Sensitivity Low SNR analysis

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10th IRAM Millimeter Interferometry School
Grenoble, 1—5 October 2018

Outline

Sensitivity

- Point source sensitivity
- Noise in images
- Brightness sensitivity

Low S/N analysis

- Continuum data
- Line data
- Examples

Noise in visibilities

- Noise on one visibility:

$$\delta S = \frac{\sqrt{2}k}{\eta A} \frac{T_{sys}}{\sqrt{\Delta t\,\Delta \nu}}$$

- Noise is uncorrelated from one baseline to another
- There are N(N-1)/2 baselines for N antennas
- So the **point source sensitivity (= average of all visibilities)** is

$$\delta S = \frac{2k}{\eta A} \frac{T_{sys}}{\sqrt{N(N-1)\,t_{int}\,\Delta \nu}}$$

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Average of all visibilities to detect a point source

But we are doing a map, ie a (weighted) Fourier Transform…

Noise in images

- The Fourier Transform is a **linear combination** of the visibilities with some rotation (phase factor) applied. How do we derive the noise in the image from that on the visibilities ?

- **Phase rotation?**
  - the correlator gives the same noise (variance) on the real and imaginary part of the complex visibility $\langle \varepsilon_r^2 \rangle = \langle \varepsilon_i^2 \rangle$
  - Real and Imaginary are uncorrelated $\langle \varepsilon_r \varepsilon_i \rangle = 0$
  - so rotation (phase factor) has **no effect on noise**

$$\varepsilon'_\mathrm{R} = \varepsilon_\mathrm{R} \cos(\phi) - \varepsilon_\mathrm{I} \sin(\phi)$$

$$\varepsilon'_\mathrm{I} = \varepsilon_\mathrm{R} \sin(\phi) + \varepsilon_\mathrm{I} \cos(\phi)$$

$$\langle {\varepsilon'_\mathrm{R}}^2 \rangle = \langle \varepsilon_\mathrm{R}^2 \rangle \cos^2(\phi) - 2\langle \varepsilon_\mathrm{R} \varepsilon_\mathrm{I} \rangle \cos(\phi)\sin(\phi) + \langle \varepsilon_\mathrm{I}^2 \rangle \sin^2(\phi) = \langle \varepsilon^2 \rangle$$

$$\langle \varepsilon'_\mathrm{R} \varepsilon'_\mathrm{I} \rangle = \langle \varepsilon_\mathrm{R}^2 \rangle \cos(\phi)\sin(\phi) - \langle \varepsilon_\mathrm{I}^2 \rangle \cos(\phi)\sin(\phi) = 0$$

Noise in images

- Noise can be estimated **at the phase center**
- In the imaging process, we combine (with some weights) the individual visibilities V_i. At the phase center:

$$I = \sum w_i V_i / \sum w_i$$

- This is a classical case of noise propagation. If **natural weights** $w_i = 1/\sigma_i^2$ we have

$$1/\sigma^2 = \sum 1/\sigma_i^2$$

- So the noise rms in the image is indeed given by:

$$\delta S = \frac{\mathbf{2k}}{\boldsymbol{\eta A}} \frac{\mathbf{T_{sys}}}{\sqrt{\mathbf{N(N-1)t_{int}}\boldsymbol{\Delta\nu}}}$$

Brightness sensitivity

- **Extended source sensitivity** is defined in terms of brightness temperatures = the (Rayleigh-Jeans) temperature of a source filling the beam and giving the same observed flux (Jy)

- Beam is $\theta_1\theta_2$ (solid angle Ω)

$$\boxed{T = \frac{\lambda^2}{2k\,\Omega} S} = \frac{\lambda^2}{2k} \frac{4ln(2)}{\pi\,\theta_1\theta_2} S$$

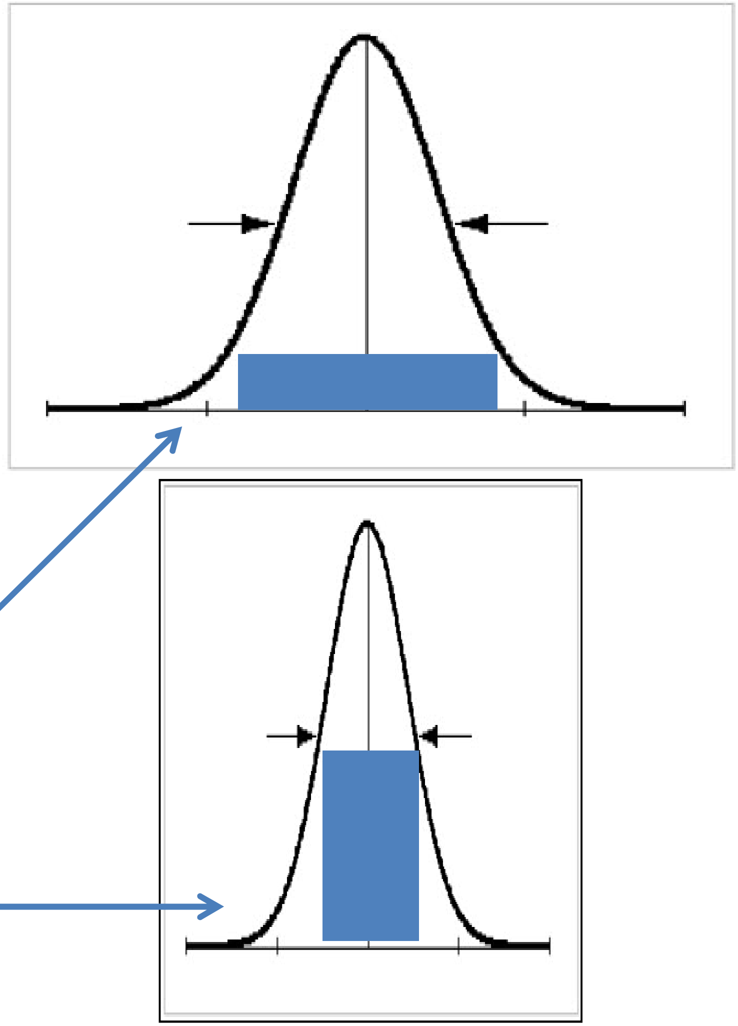
- So the brightness temperature rms is:

$$\delta T = \frac{2ln(2)\lambda^2}{k\pi} \frac{1}{\theta_1\theta_2} \delta S$$

Brightness sensitivity

- **Brightness temperature = temperature of a source filling the beam, and giving the observed flux**
- Beam x Temperature = flux
- The brightness temperature depends on the beam size

Flux of extended weak source = flux of compact bright source



Brightness sensitivity

- The point-source sensitivity (Jy/beam) does not depend on the angular resolution
- The brightness sensitivity (Kelvin) does depends on the angular resolution θ

$$\delta S = \frac{2k}{\eta A} \frac{T_{sys}}{\sqrt{N(N-1)\, t_{int}\, \Delta\nu}}$$

$$\delta T = \frac{2\ln(2)\lambda^2}{k\,\pi} \frac{1}{\theta_1 \theta_2} \delta S \propto \frac{1}{\theta^2 \sqrt{t_{int}\, \Delta\nu}}$$

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Brightness sensitivity

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THE BRIGHTNESS SENSITIVITY DEPENDS ON THE ANGULAR RESOLUTION!

