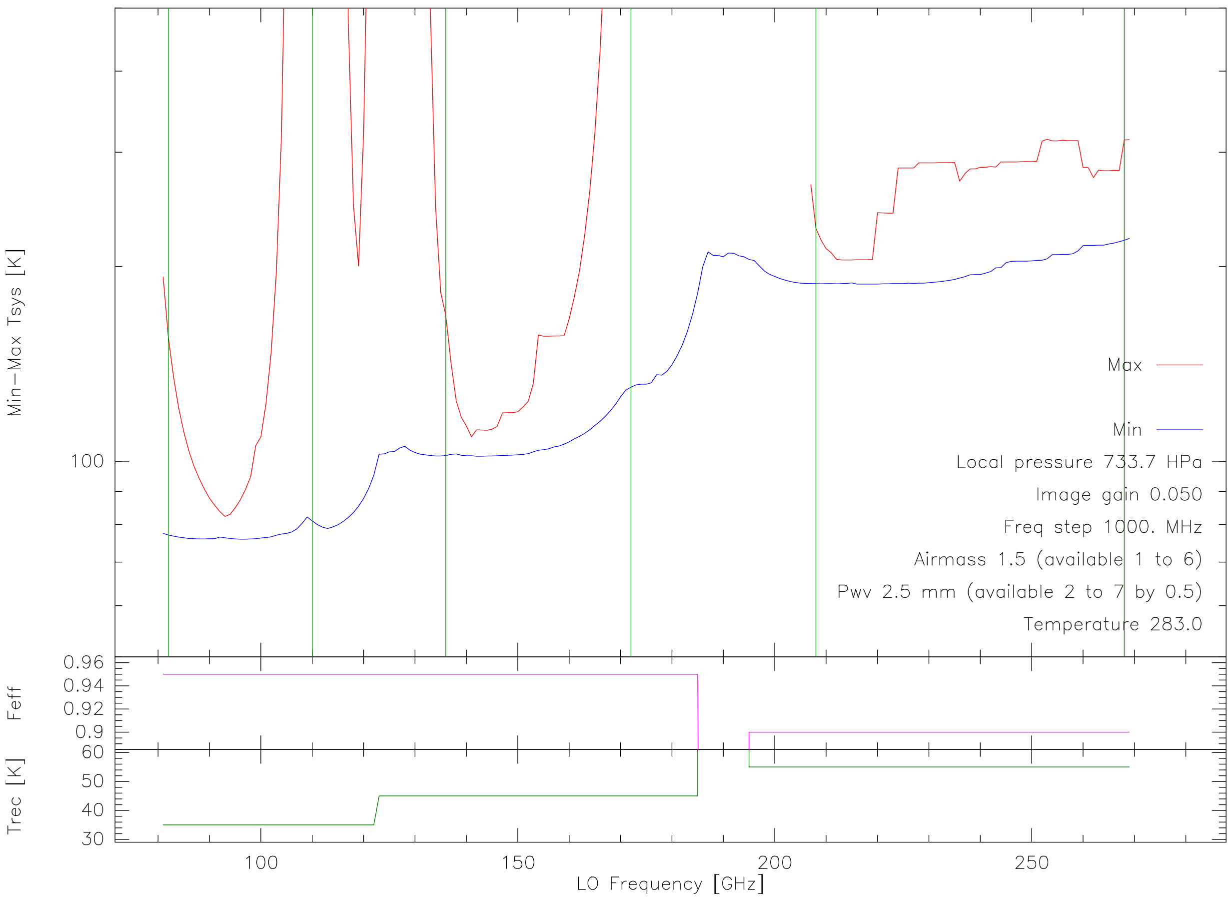

Figure 2: **Top:** Minimum and maximum T_{sys} obtained in the intermediate frequency bandwidth as a function of the local oscillator frequency used in the tuning. **Middle:** Assumed forward effiencies in the computation. **Bottom:** Assumed receiver temperatures in the computation.

receiver band (in addition to the season) because the NOEMA operation team schedule the different receiver bands according to the actual weather (high frequency bands are scheduled only during the best weather conditions).

