IRAM Memo 2015-2

NOEMA time/sensitivity estimator

J. Pety[1,2], J. Boissier[1], E. Reynier[1]

1. IRAM (Grenoble)
2. Observatoire de Paris

August, 17th 2015
Version 1.1

Abstract

This memo describes the equations used in the NOEMA time/sensitivity estimator available in the `GILDAS/ASTRO` program.

Contents

1 Generalities

1.1 The interferometric point source sensitivity

The point source sensitivity for an interferometric measurement reads

$$\sigma_{\text{Jy}} = \frac{J\,T_{\text{sys}}}{\eta_{\text{atm}}\sqrt{n_{\text{ant}}\,(n_{\text{ant}}-1)\,d\nu\,\Delta t}}, \tag{1}$$

where σ_{Jy} is the rms noise flux obtained by integration with an interferometer of n_{ant} identical antenna during Δt in a frequency resolution $d\nu$ with a system temperature given by T_{sys}. J is the Jy/K typical conversion factor of the interferometer. It reads

$$J = \eta_{\text{spec}}\,J_{\text{ant}}, \tag{2}$$

where η_{spec} is the spectrometer efficiency and J_{ant} the typical conversion factor of each interferometer antenna. J_{ant} is defined as

$$J_{\text{ant}} = \frac{2k}{F_{\text{eff}}\,S}, \tag{3}$$

where k is the Boltzman constant, F_{eff} the forward efficiency, and S the effective antenna collecting area. J characterizes the hardware, *i.e.* it assumes excellent atmospheric conditions. The atmospheric decorrelation is taken into account through an additional efficiency factor, η_{atm}, that is directly related to the atmospheric rms phase noise (ϕ_{rms}) through

$$\eta_{\text{atm}} = e^{-\frac{\phi_{\text{rms}}^2}{2}}. \tag{4}$$

Equation 1 is true only when the source is unresolved, *i.e.*, there is no effect of beam dilution. In practice this is rarely the case because the interferometer tries to resolve the source. Thus, this noise formula should be used with caution when preparing the observations. In practice, this formula is useful when one wishes to compare the sensitivity of two different interferometer. Indeed, this point source sensitivity is independent of the interferometer synthesized beam that depends on the details of the observations and, in particular, the interferometer configuration and the completeness of the Earth synthesis.

1.2 The interferometric extended source sensitivity

The sensitivity of an interferometer to an extended source reads

$$\sigma_{\text{K}} = \frac{\theta_{\text{prim}}^2}{\theta_{\text{maj}}\,\theta_{\text{min}}}\,\frac{T_{\text{sys}}}{\eta_{\text{spec}}\sqrt{n_{\text{ant}}\,(n_{\text{ant}}-1)\,d\nu\,\Delta t}}, \tag{5}$$

where σ_{K} is the rms noise brightness, θ_{prim} the half primary beam width, and θ_{maj} and θ_{min} the half beamwidth along the major and minor axes of the synthesized beam.

This formula clearly states that the sensitivity to extended sources depends on the dilution of the synthesized beam in the primary beam. For a given interferometer, the primary beamwidth is a fixed quantity while the synthesized beam is to first order proportional to the longest baseline in the current interferometer configuration. Hence, doubling the largest baseline will multiply σ_{K} by a factor $4(=2^2)$ for the same integration time or it will multiply the integration time by a factor $16(=2^4)$ in order to reach the same sensitivity. This just reflects that while the interferometer tries to mimic a single-dish antenna of same diameter as the largest baseline, all the antenna of the interferometer only fill a fraction of the total collecting area of the single-dish, this fractions decreasing with a power of two as the baseline linearly increases.

It is easy to show that σ_{K} and σ_{Jy} are linked through

$$\sigma_{\text{K}} = \frac{4\,\ln 2\,\lambda^2}{2\pi\,k\,\theta_{\text{maj}}\theta_{\text{min}}}\,\sigma_{\text{Jy}}, \tag{6}$$

where λ is the observed wavelength. In practice, time/sensitivity estimator usually computes the relationship between Δt and σ_{Jy}, and then the relationship between σ_{K} and σ_{Jy}.

Table 1: System temperatures used for NOEMA estimations in proposal mode

	Summer			Winter		
Band 1						
Freq. (GHz)	80	110	115	80	110	115
Tsys (K)	100	100	180	100	100	170
Band 2						
Freq. (GHz)	129	150	177	129	150	177
Tsys (K)	150	150	200	130	130	170
Band 3						
Freq. (GHz)	201	230	267	201	230	267
Tsys (K)	250	250	250	200	200	200
Band 4						
Freq. (GHz)	277	340	370	277	340	370
Tsys (K)	500	500	500	370	370	370