Brightness sensitivity

The brightness sensitivity (Kelvin) does depends on the angular resolution θ

Single-dish

Diameter x2

- Angular resolution x 2
- Collecting area x 4
- Point-source sens. x 4
- Brightness sens. =

Brightness sensitivity

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Interferometer

Baseline x2

- Angular resolution x2
- Collecting area =
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- Brightness sens /4

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Diameter x2

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Interferometer

Baseline x2

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- Collecting area =
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Brightness sensitivity

Example 1:

- At 1″ resolution, a source has been detected with 20 σ in only 30 min, so it will be easy to map it at 0.1″

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Example 1:

- At 1″ resolution, a source has been detected with 20 σ in only 30 min, so it will be easy to map it at 0.1″
- Really?
  - Increase resolution by 10 means reducing brightness sensitivity by 100
  - Need **10000** times more integration time to reach same brightness sensitivity, i.e. 5000 hours ~ 7 months, full-time
  - **Time $\propto$ 1/resolution^4 for a given sensitivity…**
  - If we relax sensitivity by a factor 5 (4 σ detection), still need 400 times more integration time = 200 h

Brightness sensitivity

Example 2:

- ALMA accepts projects for a given angular resolution (e.g. 1")
- But observes with 0.8"

Brightness sensitivity

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- Same brightness sensitivity? Integration time increased by **2.4** (time $\propto$ 1/resolution^4)
- Same integration time? Brightness rms increased by **1.5**

Brightness sensitivity

Example 2:

- ALMA accepts projects for a given angular resolution (e.g. 1″)
- But observes with 0.8″
- Same brightness sensitivity? Integration time increased by **2.4** (time $\propto$ 1/resolution^4)
- Same integration time? Brightness rms increased by **1.5**
  - Yes, but then, I can smooth the image, right?
  - Yes, will get 1″ resolution, but not the same brightness rms (because smoothing = downweighting long baselines = reducing integration time)

Brightness sensitivity

Conclusions: do not forget

$$\delta T \propto \frac{1}{\theta^2 \sqrt{t_{int}\, \Delta\nu}}$$

- Planning observation often means **compromizing sensitivity/time/resolutions**
- Mapping sources at (very) high angular resolution is extremely time-consuming and reserved to very bright sources

Low SNR detections

- **A nice case**
  - Observers advantage: don't have to worry about bandpass & flux calibration…
  - Theorists advantage: the data is always compatible with your favorite model

- **A necessary challenge**
  - Want to push instrument to the limits
  - So a careful analysis is necessary: when is a source detected? which parameters can be derived?

Detection

- **Do not resolve it!** Use UV_FIT (fit in the uv plane) with an appropriate source size
- Do you have some information on the absolute position? (optical, previous data, model,…)

<u>< 1/10th of beam</u>

- fix the position
- UV_FIT
- need > 3 σ to claim detection

<u>About the beam</u>

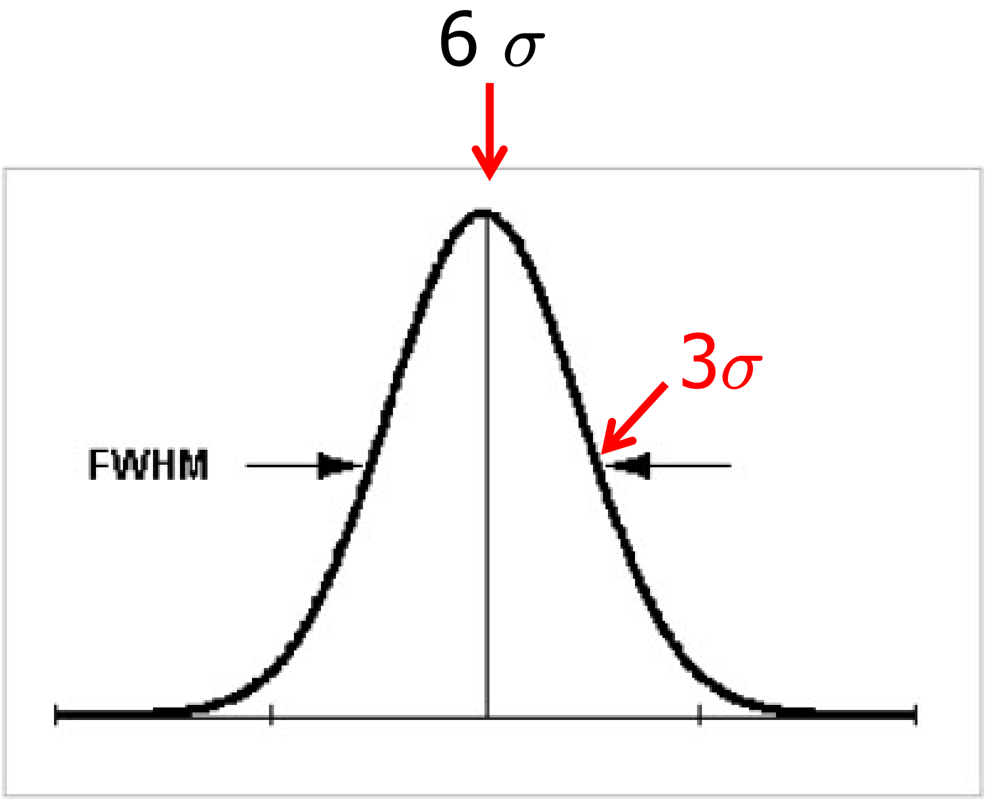
- do not fix the position
- UV_FIT
- need > 4 σ for detection

<u>Unknown</u>

- make an image to locate
- use as starting point, but do not fix the position
- UV_FIT
- need 5 σ signal for detection

Source size

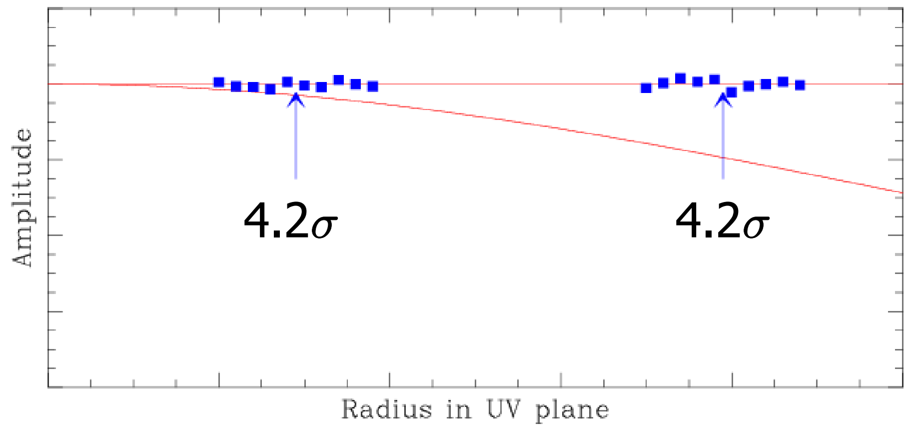
SNR < 6 σ → cannot measure any source size

