Institut de
Radioastronomie
Millimétrique

mm interferometers

Frédéric Gueth

Outline

- Image formation

  $\hookrightarrow$ Fourier optics, van-Cittert- Zernike theorem

- The ideal interferometer

  $\hookrightarrow$ geometrical delay, source size, bandwidth

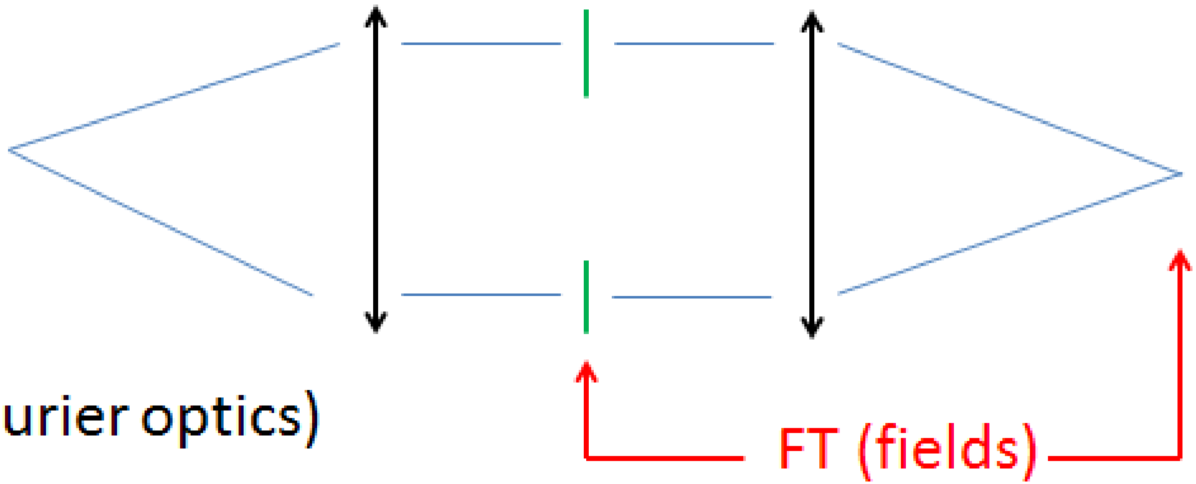
- The real interferometer

  $\hookrightarrow$ heterodyne receivers, delay tracking, correlators

- Aperture synthesis

  $\hookrightarrow$ *uv* plane, field of view

- Fraunhofer diffraction



- (Fourier optics)

- Astronomy case

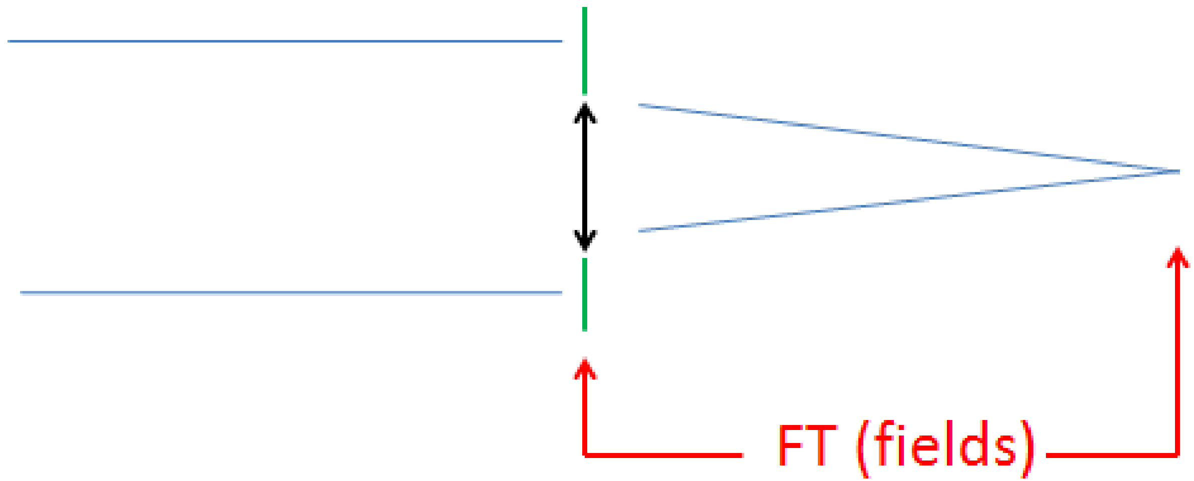


Image formation

Fourier optics

Single-dish observations (point source)

Aperture function $\rightleftharpoons$ Voltage pattern

$$\Big\downarrow |\cdot|^2$$

Power pattern $B(\ell, m)$

= Primary beam

Image formation

Fourier optics

Single-dish observations (point source)

Aperture function $\rightleftharpoons$ Voltage pattern

$\star \downarrow$ $\quad\quad$ $\downarrow |\cdot|^2$

Transfer function $T(u,v)$ $\rightleftharpoons$ Power pattern $B(\ell,m)$

Transfer function describes how **spatial frequencies** are transmitted by the telescope; it is the **autocorrelation** of the aperture

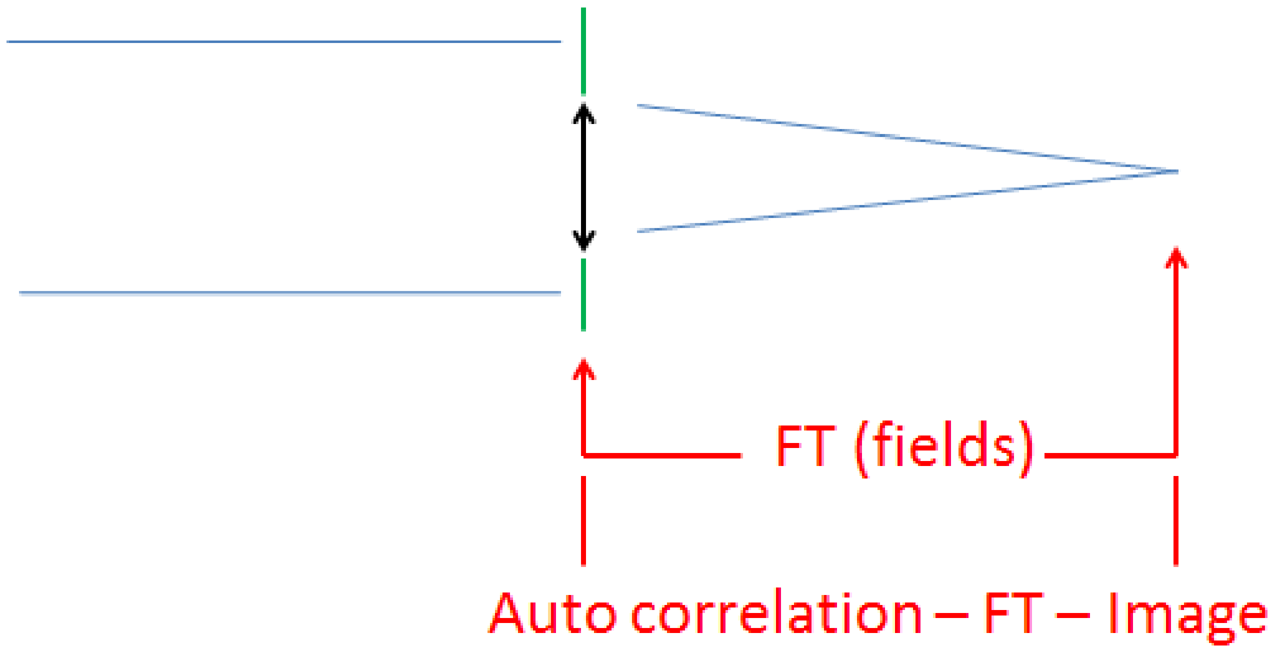

FT (fields)
Auto correlation – FT – Image

Image formation

Fourier optics

Interferometers (point source)

Aperture function $\rightleftharpoons$ Voltage pattern

$\star \downarrow$ $\qquad\qquad$ $\downarrow |\cdot|^2$

Transfer function $\mathbf{T}(\mathbf{u},\mathbf{v})$ $\rightleftharpoons$ Power pattern $B(\ell,m)$