1.3 System temperature

The system temperature is a summary of the noise added by the system. This noise comes from 1) the receiver and the optics, 2) the emission of the sky, and 3) the emission picked up by the secondary side lobes of the telescope. It is usual to approximate it (in the $T_a^\star$ scale) with

$$T_{\rm sys} = \frac{(1+G_{\rm im})\exp\{\tau_{\rm s}\,A\}}{F_{\rm eff}}\left[F_{\rm eff}\,T_{\rm atm}\,\left(1-\exp\{-\tau_{\rm s}\,A\}\right)+(1-F_{\rm eff})\,T_{\rm cab}+T_{\rm rec}\right], \tag{7}$$

where $G_{\rm im}$ is the receiver image gain, $F_{\rm eff}$ the telescope forward efficiency, $A = 1/\sin(\text{elevation})$ the airmass, $\tau_{\rm s}$ the atmospheric opacity in the signal band, $T_{\rm atm}$ the mean physical atmospheric temperature, $T_{\rm cab}$ the ambient temperature in the receiver cabine and $T_{\rm rec}$ the noise equivalent temperature of the receiver and the optics. All those parameters are easily measured, except $\tau_{\rm s}$, which depends on the amount of water vapor in the atmosphere and which is estimated by complex atmospheric models.

In the ASTRO sensitivity estimator, the system temperature is computed when using the detailed mode, while it is interpolated betweem tabulated values (see Table 1) in proposal mode. The values are different for summer and winter due to the different atmospheric characteristics.

1.4 Elapsed telescope time

The goal of a time estimator is to find the elapsed telescope time ($\Delta t_{\rm tel}$) needed to obtain a given rms noise, while a sensitivity estimator aims at finding the rms noise obtained when observing during $\Delta t_{\rm tel}$. If $\Delta t_{\rm on}$ is the total integration time spent on-source, then

$$\Delta t_{\rm on} = \eta_{\rm tel}\,\Delta t_{\rm tel}, \tag{8}$$

where $\eta_{\rm tel}$ is the efficiency of the observing mode, *i.e.* the time needed 1) to do calibrations (*e.g.* pointing, focus, temperature scale calibration), and 2) to slew the telescope between useful integrations.

The tuning of the receivers is not proportional to the total integration time but it should be added to the elapsed telescope time. A time estimator can hardly anticipate the total tuning time for a project. Indeed, one project (*e.g.* faint line detection) can request only one tuning to be used during many hours and another (*e.g.* line survey) can request a tuning every few minutes. In our case, we thus request that the estimator user add by hand the tuning time to the elapsed telescope time estimation.

1.5 The number of polarizations

Heterodyne mixers are coupled to a single linear polarization of the signal. Hence, heterodyne receivers have at least two mixers, each one sensitive to one of the two linear polarization of the incoming signal.

Both mixers are looking at the same sky position. This implies that we have to distinguish between the time spent on a given position of sky and the human elapsed time. Indeed, we will use the time spent on a given position of the sky when estimating the sensitivity, while we will give human elapsed time for the telescope and the on and off times.

If the mixers are tuned at the same frequency, the times spent on and off in the same direction of the sky will be twice the human elapsed time. We thus have to introduce the number of polarization simultaneously tuned at the same frequency, $n_{\rm pol}$, which can be set to 1 or 2. The simplest way to take into account the distinction between human time and sky time is to slightly modify the radiometer equation to take into account the number of polarization

$$\sigma_{\rm Jy} = \frac{J\, T_{\rm sys}}{\eta_{\rm spec}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\, n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{9}$$

This equation implies that $\Delta t_{\rm on}$, and $\Delta t_{\rm tel}$ will be human times.

For the current generation of receiver (2006), the two polarizations may be tuned at the same frequency or at different ones, *i.e.*, $n_{\rm pol} = 2$ or $n_{\rm pol} = 1$, respectively.

2 Observing mode

There are three main observation kinds.

Single-source, single-field observations where the telescope tracks a single source during the full integration time. This mode is used when the signal-to-noise ratio is the limiting factor.

Track-sharing, single-field observations where the telescope regularly cycles between a few close-by sources. This mode is used when the sources are so bright that the limiting factor is the Earth synthesis, not the signal-to-noise ratio.

Single-source mosaicking where the telescope regularly cycles between close-by pointings that usually follows a hexagonal compact pattern whose side is $\lambda/(2d_{\rm prim})$, where $d_{\rm prim}$ is the diameter of the interferometer antennas. This modes is used to image sources wider than the primary beam field of view.

In the following, we will work out the equations needed by the time/sensitivity estimator for observing mode.

2.1 Single-source, single-field observations

That's the simplest case. The point source sensitivity in this case is

$$\Delta t_{\rm on} = \eta_{\rm tel}\,\Delta t_{\rm tel}, \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\, T_{\rm sys}}{\eta_{\rm spec}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\, n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{10}$$

2.2 Track-sharing, single-field observations

In this case, the telescope time is equally divided between the $n_{\rm sou}$ observed sources. This yields

$$\Delta t_{\rm on} = \frac{\eta_{\rm tel}\,\Delta t_{\rm tel}}{n_{\rm sou}}, \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\, T_{\rm sys}}{\eta_{\rm spec}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\, n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{11}$$

Note that it is technically feasible to observe sources in track-sharing with different integration times. This case is not implemented yet in the sensitivity estimator and the different sensitivities should be computed independently.