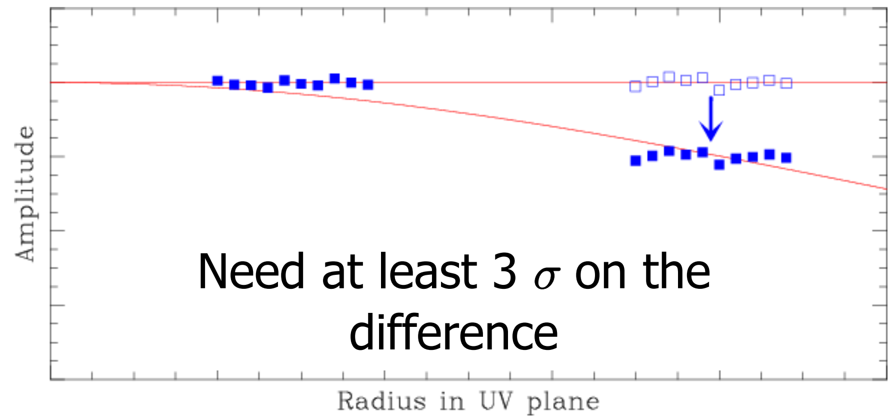


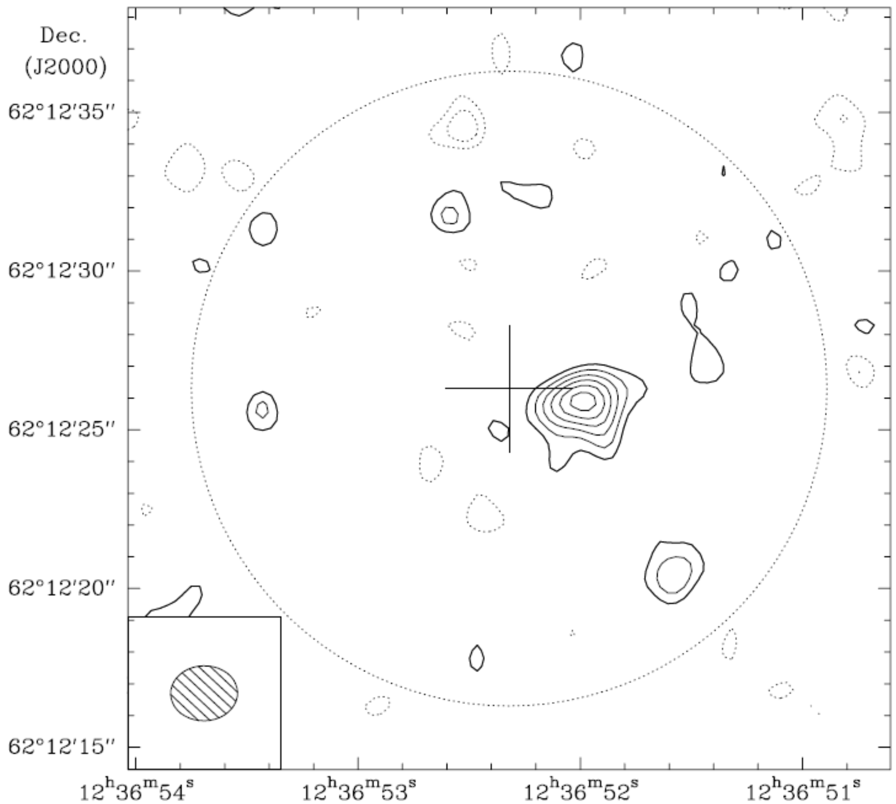
Source size

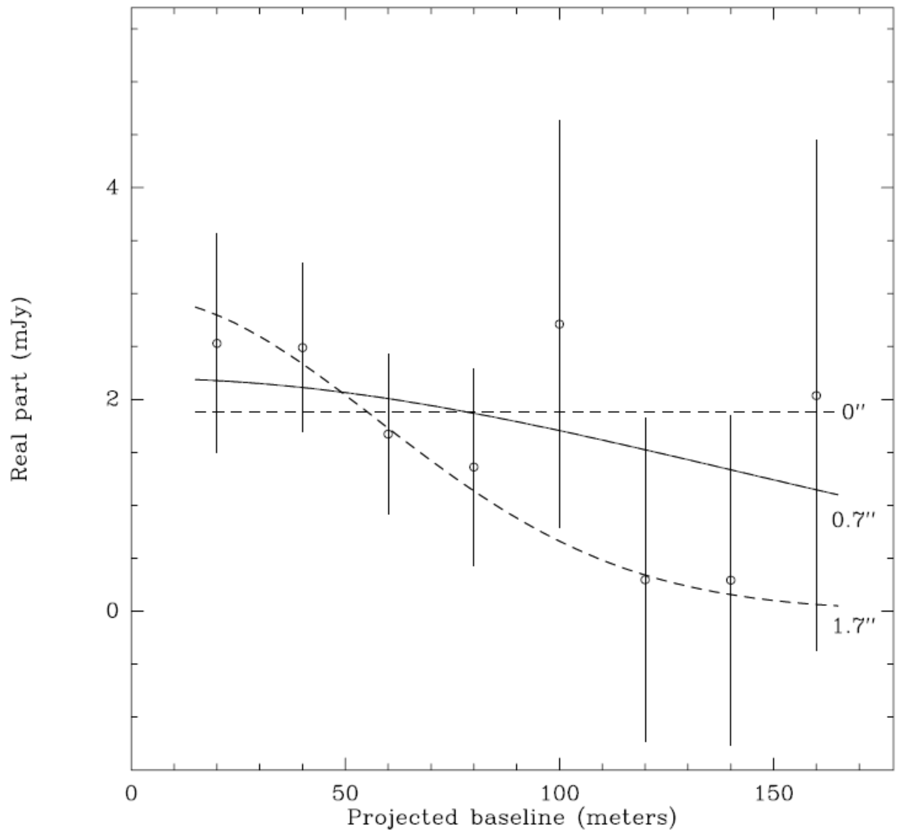
SNR < 6 σ → cannot measure any source size

- divide data in two subsets: shortest baselines on one side, longest on another
- each subset gets a 4.2 σ error on flux
- error on the difference is then just 3 σ
- so you cannot measure any difference and don't know if the source is resolved or not









Example: **HDF source** (Downes et al. 1998)

7 σ detection of the strongest source in the Hubble Deep Field.