Aperture synthesis = sample **directly the transfer function** by mesuring the spatial correlation of the incident electric field

Image formation

Fourier optics

How to measure the spatial correlation of the incident electric field? **Young's hole!**

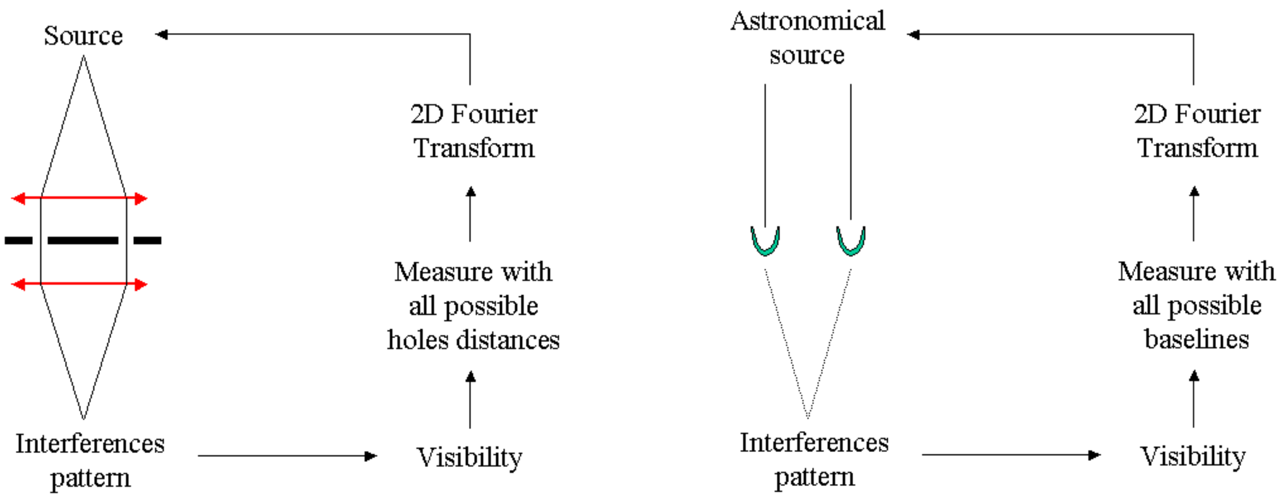


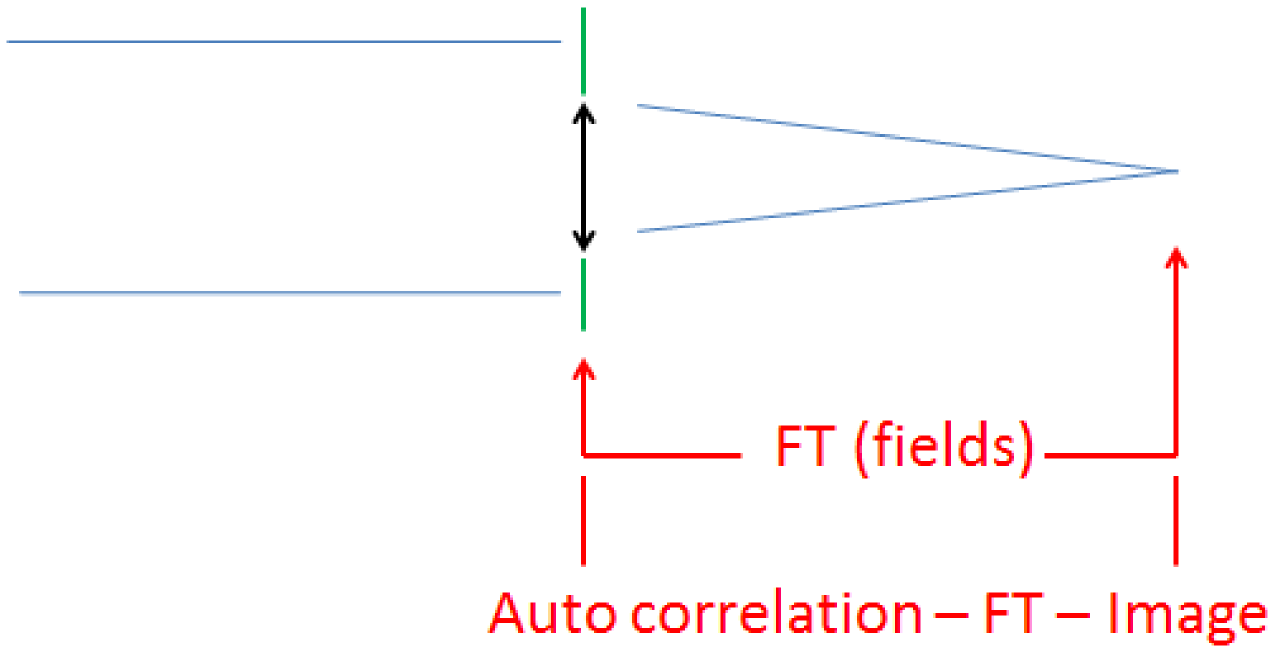
Image formation

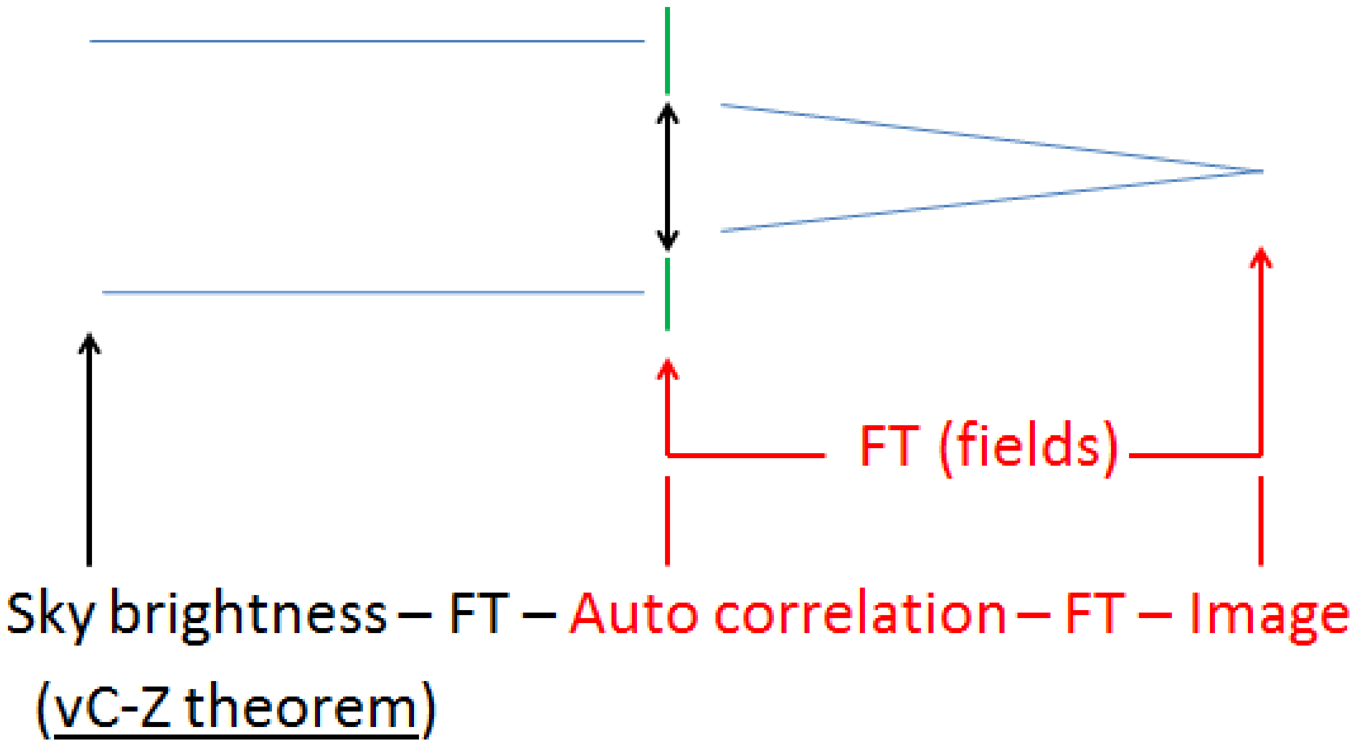
van Cittert–Zernike theorem

- Source at infinite distance; no spatial coherence; homogeneous medium between source and measure; measure in plane perp. to the line of sight

