A Sightseeing Tour of mm Interferometry

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10th IRAM Millimeter Interferometry School
Oct. 1 - 5 2018, Grenoble

Towards Higher Resolution:
I. Problem

Telescope resolution:

- $\sim \lambda/D$;
- IRAM-30m: $\sim 11''$ @ 1 mm.

Needs to:

- increase D;
- increase precision of telescope positionning;
- keep high surface accuracy.

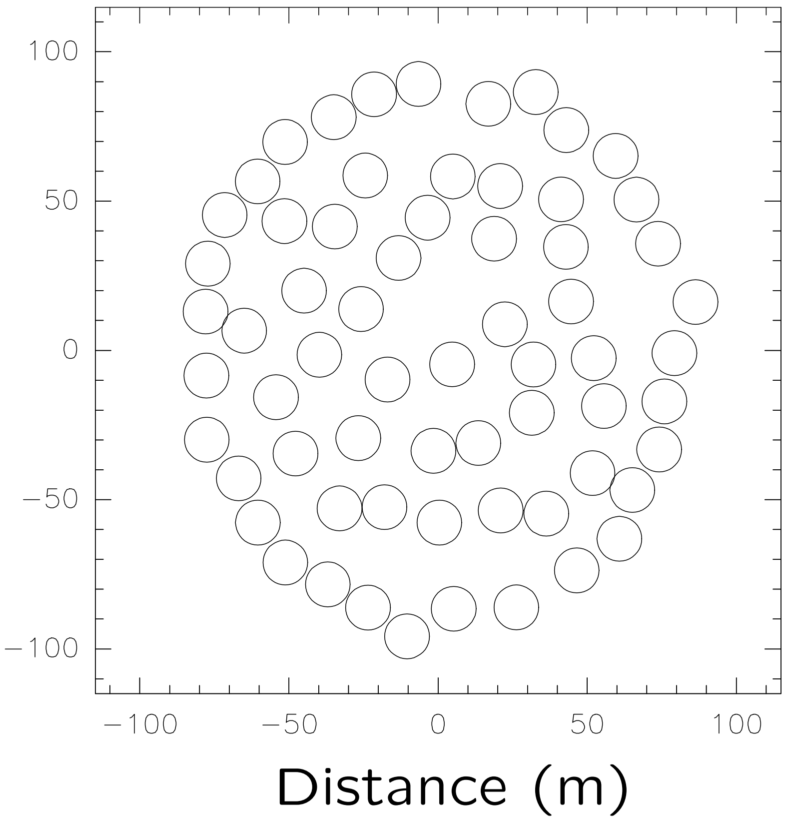
$\Rightarrow$ Technically difficult (perhaps impossible?).

Towards Higher Resolution:
II. Solution

Aperture Synthesis: Replacing a single large telescope by a collection of small telescope "filling" the large one.

$\Rightarrow$ Technically difficult but feasible.

ALMA



Vocabulary and notations:

Baseline Line segment between two antenna.

b_{ij} Baseline length between antenna i and j.

Configuration Antenna layout (*e.g.* compact configuration).

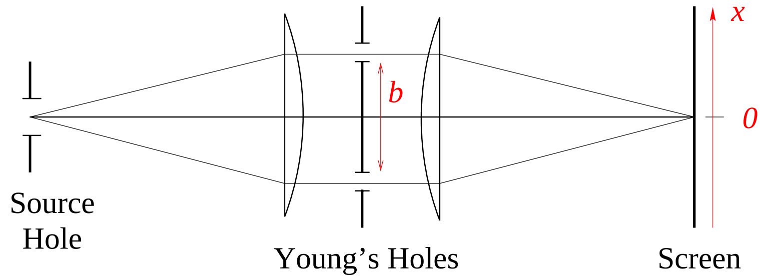
D configuration size (*e.g.* 150 m).

Primary beam resolution of one antenna (*e.g.* 27″ @ 1 mm).

Synthesized beam resolution of the array (*e.g.* 2″ @ 1 mm).

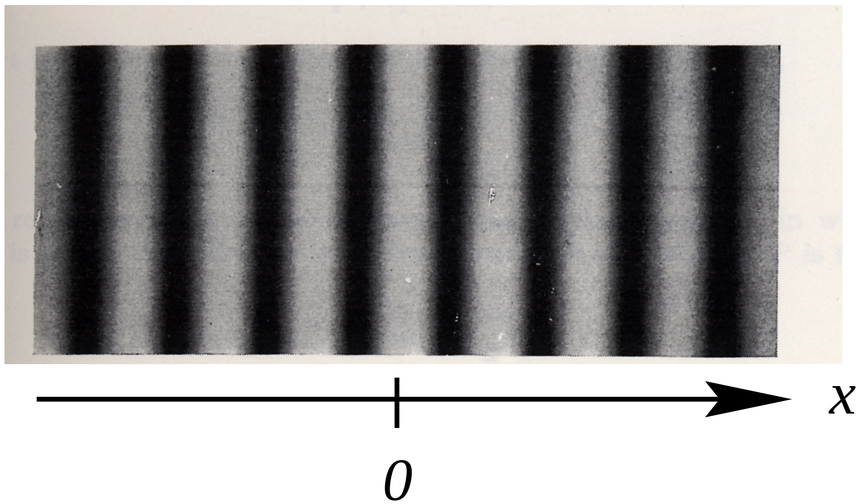
Young's Experiment

Setup



Lens $\Rightarrow$ Fraunhofer conditions (*i.e.* Plane waves as if the source were placed at infinity).

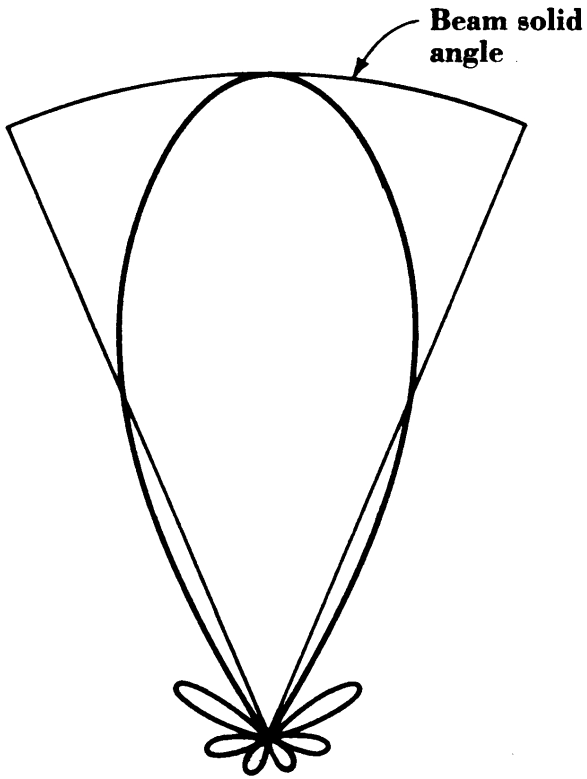
Obtained image of interference: fringes



$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{bx}{\lambda}\right)$$

with
- λ Source wavelength;
- b Distance between the two Young's holes;
- x Distance from the optical center on the screen.

Parenthesis: PSF = Diffraction Pattern = Beam Pattern



Single-Dish sensitivity
in polar coordinates.

Combination of:

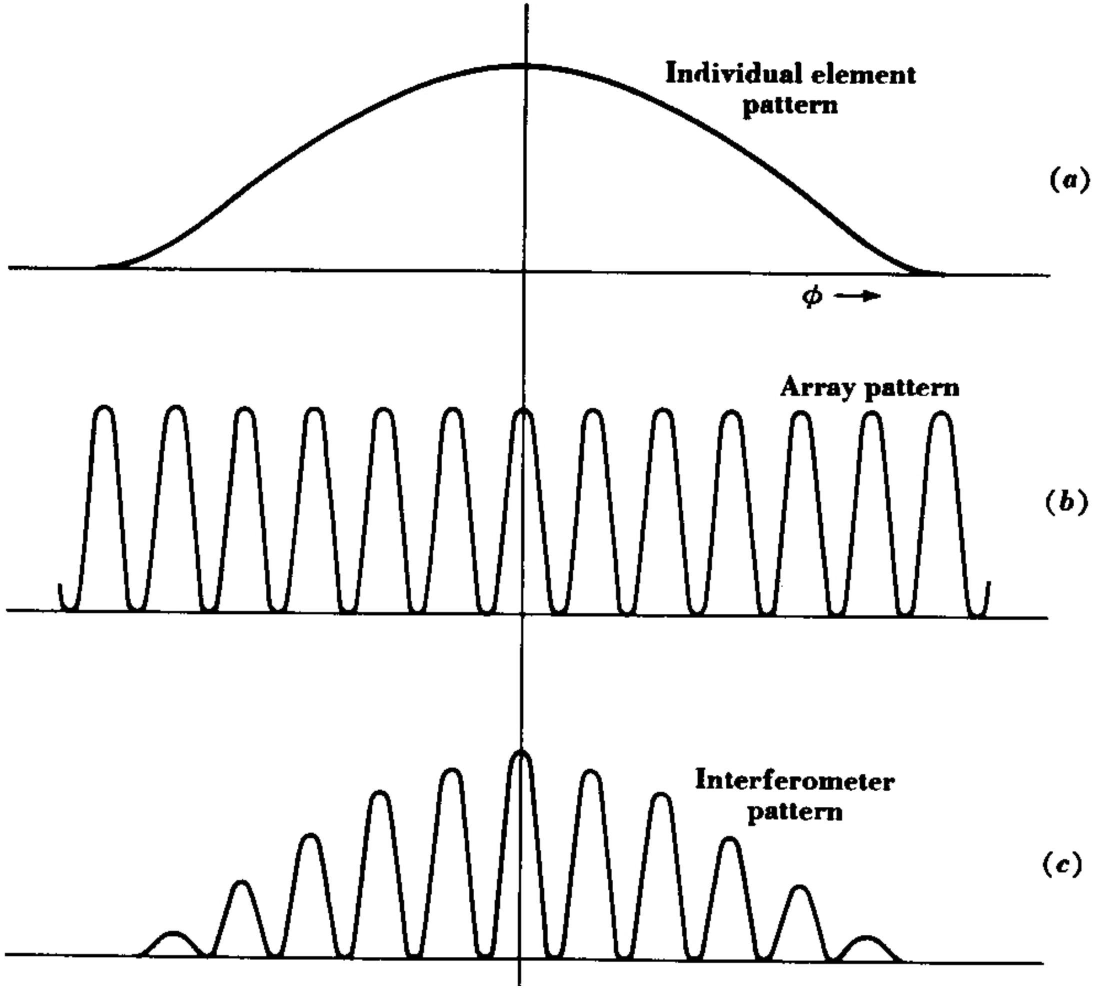
- Antenna properties;
- Optical system (*i.e.* how the waves are feeding the receiver).

Typical kind:

Optic/IR Airy function;
Radio Gaussian function.

(Lecture by C. Kramer)

Effect of the Antenna Diffraction Pattern



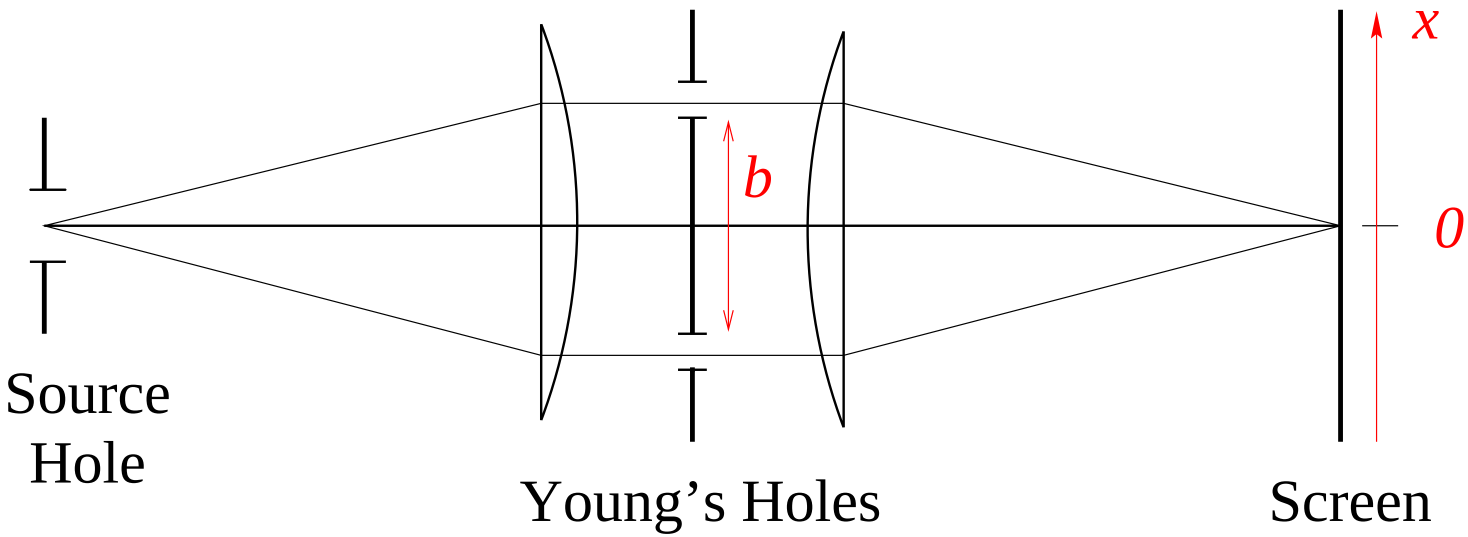
$$I(x) = B(x).\left\{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\frac{bx}{\lambda}\right)\right\}$$

Effect of the Source Hole Size: I. Description

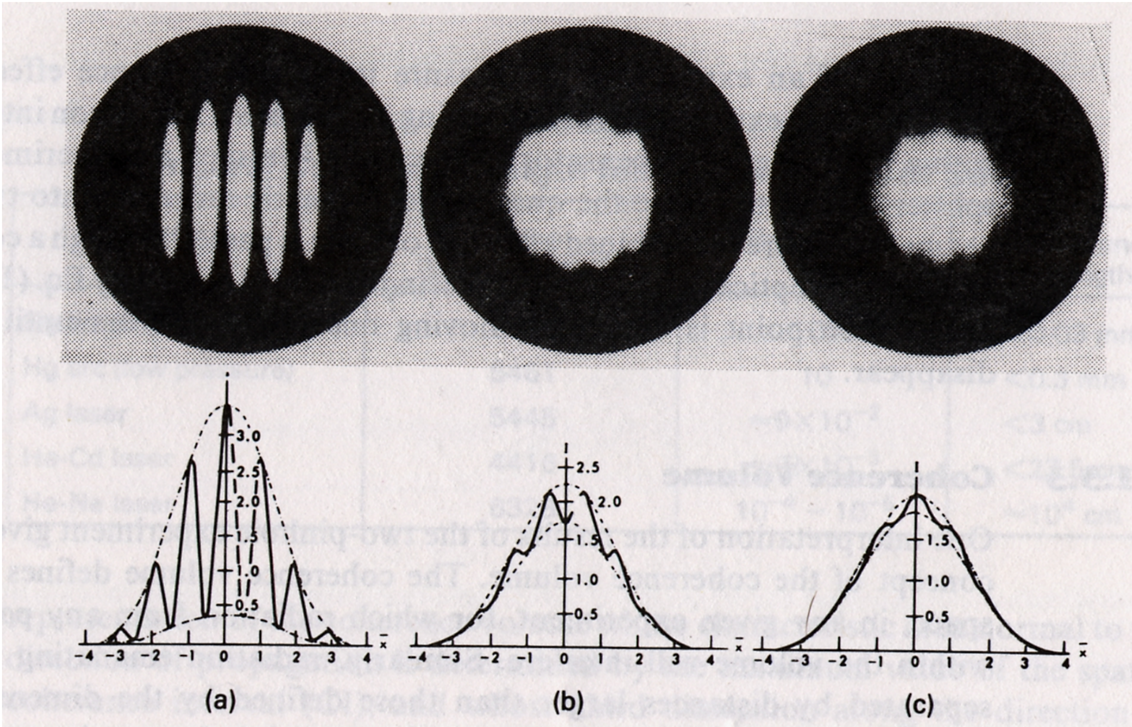
Hypothesis: Monochromatic source (but not a laser).

