Thoughts about polarization calibration with the next generation receivers at Plateau de Bure

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1. Introduction

With the next generation receivers, it will be possible to simultaneously derive all four Stokes parameters, i.e. to fully retrieve the polarization state of the incoming radiation.

In the following, the reference for horizontal and vertical polarization is the Cassegrain focus, and not the equatorial (= astronomical) coordinate system. The electrical field measured by an antenna j with the receiver mounted with its polarization 45° (clock-wise) below the horizontal axis is denoted X_j and that measured with the other receiver 45° clockwise from the vertical axis is denoted Y_j.

Due to this particular receiver orientation, Stokes Q is replaced by Stokes U, and Stokes U by –Q with respect to the textbook formulae. The convention for the total power calibration is that a receiver measuring one polarization (horizontal or vertical, respectively left – or right hand circular) receives only half of the total power of an unpolarized radiation field.

All calculations have been done using Jones calculus and the coherence matrix [1].

1.1 Quarter- wave plates inserted

The receivers couple to the circular polarization on the sky. The waveplates' fast axes coincide with the horizontal axis in the Cassegrain foci. For each baseline, we get four cross-correlation products. The complex Stokes visibilities are denoted by I, Q, U and V. A prefix "D" denotes the leakage term due to e.g. imperfections of the quarter-wave plates and beam splitters, or due to scattered radiation reaching the mixer from elsewhere, or contributions from the antennas themselves (polarization dependent gain-elevation curve).

X_1 and Y_1 are measured by different receivers of the same antenna 1 (evidently at the same frequency), coupling to right hand circular and left hand circular polarized emission, respectively.

Parallel band correlation products:

$X_1 \times X_2^* = [I + V]/2$ and $Y_1 \times Y_2^* = [I - V]/2$

Crossed band correlation products:

$X_1 \times Y_2^* = [-Q + iU + (DX_1 + DY_2^*) . I]/2$ and $Y_1 \times X_2^* = [-Q - iU + (DY_1 + DX_2^*) . I]/2$

This setup is optimized for the detection of linear polarization (i.e. Stokes Q and U), since the atmospheric contribution to the crossed band correlation products cancels (the atmosphere is unpolarized).

1.2 Quarter-wave plates removed

The receivers directly couple to perpendicular linear polarization.

Parallel band correlation products:

$X_1 \times X_2^* = [I + U] / 2$

$Y_1 \times Y_2^* = [I - U] / 2$

Crossed band correlation products:

$X_1 \times Y_2^* = [-Q + iV + (DX_1 + DY_2^*) . I] / 2$

$Y_1 \times X_2^* = [-Q - iV + (DY_1 + DX_2^*) . I] / 2$

This setup is optimized for the measurement of circular polarization (e.q. Zeeman experiments).

Conclusion of Section 1:

Quarter-wave plates are needed, in order to optimize for the detection of linear polarization. For Zeeman experiments, the filter wheel needs an empty position. To allow for an easy setup of the observations, the filterwheel should be remotely controllable from the opertors' desk.

Since the spectral bandwidth of the efficiency of quarter-wave plates is limited, more than one QWP may be needed.

The question whether it will be useful to have both RHC *and* LHC *for each* receiver will be treated in the next section.

2. Calibration of the leakage terms

The determination of the leakage terms is especially important for observations of strong, weakly polarized sources (e.g. certain masers). They can be measured by means of observations of an unpolarized source (or a source with well known polarization characteristics).

Demonstration for a three-antenna array :

$X_1 \times Y_2^* = (DX_1 + D Y_2^*) . I$ $\quad$ $Y_1 \times X_2^* = (DY_1 + D X_2^*) . I$

$X_1 \times Y_3^* = (DX_1 + D Y_3^*) . I$ $\quad$ $Y_1 \times X_3^* = (DY_1 + D X_3^*) . I$

$X_2 \times Y_3^* = (DX_2 + D Y_3^*) . I$ $\quad$ $Y_2 \times X_3^* = (DY_2 + D X_3^*) . I$

6 complex equations, 6 complex unknowns (i.e. two leakage terms per antenna)

→ the problem is fully determined (see also [2]).

Conclusion of Section 2.

In order to simultaneously retrieve all Stokes parameters, and to determine the instrumental leakage parameters, it is instrumental to have both RHC and LHC (horizontal and vertical, respectively) polarization per antenna (but not necessarily per receiver).

Possible reasons to have both RHC and LHC polarization for a given receiver :

(i) Polarimetry measurements even if the 2nd polarization receiver for a given frequency band is not available, applying a switch cycle to recover all Stokes parameters. In this “polarization switch mode”, the correlator could use all of its bandwidth capability (instead of 25 % only), e.g. for line surveys or more instantaneous sensitivity (time multiplexing vs. frequency multiplexing). However, the Stokes parameters are not measured simultaneously (i.e. slightly different UV coverage for the two parallel-mode and the two crossed-mode correlation products).