The T_{sys} can vary significantly over the large bandwitdh of the 2SB NOEMA receivers. Figure 2 shows the minimum and maximum system temperature inside the IF bandwidth for all possible local oscillator tunings. As a result, for continuum estimation, a frequency averaged T_{sys} is interpolated from a pre-computed table in PROPOSAL mode. The input frequency in that case is the LO frequency of the tuning (see Fig. 3). The averaging is done such as $1/<T_{\mathrm{sys}}>^2 = 1/N\sum 1/T_{\mathrm{sys}}^2$. This is not implemented in DETAILED mode, due to the prohibitive computing cost of the atmospheric model over the $2\times 8\,\mathrm{GHz}$.

1.4 The number of polarizations

All NOEMA antennas are equipped with dual polarization receivers. They measure the signal coming from the pointed direction in two perpendicular polarizations in the same frequency range. For the current generation of receiver (2006) and correlators, one or two polarizations are processed by the correlators, depending on the project settings. We thus have to introduce the number of polarizations n_{pol}, which can

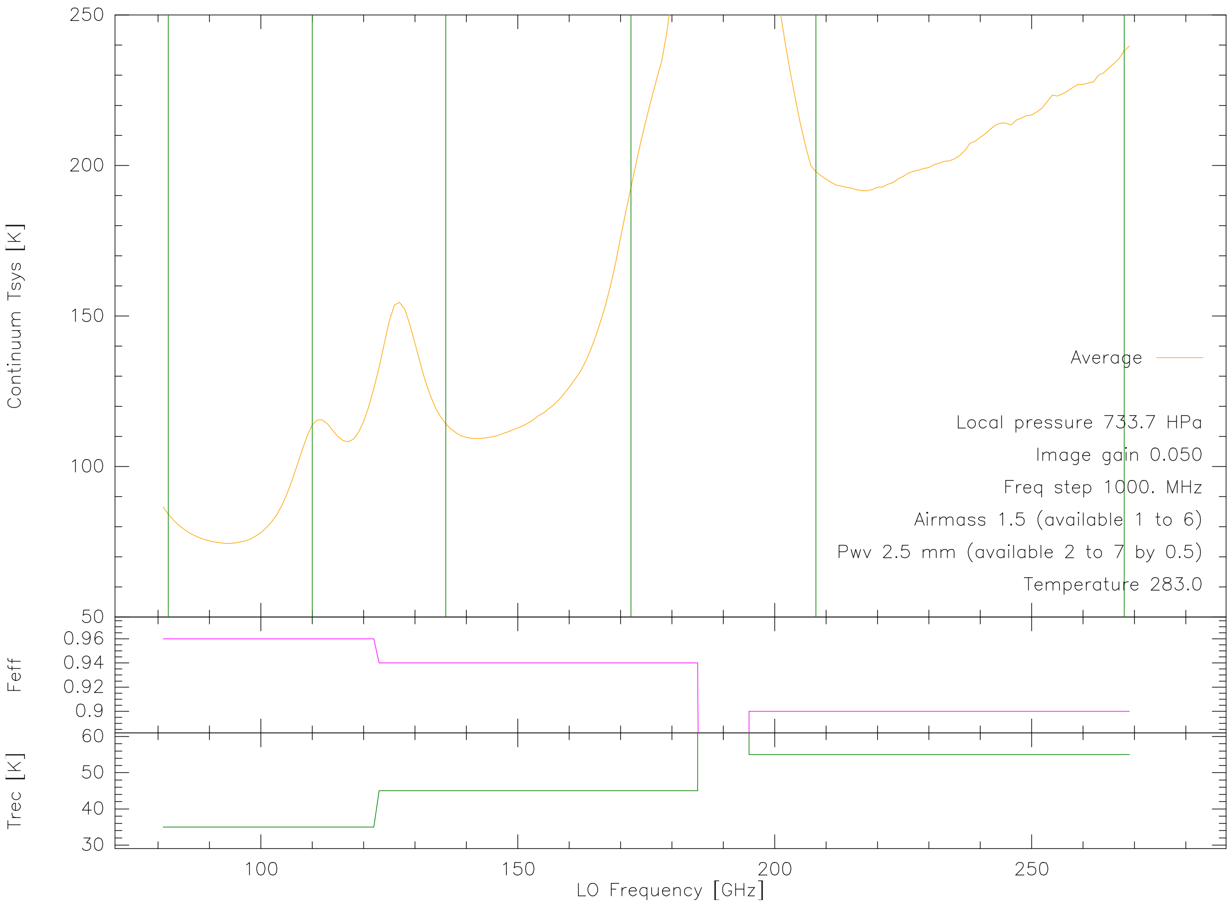

Figure 3: **Top:** Averaged "continuum" $T_{\rm sys}$ as a function of the local oscillator frequency used in the tuning. **Middle:** Assumed forward effiencies in the computation. **Bottom:** Assumed receiver temperatures in the computation.

be set to 1 or 2 and insert it in the radiometer equation with

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \quad (8)$$

2 Observing mode and elapsed telescope time

The goal of a time estimator is to find the elapsed telescope time ($\Delta t_{\rm tel}$) needed to obtain a given rms noise, while a sensitivity estimator aims at finding the rms noise obtained when observing during $\Delta t_{\rm tel}$. The total integration time spent on-source $\Delta t_{\rm on}$ is shorter than the elapsed telescope time due to several factors.

1. At the beginning of a project a significant time ($\Delta t_{\rm tune} \sim 40$ minutes) is spent in receiver tuning and calibration observations before observing the actual astronomical target. This means that even for a very short ON source time, a project cannot be shorter than $\Delta t_{\rm tune}$.

2. After this initial phase, the observing mode does not dedicate 100% of the time to the astronomical target. Part of the time is spent for calibration (pointing, focus, atmospheric calibration,...) and to slew the telescopes between useful integrations.