Description:

- The Source Hole Size is increased.
- Everything else is kept equal.



Effect of the Source Hole Size:
II. Results



Fringes disappear! $\Rightarrow$ $\begin{cases} \text{Fringe contrast is linked to the} \\ \text{spatial properties of the source.} \end{cases}$

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

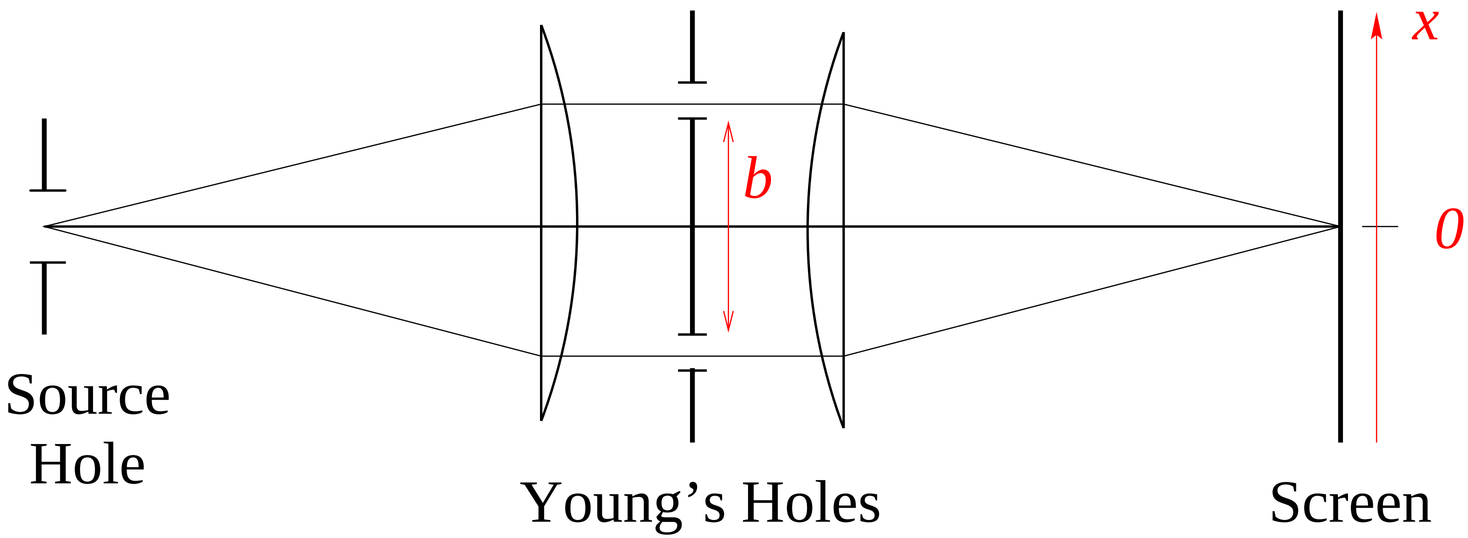
Effect of the Distance Between Young's Holes: I. Description

Hypothesis:

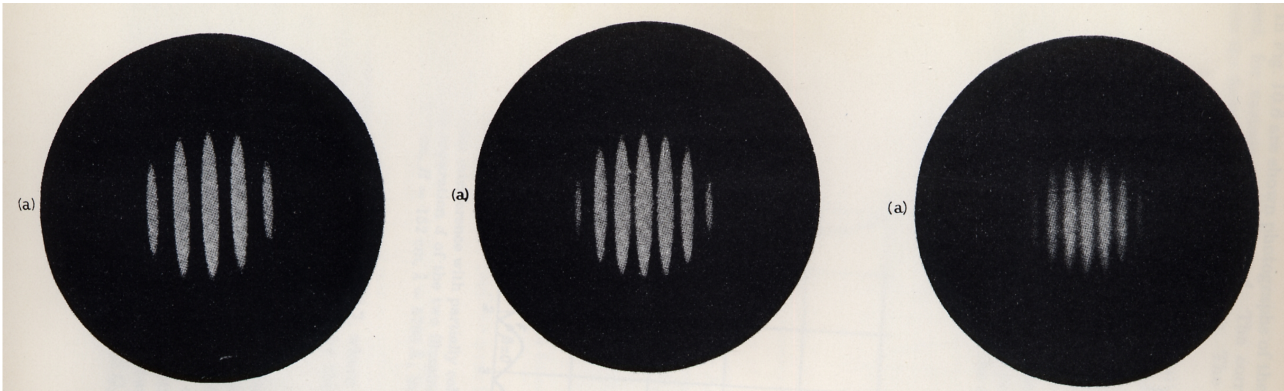
- Monochromatic source (but not a laser).
- The source hole is a circular disk.

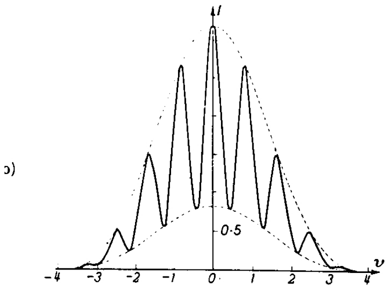
Description:

- The distance between the two Young's holes is increased.
- Everything else is kept equal (in particular the hole size).

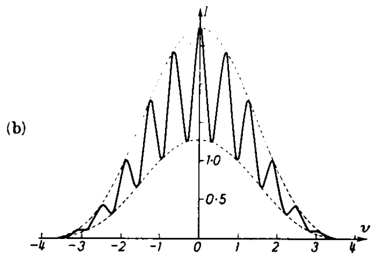


Effect of the Distance Between Young's Holes: II. Results

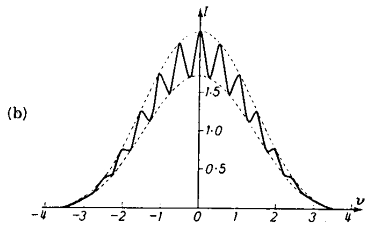




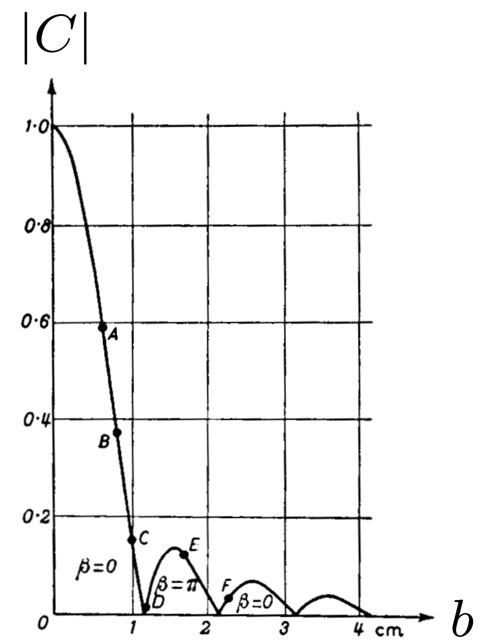
(A): $d = 0{\cdot}6$ cm
$|\mu_{12}| = 0{\cdot}593$, $\beta_{12} = 0$



(B): $d = 0{\cdot}8$ cm
$|\mu_{12}| = 0{\cdot}361$, $\beta_{12} = 0$

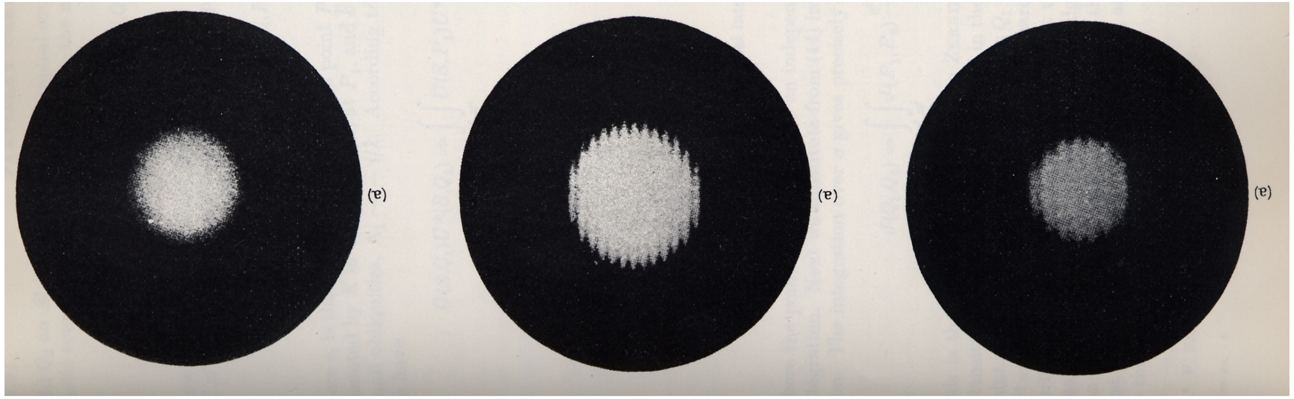


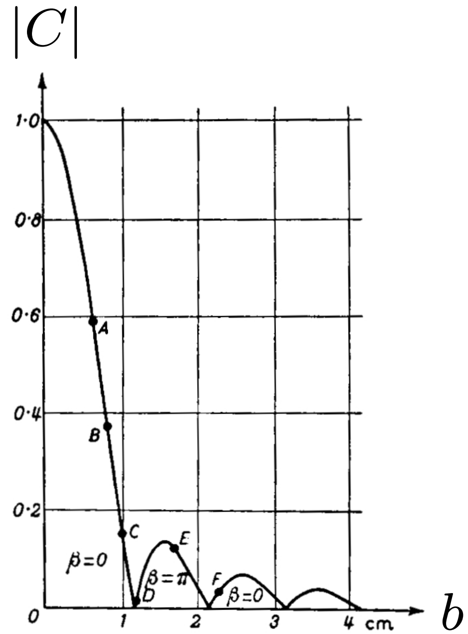
(C): $d = 1$ cm
$|\mu_{12}| = 0{\cdot}146$, $\beta_{12} = 0$

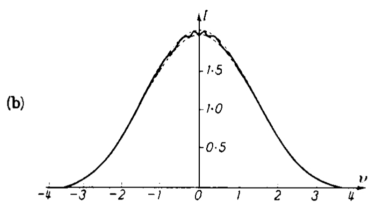


$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

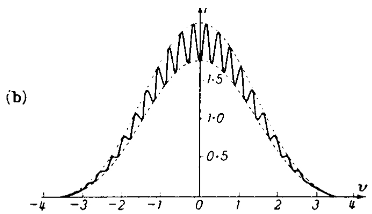
Effect of the Distance Between Young's Holes: II. Results (Continued)



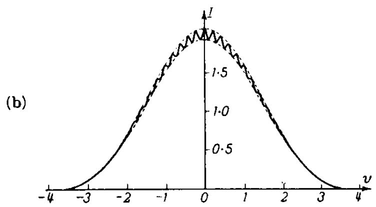




(D): d = 1·2 cm
$|\mu_{12}|$ = 0·015, $\beta_{12} = \pi$



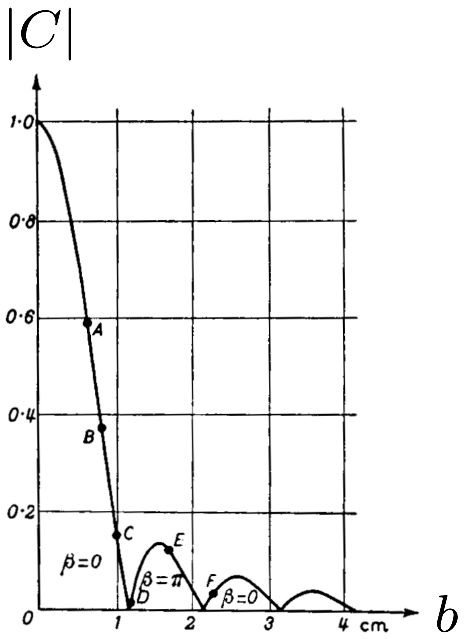
(E): d = 1·7 cm
$|\mu_{12}|$ = 0·123, $\beta_{12} = \pi$



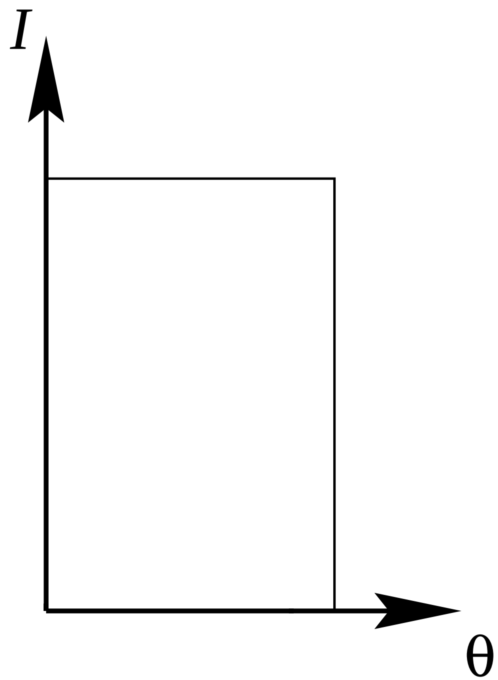
(F): d = 2·3 cm
$|\mu_{12}|$ = 0·035, $\beta_{12} = 0$

$$I(x) = I_1 + I_2 + 2\sqrt{I_1 I_2}|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right) \text{ with } |C| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

Measured Curve = 2D Fourier Transform of the Source



$$\frac{J_1(b)}{b} \overset{\text{2D FT}}{\rightleftharpoons} \text{Heaviside}(\theta)$$



Source = Uniformly illuminated disk.

Theoretical Basis of the Aperture Synthesis

The van Citter-Zernike theorem

$$V_{ij}(b_{ij}) = C_{ij}(b_{ij}).I_{\text{tot}} \overset{\text{2D FT}}{\rightleftharpoons} B_{\text{primary}}.I_{\text{source}}$$

- Young's holes = Telescopes;
- Signal received by telescopes are combined by pairs;
- Fringe visibilities are measured.

⇒ One Fourier component of the source (*i.e.* one visibility) is measured by baseline (or antenna pair).

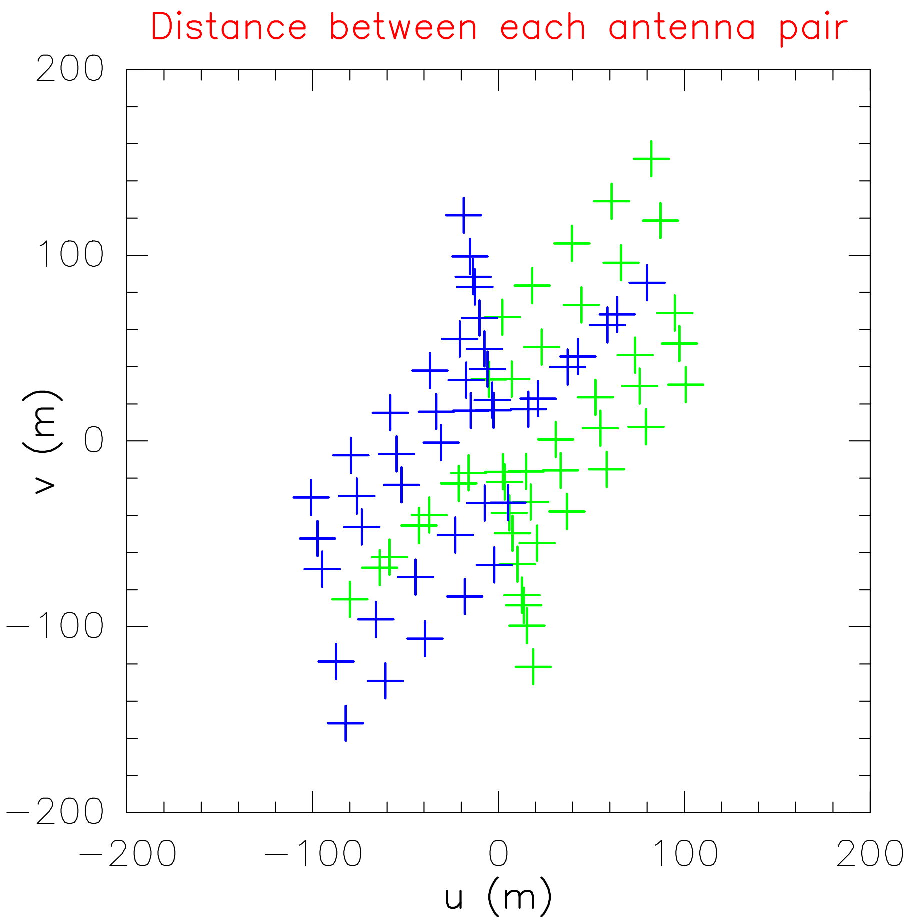
⇒ Each baseline lenght b_{ij} = a spatial frequency.