(ii) The observers have dual polarization receivers, but want to observe simultaneously the polarization of two spectral lines that do not fit into the IF range (or e.g. measure spectral indices of polarized continuum emission, e.g. to determine a rotation measure). Since dual-frequency observations within a single frequency band are not foreseen (both receivers share the same LO signal), this is not an issue here (the observing time will be split in those cases).

The conclusion is that ONE circular polarization per receiver (of a given frequency band) is probably sufficient.

3. Crossed band polarization calibration

The difficulty with the delay and the RF bandpass calibration is that most point-like calibrator sources are at most weakly polarized. I consider two limiting cases :

3.1 A strong, polarized calibrator is available: usual RF calibration

The delay calibration is done in the usual way. The complex gains for a baseline are as usual decomposed into antenna-dependent terms, and the latter are themselves decomposed into frequency dependent and time dependent factors.

Here I treat the frequency dependent factors only.

Both the parallel- and crossed-mode delay and bandpass calibration could be done as usual, integrating sufficiently long to ensure a sufficient signal/noise ratio in the crossed-mode calibration (see also [2]).

Time estimate :

A phase noise of 1 degree requires a signal-to-noise ratio of 60. I assume a 10% loss due to atmospheric decorrelation, an antenna efficiency of 22 Jy/K, and a SSB system temperature of 150 K (lower under good weather conditions and with new generation receivers). A spectral channel width of 5 MHz is sufficient to resolve the usual phase variations across the RF band. For a strongly polarized source like BL Lac (2200+420, mean flux 2.4 Jy, max. linear polarization = 10%), we need 23 hours observing time ! For a stronger source like 3C273 (mean flux 8 Jy), with a weaker linear polarization (4.5 % mean over 2 years), the bandpass calibration still needs 10 hours. Note that a survey of AGN polarization (Thum et al., 2003) shows that linear polarizations in excess of 10 % are rather the exception. Furthermore, these calibrators are not always available.

Conclusion of Section 3.1 :

An excellent bandpass calibration in all four cross correlation products (two in parallel mode, two in crossed mode) will thus take a prohibitively long observing time.

3.2 Only weakly polarized RF calibrators available

Fortunately, the crossed-band bandpass calibration can be derived from the parallel-mode bandpass calibrations, and a dedicated calibration using the reference antenna only.

The crossed-band bandpass calibration can be determined by measuring the phase difference between the receivers of the antenna used as phase reference. Since no sufficiently strongly polarized RF calibrator is available, this can only be done by injecting a polarized signal into both horns of the dual-polarization receiver (i.e.by virtue of a grid in front of the sky or a calibration load, oriented at 45 degrees with respect to the vertical in the Cassegrain focus).

Demonstration for baseline 12 (leakage terms already removed) :

Parallel mode phase calibration (easily measurable on an unresolved source, as usual) :

$X_1 \times X_2{}^* = g_{X,1} \,.\, g_{X,1} \,.\, \exp[i\ (\varphi_{X,1} - \varphi_{X,2})] \,.\, \{I + U\}/2$,

$Y_1 \times Y_2{}^* = g_{Y,1} \,.\, g_{Y,2} \,.\, \exp[i\ (\varphi_{Y,1} - \varphi_{Y,2})] \,.\, \{I - U\}/2$

for receivers coupling to linear polarization (otherwise the final brackets have to be replaced by

I + V and I – V, respectively). Then the gain phase differences

$\Delta\varphi_X = \varphi_{X,1} - \varphi_{X,2}$ and $\Delta\varphi_Y = \varphi_{Y,1} - \varphi_{Y,2}$

are known, as well as the gain amplitude products $g_{X,1}.g_{X,2}$ and $g_{Y,1}.g_{Y,2}$.

The crossed –band correlation products (receivers coupling to linear polarization) would be

$X_1 \times Y_2{}^* = g_{X,1} .\ g_{Y,2} \,.\, \exp[i\ (\varphi_{X,1} - \varphi_{Y,2})] \,.\, \{-Q + iV\}/2$ and

$Y_1 \times X_2{}^* = g_{Y,1} \,.\, g_{X,2} \,.\exp[i\ (\varphi_{Y,1} - \varphi_{X,2})] \,.\, \{-Q - iV\}/2$, respectively.