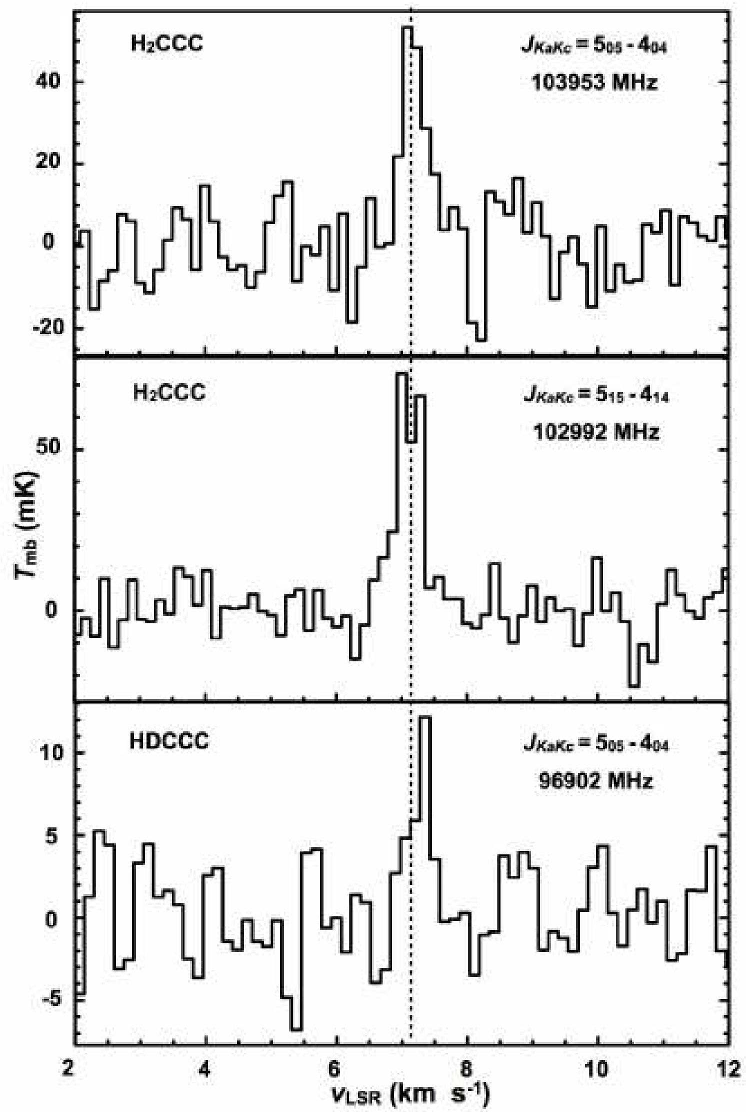
Attempts to derive a size not succesfull.
Can be unresolved, can be as large as the synthesized beam...

Lines: things get worse

- **Line velocity unknown**: observer will select the brightest part of the spectrum ➔ bias
- **Line width unknown**: observer may limit the width to brightest part of the spectrum ➔ another bias
- If **position is unknown**, it is determined from the integrated area map made from the tailored line window specified by the astronomer. This gives a biased total flux.
- Any speculated spatial extension will increase the total flux, by enlarging the selected image region (same effect as the tailored line window).

- **Net result = 1 to 2 σ** positive bias on integrated line flux.
- *Things get really messy if a continuum is superposed to the weak line...*

Lines: things get worse



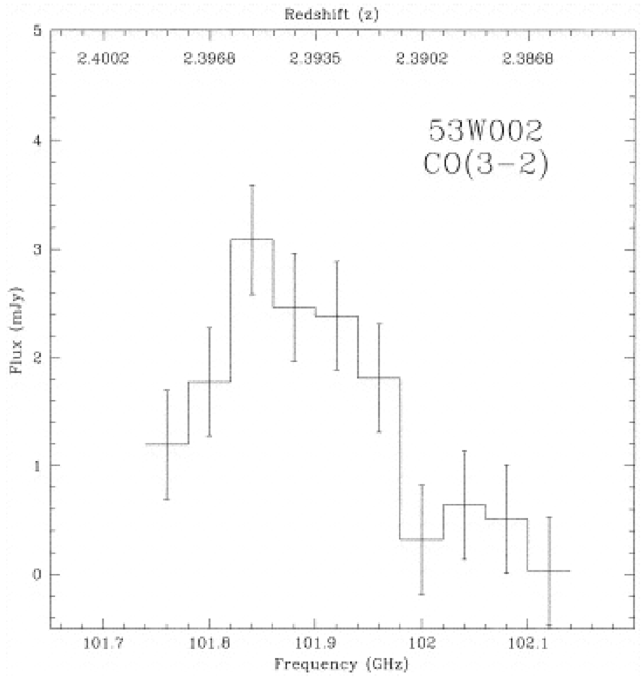
Weak line analysis

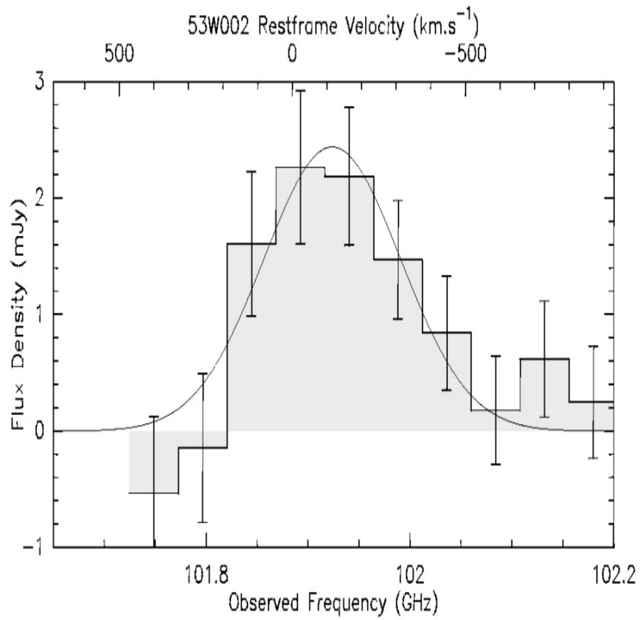
- Do not believe velocity gradient unless proven at a 6 σ level in each channel. Remember that position accuracy per channel is the beamwidth divided by the signal-to-noise ratio...
- Do not believe source size unless S/N > 10 (or better)
- Expect line widths to be very inaccurate
- Expect integrated line intensity to be positively biased by 1 to 2 σ
- Even more biased if source is extended

Example

Examples are numerous, specially for high redshift CO, e.g. **53 W002** :

- OVRO (S. et al. 1997) claims an extended source, with velocity gradient. Yet the total line flux is 1.5 +- 0.2 Jy.km/s i.e. (at best) only 7 σ .
- PdBI (A. et al. 2000) finds a line flux of 1.20 +- 0.15 Jy.km/s, no source extension, no velocity gradient, different line width and redshift.
- Note that the line fluxes agree within the errors…