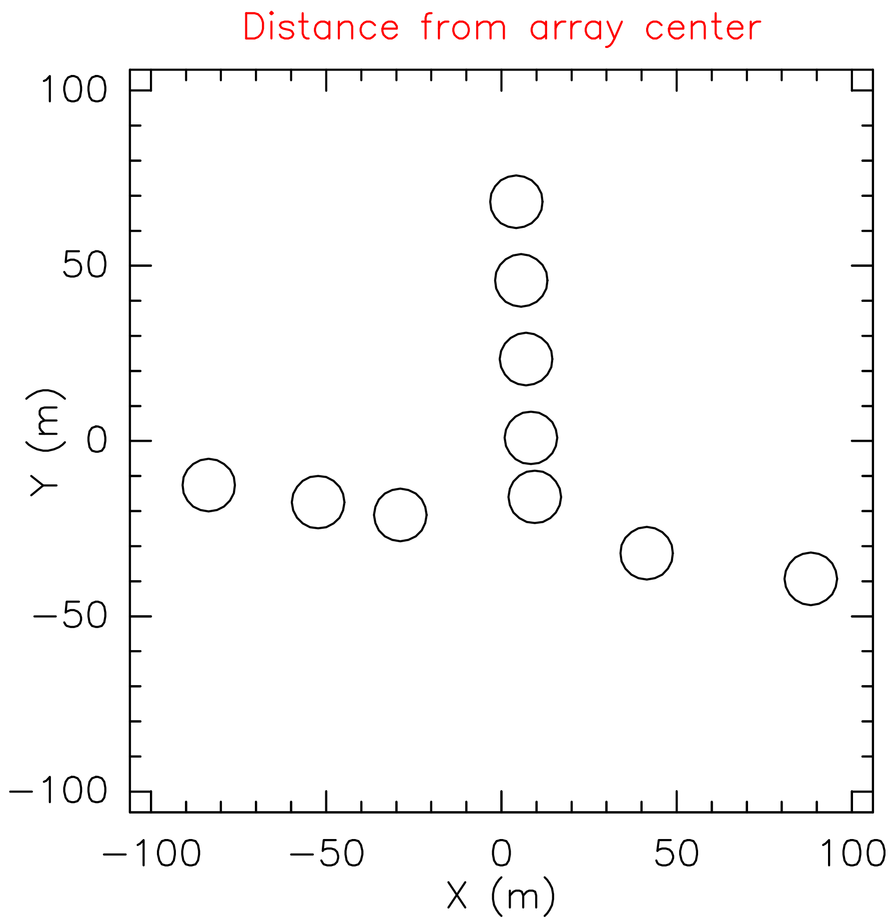
⇒ Convention: Spatial frequencies are measured in meter.

An Example: NOEMA in 2018

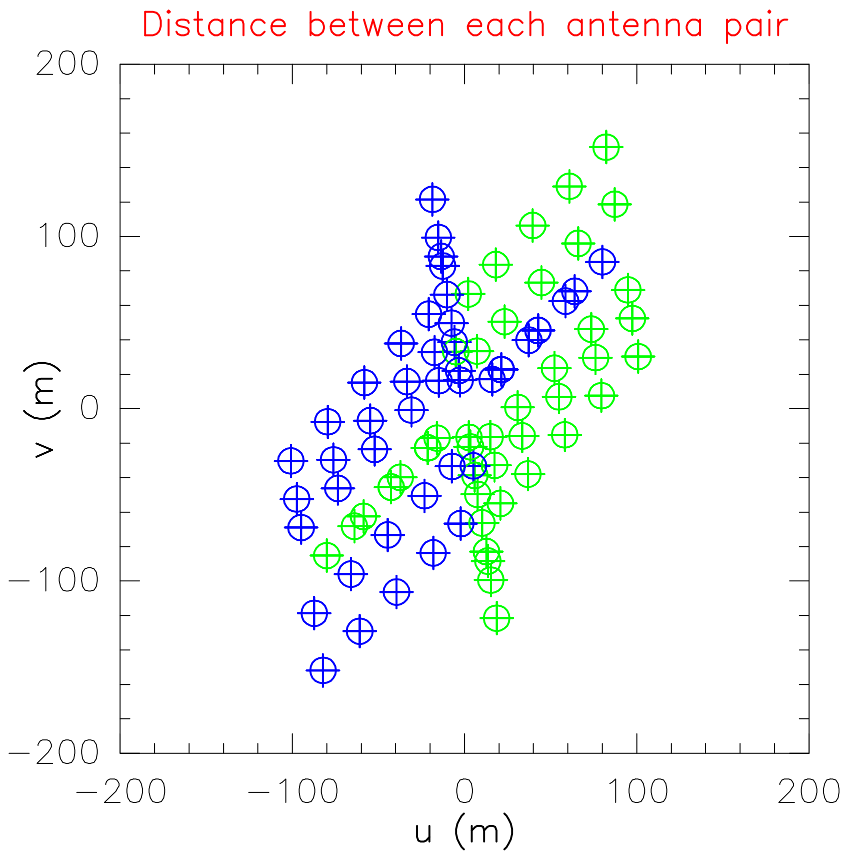
Number of baselines: $N(N-1) = 90$ for $N = 10$ antennas.

Convention: Fourier plane $=$ uv plane.





Each Visibility is a Weighted Sum of the Fourier Components of the Source



$$V_{ij}(b_{ij}) \overset{\text{2D FT}}{\rightleftharpoons} B_{\text{primary}} . I_{\text{source}}$$

i.e. $V_{ij}(b_{ij}) = \left\{ \tilde{B}_{\text{primary}} * \tilde{I}_{\text{source}} \right\} (b_{ij})$

with $\tilde{B}_{\text{primary}}$ a Gaussian of FWHM=15 m.

$\Rightarrow$ Indirect information on the source (important for mosaicing).

Mathematical Properties of Fourier Transform

1 Fourier Transform of a product of two functions
= convolution of the Fourier Transform of the functions:

$$\text{If } (F_1 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} \text{and} F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_2}), \text{ then } F_1.F_2 \overset{\text{FT}}{\rightleftharpoons} \tilde{F_1} * \tilde{F_2}.$$

2 Sampling size $\overset{\text{FT}}{\rightleftharpoons}$ Image size.

3 Bandwidth size $\overset{\text{FT}}{\rightleftharpoons}$ Pixel size.

4 Finite support $\overset{\text{FT}}{\rightleftharpoons}$ Infinite support.

5 Fourier transform evaluated at zero spacial frequency
= Integral of your function.

$$V(u=0, v=0) \overset{\text{FT}}{\rightleftharpoons} \sum_{ij \in \text{image}} I_{ij}.$$

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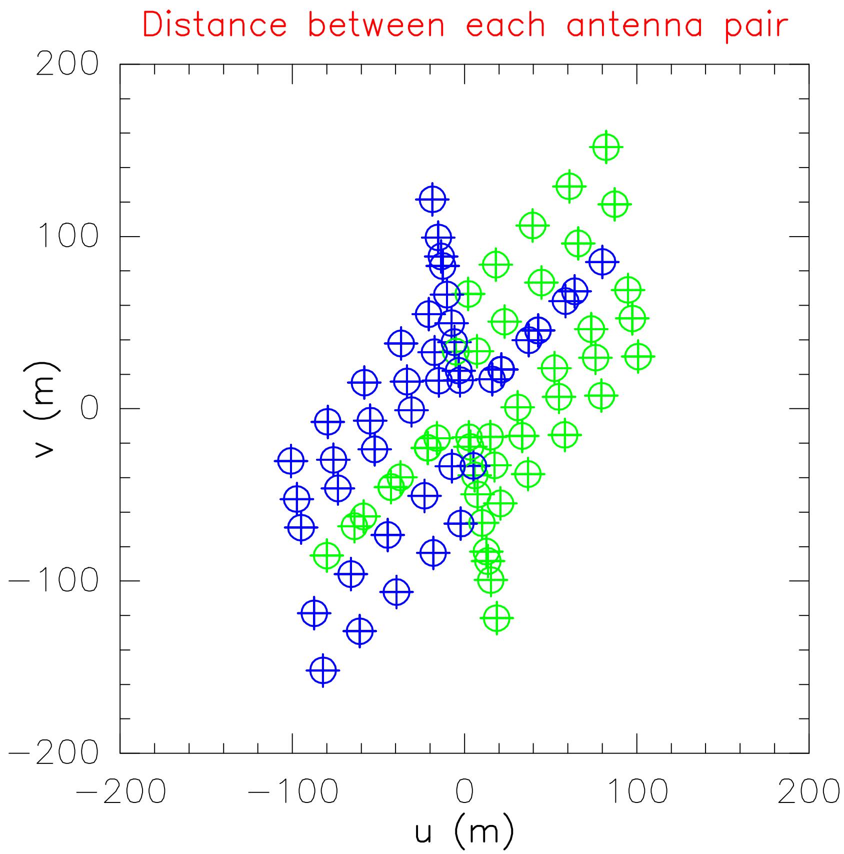

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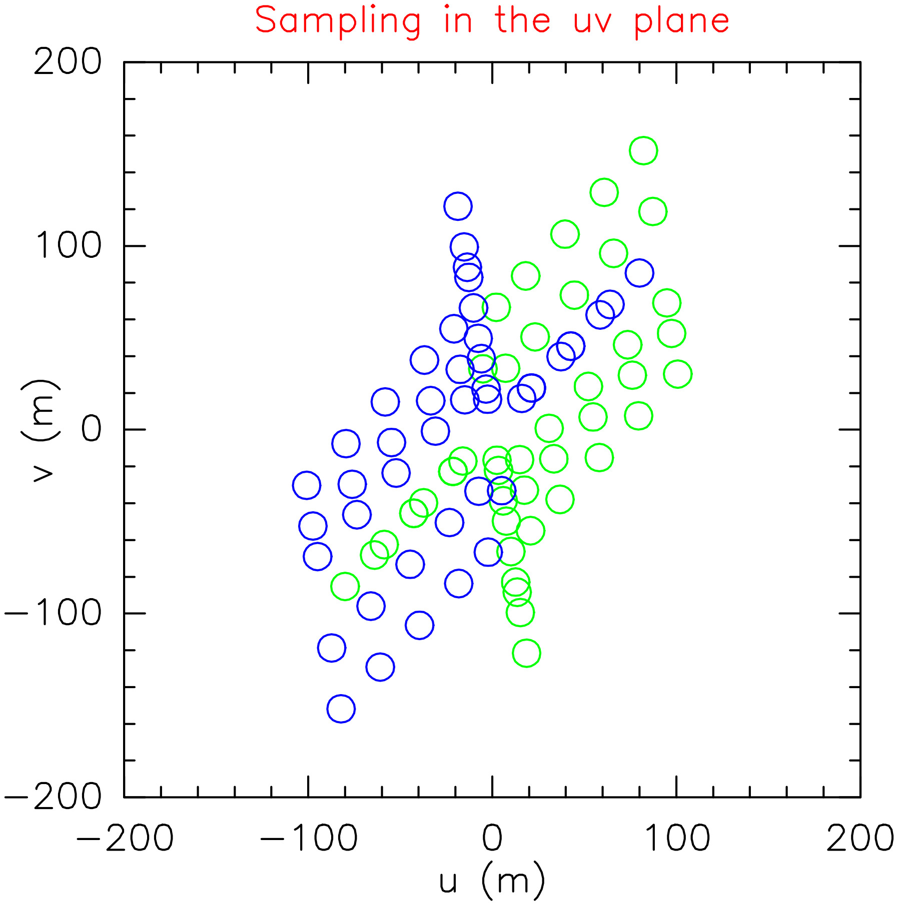
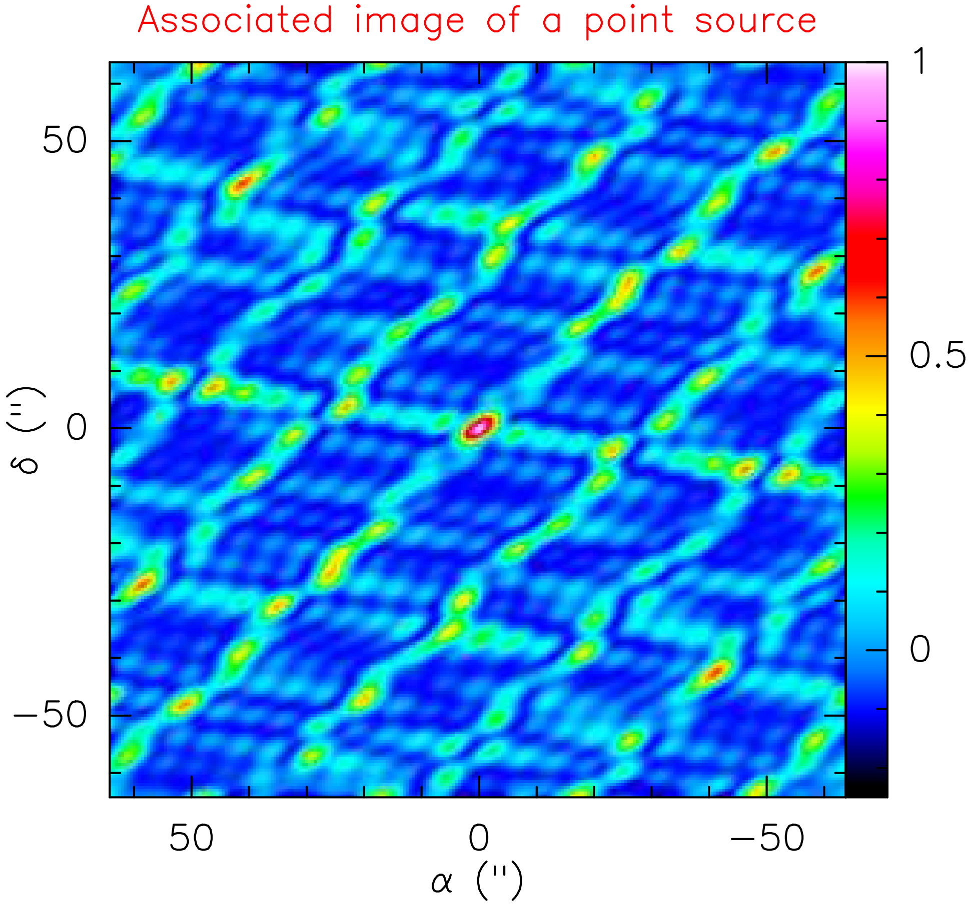
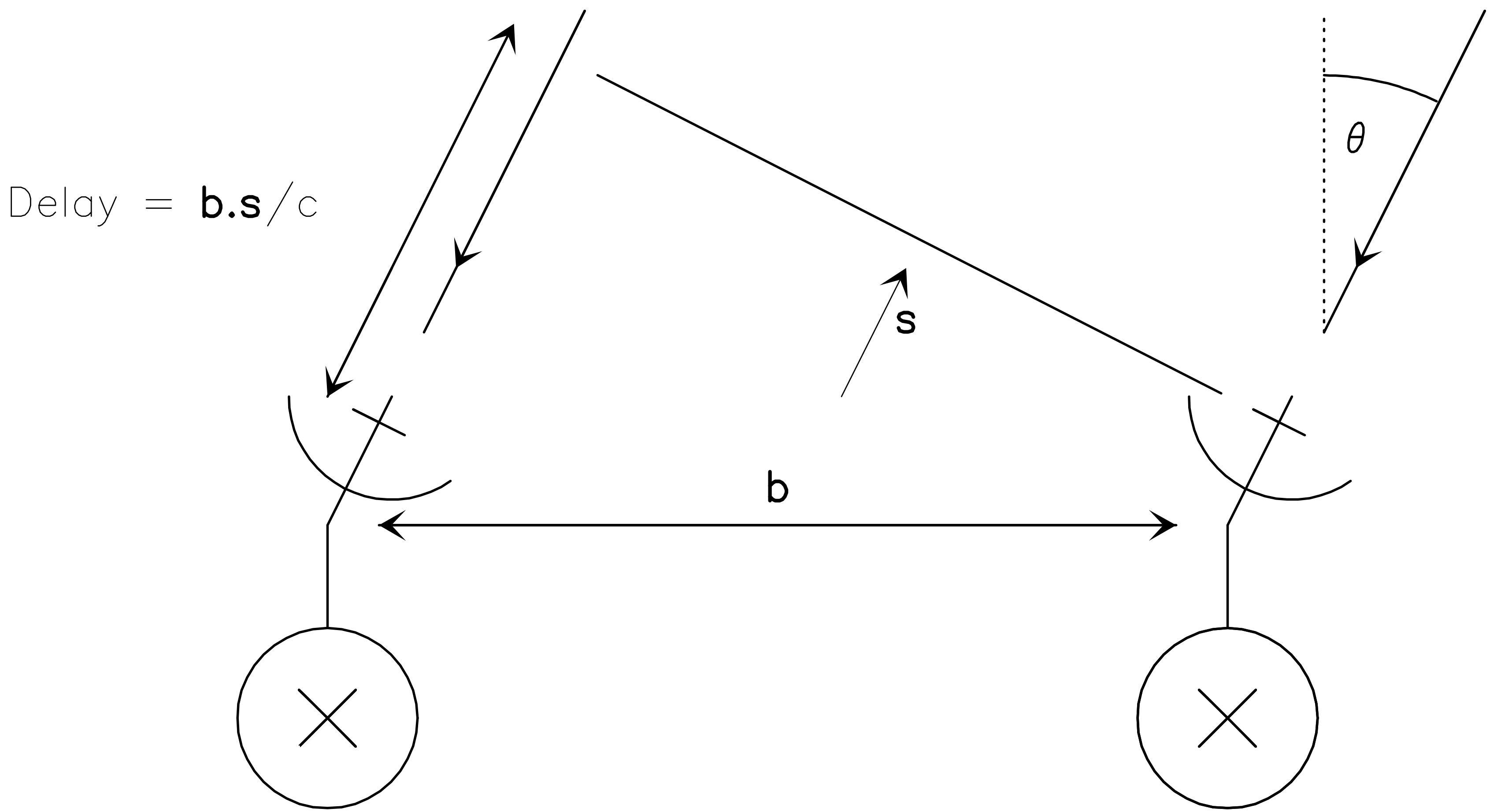