However, it is readily shown that the crossed-band RF calibration can be retrieved from the parallel band RF calibrations for $\Delta\varphi_X$ an $\Delta\varphi_Y$ (i.e. the usual parallel mode calibrations) and a crossed-band correlation between both polarizations of ONE reference antenna (here antenna 2, bracketed terms):

$\Phi_{X,1} - \varphi_{Y,2} = \varphi_{X,1} - \varphi_{X,2} + \varphi_{X,2} - \varphi_{Y,2} = \Delta\varphi_X + [\varphi_{X,2} - \varphi_{Y,2}]$ and

$\varphi_{Y,1} - \varphi_{X,2} = \varphi_{Y,1} - \varphi_{Y,2} + \varphi_{Y,2} - \varphi_{X,2} = \Delta\varphi_Y - [\varphi_{X,2} - \varphi_{Y,2}]$, likewise,

for the gain phase differences, respectively for the gain amplitude ratios :

$g_{X,1} .\ g_{Y,2} = [g_{X,1} .\ g_{X,2}] . [g_{Y,2} / g_{X,2}]$ respectively $g_{Y,1} .\ g_{X,2} = [g_{Y,1} .\ g_{Y,2}] . [g_{X,2} / g_{Y,2}]$

Conclusion of Section 3 :

The crossed mode bandpass calibration can be derived from the usual parallel mode bandpass calibration on an unpolarized source, and a dedicated measurement (using a polarizing grid) of the bandpass phase difference and gain ratio between the two receivers of the phase reference antenna. The correlator MUST be able to cross-correlate the horizontal and vertical polarization of the receivers in the reference antenna at least once per project (= tuning). There are basically two ways to calibrate, for a given antenna, the RHC polarized receiver against its LHC polarized counterpart :

(i) Using the strongest RF calibrators on the sky. The experience from the 30m telescope tells us that the phase variations of the bandpass phase on small frequency scales (of the order of a few 10 MHz) mainly originate from the IF part (since the same LO reference is used for both receivers). At the Plateau de Bure observatory, the IF bandpass calibration is done internally (noise source injection). The remaining RF part can be expected to vary smoothly (e.g. like a 3^{rd} order polynomial across a few 100 MHz bandwidth). For spectral channels binned to 100 MHz widths, the time estimates of Section 1.2 drop to affordable 1.2 hours for 2200+420 (respectively 0.5 hours for 3C273).

(ii) If the RF phase of the crossed-band correlation product of a given antenna does not vary smoothly, as assumed in (i), we have to use an artifical polarization signal within the receiver cabin. This is possible since the atmospheric contribution is, though chromatic, unpolarized, and since the instrumental contribution from the antenna is, though polarized, achromatic (and will be calibrated by a dedicated observation). Thus, both contributions do not matter for delay and bandpass calibration of the crossed-band mode.

The best method tested with the correlation polarimetry at the 30m telescope is a (remotely controlled) polarizing grid in front of the (common) cold load, and to cross-correlate horizontal with vertical polarization (no quarter-wave plates). The phase calibration then proceeds like a hot /cold load calibration, but with a strongly polarized (almost 100 %) signal in the cold load subscan. A polarizing grid does not make sense in front of the hot load : Although the transmitted component has a clean polarization, the reflected one will be scattered in the receiver cabin, and yields an ambient signal spoiled by the other polarization.

At the Plateau de Bure, the filter wheel will have an empty position (= sky), two positions for the quarter-wave plates, a hot load (different for both receivers), and a mirror looking at the cold load. The only place to insert a grid is thus in front of the cold load or in front of the sky. For this, an additional device maybe needed (e.g. another wheel allowing one position for transmission, and one position for transmission through a polarizing medium).

Reasons to have a polarizing grid for each antenna :

An artificially polarized signal can be produced by an horizontally or vertically mounted *and* remotely removable polarizing grid used in transmission against the cold load (more stable total power) or the sky.

For an ideal grid (e.g. q = 100 %, u = v = 0), the response is (the index R stands for reference antenna)

$$X_R \times Y_R^* = - g_{X,R} . g_{Y,R} . \exp [i (\varphi_{X,R} - \varphi_{Y,R})] / 2$$

Each antenna should be equipped with such a calibration grid, for two reasons :

In summertime, the interferometer may lack one or more antennas for maintenance, thus sometimes that containing the calibration grid. Scheduling may then become difficult, especially if combined with sun avoidance restrictions.

There is yet another reason : Re-writing the crossed-band calibration for a baseline that does not contain the reference antenna (here antenna 2), we find

$$\varphi_{X,1} - \varphi_{Y,3} = [\varphi_{X,1} - \varphi_{Y,2}] + [\varphi_{Y,2} - \varphi_{Y,3}] = [\varphi_{X,1} - \varphi_{X,2}] + [\varphi_{X,2} - \varphi_{Y,2}] + [\varphi_{Y,2} - \varphi_{Y,3}]$$

where the first and third bracketed term are measured by the usual parallel-band calibration, and the second one is the crossed-mode calibration for the reference antenna. However, the phase difference $\varphi_{X,1} - \varphi_{Y,3}$ is now decomposed into three phase difference terms, leading to phase errors systematically larger by a factor of $\sqrt{3/2}$ = 1.2 for baselines not containing the reference antenna, with respect to those containing it.

References

[1] Wiesemeyer, H. IRAM technical report 2001-1 (see http://www.iram.fr/GENERAL/reports)

[2] Fomalont, E.B. & Perley, R.A., 1989, in "Synthesis Imaging in Radio Astronomy", eds. R.A. Perley et al., ASP Conf . Series Vol. 6, p.109

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