Spatial autocorrelation of measured field = FT(source brightness)

- $< E(x_1)E(x_2) > \rightleftharpoons S(\alpha)$
- $S(\alpha) =$ brightness distribution
- $< E(x_1)E(x_2) >=$ spatial correlation of incoming field, depends only on $u = x_1 - x_2 =$ spatial frequency


FT (fields)
Auto correlation – FT – Image


FT (fields)
Sky brightness – FT – Auto correlation – FT – Image
(vC-Z theorem)

Implementing the van Cittert–Zernike theorem

1. Build a device that measures the spatial autocorrelation of the incoming signal
2. Do it for all possible spatial frequencies
3. Take the FT and get an image of the brightness distribution

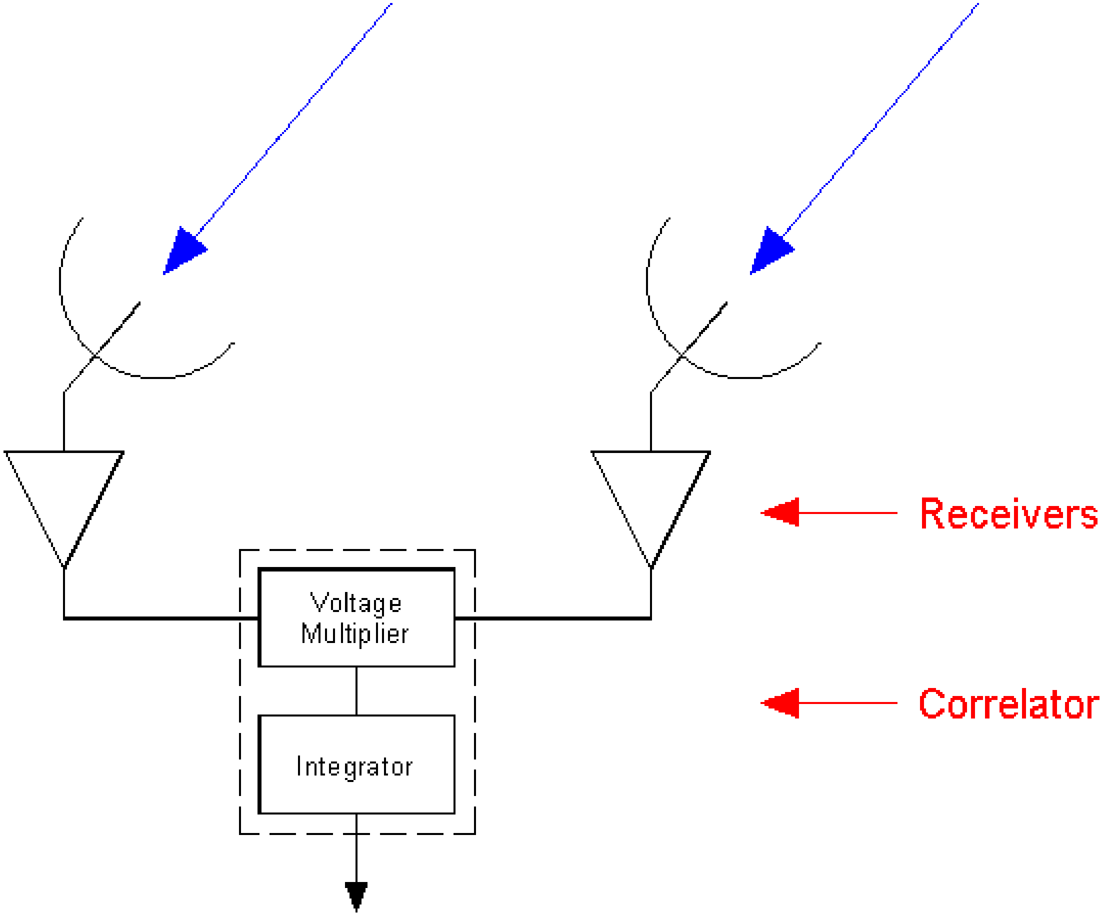
Image formation

Implementing the van Cittert–Zernike theorem

1. Build a device that measures the spatial autocorrelation of the incoming signal $\longrightarrow$ **2-elements interferometer**
2. Do it for all possible spatial frequencies $\longrightarrow$ **N antennas**
3. Take the FT and get an image of the brightness distribution $\longrightarrow$ **software**
4. Process the image because of incomplete uv coverage (2.)

The ideal interferometer

Sketch



The ideal interferometer

Measurements

- The heterodyne receiver measures the incoming electric field $E\cos(2\pi\nu t)$
- The correlator is a multiplier followed by a time integrator:

$$r = < E_1 \cos(2\pi\nu t)\, E_2 \cos(2\pi\nu t) > = E_1 E_2$$

- We have measured the spatial correlation of the signal!

The ideal interferometer

Measurements

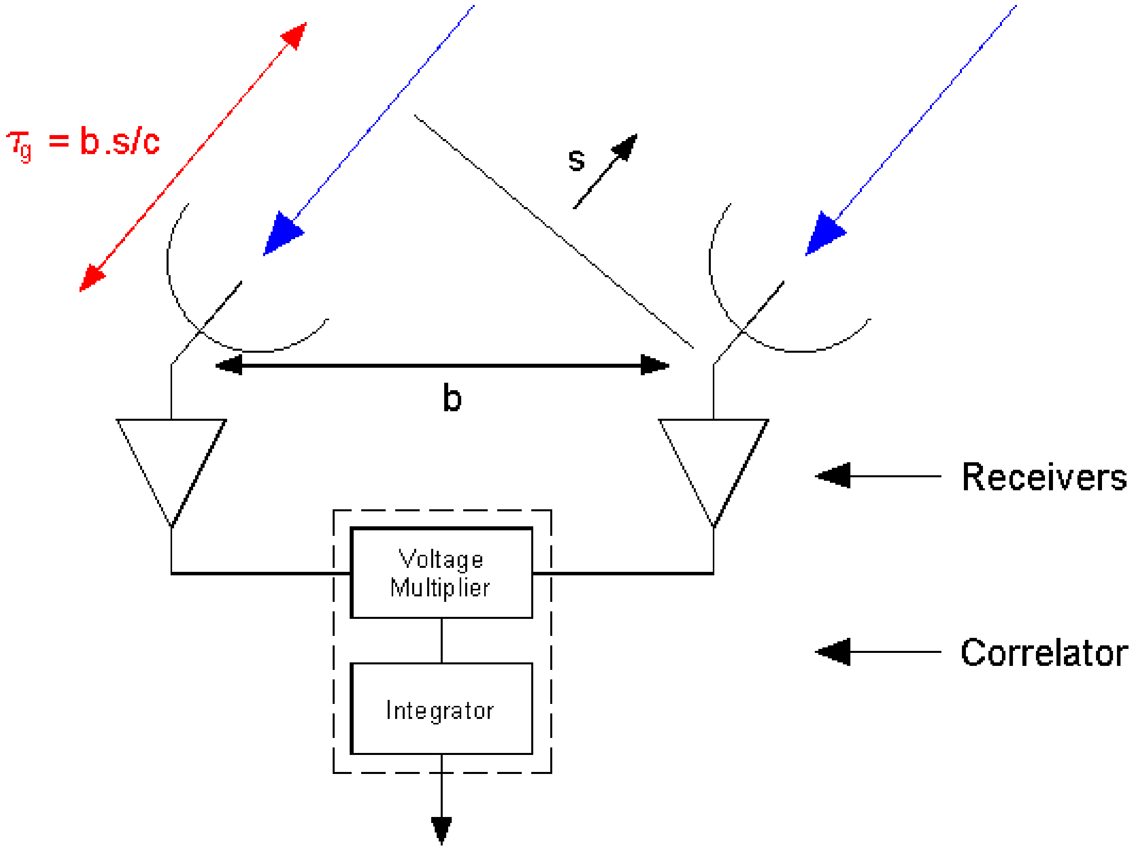
- The heterodyne receiver measures the incoming electric field $E\cos(2\pi\nu t)$
- The correlator is a multiplier followed by a time integrator:

$$r = < E_1 \cos(2\pi\nu t)\, E_2 \cos(2\pi\nu t) > = E_1 E_2$$

- We have measured the spatial correlation of the signal!
- **But we have forgotten the geometrical delay**

