IRAM Memo 2015-2

NOEMA time/sensitivity estimator

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July, 12th 2018
Version 2.2

Abstract

This memo describes the equations used in the NOEMA time/sensitivity estimator available in the `GILDAS/ASTRO` program.

Contents

1 Generalities

1.1 The interferometric point source sensitivity

The point source sensitivity for an interferometric measurement reads

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,\Delta t}}, \tag{1}$$

where $\sigma_{\rm Jy}$ is the rms noise flux obtained by integration with an interferometer of $n_{\rm ant}$ identical antenna during Δt in a frequency resolution $d\nu$ with a system temperature given by $T_{\rm sys}$. J is the Jy/K typical conversion factor of the interferometer. It reads

$$J = \eta_{\rm spec}\,J_{\rm ant}, \tag{2}$$

where $\eta_{\rm spec}$ is the spectrometer efficiency and $J_{\rm ant}$ the typical conversion factor of each interferometer antenna. $J_{\rm ant}$ is defined as

$$J_{\rm ant} = \frac{2k}{S}, \tag{3}$$

where k is the Boltzman constant, and S the effective antenna collecting area (eq. 3-113 in Kraus , 1982).

J characterizes the hardware, *i.e.* it assumes excellent atmospheric conditions. The atmospheric decorrelation is taken into account through an additional efficiency factor, $\eta_{\rm atm}$, that is directly related to the atmospheric rms phase noise ($\phi_{\rm rms}$) through

$$\eta_{\rm atm} = e^{-\frac{\phi_{\rm rms}^2}{2}}. \tag{4}$$

Equation 1 is true only when the source is unresolved, *i.e.*, there is no effect of beam dilution. In practice this is rarely the case because the interferometer tries to resolve the source. Thus, this noise formula should be used with caution when preparing the observations. In practice, this formula is useful when one wishes to compare the sensitivity of two different interferometer. Indeed, this point source sensitivity is independent of the interferometer synthesized beam that depends on the details of the observations and, in particular, the interferometer configuration and the completeness of the Earth synthesis.

1.2 The interferometric extended source sensitivity

The sensitivity of an interferometer to an extended source reads

$$\sigma_{\rm K} = \frac{\theta_{\rm prim}^2}{\theta_{\rm maj}\,\theta_{\rm min}}\,\frac{T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,\Delta t}}, \tag{5}$$

where $\sigma_{\rm K}$ is the rms noise brightness, $\theta_{\rm prim}$ the half primary beam width, and $\theta_{\rm maj}$ and $\theta_{\rm min}$ the half beamwidth along the major and minor axes of the synthesized beam.

This formula clearly states that the sensitivity to extended sources depends on the dilution of the synthesized beam in the primary beam. For a given interferometer, the primary beamwidth is a fixed quantity while the synthesized beam is to first order proportional to the longest baseline in the current interferometer configuration. Hence, doubling the largest baseline will multiply $\sigma_{\rm K}$ by a factor $4(=2^2)$ for the same integration time or it will multiply the integration time by a factor $16(=2^4)$ in order to reach the same sensitivity. This just reflects that while the interferometer tries to mimic a single-dish antenna of same diameter as the largest baseline, all the antenna of the interferometer only fill a fraction of the total collecting area of the single-dish, this fractions decreasing with a power of two as the baseline linearly increases.

It is easy to show that $\sigma_{\rm K}$ and $\sigma_{\rm Jy}$ are linked through

$$\sigma_{\rm K} = \frac{4\,\ln 2\,\lambda^2}{2\pi\,k\,\theta_{\rm maj}\theta_{\rm min}}\,\sigma_{\rm Jy}, \tag{6}$$

where λ is the observed wavelength. In practice, time/sensitivity estimator usually computes the relationship between Δt and $\sigma_{\rm Jy}$, and then the relationship between $\sigma_{\rm K}$ and $\sigma_{\rm Jy}$.