Incomplete uv plane coverage ⇒ difficult to make a reliable image (Lectures by C. Herrera, and J. Pety).

Earth Rotation and Super Synthesis

Precision: Spatial frequencies = baseline lengths projected onto a plane perpendicular to the source mean direction.



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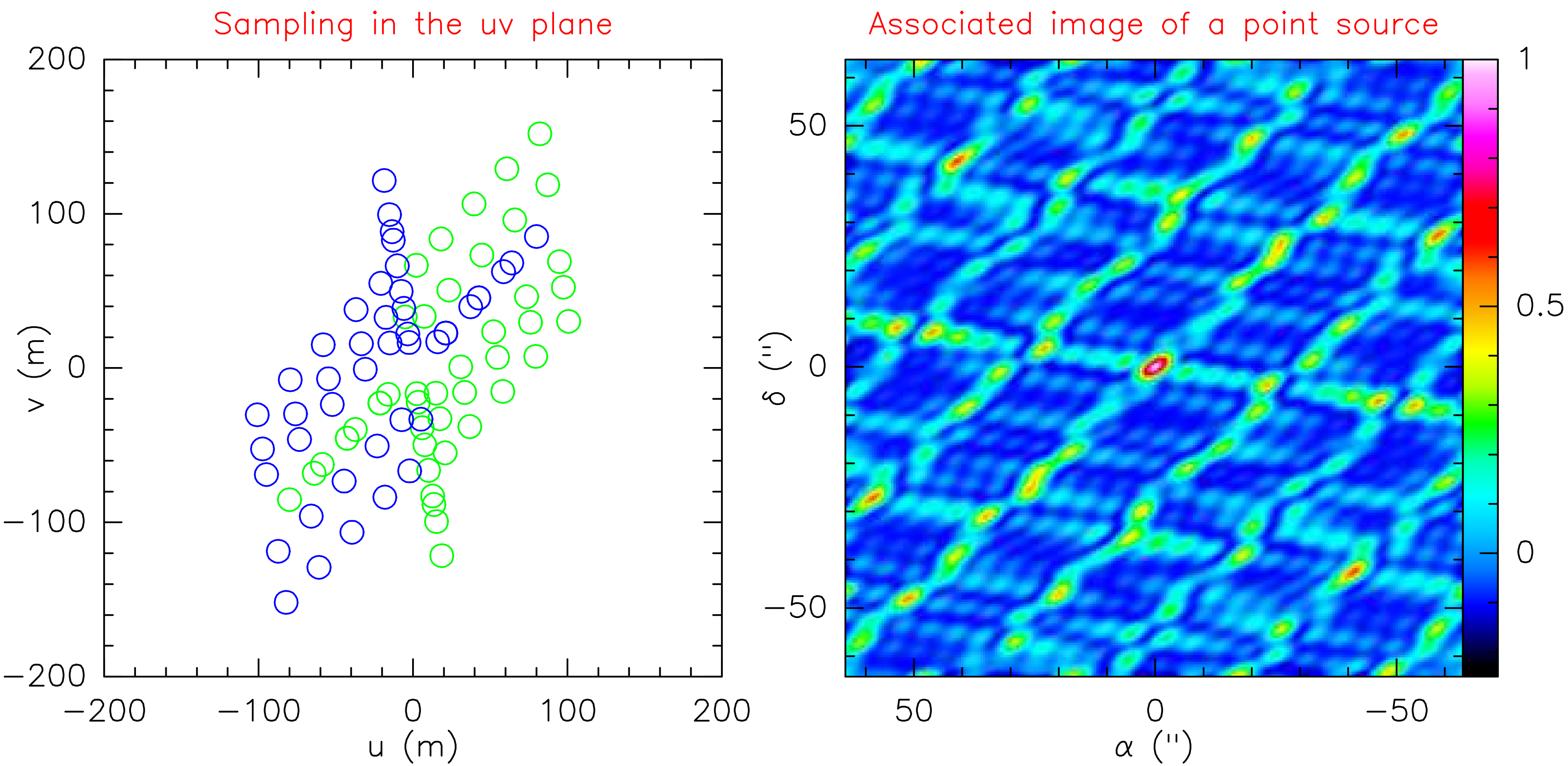
Advantage: Possibility to measure different Fourier components without moving antennas!



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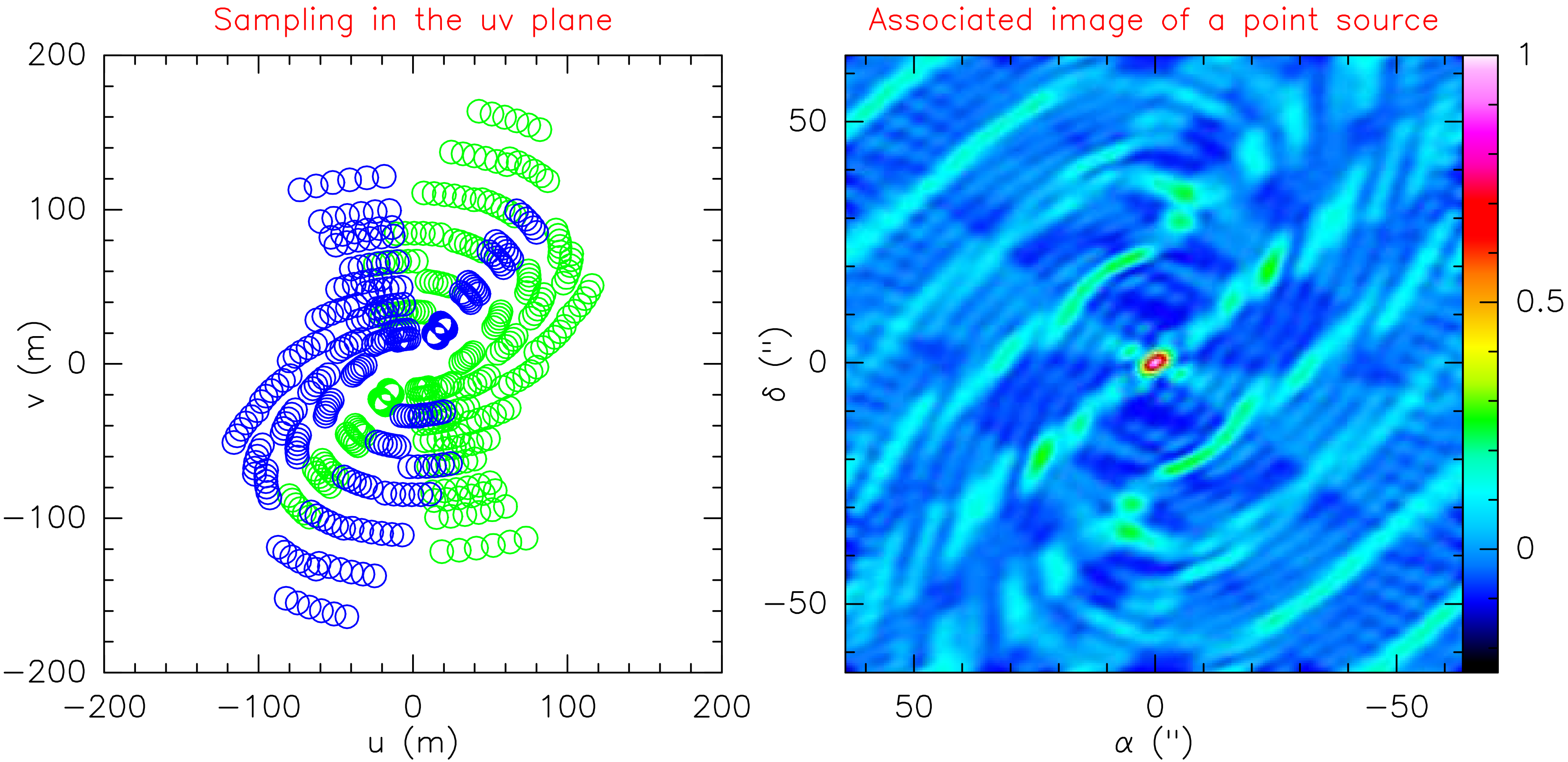

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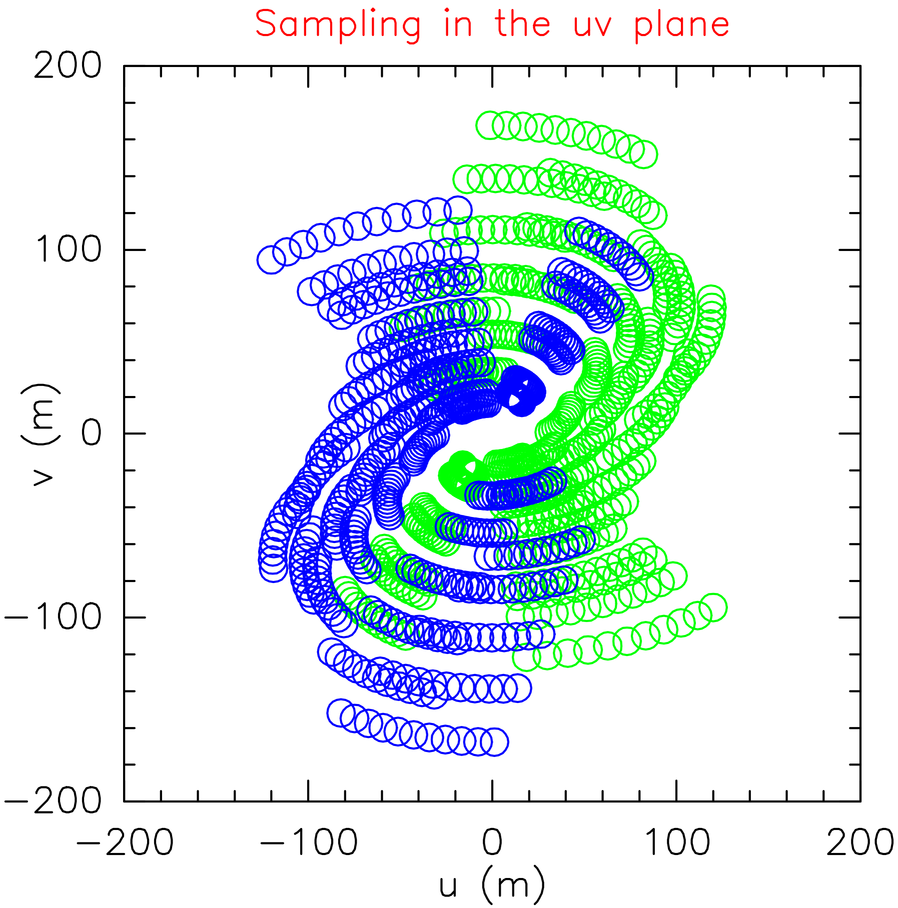