The ideal interferometer

Sketch



The ideal interferometer

Measurements

- There is a **geometrical delay** τ_g between the two antennas $\longrightarrow$ **more complex** experiment than the Young's holes
- Correlator output:

$$r = < E_1 \cos(2\pi\nu t)\, E_2 \cos(2\pi\nu t) > = E_1 E_2$$

$$\begin{aligned} r &= < E_1 \cos(2\pi\nu(t-\tau_g))\, E_2 \cos(2\pi\nu t) > \\ &= E_1 E_2 \cos(2\pi\nu\tau_g) \end{aligned}$$

The ideal interferometer

Measurements

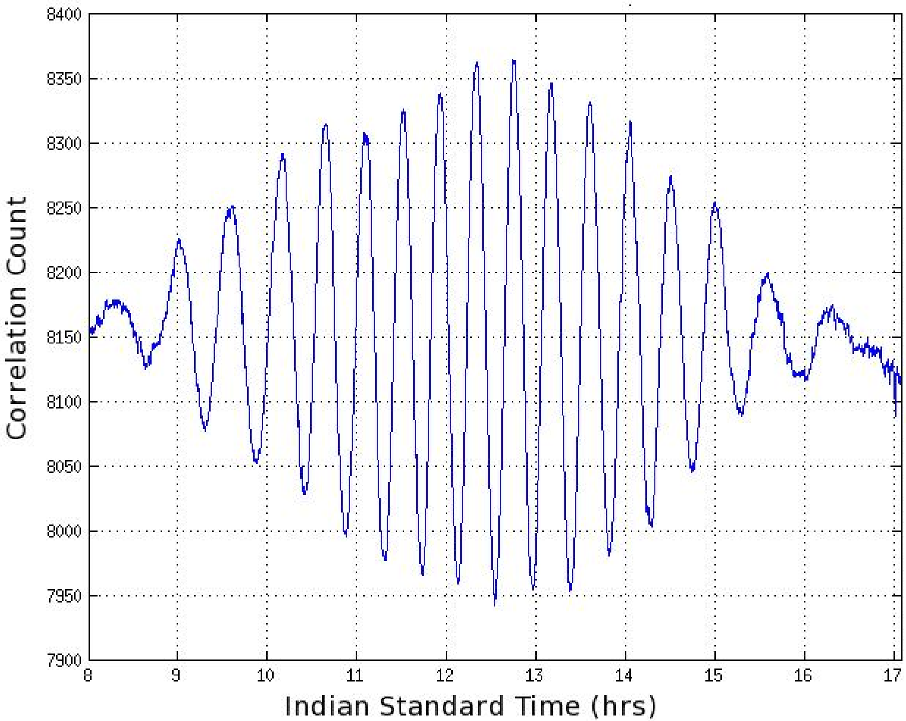
- Correlator output: $r = E_1 E_2 \cos(2\pi\nu\tau_g)$
- τ_g varies slowly with time (Earth rotation) $\longrightarrow$ **fringes**
- Natural fringe rate:

$$\tau_g = \frac{\mathbf{b.s}}{c} \qquad\qquad \nu\,\frac{d\tau_g}{dt} \simeq \Omega_{earth}\,\frac{\mathrm{b}\nu}{c}$$

~ 50 Hz for $b = 800$ m and $\nu = 250$ GHz

The ideal interferometer

Measurements



The ideal interferometer

Measurements

- Correlator output: $r = E_1 E_2 \cos(2\pi\nu\tau_g)$
- τ_g varies slowly with time (Earth rotation) $\longrightarrow$ **fringes**
- τ_g is **known** from the antenna position, source direction, time $\longrightarrow$ could be corrected
- Are we done?

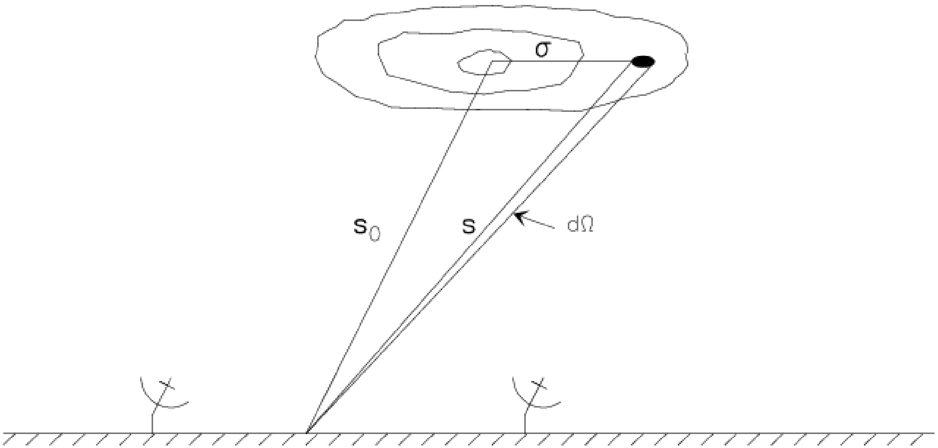
The ideal interferometer

Measurements

- Correlator output: $r = E_1 E_2 \cos(2\pi\nu\tau_g)$
- τ_g varies slowly with time (Earth rotation) $\longrightarrow$ **fringes**
- τ_g is **known** from the antenna position, source direction, time $\longrightarrow$ could be corrected
- Are we done?
- No... the source is **not a point source**

The ideal interferometer

Source size



$\mathbf{s} = \mathbf{s_o} + \boldsymbol{\sigma}$

Power received from $d\Omega$:
$A(\mathbf{s})I(\mathbf{s})d\Omega$
$A(s)$ = beam
$I(s)$ = source

Correlator output: $r = E_1 E_2 \cos(2\pi\nu\tau_g)$

$$r = A(\mathbf{s})I(\mathbf{s})d\Omega\cos(2\pi\nu\tau_g(\mathbf{s}))$$

The ideal interferometer

Source size

- Correlator output integrated over source:

$$R = \int_{Sky} A(\mathbf{s}) I(\mathbf{s}) \cos(2\pi\nu\tau_g(\mathbf{s}))\, d\Omega$$
$$= |V| \cos(2\pi\nu\tau_g - \varphi_{\mathrm{V}})$$

- **Complex visibility:**

$$V = |V| e^{i\varphi_{\mathrm{V}}} = \int_{Sky} A(\sigma) I(\sigma) e^{-2i\pi\nu\mathbf{b}.\sigma/c} d\Omega$$

The ideal interferometer

Source size

$$
\begin{aligned}
R &= \int_{Sky} A(\mathbf{s}) I(\mathbf{s}) \cos(2\pi\nu \mathbf{b}.\mathbf{s}/c)\, d\Omega \\
&= \cos\left(2\pi\nu \frac{\mathbf{b}.\mathbf{s}_o}{c}\right) \int_{Sky} A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \cos(2\pi\nu \mathbf{b}.\boldsymbol{\sigma}/c) d\Omega \\
&- \sin\left(2\pi\nu \frac{\mathbf{b}.\mathbf{s}_o}{c}\right) \int_{Sky} A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \sin(2\pi\nu \mathbf{b}.\boldsymbol{\sigma}/c) d\Omega \\
&= \cos\left(2\pi\nu \frac{\mathbf{b}.\mathbf{s}_o}{c}\right) |V| \cos\varphi_{\mathrm{V}} - \sin\left(2\pi\nu \frac{\mathbf{b}.\mathbf{s}_o}{c}\right) |V| \sin\varphi_{\mathrm{V}} \\
&= |V| \cos(2\pi\nu\tau_g - \varphi_{\mathrm{V}})
\end{aligned}
$$