As of Gildas Jul17 release, the input time of the ASTRO sensitivity estimator is telescope time. The actual on source time is then computed taking into account those two points.

This computation depends on the observing mode. There are three main observation kinds.

Single-source, single-field observations where the telescope tracks a single source during the full integration time. This mode is used when the signal-to-noise ratio is the limiting factor.

Track-sharing, single-field observations where the telescope regularly cycles between a few close-by sources. This mode is used when the sources are so bright that the limiting factor is the Earth synthesis, not the signal-to-noise ratio.

Single-source mosaicking where the telescope regularly cycles between close-by pointings that usually follows a hexagonal compact pattern whose side is $\lambda/(2d_{\rm prim})$, where $d_{\rm prim}$ is the diameter of the interferometer antennas. This modes is used to image sources wider than the primary beam field of view.

In the following, we will work out the equations needed by the sensitivity estimator for each of these observing modes.

2.1 Single-source, single-field observations

That's the simplest case. The point source sensitivity in this case is

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}} \tag{9}$$

where $\Delta t_{\rm on}$ is the time spent on-source. It is related to the total elapsed telescope time $\Delta t_{\rm tel}$ through:

$$\Delta t_{\rm on} = \frac{\Delta t_{\rm obs}}{\eta_{\rm tel}} \tag{10}$$

$\Delta t_{\rm obs}$ is the time actually spent in observations (i.e. after instrument tuning and calibration). We have $\Delta t_{\rm obs} = \Delta t_{\rm tel} - n_{\rm tracks} \times \Delta t_{\rm tune}$. As described above $\Delta t_{\rm tune}$ is estimated to be 40 min (based on history of observed projects). $n_{\rm track}$ is the number of tracks to be observed to reach the requested telescope time. It is computed as $n_{\rm track} = \frac{\Delta t_{\rm tel}}{\Delta t_{\rm track}}$ where $\Delta t_{\rm track}$is the typical duration of a track, which depends on the source declination:

- 8 hours for a delination larger than 0 deg.
- 0 hours for a declination lower than -30 deg.
- A linear interpolation with the declination between these two values.