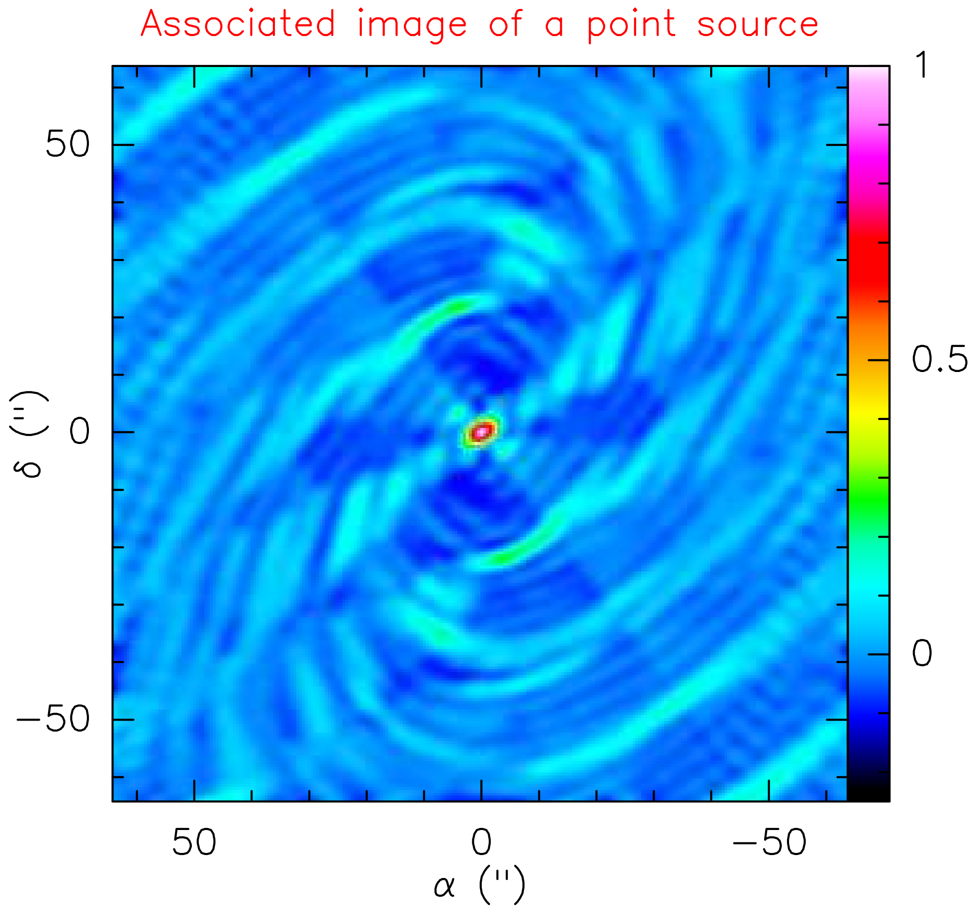

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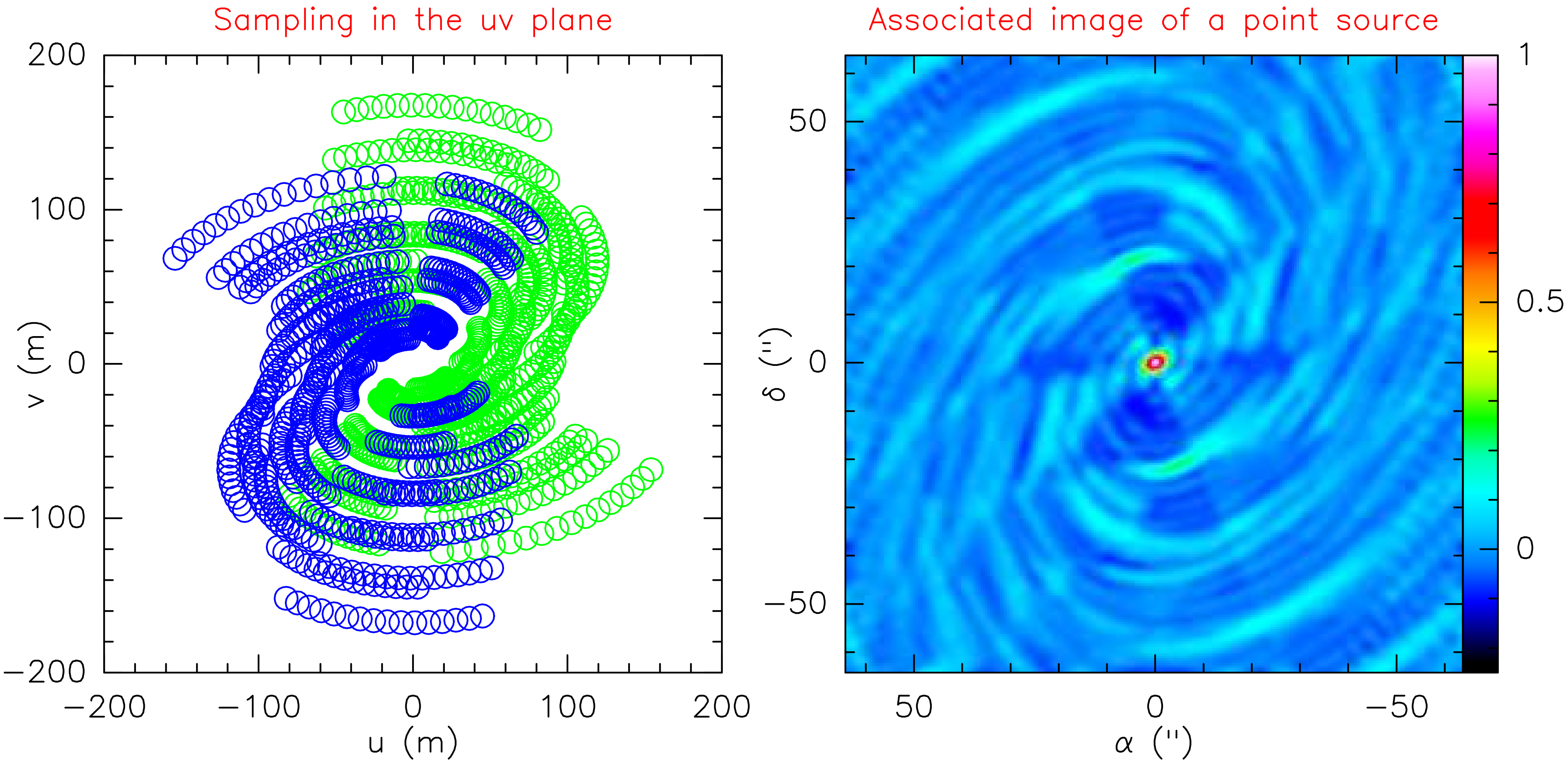

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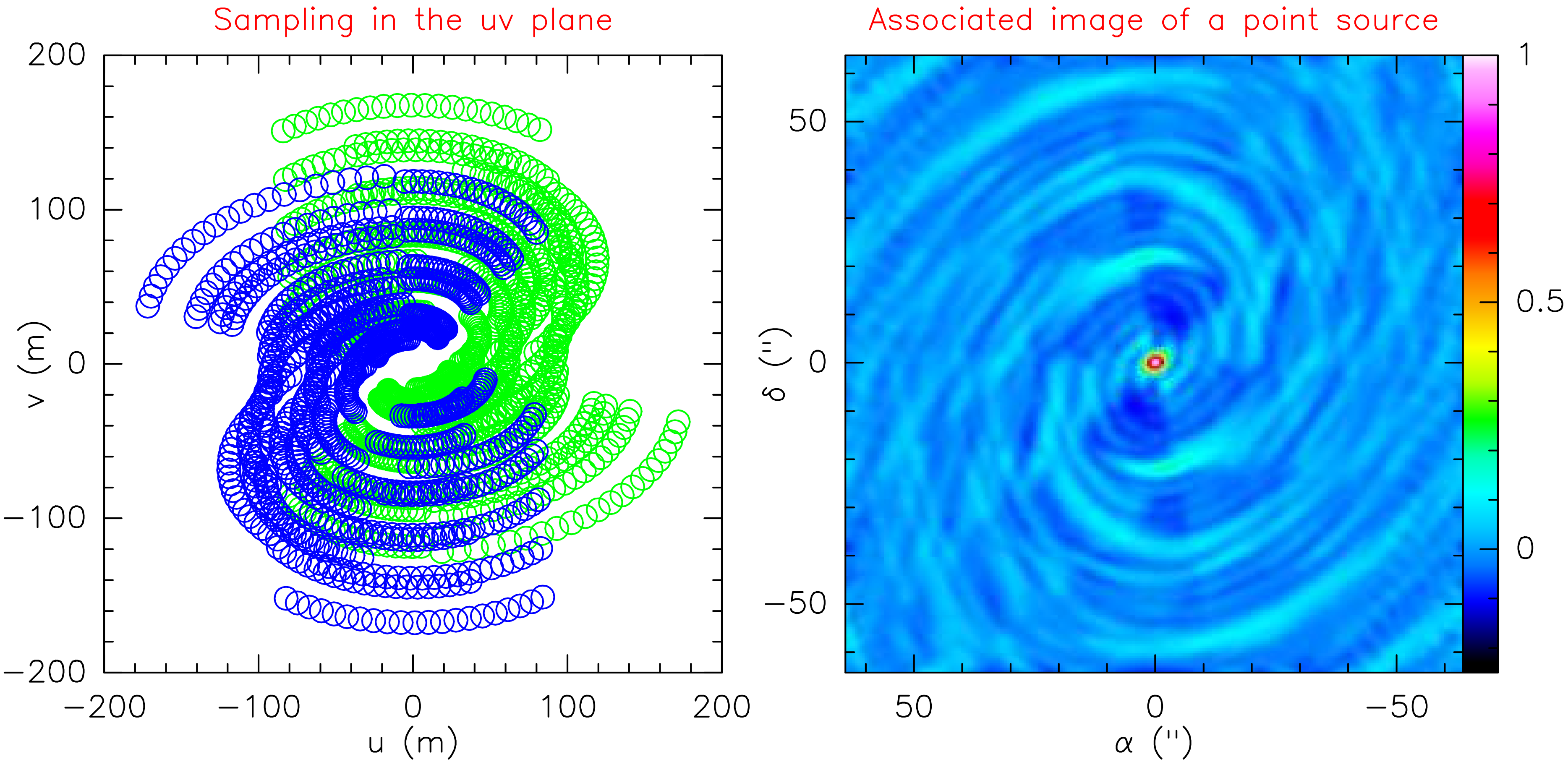

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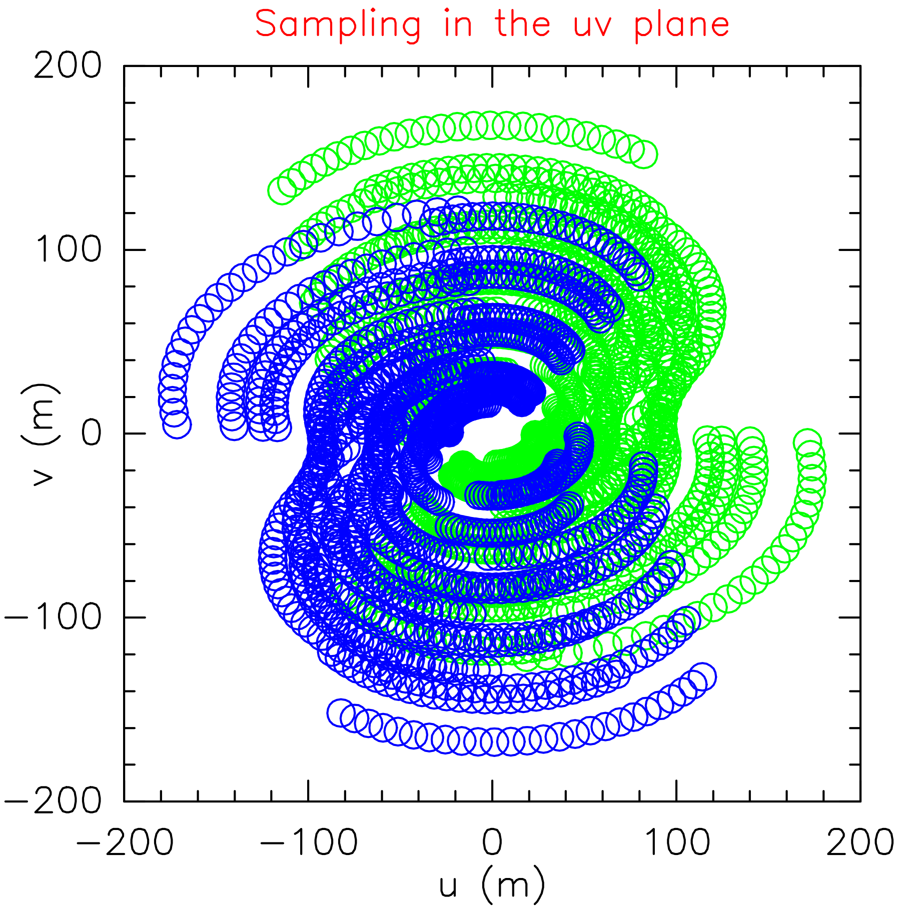

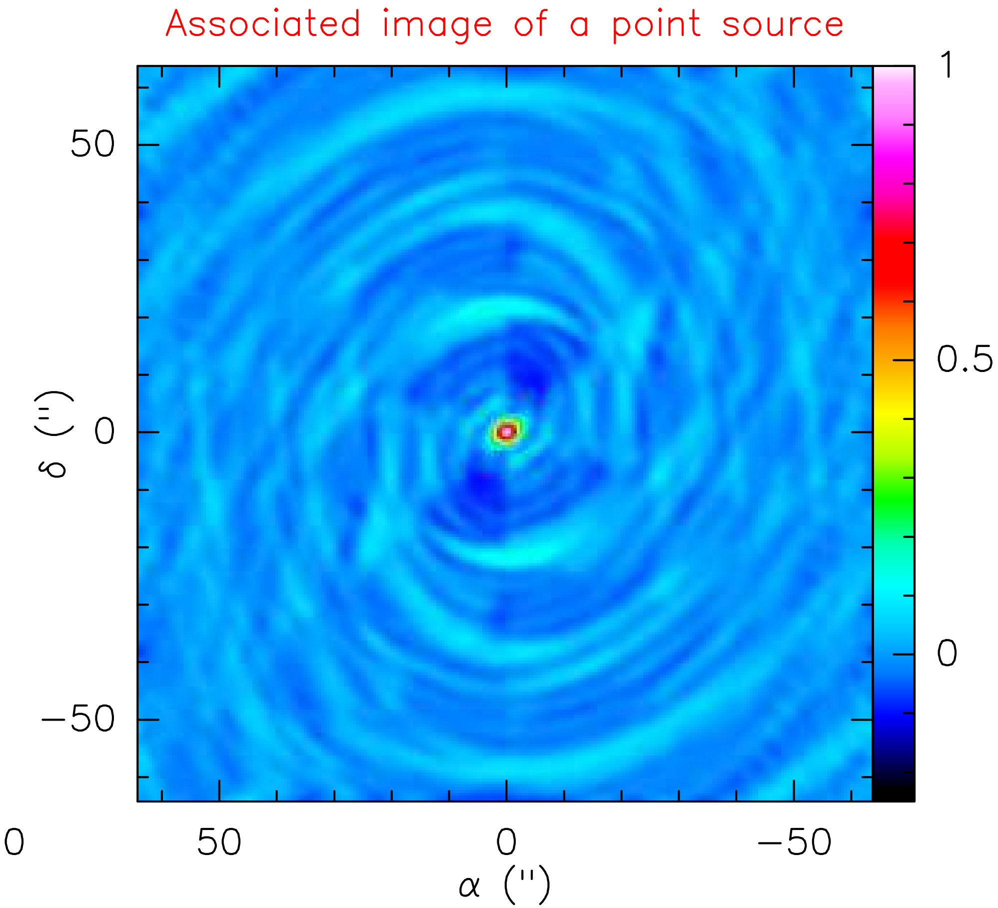

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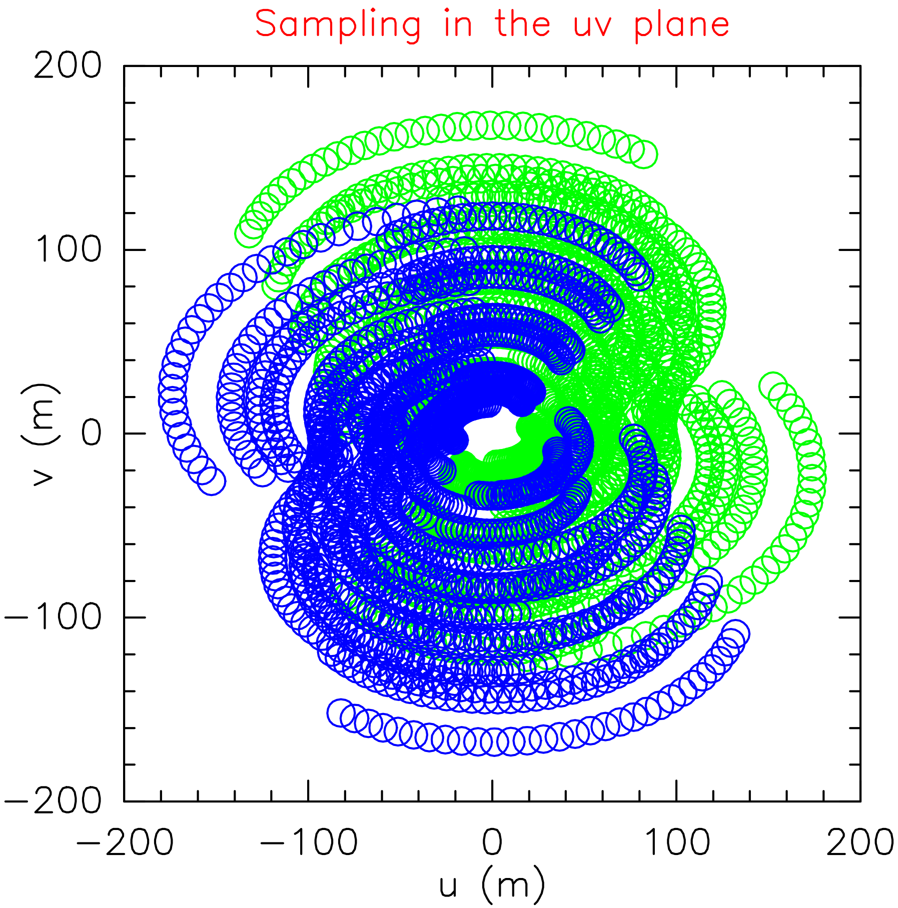