The ideal interferometer

Summary

- Correlator output:

$$r = \; < E_1 \cos(2\pi\nu t)\, E_2 \cos(2\pi\nu t) > = E_1 E_2$$

$$r = E_1 E_2 \cos(2\pi\nu\tau_g) \qquad \longleftarrow \text{delay}$$

$$R = |V| \cos(2\pi\nu\tau_g - \varphi_V) \qquad \longleftarrow \text{source size}$$

- Complex visibility V resembles a Fourier Transform:

$$V = |V| e^{i\varphi_V} = \int_{Sky} A(\sigma) I(\sigma) e^{-2i\pi\nu \mathbf{b}.\sigma/c} d\Omega$$

The ideal interferometer

Summary

- Correlator output:

$$r \;=\; < E_1 \cos(2\pi\nu t)\, E_2 \cos(2\pi\nu t) > = E_1\, E_2$$
$$r \;=\; E_1 E_2 \cos(2\pi\nu\tau_g) \qquad \longleftarrow \text{delay}$$
$$R \;=\; |V| \cos(2\pi\nu\tau_g - \varphi_V) \qquad \longleftarrow \text{source size}$$

- **3D version of van Cittert–Zernike**
  - We do **not** measure $r = FT(I)$
  - We measure R = something related to V, which resembles the FT(I)

The ideal interferometer

Bandwidth

- Next problem: **bandwidth** (the source is **not** monochromatic)
- Integrating over a finite bandwidth $\Delta\nu$

$$
\begin{aligned}
R &= \frac{1}{\Delta\nu}\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} |V| \cos(2\pi\nu\tau_g - \varphi_V)\, d\nu \\
&= |V| \cos(2\pi\nu_0\tau_g - \varphi_V)\, \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g}
\end{aligned}
$$

- The fringe visibility is attenuated by a $\sin(x)/x$ envelope (= bandwidth pattern) which falls off rapidly

The ideal interferometer

Summary

- Correlator output:

$$
\begin{aligned}
r &= \; < E_1 \cos(2\pi \nu t)\, E_2 \cos(2\pi \nu t) > \, = E_1 E_2 \\
r &= \; E_1 E_2 \cos(2\pi \nu \tau_g) && \longleftarrow \text{delay} \\
R &= \; |V| \cos(2\pi \nu \tau_g - \varphi_{\rm V}) && \longleftarrow \text{source size} \\
R &= \; |V| \cos(2\pi \nu_0 \tau_g - \varphi_{\rm V}) \, \frac{\sin(\pi \Delta \nu \tau_g)}{\pi \Delta \nu \tau_g} && \longleftarrow \text{bandwidth}
\end{aligned}
$$

- We measure R, which is related to V, which resembles the FT(I). R strongly depends on τ_g.

The ideal interferometer

Delay correction

$$R = |V| \cos(2\pi \nu_0 \tau_g - \varphi_V) \frac{\sin(\pi \Delta\nu \tau_g)}{\pi \Delta\nu \tau_g}$$

- τ_g varies with time because of the Earth rotation $\longrightarrow$ rapid decrease of R (1% for a path length difference of $\sim$2 cm and $\Delta\nu = 1$GHz)
- Tracking a source requires the **compensation of the geometrical delay**
- Inteferometry requires temporal coherence!

The ideal interferometer

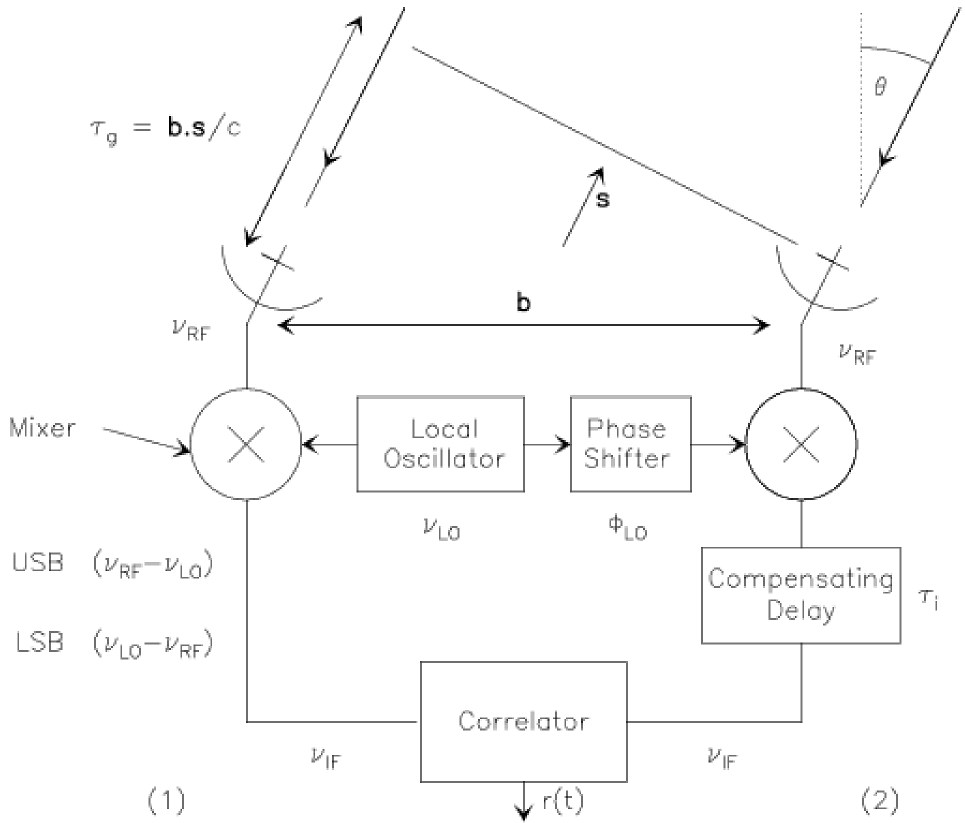
Delay correction

$$R = |V| \cos(2\pi\nu_0\tau_g - \varphi_V) \frac{\sin(\pi\Delta\nu\tau_g)}{\pi\Delta\nu\tau_g}$$

- Tracking a source requires the **compensation of the geometrical delay**
- This can be achieved by introducing an **instrumental delay** in the correlator
- If delay is compensated, we can measure $R = |V| \cos(\varphi_V)$

The real interferometer

Sketch



The real interferometer
Heterodyne detection

- In the receiver **mixer**, the incident electic field is combined with a **local oscillator** signal

$$U(t) = E\cos(2\pi\nu t + \varphi)$$
$$U_{\mathrm{LO}}(t) = E_{\mathrm{LO}}\cos(2\pi\nu_{\mathrm{LO}} t + \varphi_{\mathrm{LO}})$$
$$\nu_{\mathrm{LO}} \simeq \nu$$

- to produce the **intermediate frequencies**

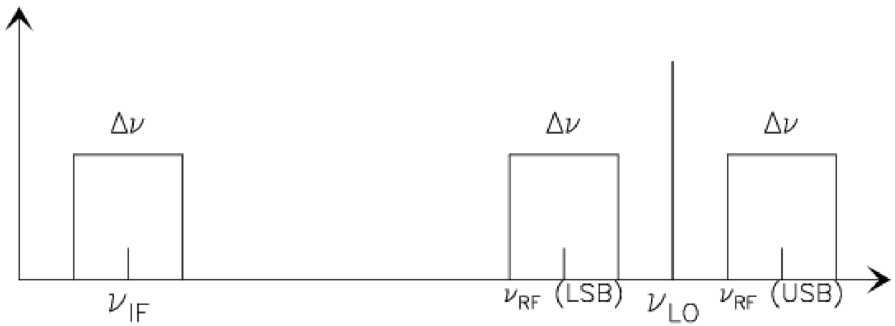
$$\nu_{\mathrm{IF}} - \Delta\nu/2 \leq |\nu - \nu_{\mathrm{LO}}| \leq \nu_{\mathrm{IF}} + \Delta\nu/2$$