2.3 Mosaicking

In this case, the telescope time is equally divided between the n_{point} pointings used to cover the full extent of the source. It thus seems similar to the track-sharing, single-field observations. However, there are two subtleties.

1. The processing (imaging and deconvolution) of a mosaic implies a division by the primary beam of the interferometer. As the primary beam is to first order a Gaussian decreasing to zero, this implies that the noise of the mosaic will vary over the field of view. In particular it increases sharply at the edges of the field of view.

2. The cycling of the pointings of the mosaic is done to Nyquist sample the observed field of view. This implies that there is an important redundancy between the pointings, contrary to track sharing where the sources are supposed to be fully independent on the sky. For instance, when mosaicking with a hexagonal compact pattern, each line of sight will be observed by 7 contiguous pointings, except at the mosaic edges.

The time/sensitivity estimator will thus have to link the elapsed telescope time to cover the whole mapped region to the sensitivity in each independent resolution element. To do this, we need to introduce

- A_{map} and A_{beam}, which are respectively the area of the map and the area of the resolution element. The map area is a user input while the resolution area is linked to the telescope full width at half maximum (θ) by

$$A_{\text{beam}} = \frac{0.8\,\pi\,\theta^2}{4\,\ln(2)}, \tag{12}$$

  The 0.8 factor represents the truncation of the beam at 20% of its maximum, which is performed during the imaging process. Three tests can be checked on A_{map}:

  1) A_{map} must be larger than 2 times A_{beam} (below this we advise to use the track sharing mode with two independent fields);

  2) A_{map} must be smaller than a limit defined by to the shortest integration time achievable with NOEMA ($A^{\text{uv}}_{\text{max}}$). The distance covered by a visibility in the uv-plane during an integration should always smaller than the distance associated to tolerable aliasing (see Pety and Rogrígues-Fernández 2010 for more details). This can be written as the following condition (Eq. C.3 in the article):

$$\frac{\delta t}{1s} << \frac{6900}{\theta_{alias}/\theta_{syn}} \tag{13}$$

  where δt is the integration time, θ_{alias} the map angular size, and θ_{syn} the angular resolution.

  For a given angular resolution, the interferometer minimum integration time corresponds to a maximal map size according to:

$$A^{\text{uv}}_{\text{max}} = \frac{6900 \times \theta_{syn}}{\delta t_{min}\,\eta} \tag{14}$$

  where η is a ad-hoc integer set to 5 to ensure the condition defined in Eq. 13.

  3) A_{map} must also be smaller than a limit related to the maximum number of fields observable in a given time with NOEMA ($A^{\text{cycle}}_{\text{max}}$). Presently, we assume that all the pointings should be covered within 1.5 cycle on source between two calibrators, i.e. $\sim$35 min.

  The minimum integration time of NOEMA is 10s. However, the slewing time between two positions being 8s, it is recommanded, in order to limit overheads, to spend at least 20s per position (i.e. 2×10s if a short integration time is needed to verify Eq. 13). As a result, the number of fields that can be covered is:

$$n^{\text{cycle}}_{\text{max}} = 35 \times 60/(20 + 8) = 75. \tag{15}$$

Assuming a standard sampling for the mosaic this corresponds to $75\times4/7$ independent beams and we have:

$$A_{\rm max}^{\rm cycle} = n_{\rm max}^{\rm cycle}\,\frac{4}{7}\,A_{\rm beam} \sim 43\,A_{\rm beam} \tag{16}$$

This $A_{\rm max}^{\rm cycle}$ is a technical limitation for a given observing track. Larger maps can be built putting together different sub-maps observed in different tracks.

- The number of independent measurements ($n_{\rm beam}$) in the final map which is given by

$$n_{\rm beam} = \frac{A_{\rm map}}{A_{\rm beam}}. \tag{17}$$

Because of the redundancy, we must have $n_{\rm beam} < n_{\rm point}$.

The on-source time is then shared between $n_{\rm beam}$ independent measurements. This yields

$$\Delta t_{\rm on} = \frac{\eta_{\rm tel}\,\Delta t_{\rm tel}}{n_{\rm beam}} \quad \text{and} \quad \sigma_{\rm Jy} = \frac{J\,T_{\rm sys}}{\eta_{\rm spec}\sqrt{n_{\rm ant}\,(n_{\rm ant}-1)\,d\nu\,n_{\rm pol}\,\Delta t_{\rm on}}}. \tag{18}$$

Note that $\eta_{\rm tel}$ must be sligthly smaller for a mosaic than for a single-field, single-source observation because the telescope have to slew between the fields, increasing the overheads. But this is a second order effect. Finally, this noise estimate is correct at any point of the mosaic that is covered by the same number of pointings (in particular, the mosaic center). It will be higher at the mosaic edges.