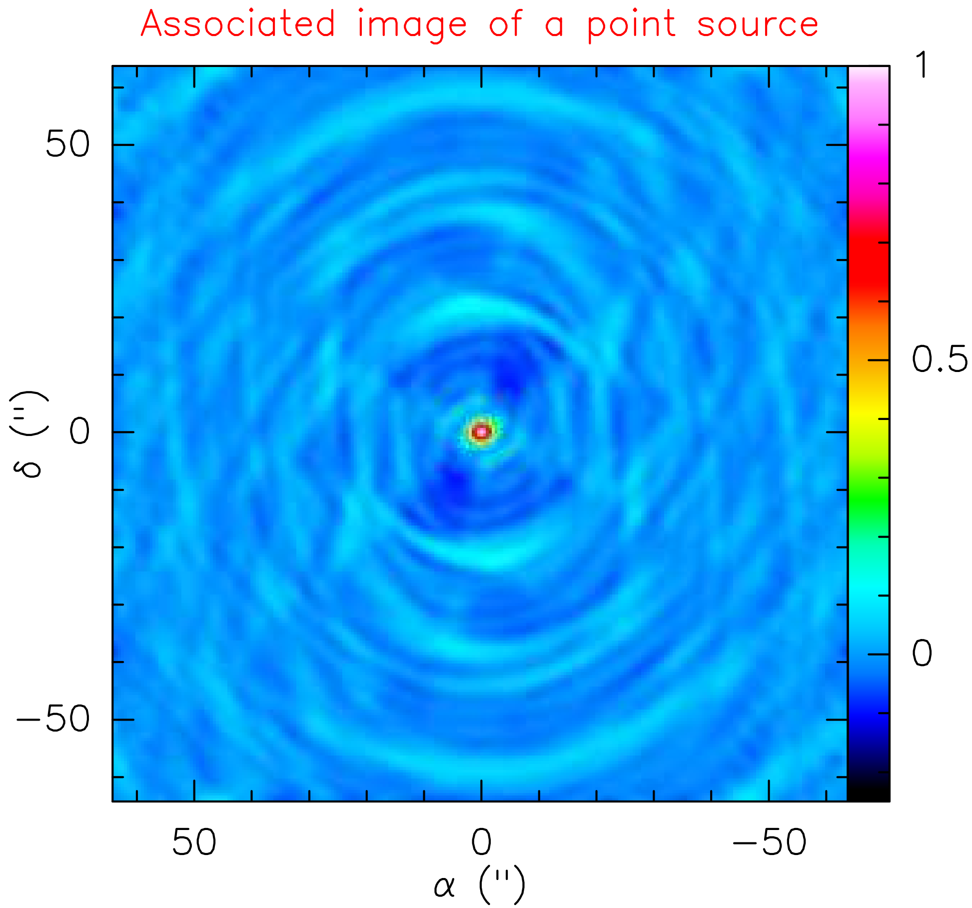

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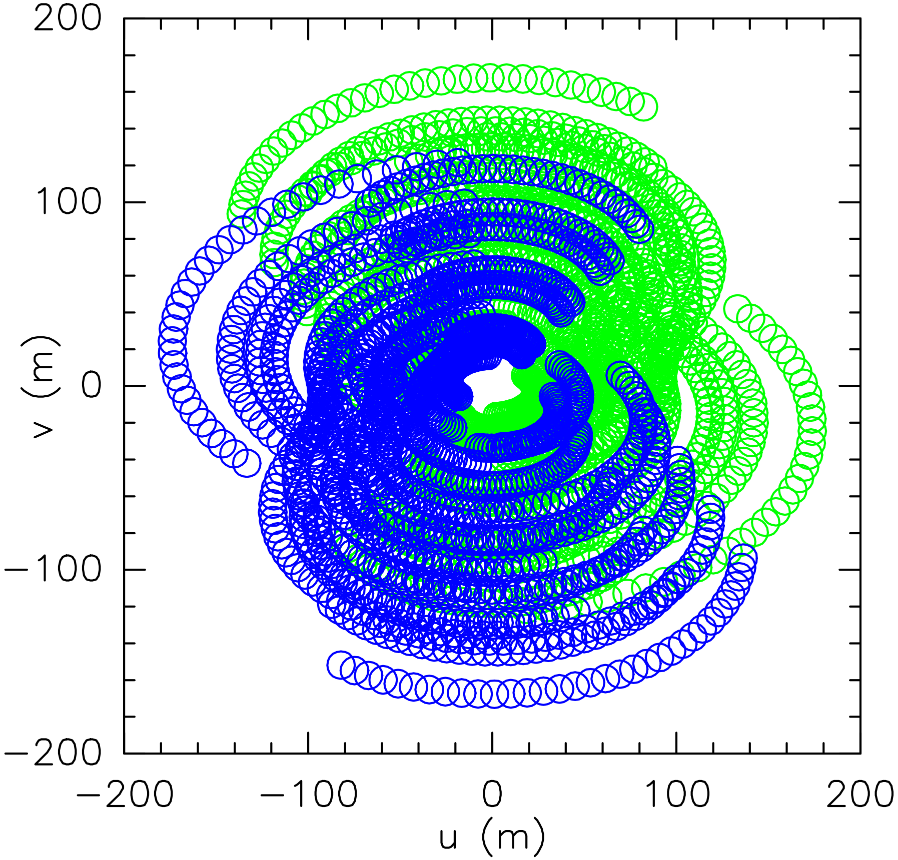
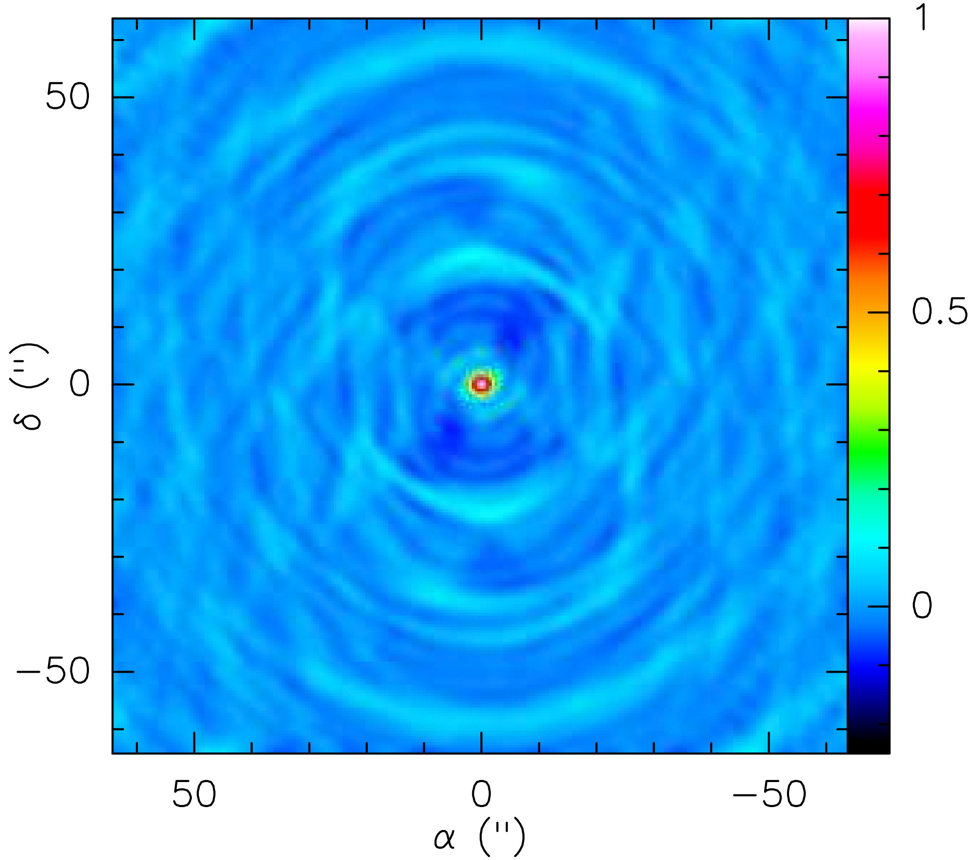
Sampling in the uv plane



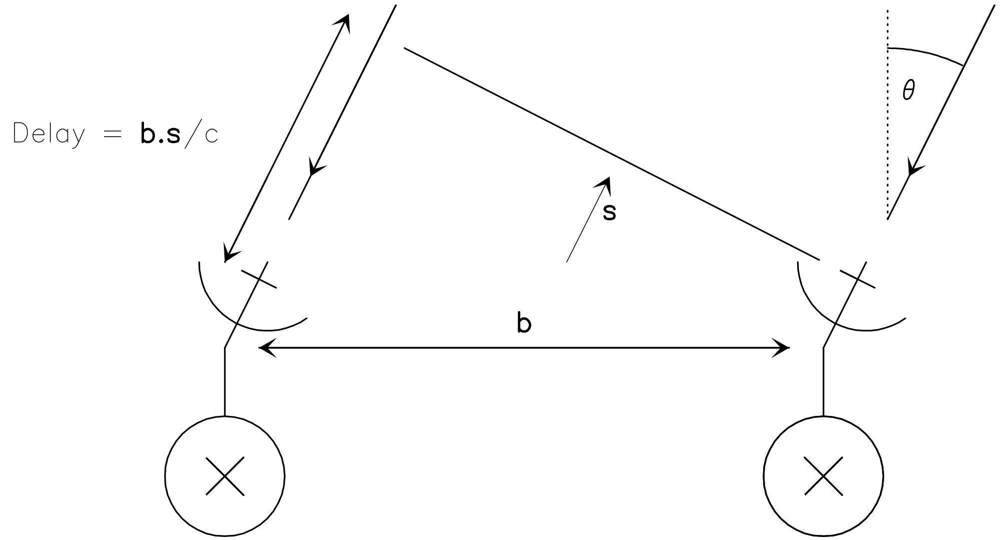
Associated image of a point source



Delay Correction: I. Why?

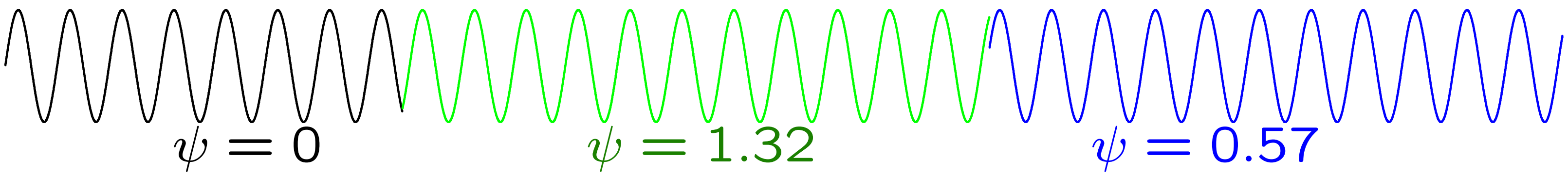
Real life: Source not at zenith.

$\Rightarrow$ { Wave plane arrives at different moment on each antenna.



Temporal coherence:

- $E(t) = E_0 \cos(\omega t + \psi)$
- Temporally Incoherent Source = random phase changes.
- Coherence time: mean time over which wave phase = constant.



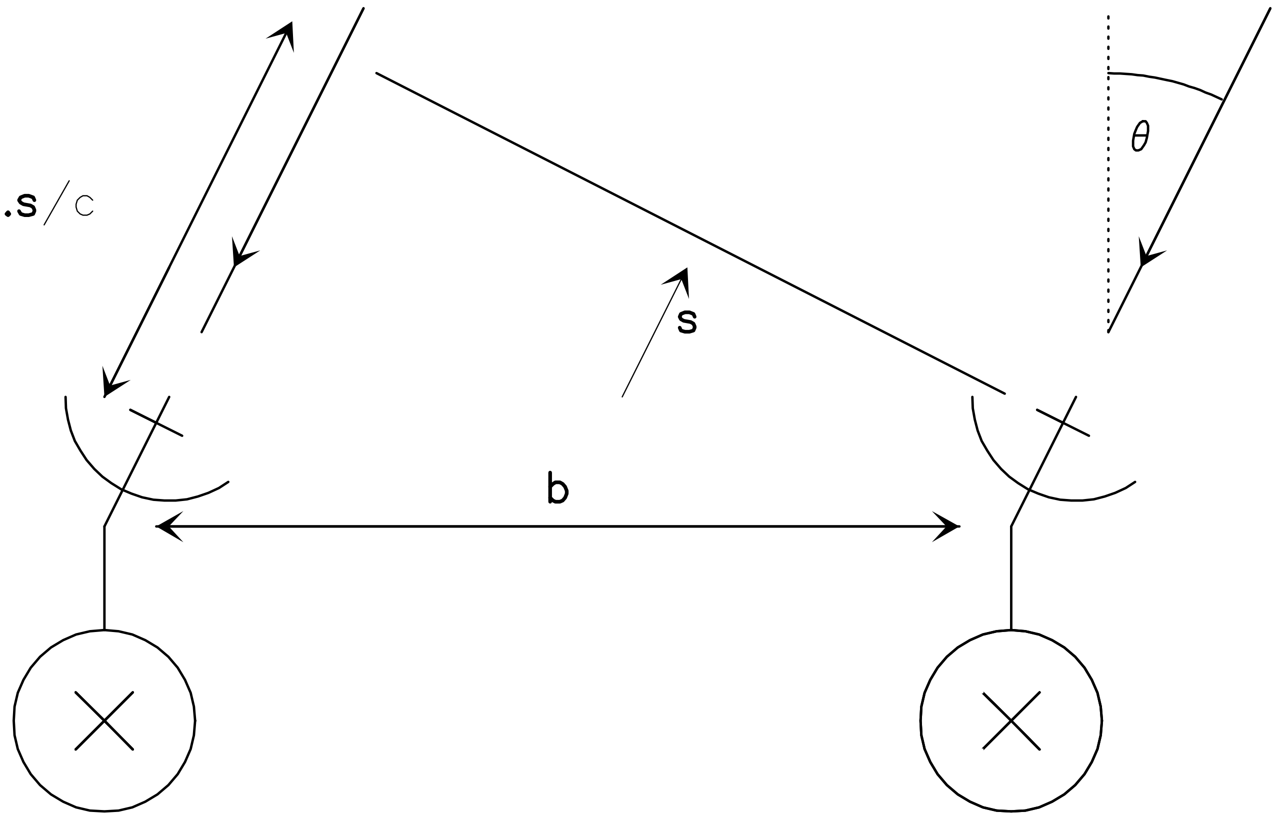
Problem: (Coherence time $\lesssim$ delay) $\Rightarrow$ fringes disappear!

Delay Correction: II. Earth rotation

Earth rotation:

- Advantage: Super synthesis;
- Inconvenient: Delay correction varies with time!