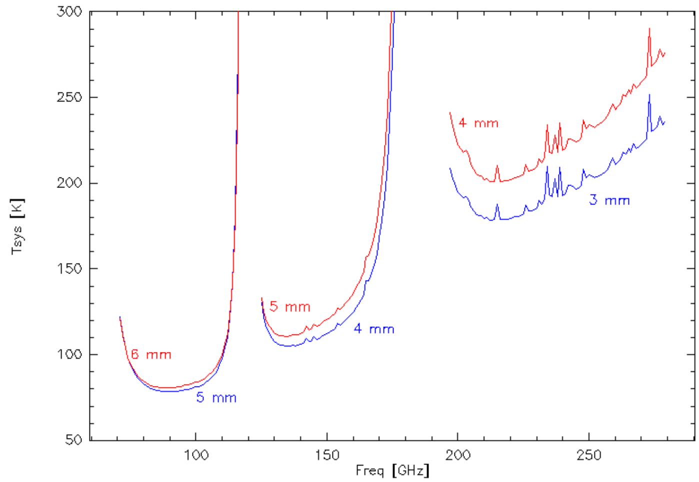

Figure 1: Summer (red) and Winter (blue) semester T_{sys} for a source at Dec=20 degrees (i.e. maximum elevation 65 deg). The numbers indicate the precipitable water vapour (PWV) values assumed in the computation.

1.3 System temperature

The system temperature is a summary of the noise added by the system. This noise comes from 1) the receiver and the optics, 2) the emission of the sky, and 3) the emission picked up by the secondary side lobes of the telescope. It is usual to approximate it (in the $T_a^\star$ scale) with

$$T_{sys} = \frac{(1 + G_{im}) \exp\{\tau_s A\}}{F_{eff}} \left[F_{eff}\, T_{atm}\, (1 - \exp\{-\tau_s A\}) + (1 - F_{eff})\, T_{cab} + T_{rec}\right], \quad (7)$$

where G_{im} is the receiver image gain, F_{eff} the telescope forward efficiency, $A = 1/\sin(\text{elevation})$ the airmass, τ_s the atmospheric opacity in the signal band, T_{atm} the mean physical atmospheric temperature, T_{cab} the ambient temperature in the receiver cabine and T_{rec} the noise equivalent temperature of the receiver and the optics. All those parameters are easily measured, except τ_s, which depends on the amount of water vapor in the atmosphere and which is estimated by complex atmospheric models.

In the ASTRO sensitivity estimator in detailed mode, the system temperature is computed using an atmospheric model (ATMOSPHERE command). In proposal mode, the T_{sys} is interpolated in frequency and airmass from tabulated values (see Fig. 1). The airmass is estimated using the maximum elevation of a source at the chosen Declination. The values are different for summer and winter due to the different atmospheric characteristics.

The T_{sys} can vary significantly over the large bandwitdh of the 2SB NOEMA receivers. As as result, for continuum estimation, a frequency averaged T_{sys} is interpolated from a pre-computed table in PROPOSAL mode. The input frequency in that case is the LO frequency of the tuning. The averaging is done quadratically. This is not implemented in DETAILED mode, due to tdue do the prohibitive computing cost of the atmospheric model over the 2x8 GHz.

1.4 Elapsed telescope time

The goal of a time estimator is to find the elapsed telescope time (Δt_{tel}) needed to obtain a given rms noise, while a sensitivity estimator aims at finding the rms noise obtained when observing during Δt_{tel}.

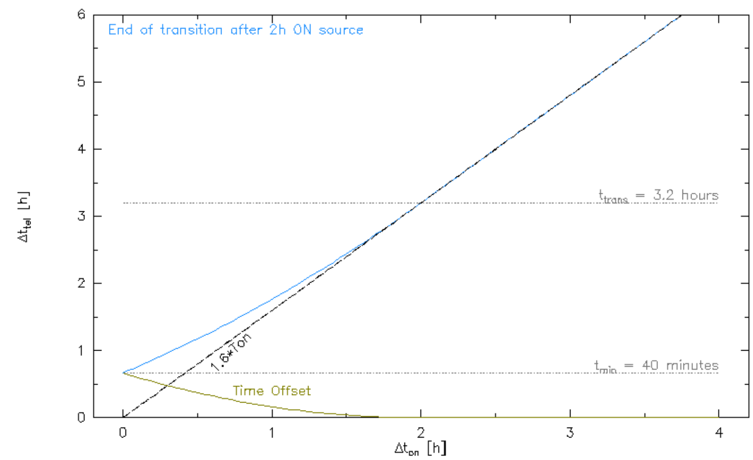

Figure 2: Relation between $\Delta t_{\rm tel}$ and $\Delta t_{\rm on}$ as implemented in ASTRO and PMS sensitivity estimators

The total integration time spent on-source $\Delta t_{\rm on}$ is shorter than the elapsed telescope time due to several factors:
1. At the beginning of a project a significant time ($t_{min} \sim 40$ minutes) is spent in receiver tuning and calibration observations before observing the actual astronomical target. This means that even for a very short ON source time, a project cannot be shorter than t_{min}.
2. After this initial phase, the observing mode does not dedicate 100% of the time to the astronomical target. Part of the time is spent for calibration (pointing, focus, atmospheric calibration,...) and to slew the telescopes between useful integrations.