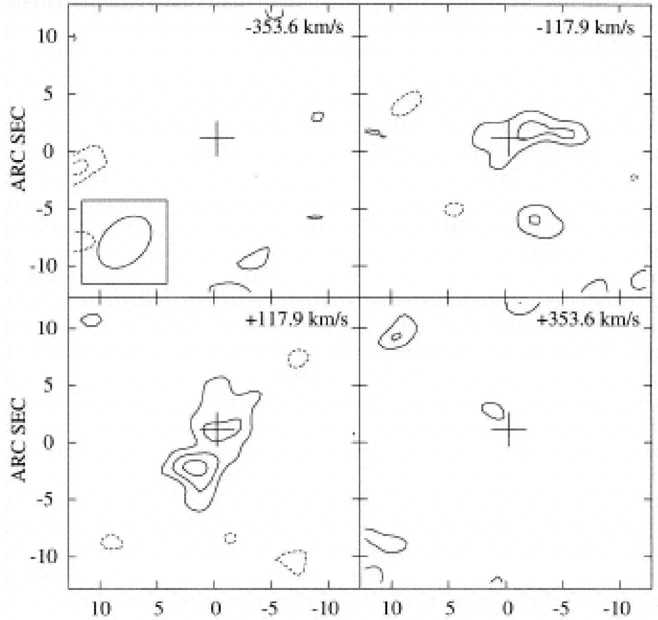


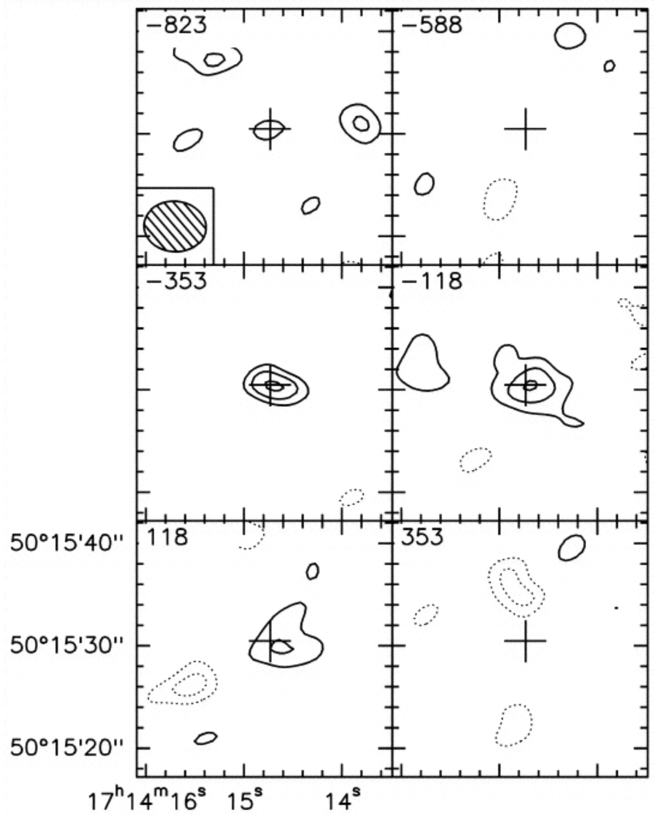


Example

Remark(s)

- But the images (contours) look convincing!
- Answer : beware of visually confusing contours which start at 2 σ (sometimes even 3) but are spaced by 1 σ