Delay = **b**.**s**/c

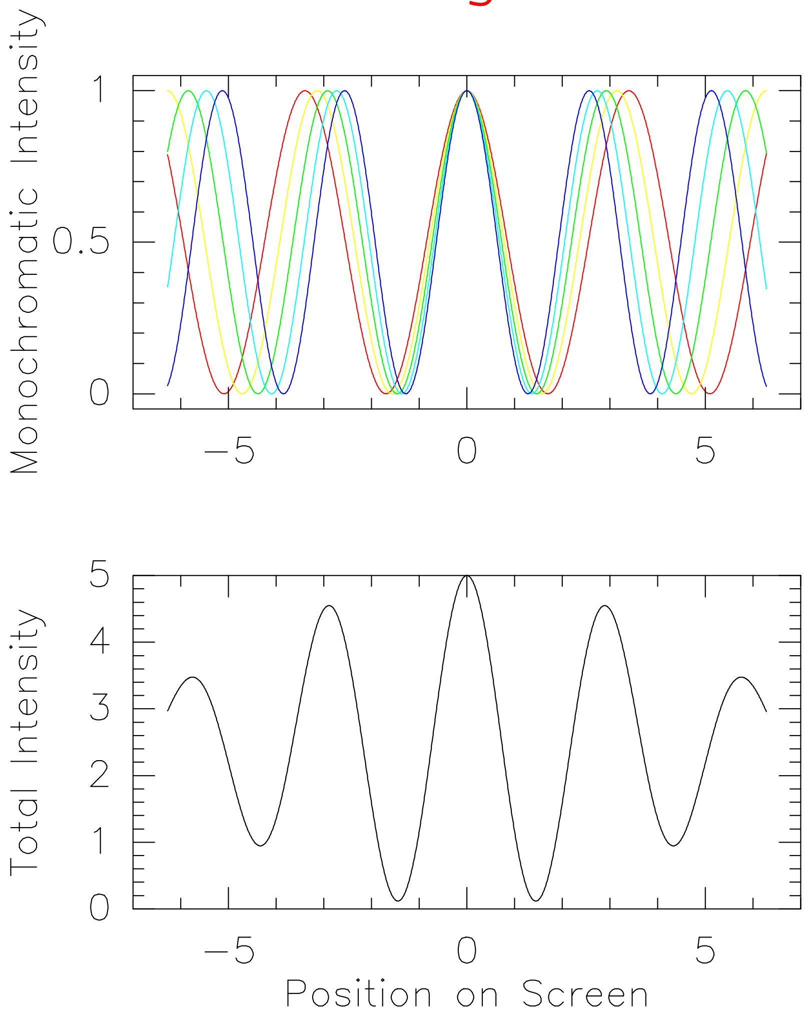


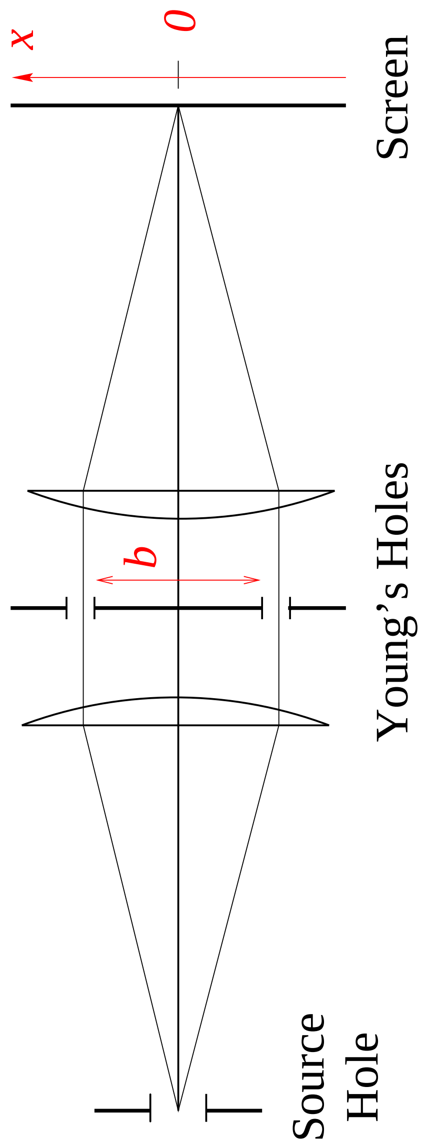
Delay Correction: III. Finite Bandwidth

Real life: Observation of finite bandwidth ⇒ polychromatic light.

Perfect delay correction

⇒ White fringes in 0.





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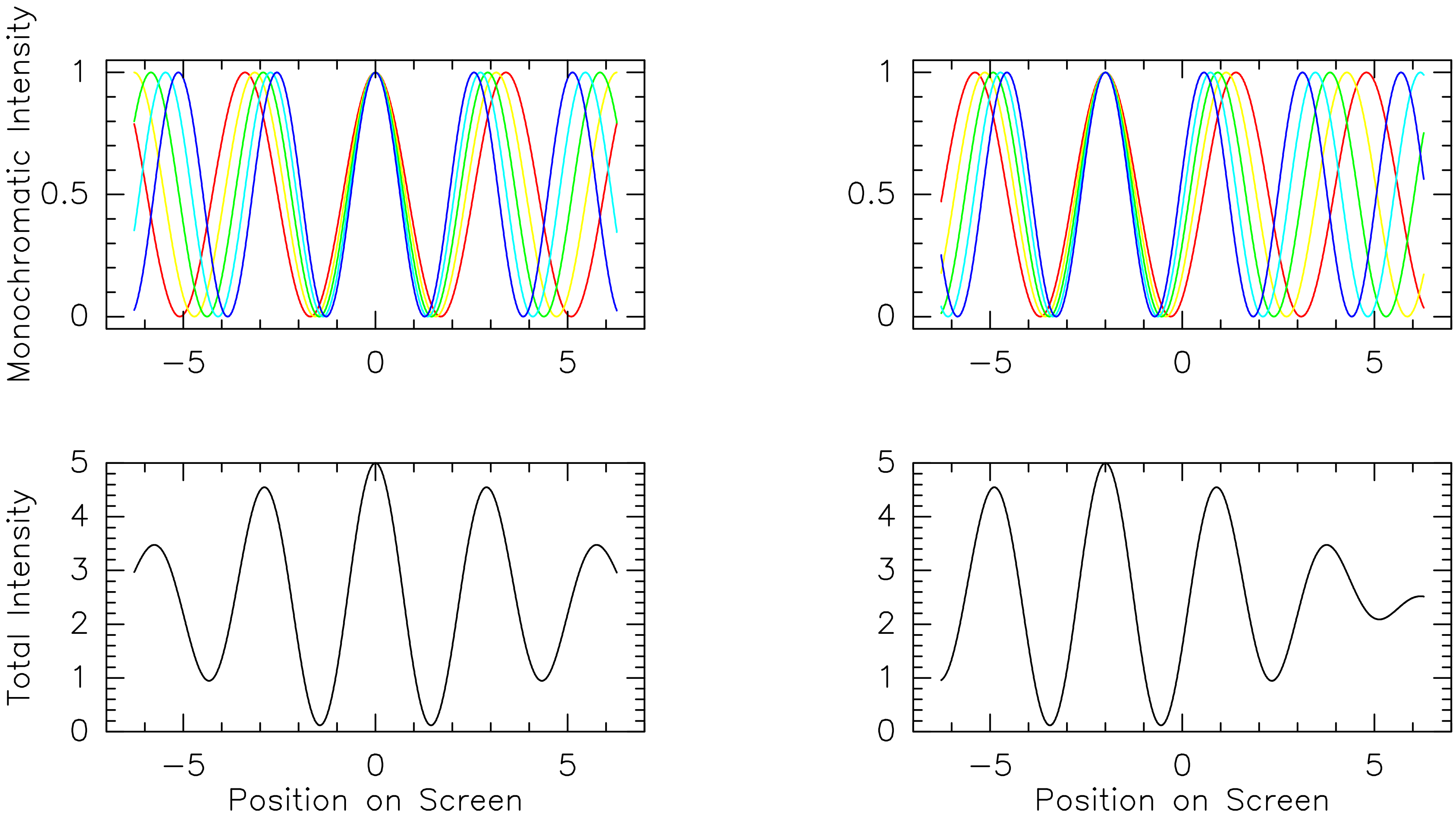
Perfect delay correction
⇒ White fringes in 0.

Worse and worse delay correction.
⇒ Translation of the fringe pattern.
⇒ Fringes seem to disappear.



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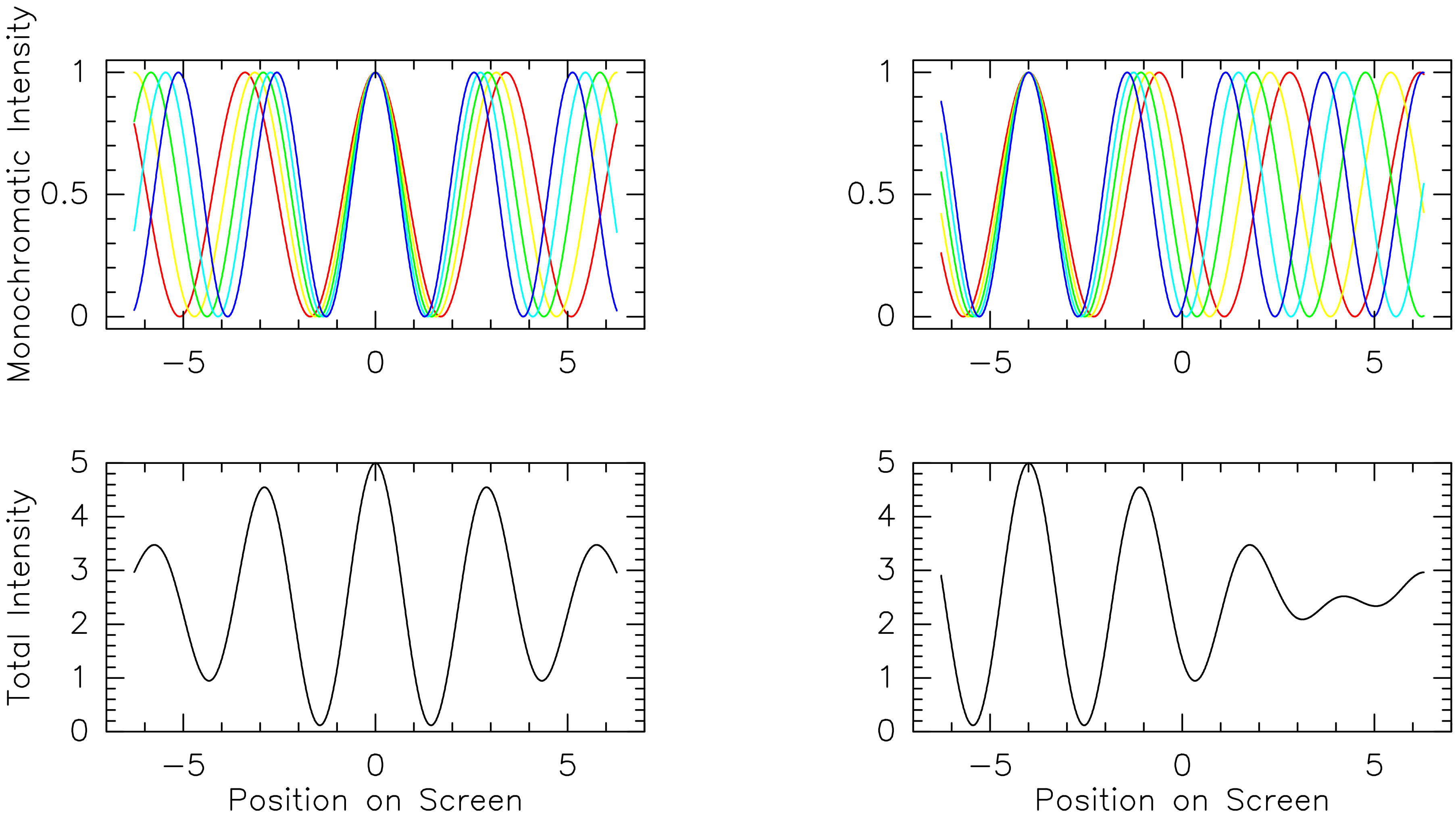

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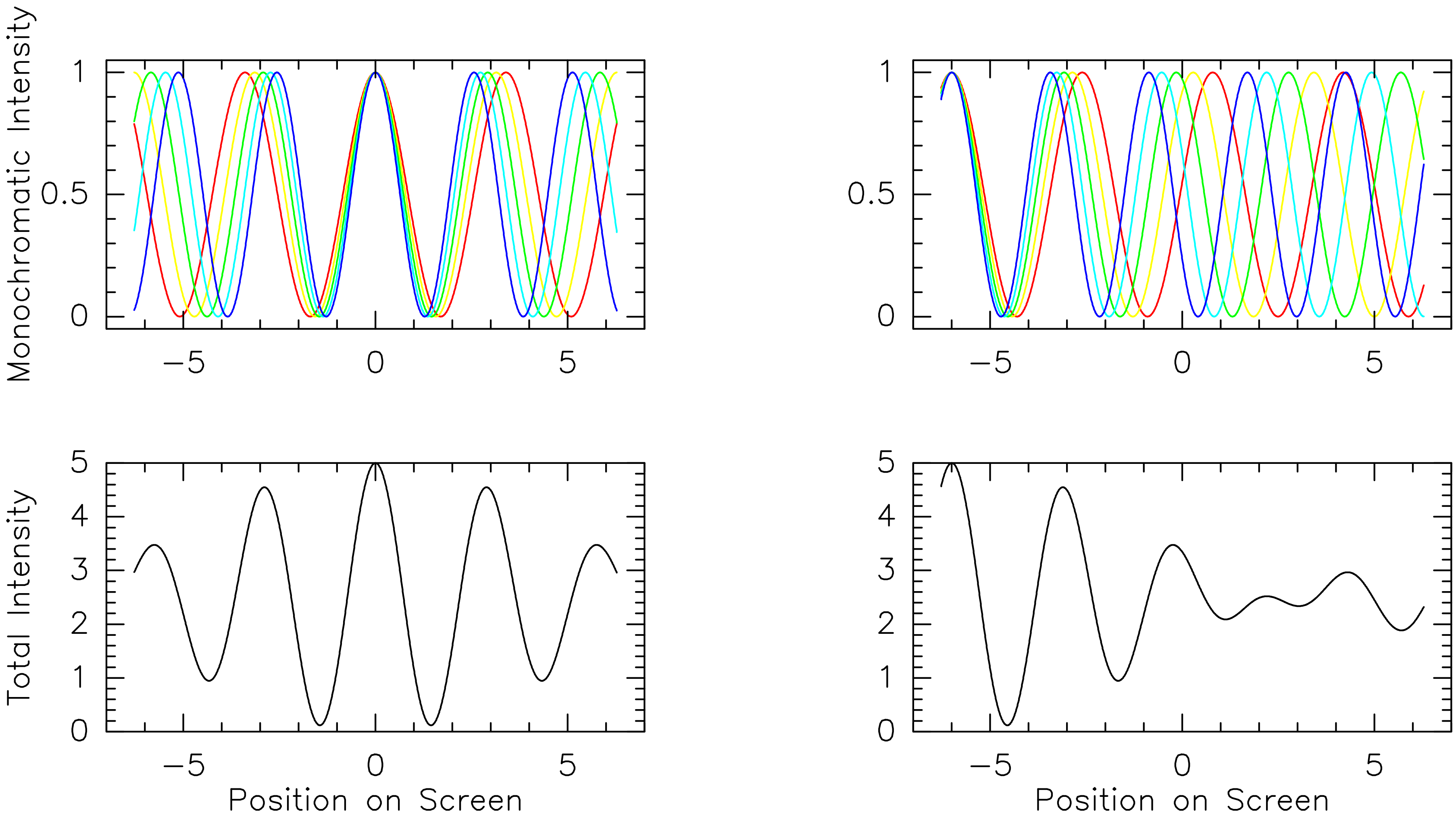