As of Gildas Jul17 release, the input time of the ASTRO sensitivity estimator is telescope time. The actual on source time is then computed taking into account those two points.

For long projects the relation between on source and telescope time is obtained the following:

$$\Delta t_{\rm tel} = \eta_{\rm tel}\, \Delta t_{\rm on}, \tag{8}$$

where $\eta_{\rm tel}$ quantifies the efficiency of the observing mode, estimated to be about 1.6. This is valid when $\Delta t_{\rm tel}$ is on the order of a a transition time $t_{trans} \sim 3.2$ h (i.e. $\Delta t_{\rm on} \sim 2$h)

For shorter projects, an additional time offset is included to take into account the inition calibrations/tuning. The time offset depends on the total time: for very short times, it is on the order of t_{min} =40 minutes. It decreases following a quadratic low when the total time increases and is null at $\Delta t_{\rm tel} = t_{trans}$ =3.2 h. The exact equation is given below (Eq. 9) and is illustrated in Fig. 2.

$$\Delta t_{\rm tel} = \eta_{\rm tel}\, \Delta t_{\rm tel} \, + t_{min}\, (\frac{t_{trans} - \Delta t_{\rm on}}{t_{trans}})^2 \tag{9}$$

1.5 The number of polarizations

All NOEMA antennas are equipped with dual polarization receivers. They measure the signal coming from the pointed direction in two perpendicular polarizations in the same frequency range. For the current generation of receiver (2006) and correlators, one or two polarizations are processed by the correlators, depending on the project settings. We thus have to introduce the number of polarizations $n_{\rm pol}$, which can be set to 1 or 2 and insert it in the radiometer equation with:

$$\sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{10}$$

2 Observing mode

There are three main observation kinds.

Single-source, single-field observations where the telescope tracks a single source during the full integration time. This mode is used when the signal-to-noise ratio is the limiting factor.

Track-sharing, single-field observations where the telescope regularly cycles between a few close-by sources. This mode is used when the sources are so bright that the limiting factor is the Earth synthesis, not the signal-to-noise ratio.

Single-source mosaicking where the telescope regularly cycles between close-by pointings that usually follows a hexagonal compact pattern whose side is $\lambda/(2d_{\rm prim})$, where $d_{\rm prim}$ is the diameter of the interferometer antennas. This modes is used to image sources wider than the primary beam field of view.

In the following, we will work out the equations needed by the time/sensitivity estimator for observing mode.

2.1 Single-source, single-field observations

That's the simplest case. The point source sensitivity in this case is

$$\Delta t_{\rm on} = \eta_{\rm tel}\,\Delta t_{\rm tel}, \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{11}$$

2.2 Track-sharing, single-field observations

In this case, the telescope time is equally divided between the $n_{\rm sou}$ observed sources. This yields

$$\Delta t_{\rm on} = \frac{\eta_{\rm tel}\,\Delta t_{\rm tel}}{n_{\rm sou}}, \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{12}$$

Note that it is technically feasible to observe sources in track-sharing with different integration times. This case is not implemented yet in the sensitivity estimator and the different sensitivities should be computed independently.

2.3 Mosaicking

In this case, the telescope time is equally divided between the $n_{\rm point}$ pointings used to cover the full extent of the source. It thus seems similar to the track-sharing, single-field observations. However, there are two subtleties.

1. The processing (imaging and deconvolution) of a mosaic implies a division by the primary beam of the interferometer. As the primary beam is to first order a Gaussian decreasing to zero, this implies that the noise of the mosaic will vary over the field of view. In particular it increases sharply at the edges of the field of view.