The real interferometer

Heterodyne detection

- The receiver output is

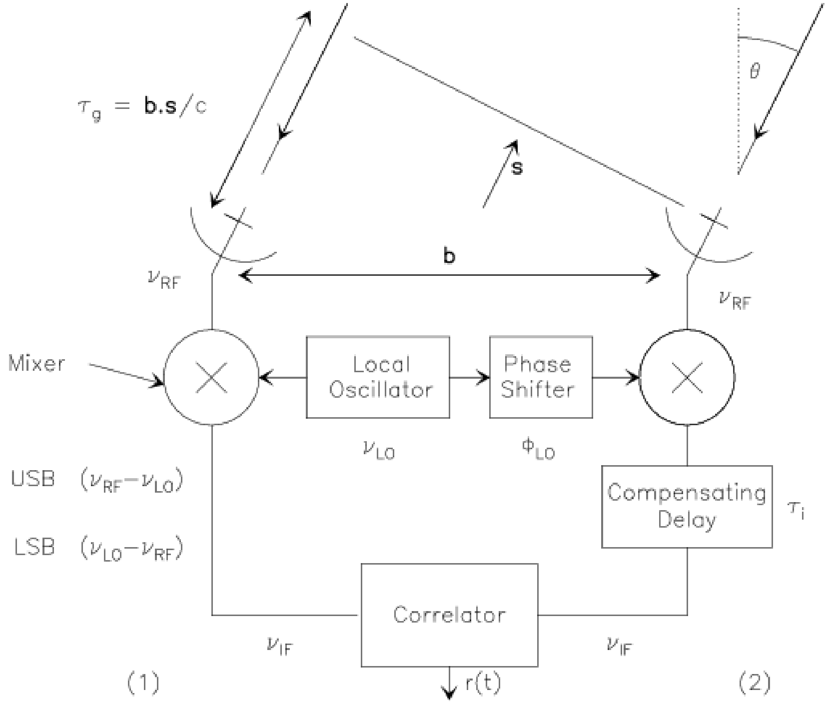
$$I(t) \propto E\, E_{\rm LO} \cos\left(\pm\left(2\pi(\nu-\nu_{\rm LO})t+\varphi-\varphi_{\rm LO}\right)\right)$$



LSB **USB**

PdBI: $\nu_{\rm IF}$ = 4–8 GHz, ALMA: 4–8 GHz, NOEMA: 4–12 GHz

The real interferometer
Delay tracking



- A compensating delay is introduced in one of the branch of the interferometer, **on the IF signal**
- Equivalent to the delay lines in IR interferometers

The real interferometer

Delay tracking

- Phases of the two signals (USB):

$$\varphi_1 = 2\pi\nu\tau_g \qquad \varphi_1 = 2\pi\nu\tau_g = 2\pi(\nu_{\mathrm{LO}} + \nu_{\mathrm{IF}})\tau_g$$

$$\varphi_2 = 0 \qquad \varphi_2 = 2\pi\nu_{\mathrm{IF}}\tau_i$$

- Correlator output:

$$R = |V|\cos(2\pi\nu\tau_g - \varphi_{\mathrm{V}})$$

$$R = |V|\cos(\varphi_1 - \varphi_2 - \varphi_{\mathrm{V}})$$

$$R = |V|\cos(2\pi\nu_{\mathrm{LO}}\tau_g - \varphi_{\mathrm{V}})$$

The real interferometer

Fringe Stopping

- Delay tracking not enough because applied on the IF
- Solution: in addition to delay tracking, **rotate the phase of the local oscillator** such that at any time:

$$\varphi_{\mathrm{LO}}(t) = 2\pi \nu_{\mathrm{LO}} \tau_g(t)$$

- τ_g is computed for a reference position = **phase center**
- Phase center = pointing center in practice, though not mandatory

The real interferometer

Fringe stopping

- Phases of the two signals (USB):

$$\varphi_1 = 2\pi\nu\tau_g = 2\pi(\nu_{\rm LO} + \nu_{\rm IF})\tau_g$$
$$\varphi_2 = 2\pi\nu_{\rm IF}\tau_i + \varphi_{\rm LO}$$
$$\varphi_{\rm LO} = 2\pi\nu_{\rm LO}\tau_g$$

- Correlator output:

$$R = |V|\cos(\varphi_1 - \varphi_2 - \varphi_{\rm V})$$
$$R = |V|\cos(-\varphi_{\rm V})$$

The real interferometer

Complex correlator

- After fringe stopping:

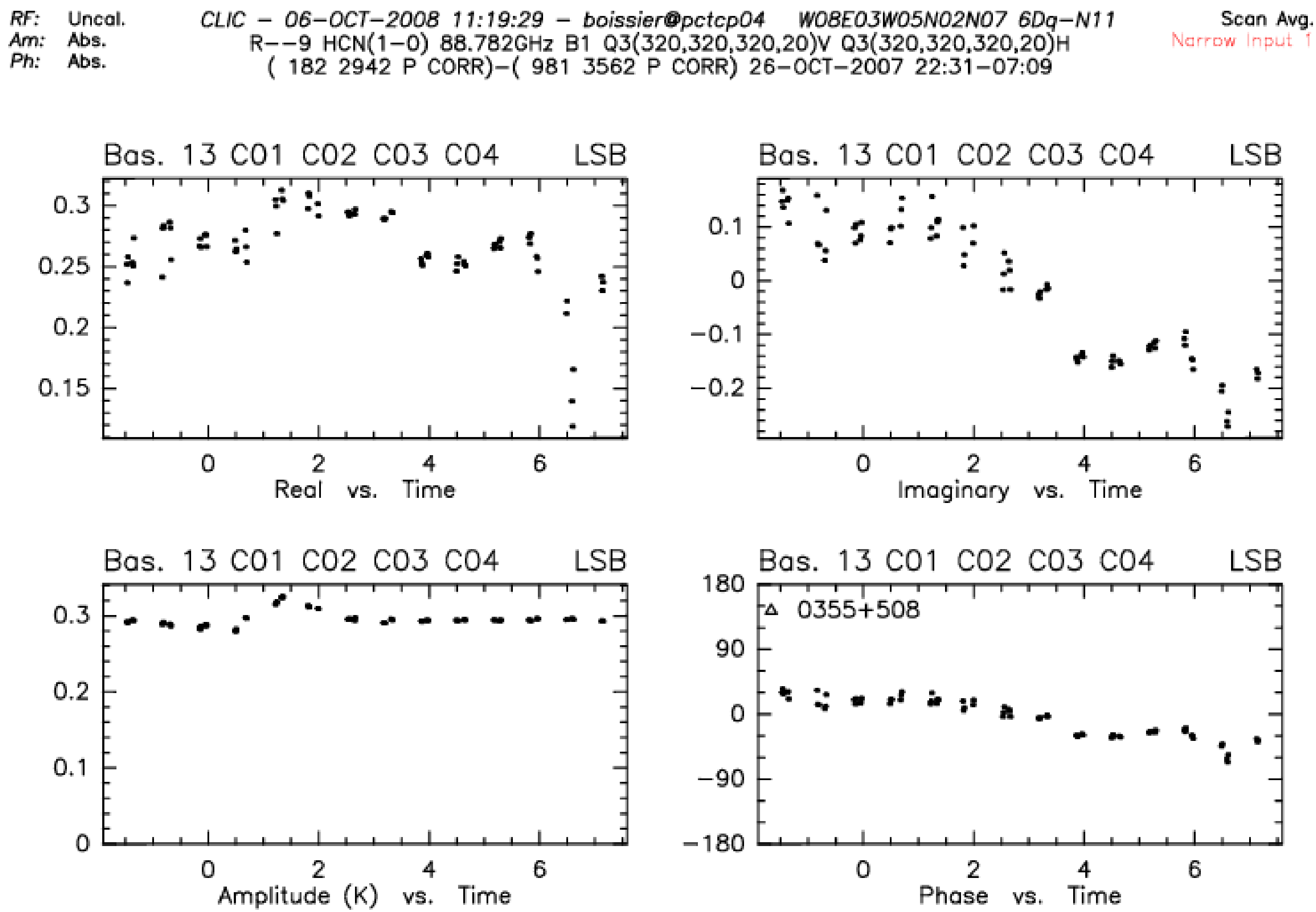
$$R = |V|\cos(-\varphi_{\rm V})$$

- The corrections were so good that there is **no time or delay dependance** any more $\longrightarrow$ cannot measure $|V|$ and $\varphi_{\rm V}$ separately.
- A second correlator is necessary, with one signal phase shifted by $\pi/2$: $R_i = |V|\sin(-\varphi_{\rm V})$
- **The complex correlator measures directly the visibility**