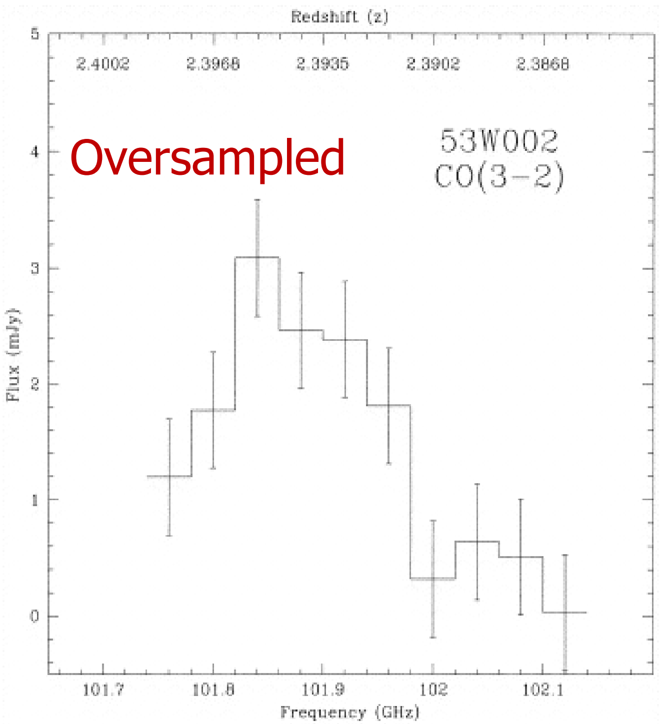


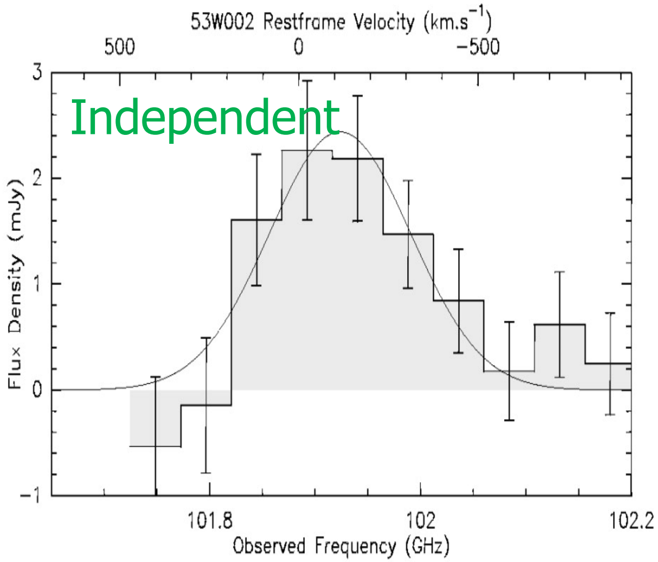
Example

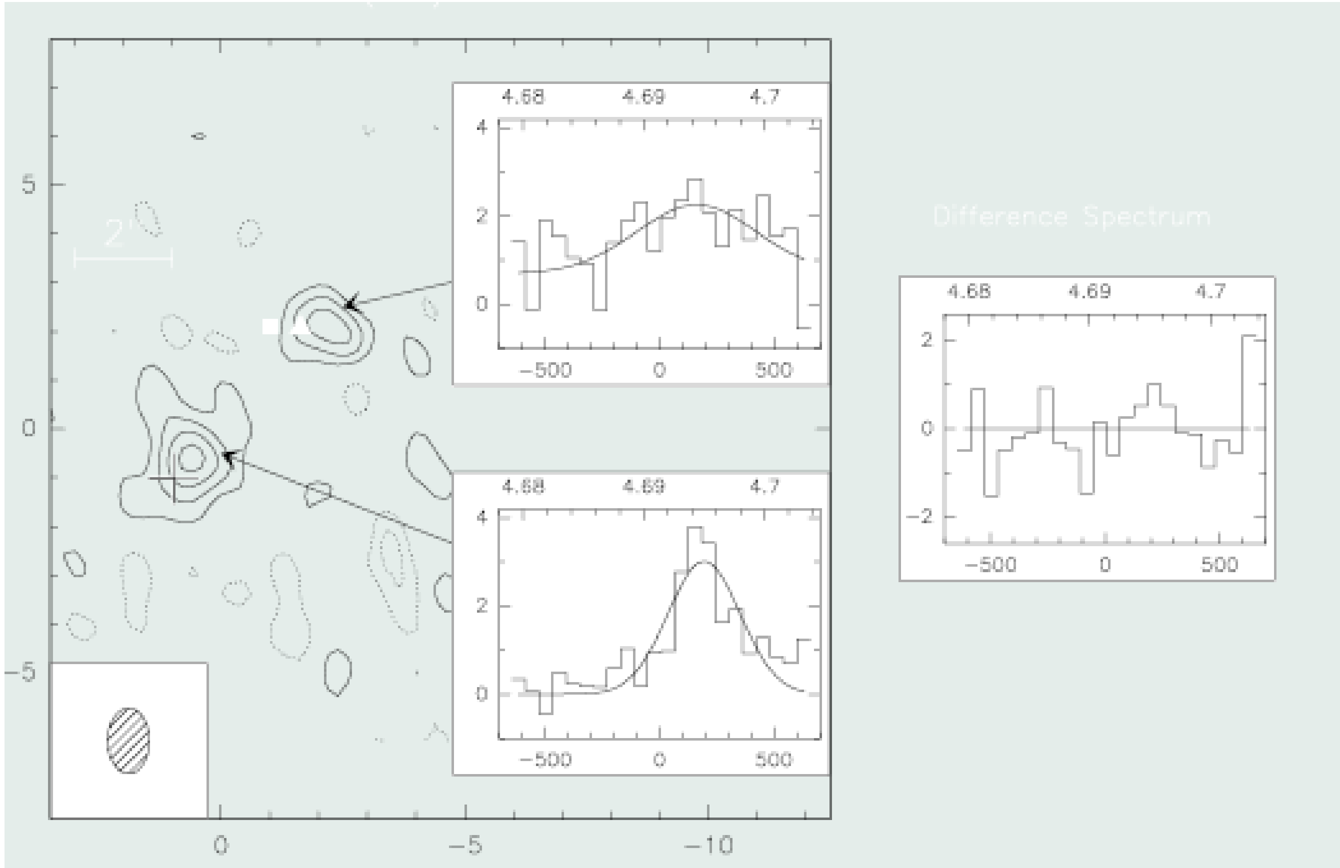
Remark(s)

- But the images (contours) look convincing!
- Answer : beware of visually confusing contours which start at 2 σ (sometimes even 3) but are spaced by 1 σ

- But the spectrum looks convincing, too !
- Answer : beware of visually confusing spectra, which are oversampled by a factor 2. The noise is then not independent between adjacent channels.





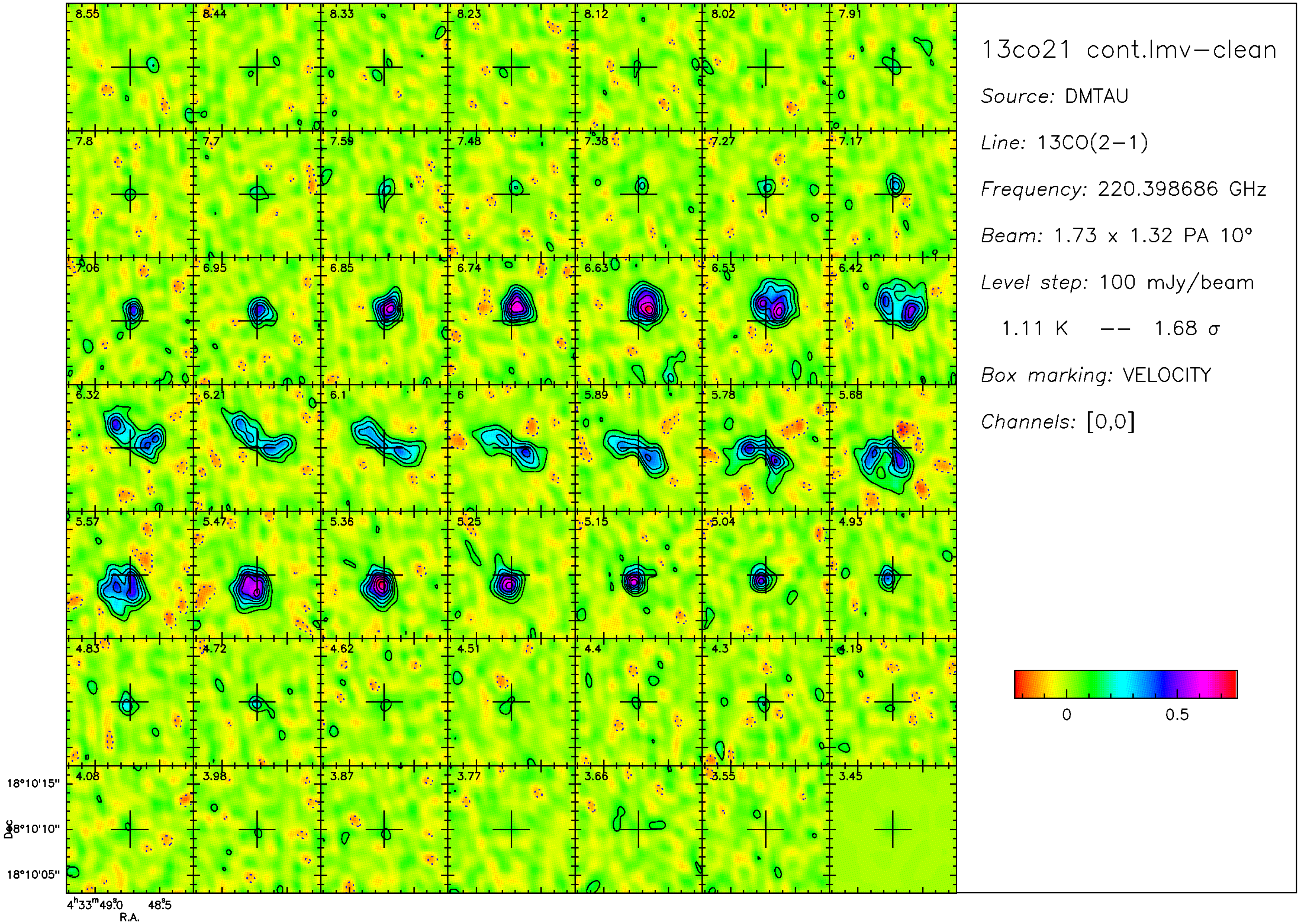


- Contour map of dust emission at 1.3 mm, with 2 σ contours
- The inserts are redshifted CO(5-4) spectra
- A weak continuum (measured independently) exist on the Northern source
- The rightmost insert is a difference spectrum (with a scale factor applied, and continuum offset removed): **No SIGNIFICANT PROFILE DIFFERENCE!**
- i.e. **no Velocity Gradient** measured.