2. The cycling of the pointings of the mosaic is done to Nyquist sample the observed field of view. This implies that there is an important redundancy between the pointings, contrary to track sharing where the sources are supposed to be fully independent on the sky. For instance, when mosaicking with a hexagonal compact pattern, each line of sight will be observed by 7 contiguous pointings, except at the mosaic edges.

The time/sensitivity estimator will thus have to link the elapsed telescope time to cover the whole mapped region to the sensitivity in each independent resolution element. To do this, we need to introduce

- A_{map} and A_{beam}, which are respectively the area of the map and the area of the resolution element. The map area is a user input while the resolution area is linked to the telescope full width at half maximum (θ) by

$$A_{\mathrm{beam}} = \frac{0.8\,\pi\,\theta^2}{4\,\ln(2)}, \tag{13}$$

The 0.8 factor represents the truncation of the beam at 20% of its maximum, which is performed during the imaging process. Three tests can be checked on A_{map}:

1) A_{map} must be larger than 2 times A_{beam} (below this we advise to use the track sharing mode with two independent fields);

2) A_{map} must be smaller than a limit defined by to the shortest integration time achievable with NOEMA ($A^{\mathrm{uv}}_{\mathrm{max}}$). The distance covered by a visibility in the uv-plane during an integration should always smaller than the distance associated to tolerable aliasing (see Pety and Rogrígues-Fernández 2010 for more details). This can be written as the following condition (Eq. C.3 in the article):

$$\frac{\delta t}{1s} << \frac{6900}{\theta_{alias}/\theta_{syn}} \tag{14}$$

where δt is the integration time, θ_{alias} the map angular size, and θ_{syn} the angular resolution.

For a given angular resolution, the interferometer minimum integration time corresponds to a maximal map size according to:

$$A^{\mathrm{uv}}_{\mathrm{max}} = \frac{6900 \times \theta_{syn}}{\delta t_{min}\,\eta} \tag{15}$$

where η is a ad-hoc integer set to 5 to ensure the condition defined in Eq. 14.

3) The area covered in a single track must also be smaller than a limit related to the maximum number of fields observable in a given time with NOEMA ($A^{\mathrm{cycle}}_{\mathrm{max}}$). Presently, we assume that all the pointings should be covered within 2 cycles on source between two calibrators, i.e. ~45 min.

The minimum integration time of NOEMA is 10s. As a result, the number of fields that can be covered is:

$$n^{\mathrm{cycle}}_{\mathrm{max}} = 45 \times 60/(10+8) = 150. \tag{16}$$

Assuming a standard sampling for the mosaic this corresponds to 150×4/7 independent beams and we have:

$$A^{\mathrm{cycle}}_{\mathrm{max}} = n^{\mathrm{cycle}}_{\mathrm{max}}\,\frac{4}{7}\,A_{\mathrm{beam}} \sim 85\,A_{\mathrm{beam}} \tag{17}$$

This $A^{\mathrm{cycle}}_{\mathrm{max}}$ is a technical limitation for a given observing track. Larger maps can be built putting together different sub-maps observed in different tracks. This does not affect the final sensitivity and the number of needed sub-maps is computed by the estimator.

- The number of independent measurements (n_{beam}) in the final map which is given by

$$n_{\mathrm{beam}} = \frac{A_{\mathrm{map}}}{A_{\mathrm{beam}}}. \tag{18}$$

Because of the redundancy, we must have $n_{\mathrm{beam}} < n_{\mathrm{point}}$.

The on-source time is then shared between $n_{\rm beam}$ independent measurements. This yields

$$\Delta t_{\rm on} = \frac{\eta_{\rm mos}\,\Delta t_{\rm tel}}{n_{\rm beam}} \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm atm}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{19}$$

Note that $\eta_{\rm mos}$ (observing efficiency in mosaic mode) must be larger than $\eta_{\rm tel}$ (efficiency for standard projects) because the telescope has to slew between the fields, increasing the overheads. The efficiency in mosaic mode (for projects longer than t_{trans}) can be related to the standard efficiency with the equation:

$$\eta_{\rm mos} = \eta_{\rm tel} + \frac{(n_{point}-1)\times 2.5\times 8.0}{3600} \tag{20}$$

Finally, this noise estimate is correct at any point of the mosaic that is covered by the same number of pointings (in particular, the mosaic center). It will be higher at the mosaic edges.

References

Kraus, J. D., in McGraw-Hill 1982 (1966), Radio Astronomy.