The real interferometer

Complex correlator

- The correlator measures the real and imaginary parts of the visibility. **Amplitude and phases are computed off-line.**
- Amplitude and phases have more physical sense
  - Visibility amplitude = **correlated flux**
  - The atmosphere adds a **phase** to the incoming signals $\longrightarrow$ measured phase = visibility + $\varphi_1 - \varphi_2$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Am: Abs.
Ph: Abs.
CLIC – 06-OCT-2008 11:19:29 – boissier@pctcp04 W08E03W05N02N07 6Dq-N11
R--9 HCN(1-0) 88.782GHz B1 Q3(320,320,320,20)V Q3(320,320,320,20)H
(182 2942 P CORR)-(981 3562 P CORR) 26-OCT-2007 22:31-07:09
Scan Avg.
Narrow Input 1
Bas. 13 C01 C02 C03 C04 LSB
Real vs. Time
Bas. 13 C01 C02 C03 C04 LSB
Imaginary vs. Time
Bas. 13 C01 C02 C03 C04 LSB
Amplitude (K) vs. Time
Bas. 13 C01 C02 C03 C04 LSB
△ 0355+508
Phase vs. Time

Implementing the van Cittert–Zernike theorem

1. Build a device that measures the spatial autocorrelation of the incoming signal $\longrightarrow$ **2-elements interferometer**
2. Do it for all possible spatial frequencies $\longrightarrow$ **N antennas**
3. Take the FT and get an image of the brightness distribution $\longrightarrow$ **software**
4. Process the image because of incomplete uv coverage (2.)

Aperture synthesis

Complex visibility

- Complex visibility:

$$V = |V| e^{i\varphi_V} = \int_{Sky} A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) e^{-2i\pi\nu \mathbf{b}.\boldsymbol{\sigma}/c} d\Omega$$

- Going from 3-D to 2-D? ...some algrebra...
- OK providing that:

$$(\text{max. field of view})^2 \times \text{max. baseline} \ll 1$$

$$\Longrightarrow \frac{(\text{max. field of view})^2}{\text{resolution}} \ll 1$$

Aperture synthesis

Complex visibility

$$V(u,v) = \int_{Sky} A(\ell,m) I(\ell,m) e^{-2i\pi\nu(u\ell+vm)} d\Omega$$

- *uv* plane is perpendicular to the source direction, **fixed wrt source** $\longrightarrow$ **back to van Cittert-Zernike theorem**
- Price: limit on the field of view
- Approximation **ok in (sub)mm domain**, problem at wavelengths $>$ cm, maybe with ALMA (long baselines, short frequencies)

Aperture synthesis

Field of view

- Values for Plateau de Bure

θ_s	ν (GHz)	2-D Field	0.5 GHz Bandwidth	1 Min Averaging	Primary Beam
$5''$	80	$5'$	$80''$	$2'$	$60''$
$2''$	80	$3.5'$	$30''$	$45''$	$60''$
$2''$	230	$3.5'$	$1.5'$	$45''$	$24''$
$0.5''$	230	$1.7'$	$22''$	$12''$	$24''$

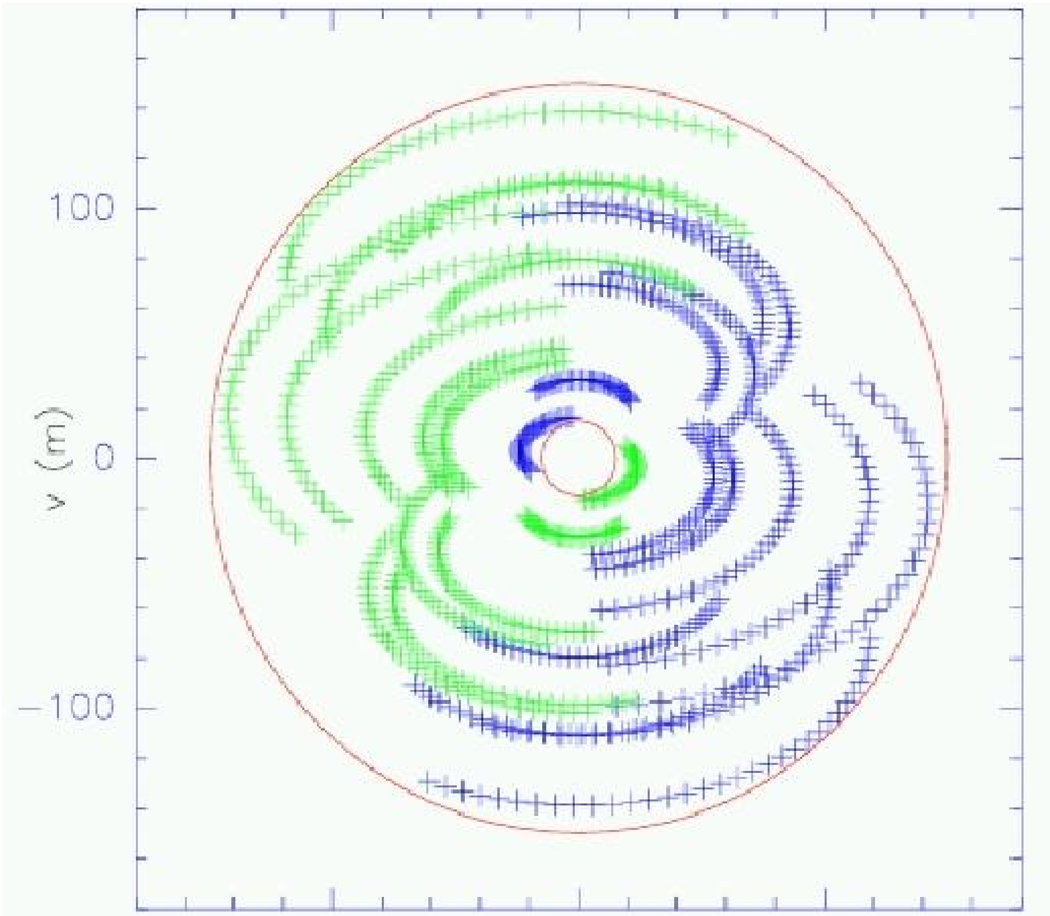
- Problem with 2D field: software; with bandwith: split the data for imaging; with time averaging: dump faster.
- **Primary beam is the main limit on the FOV**

Aperture synthesis

uv plane

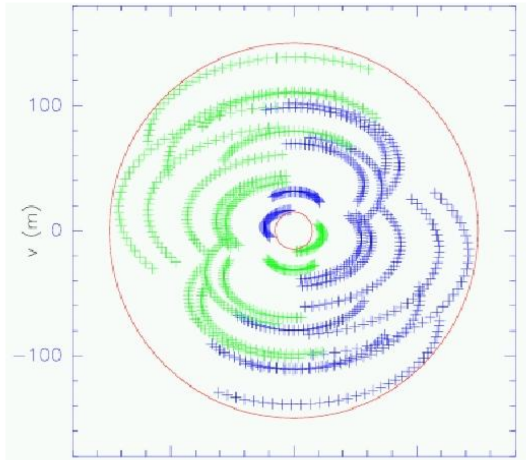
- *uv* plane is perpendicular to the source direction, **fixed wrt source**
- (u,v) is the 2–antennas **vector** baseline projected on the plane perpendicular to the source
- (u,v) are **spatial frequencies**
- ... Earth rotation ... (spherical trigonometry) ...
- (u,v) describe an **ellipse** in the *uv* plane (for $\delta = 0\,$deg, a line)