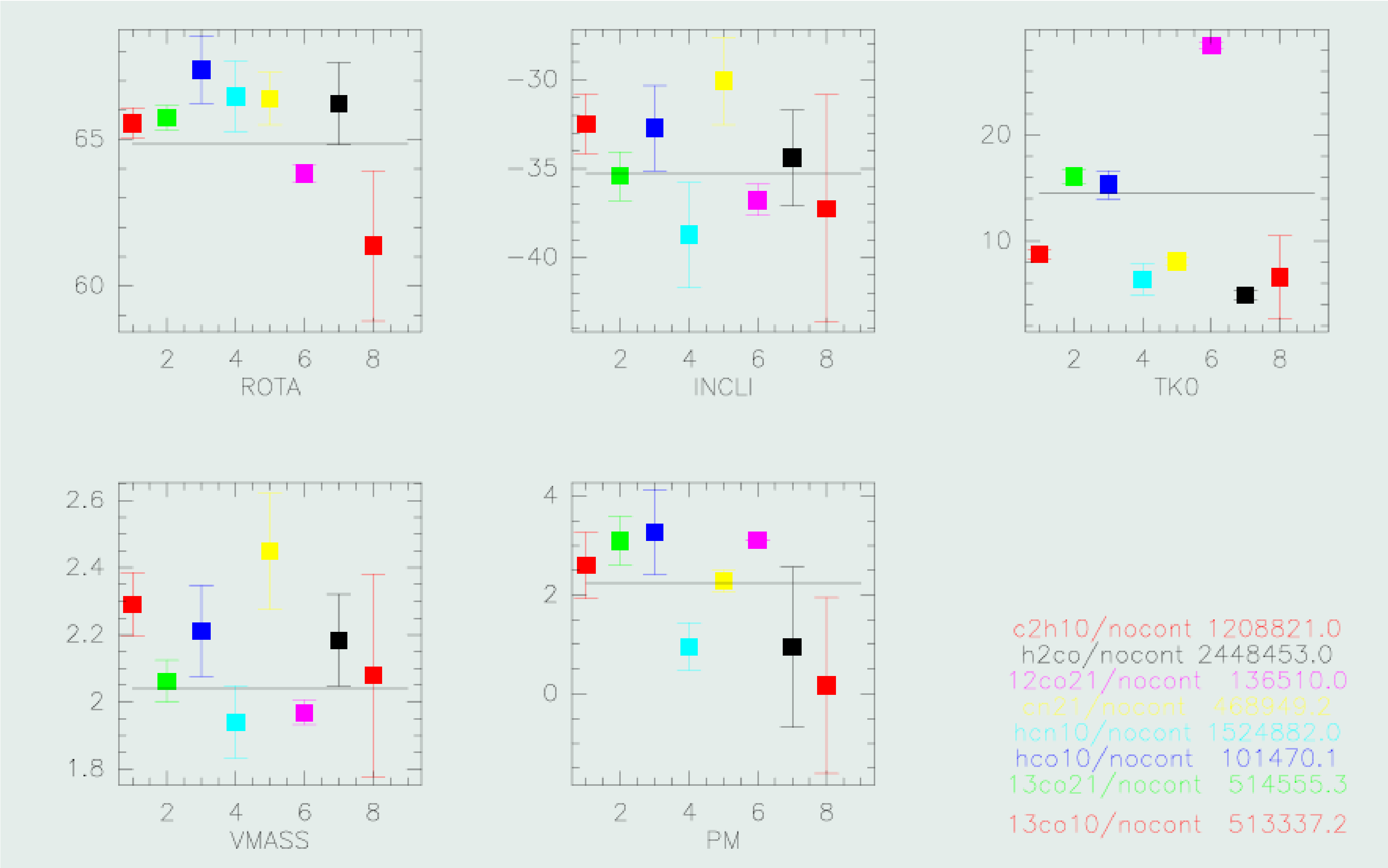
How to analyze weak lines?

Perform a statistical analysis (e.g. χ^2, or other statistical test)
comparing model prediction to observations, i.e. visibilities

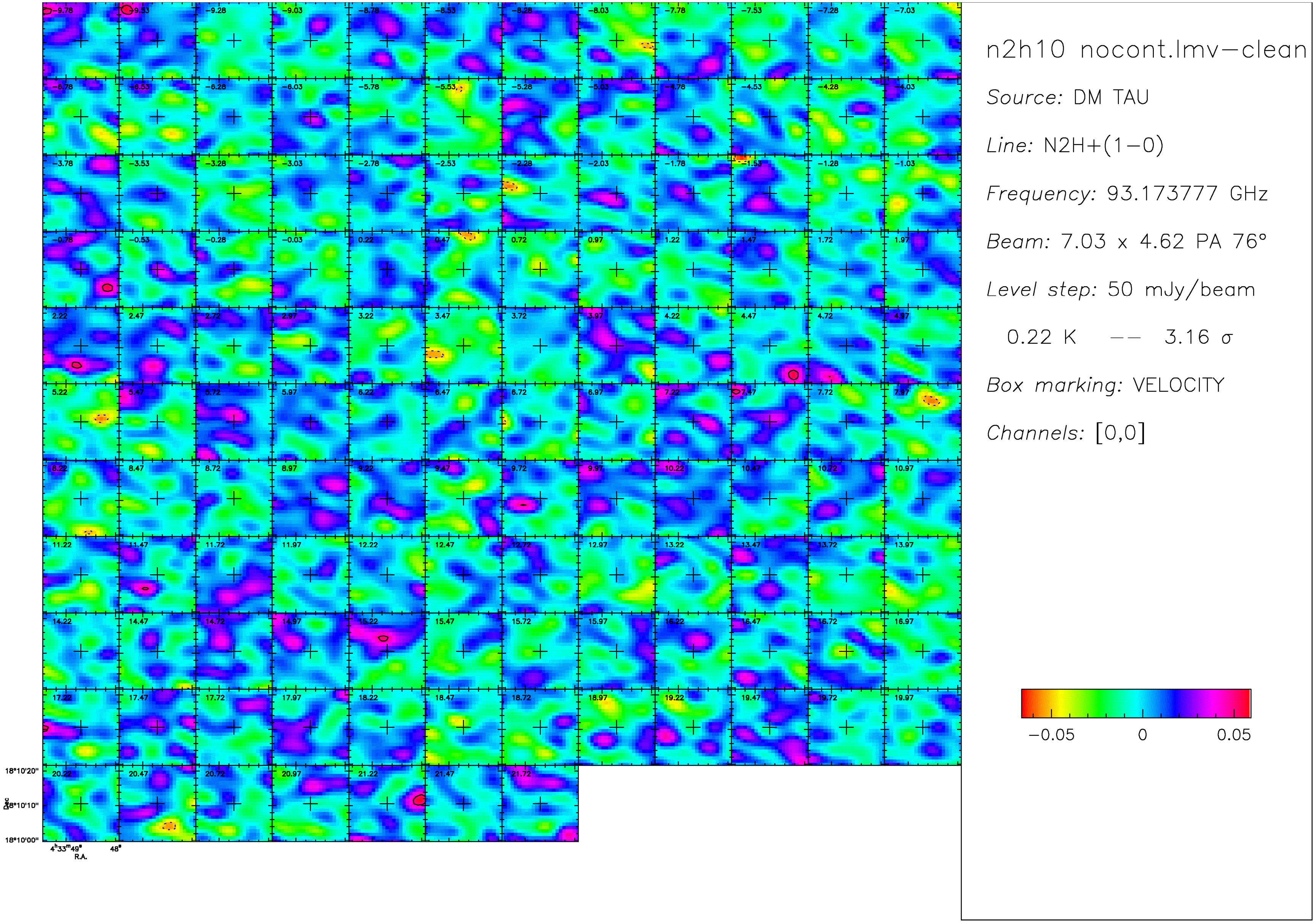
- Physical model of the source, with limited number of free parameters
- Predict visibilities
  - *The GILDAS software offer tools to compute visibilities from an image / data cube (task UV_FMODEL)*
- Beware of various subtle effect, eg primary beam, correlated (original) channels
- Appropriate statistical tests to constrain input parameters
- This can actually provide a better estimate of the noise level than the prediction given by the weights.