Finally $\eta_{\rm tel}$ quantifies the efficiency of the observing, not taking into account the initial calibrations and tuning. It is estimated to be about 1.6. Note that the exact value will depend on several parameters such as the number of calibrators and the distance between the source and the calibrator(s).

2.2 Track-sharing, single-field observations

In this case, the telescope time is equally divided between the $n_{\rm sou}$ observed sources. This yields

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}} \tag{11}$$

$$\text{with} \quad \Delta t_{\rm on} = \frac{\Delta t_{\rm tel} - n_{\rm tracks} \times \Delta t_{\rm tune}}{\eta_{\rm tel} \times n_{\rm sou}} \tag{12}$$

Note that it is technically feasible to observe sources in track-sharing with different integration times. This case is not implemented yet in the sensitivity estimator and the different sensitivities should be computed independently.

2.3 Mosaicking

Mosaicking is a particular case of wide-field imaging: The user wishes to observe a given field of view larger than the primary beam size with a sensitivity as uniform as possible. The on-source time is thus equally divided between the independent primary beams in the targeted field of view. To first order, we thus yield

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}} \tag{13}$$

$$\text{with} \quad \Delta t_{\rm on} = \frac{\Delta t_{\rm tel}\, - \,n_{\rm tracks}\times\Delta t_{\rm tune}}{\eta_{\rm tel}\times n_{\rm beam}} \quad \text{where} \quad n_{\rm beam} = \frac{A_{\rm map}}{A_{\rm beam}}, \tag{14}$$

where $n_{\rm beam}$ is the number of independent resolution elements, $A_{\rm map}$ is the area of the targeted field of view, and $A_{\rm beam}$is the area of the resolution element. The $A_{\rm map}$ area is a user input while the $A_{\rm beam}$ area is linked to the telescope full width at half maximum (θ) by

$$A_{\rm beam} = \frac{0.8\,\pi\,\theta^2_{\rm prim}}{4\,\ln(2)}, \tag{15}$$

The 0.8 factor represents the truncation of the beam at 20% of its maximum, which is performed during the imaging process.

There are several subtleties in this computation.

- $A_{\rm map}$ must be larger than 2 times $A_{\rm beam}$ (below this we advise to use the track sharing mode with two independent fields).
- The processing (imaging and deconvolution) of a mosaic implies a division by the primary beam of the interferometer. As the primary beam is to first order a Gaussian decreasing to zero, this implies that the noise of the mosaic will vary over the field of view. In particular it increases sharply at the edges of the field of view. In other words, Eq. 13 does not apply to the mosaic edges!
- The cycling of the pointings of the mosaic should ensure Nyquist sampling of the observed field of view. This implies that there is an important redundancy between the pointings, contrary to track sharing where the sources are supposed to be fully independent on the sky. For instance, when mosaicking with a hexagonal compact pattern, each line of sight will be observed by 7 contiguous pointings, except at the mosaic edges. It can thus been shown that the number of mosaic pointings, $n_{\rm point}$, is related to the number of independent elements through

$$n_{\rm point} = n_{\rm beam}\left(\frac{7}{4}\right)^2, \tag{16}$$

  for a correctly sampled mosaic. Equation 13 is only valid inside a correctly sampled mosaic.
- The pointings of a mosaic must be observed in short time cycles to ensure that all pointings are observed with similar weather conditions and that they share similar *uv* coverage. This minimizes the shift-variant part of the interferometer wide-field imaging response. This calls for the shortest possible integration time per pointing. However, the interferometer takes time to slew from one pointing to the next one without integrating. As a result, the observing efficiency $\eta_{\rm tel}$ is degraded in the cases of mosaics and we have another relationship between the elapsed telescope time and the on-source time as:

$$\Delta t_{\rm on} = \frac{\Delta t_{\rm tel}\, - \,n_{\rm tracks}\times\Delta t_{\rm tune}}{\eta_{\rm tel}\eta_{\rm mos}\times n_{\rm beam}} \quad \text{with} \quad \eta_{\rm mos} = \frac{\Delta t + \Delta t_{\rm slew}}{\Delta t}, \tag{17}$$

  where Δt is the integration time per pointing and $\Delta t_{\rm slew}$ is the time to slew between two consecutive pointings. Having a large integration time per pointing compared to $\Delta t_{\rm slew}$ will decrease the mosaicking overhead. This requirement is in sharp contrast with the previous one, namely the need to homogenize the interferometer wide-field response. The best compromise comes from two different considerations.

1. The smallest integration time is set by the acquisition system (for instance, the maximum achievable data rate). In pratice, we enforce that

$$\Delta t_{\mathrm{min}} = 10\,\mathrm{sec}. \tag{18}$$

2. The distance covered by a visibility in the uv-plane during an integration should always smaller than the distance associated to tolerable aliasing (see Pety and Rodríguez-Fernández 2010 for more details). This can be written as the following condition (Eq. C.3 in this article)

$$\frac{\Delta t}{1s} << \frac{6900}{\theta_{\mathrm{alias}}/\theta_{\mathrm{syn}}}, \tag{19}$$

where θ_{alias} is the map angular size, and θ_{syn} the angular resolution. For a given angular resolution, the interferometer minimum integration time corresponds to

$$\Delta t_{\mathrm{min}} \leq \frac{1}{\eta}\,\frac{6\,900}{1\,\mathrm{sec}}\,\sqrt{\frac{\theta_{\mathrm{maj}}\theta_{\mathrm{min}}}{A_{\mathrm{map}}}}, \tag{20}$$

where η is a ad-hoc integer set to 5 to ensure the condition defined in Eq. 19.

As the typical slew time between two pointings is $\Delta t_{\mathrm{slew}} = 11\,\mathrm{sec}$, we yield that

$$1 \leq \eta_{\mathrm{mos}} \leq 2.3 \tag{21}$$

- If the time to cycle all the pointings, $\Delta t_{\mathrm{cycle}}$, is set to 45 minutes, we yield that the maximum number of pointing per track is

$$n^{\mathrm{max}}_{\mathrm{point/track}} = \frac{\Delta t_{\mathrm{cycle}}}{\Delta t_{\mathrm{min}} + \Delta t_{\mathrm{slew}}} = 130. \tag{22}$$

- Finally, if the PI wishes to observe an area that will require more that 130 pointings per independent track, the estimator will ask to either increase the requested elapsed telescope time or to decrease the requested field-of-view area. The computation is done as follows.

  1. The number of track is then computed as described in section 2.1.
  2. The number of point per track is then $n_{\mathrm{point/track}} = \left(\frac{7}{4}\right)^2 \frac{n_{\mathrm{beam}}}{n_{\mathrm{track}}}$. This value must be lower than $n^{\mathrm{max}}_{\mathrm{point/track}}$.

In summary, the sensitivity of a Nyquist sampled mosaic is

$$\sigma_{\mathrm{Jy}} = \frac{J\,T_{\mathrm{sys}}}{\eta_{\mathrm{atm}}\sqrt{n_{\mathrm{ant}}\,(n_{\mathrm{ant}}-1)\,d\nu\,n_{\mathrm{pol}}\,\Delta t_{\mathrm{on}}}} \tag{23}$$

$$\text{with} \quad \Delta t_{\mathrm{on}} = \frac{\Delta t_{\mathrm{tel}} - n_{\mathrm{tracks}} \times \Delta t_{\mathrm{tune}}}{\eta_{\mathrm{tel}}\eta_{\mathrm{mos}}n_{\mathrm{beam}}} \tag{24}$$

References

Kraus, J. D., in McGraw-Hill 1982 (1966), Radio Astronomy.