Aperture synthesis
uv plane coverage



Aperture synthesis

Image formation



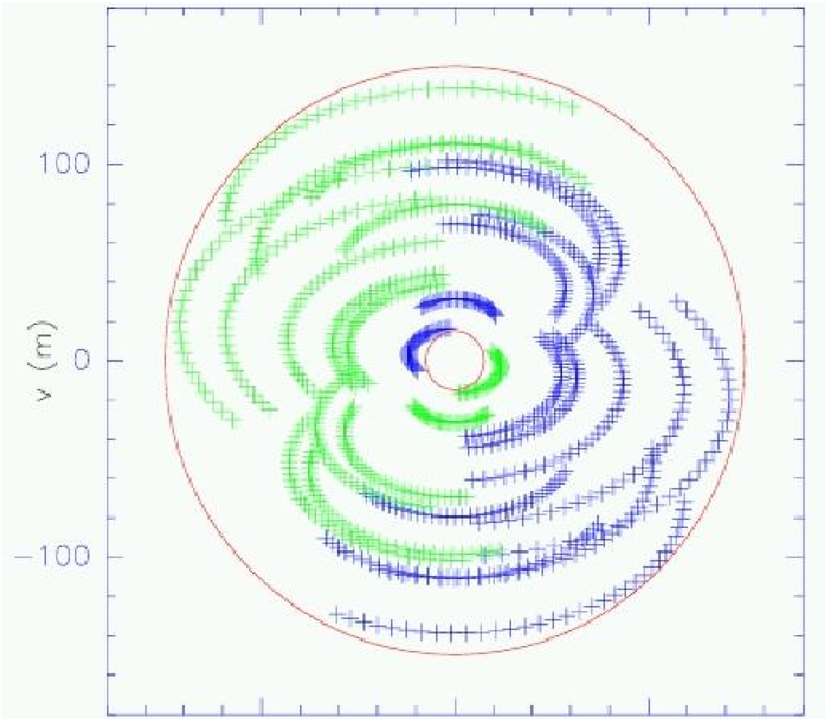
Measurements = uv plane sampling × visibilities

After FT: dirty map = dirty beam ∗ (prim. beam × sky)

The FT of the *uv* plane coverage gives the dirty beam = the PSF of the observations

Aperture synthesis

Image formation



Max. base-line gives the angular resolution

The real interferometer

Spectroscopy

- van Cittert–Zernike theorem is in the **space** domain
- There is a similar theorem in the **time** domain: the **Wiener-Kichnine** theorem
  - temporal autocorrelation of $S(t)$ = FT(spectra)

    $S(t_1)\,S(t_2) = \Sigma(\tau) \rightleftharpoons S(\nu)$
  - implementation: FT spectrometers

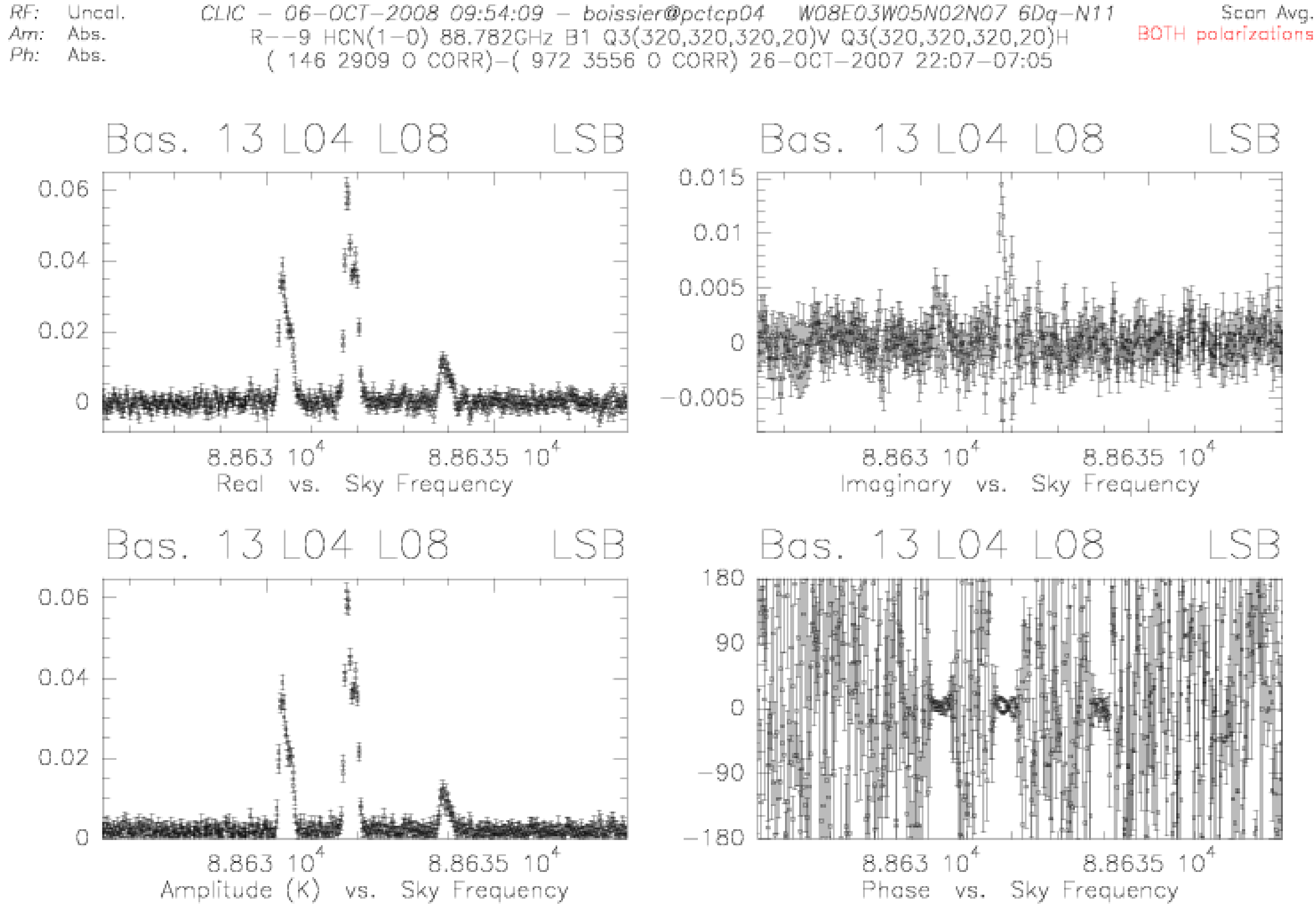
The real interferometer

Spectroscopy

- Calculate the correlation between two antennas with a **delay** between the two signals
- Do it for several delays $\delta\tau \longrightarrow$ measurement of the **temporal correlation** $\longrightarrow$ FT to get the spectra:

$$V(u,v,\nu) = \int V(u,v,\tau) e^{-2i\pi\tau\nu} d\nu$$

- This has nothing to do with geometrical delay compensation: $\delta\tau \sim 1/\delta\nu$ here
- Mixed up implementation in correlator software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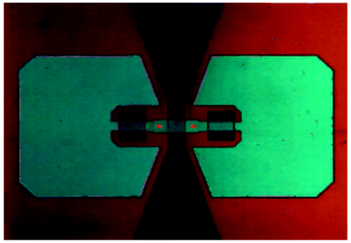
(146 2909 0 CORR)-(972 3556 0 CORR) 26-OCT-2007 22:07-07:05
Scan Avg.
BOTH polarizations
Bas. 13 L04 L08 LSB
0.06
0.04
0.02
0
8.863 10^4
8.8635 10^4
Real vs. Sky Frequency
Bas. 13 L04 L08 LSB
0.015
0.01
0.005
0
−0.005
8.863 10^4
8.8635 10^4
Imaginary vs. Sky Frequency
Bas. 13 L04 L08 LSB
0.06
0.04
0.02
0
8.863 10^4
8.8635 10^4
Amplitude (K) vs. Sky Frequency
Bas. 13 L04 L08 LSB
180
90
0
−90
−180
8.863 10^4
8.8635 10^4
Phase vs. Sky Frequency

Summary

Many other instrumental issues

- Phase lock systems to control φ_{LO}
- Real-time monitoring and correction of the phase offset in the cables or fibers
- Complex phase switching is used to cancel offsets, separate/reject side bands, ...
- Antenna position measurements, to get the delay, u, v
- Antenna deformations, e.g. thermal expansion (delay)
- Accurate focus measurements (delay)
- Atmospheric phase monitoring
- ...




S/N 01D01
ALTERA


WideX 01
WideX 02
WideX 03
WideX 04