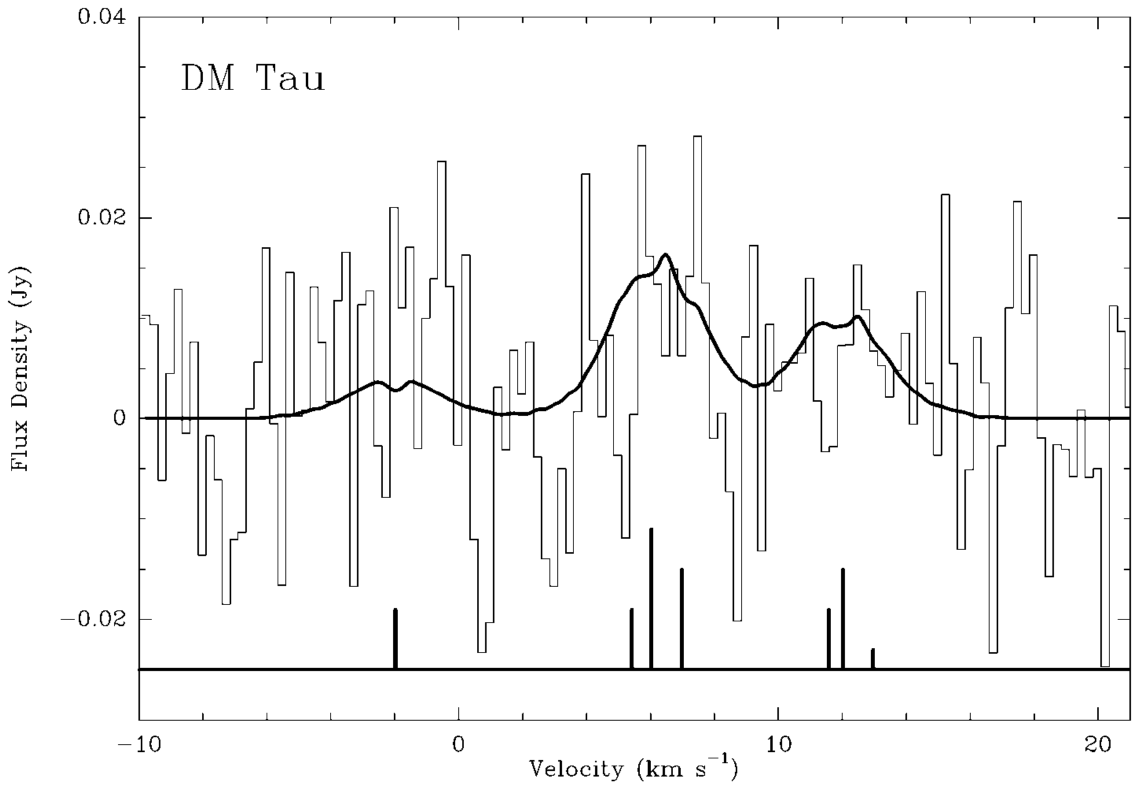
A typical data cube showing 13CO emission in a protoplanetary disk. It has quite decent S/N, and one can recognize the rotation pattern of a Keplerian disk



χ^2 analysis in the UV plane (5 disk parameters, for 8 disks)



A (really) low Signal to Noise image of the protoplanetary disk of DM Tau in the main group of hyperfine components of the N_2H^+ 1-0 transition.



Best fit integrated profile for the N_2H^+ 1-0 line, derived from a χ^2 analysis in the UV plane, using a line radiative transfer model for proto-planetary disks, assuming power law distributions, and taking into account the hyperfine structure (Dutrey et al. 2007).

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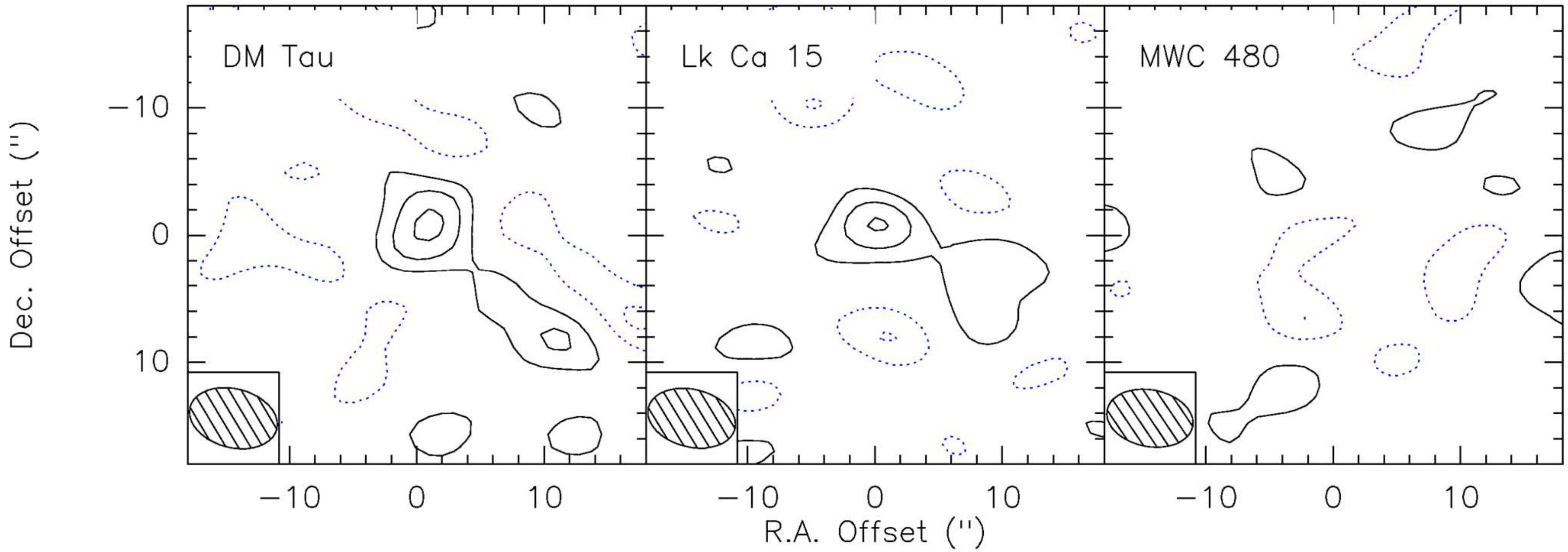

- Maps of the integrated N_2H^+ 1-0 line emission, using the best profile derived from the Â² analysis in the UV plane as a (velocity) smoothing kernel (optimal filtering).
- 7 σ detection for DM Tau, 6 σ detection for LkCa 15