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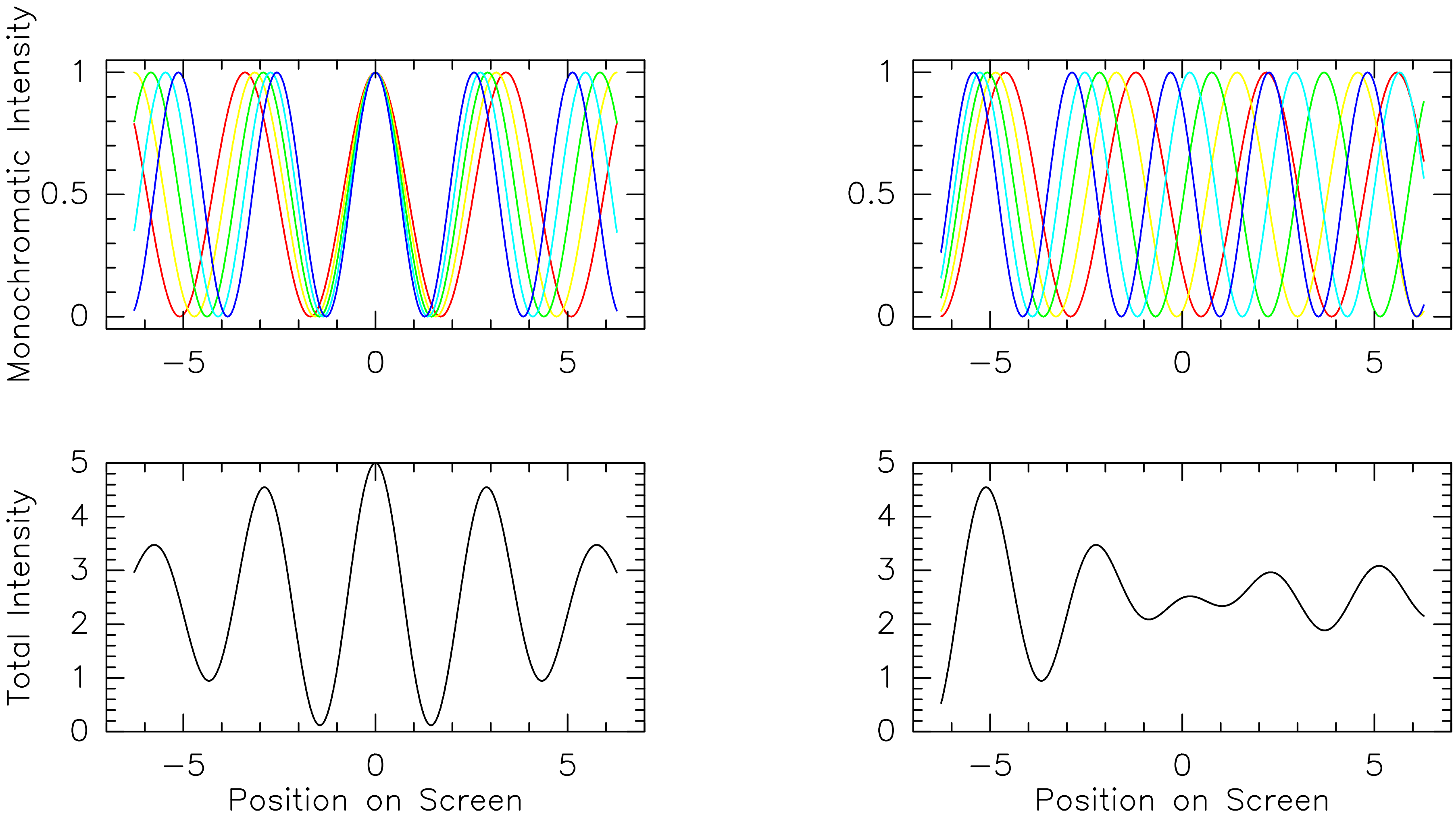

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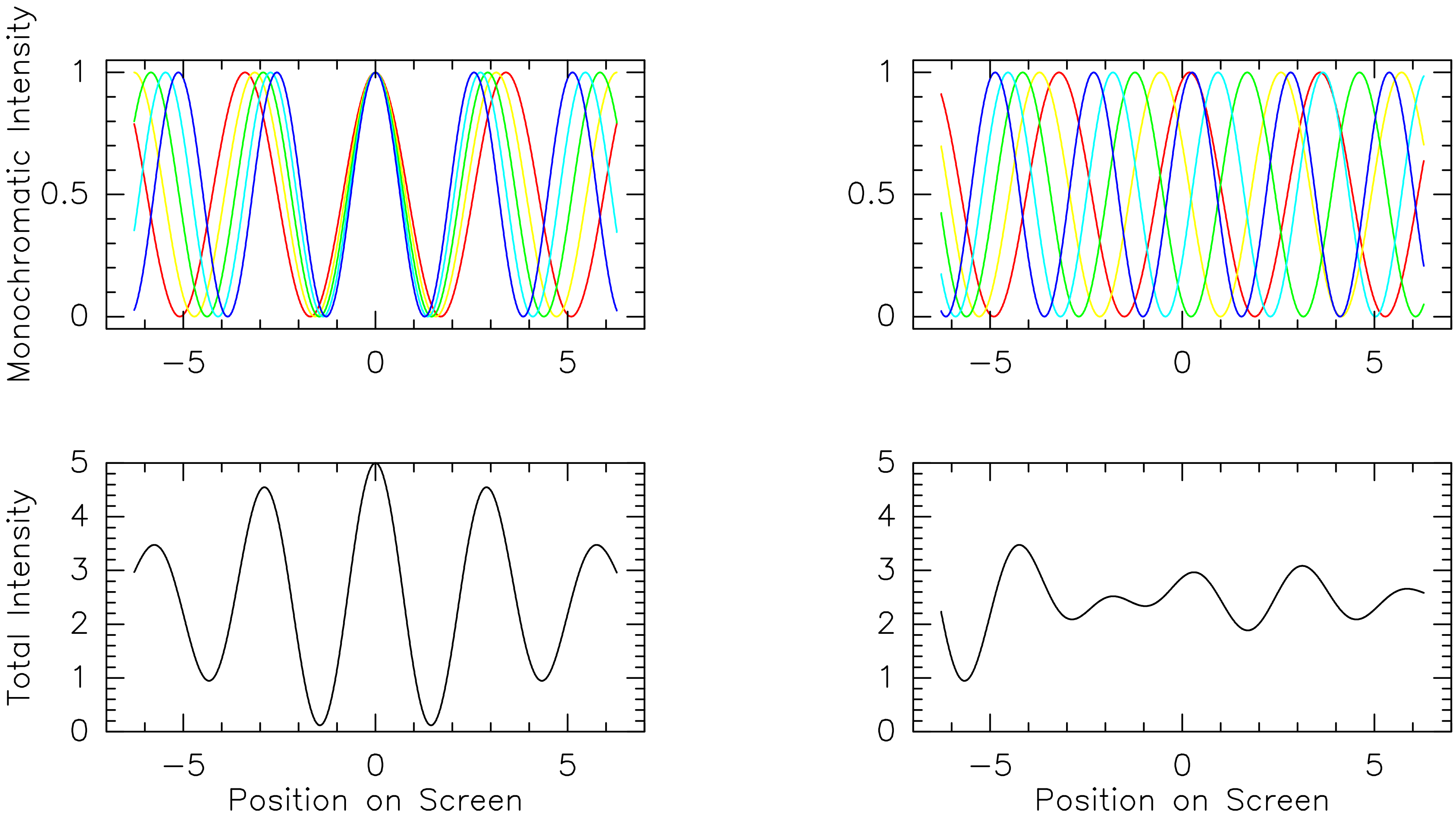
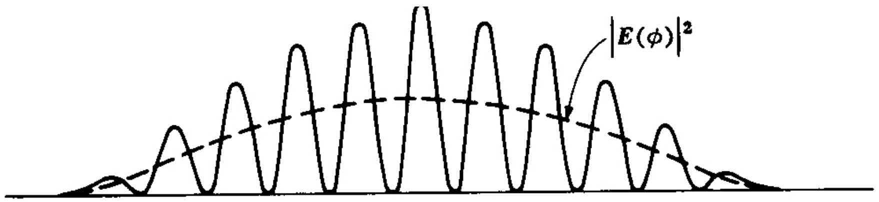
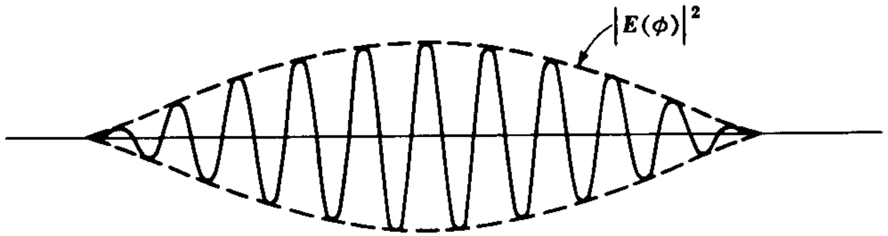
⇒ Translation of the fringe pattern.
⇒ Fringes seem to disappear.



Optic vs Radio Interferometer: I. Measurement Method

		Optic	Radio
Detector	Kind	Quadratic	Linear (Heterodyne)
	Observable	$I = \|EE^\star\|$	$\|E\| \exp{(i\psi)}$
Measure	Method	Optical fringes	Electronic correlation
	Quantity	$\|C\| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$	$\|V\| \exp(i\phi_V) = < E_1.E_2 >$
Interferometer kind		Additive	Multiplicative



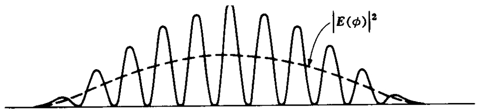


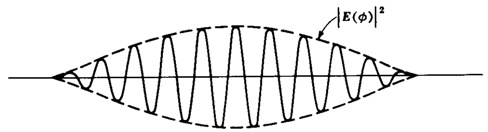
$$I_1 + I_2 + 2\sqrt{I_1 I_2}\,|C| \cos\left(\frac{bx}{\lambda} + \phi_C\right)$$

$$\overbrace{|E_1|\,|E_2|\,|C|}^{|V|} \cos\left(\frac{bx}{\lambda} + \overbrace{\phi_C}^{\phi_V}\right)$$

(Heterodyne: lectures by F. Gueth and V.Piétu)

		Optic	Radio
Detector	Kind	Quadratic	Linear (Heterodyne)
	Observable	$I = \|EE^\star\|$	$\|E\| \exp{(i\psi)}$
Measure	Method	Optical fringes	Electronic correlation
	Quantity	$\|C\| = \frac{I_{\text{max}}-I_{\text{min}}}{I_{\text{max}}+I_{\text{min}}}$	$\|V\| \exp(i\phi_V) = < E_1.E_2 >$
Interferometer kind		Additive	Multiplicative



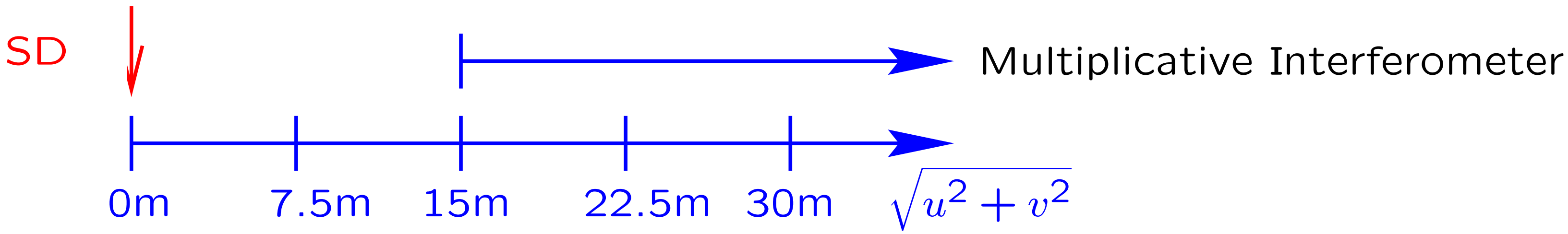


Multiplicative Interferometer

Avantage: all offsets are irrelevant ⇒ Much easier;

Inconvenient: Radio interferometer = bandpass instrument;

⇒ Low spatial frequencies are filtered out.



Optic vs Radio Interferometer: II. Atmospheric Influence

Atmosphere emits and absorbs:

Signal = Transmission x Source + Atmosphere.

- Optic: $\left.\begin{cases}\text{Source} \gg \text{Atmosphere} \\ \text{Transmission} \sim 1\end{cases}\right\} \Rightarrow$ transparent;
- Radio: $\left.\begin{cases}\text{Source} \ll \text{Atmosphere} \\ \text{Transmission can be small}\end{cases}\right\} \Rightarrow$ fog.

Good news: Atmospheric noise uncorrelated

$\Rightarrow$ Correlation suppresses it!

Bad news: Transmission depends on weather and frequency.

$\Rightarrow$ Astronomical sources needed to calibrate the flux scale! (Lecture by M. Krips)

Atmosphere is turbulent: $\Rightarrow$ Phase noise (Lectures by M. Bremer and V. Piétu).

Timescale of atmospheric phase random changes:

- Optic: 10-100 milli secondes;
- Radio: 10 minutes.

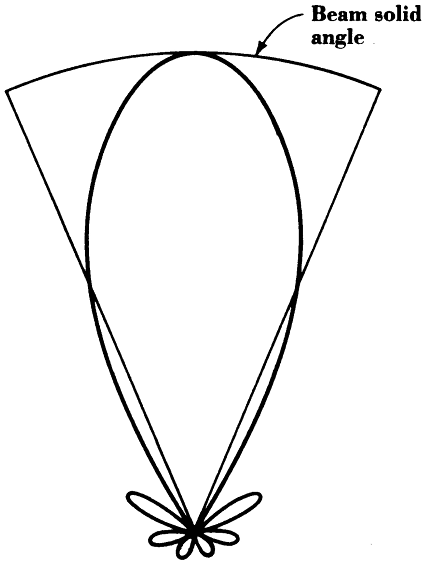
$\Rightarrow$ Radio permits phase calibration on a nearby point source (*e.g.* quasar).

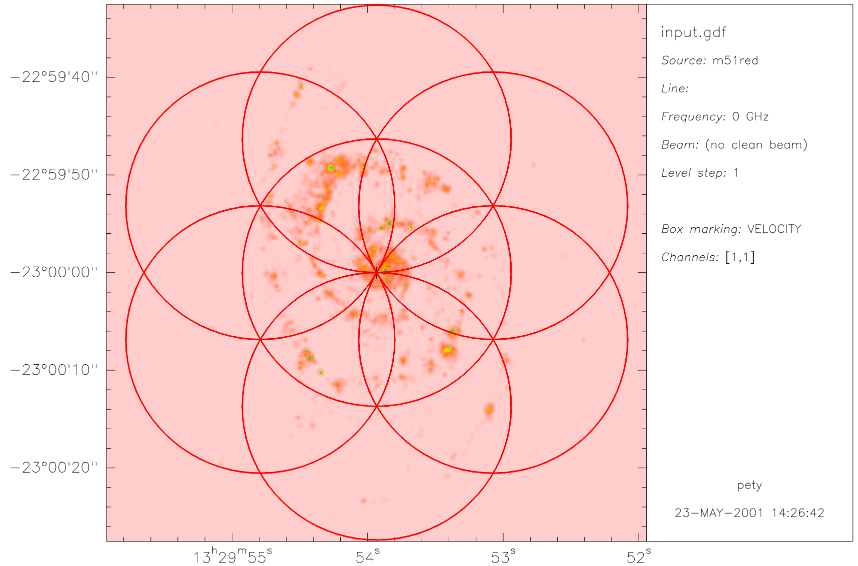
Instantaneous Field of View

One pixel detector:

- Single Dish: one image pixel/telescope pointing;
- Interferometer: numerous image pixels/telescope pointing
  - Field of view = Primary beam size;
  - Image resolution = Synthesized beam size.

Wide-field imaging: ⇒ mosaicing (Lecture by J. Pety).





Conclusion

mm interferometry:

- A bit more of theory;
- Lot's of experimental details (*e.g.* lecture by J. Boissier, and A. Castro–Carrizo).

Why caring about technical details: Some of them must be understood to know whether you can trust your data.

By the end of this week,
you should be ready to use NOEMA & ALMA!
(Lectures by C. Lefevre, M. Krips, and E. Chapillon)

Bibliography

- "Synthesis Imaging". Proceedings of the NRAO School. R. Perley, F. Schwab and A. Bridle, Eds.
- "Proceedings from IMISS2", A. Dutrey Ed.
- "Interferometry and Synthesis in Radio Astronomy", R. Thompson, J. Moran and G. W. Swenson, Jr.

Photographic Credits

- M. Born & E. Wolf, "Principles of Optics".
- J. W. Goodman, "Statistical Optics".
- J. D. Kraus, "Radio Astronomy".

Lexicon

- Beam: Antenna diffraction pattern.
- Primary Beam: Instantaneous field of view (Single-Dish Beam).
- Synthesized Beam: Image resolution (Interferometer Beam).
- Configuration: Antenna layout of interferometer.
- Baseline: Distance between two antenna.
- uv-plane: Fourier plane.
- Visibilities: $\sim$ Fourier components of the source.
- Fringe stopping: Temporal variation of delay correction needed to avoid translation of the white fringe.
- Heterodyne: Principle of linear detection.
- Correlator: Where visibilities are measured by correlation of signal coming from pairs of antenna.